Statics - TAM 210 & TAM 211

Lecture 23

March 12, 2018

Chap 7.3

Announcements

- Upcoming deadlines:
- Monday (3/12)
  - Mastering Engineering Tutorial 9
- Tuesday (3/13)
  - PL HW 8
- Quiz 5 (3/14-16)
  - Sign up at CBTF
  - Up thru and including Lecture 22 (Shear Force & Bending Moment Diagrams), although review/new material from today's lecture will be helpful.
- Last lecture for TAM 210 students (3/30)
- Written exam (Thursday 4/5, 7-9pm in 1 Noyes Lab)
  - Conflict exam (Monday 4/2, 7-9pm)
    - **Must make arrangements with Prof. H-W by Friday 3/16**
  - DRES accommodation exam. Make arrangements at DRES. Must tell Prof. H-W

Chapter 7: Internal Forces

Goals and Objectives

- Determine the internal loadings in members using the method of sections

- Generalize this procedure and formulate equations that describe the internal shear force and bending moment throughout a member

- Be able to construct or identify shear a force nd bending moment diagrams for beams when distributed loads, concentrated forces, and/or concentrated couple moments are applied

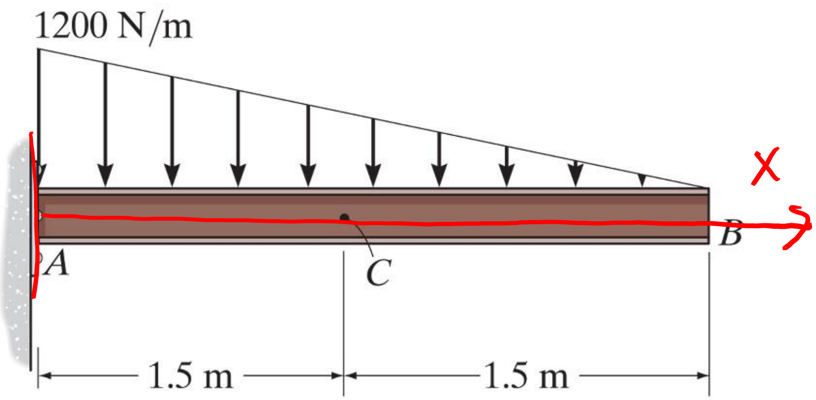
Recap: Shear Force and Bending Moment Diagrams

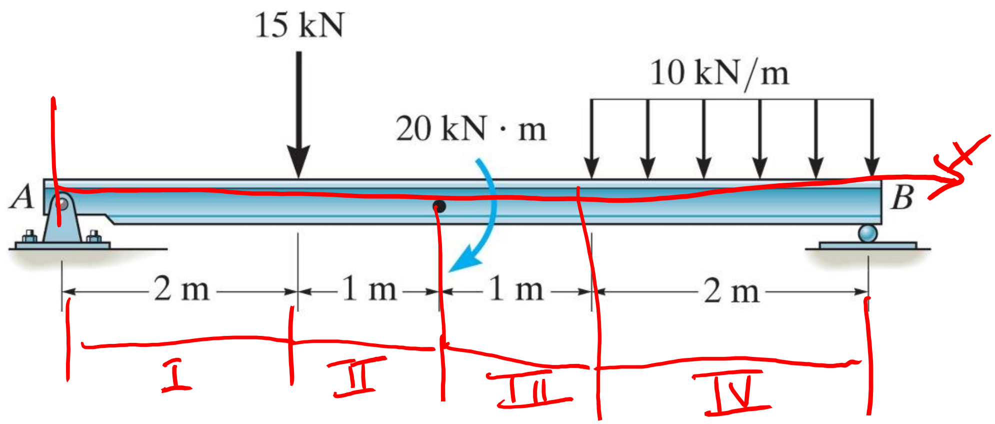
Goal: provide detailed knowledge of the variations of internal shear force and bending moments (V and M) throughout a beam when perpendicular distributed loads, concentrated forces, and/or concentrated couple moments are applied.

Normal forces (N) in such beams are zero, so we will not consider normal force diagrams.

Procedure

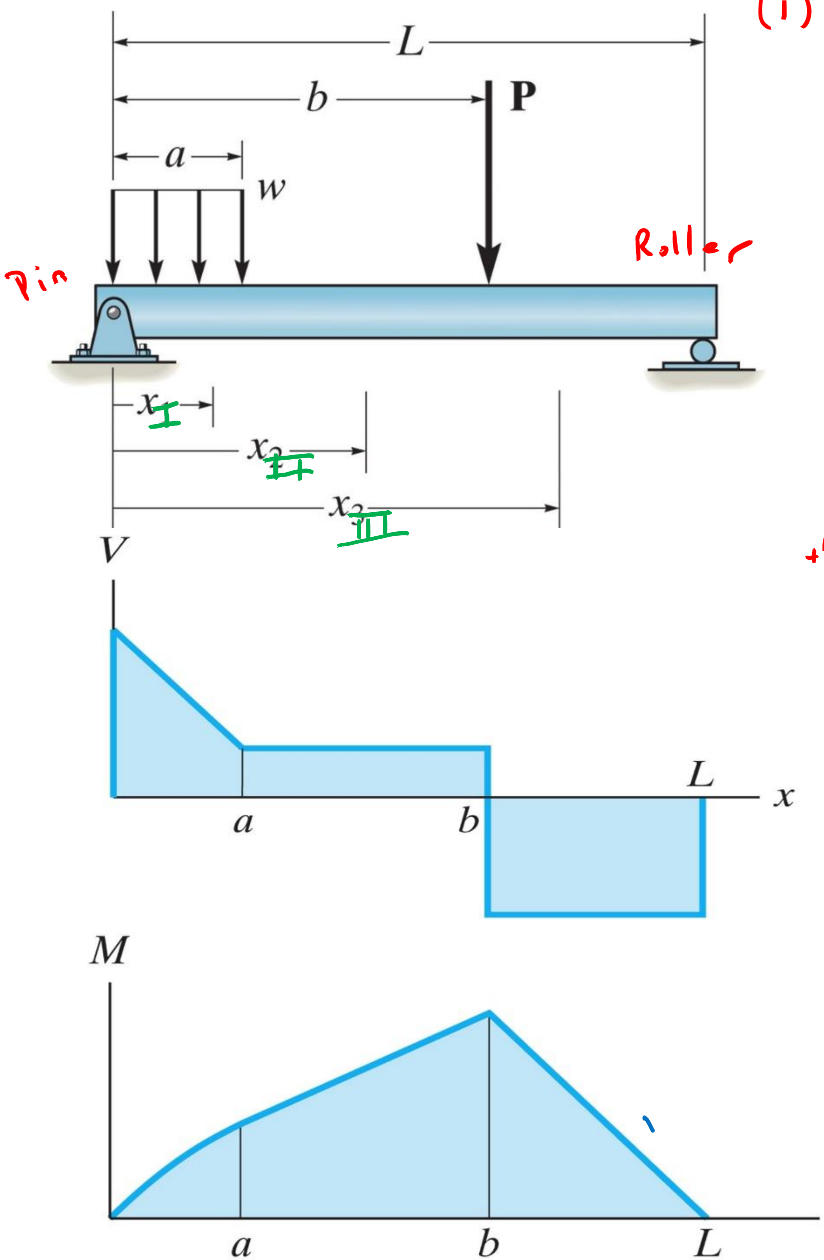
1. Find support reactions (free-body diagram of entire structure)
2. Specify coordinate x (start from left)
3. Divide the beam into sections according to loadings
4. Draw FBD of a section
5. Apply equations of equilibrium to derive V and M as functions of x: $V(x)$, $M(x)$



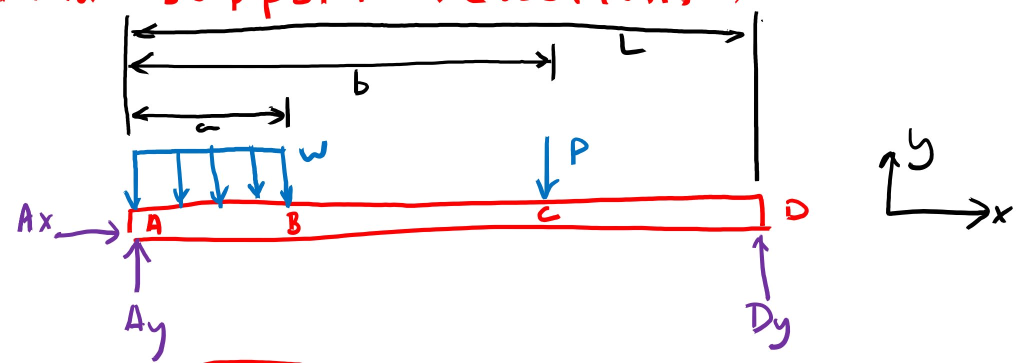


Recap: Explore and re-create the shear force and bending moment diagrams for the beam.

Example: single concentrated load, rectangular distributed load



(1) Find support reactions:



ΣF_x : $\boxed{A_x = 0}$

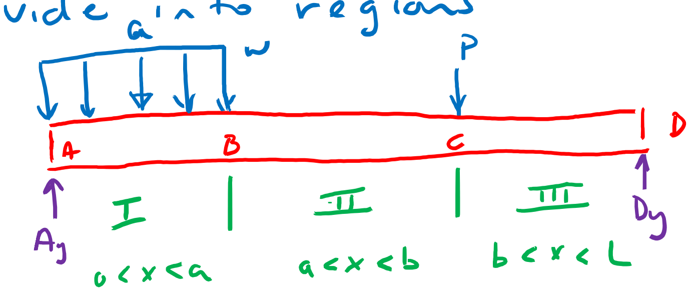
ΣF_y : $A_y + D_y - aw - P = 0$

$+\circlearrowleft\ \Sigma M_A$: $-\left(\frac{a}{2}\right)aw - b\cdot P + L\cdot D_y = 0$

$D_y = \frac{1}{L}\left(\frac{a^2 w}{2} + bP\right) \equiv \text{constant}$

$A_y = aw + P - D_y = aw\left(1 - \frac{a}{2L}\right) + P\left(1 - \frac{b}{L}\right)$ $\equiv \text{constant}$

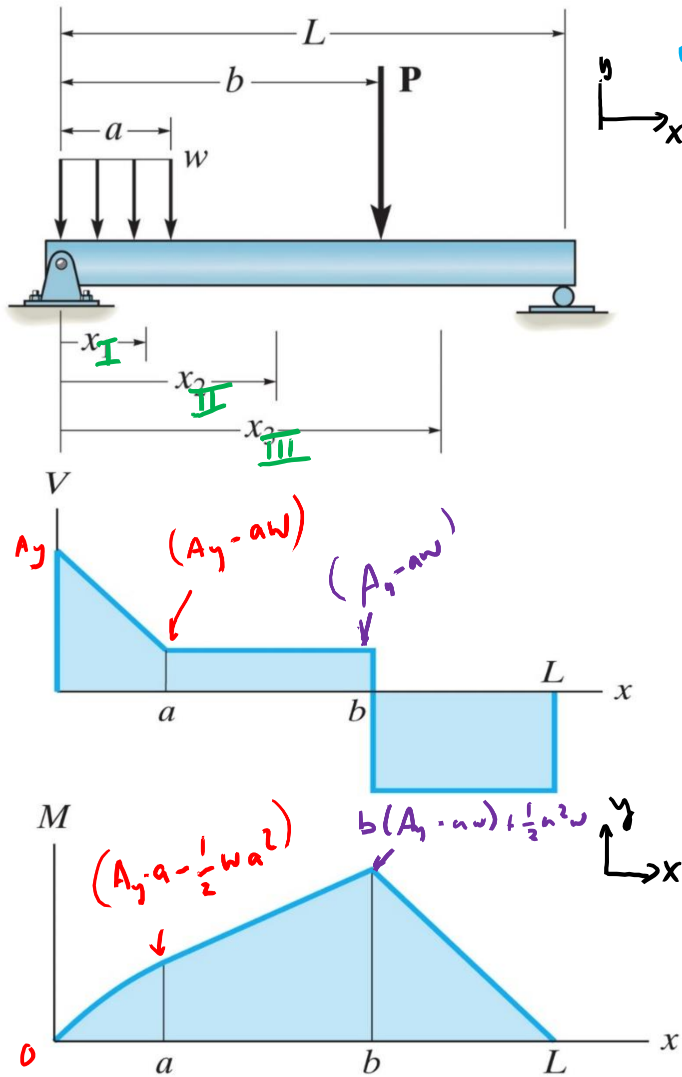
(2) Divide into regions



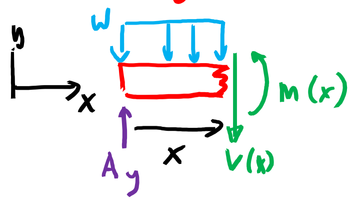
3 regions, 3 FBDs

Recap: Explore and re-create the shear force and bending moment diagrams for the beam.

Example: single concentrated load, rectangular distributed load



Region I: $0 < x < a$



$\Sigma F_y: \; A_y - V(x) - x \cdot w = 0$

$$V(x) = A_y - xw$$

linear w/slope w

$+\circlearrowleft M_A: \; M(x) - \left(\frac{x}{2}\right)(xw) - x \cdot V(x) = 0$

$M(x) = \frac{x^2 w}{2} + x(A_y - xw)$

$$\therefore \; M(x) = A_y x - \frac{1}{2} w x^2$$

quadratic

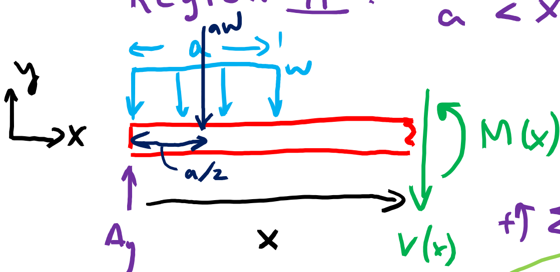
BC's:

$x = 0: \; V(0) = A_y, \quad M(0) = 0$

$x = a^{(-)}: \; V(a) = A_y - aw, \quad M(a) = A_y \cdot a - \frac{1}{2} w a^2$

Compare to values labeled on V & M diagrams to left

Region II: $a < x < b$



$\Sigma F_y: \; A_y - aw - V(x) = 0$

$$V(x) = A_y - aw$$

constant

$+\circlearrowleft \Sigma M_x: \; M(x) - x \cdot A_y - \left(x - \frac{a}{2}\right) aw = 0$

$$M(x) = x(A_y - aw) + \frac{1}{2} a^2 w$$

linear w/slope $(A_y - aw) = V(x)$

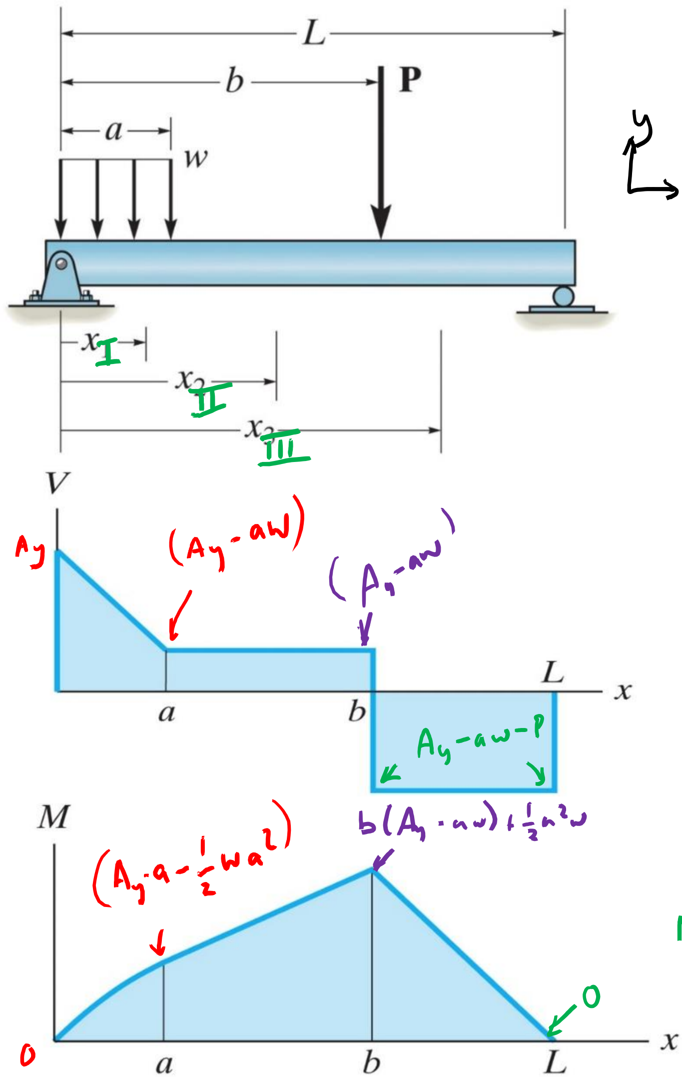
Note used pt x to sum moments

BC's: $x = a^{(+)}: \; V(a^{(+)}) = A_y - aw, \; M(a) = A_y a - \frac{1}{2} w a^2$

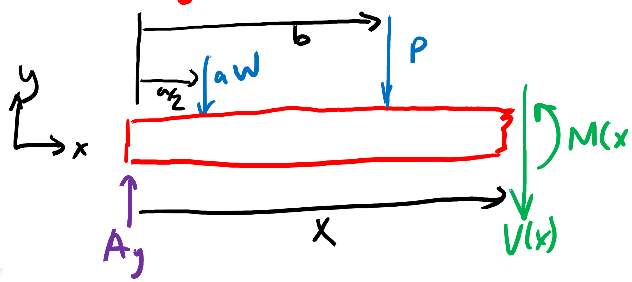
$x = b^{(-)}: \; V(b^{(-)}) = A_y - aw, \; M(b^{(-)}) = b(A_y - aw) + \frac{1}{2} a^2 w$

Recap: Explore and re-create the shear force and bending moment diagrams for the beam.

Example: single concentrated load, rectangular distributed load



Region III : $b < x < L$



$\Sigma F_y: A_y - aw - P - V(x) = 0$

$$V(x) = A_y - aw - P$$

constant

$+\circlearrowleft \Sigma M_x : M(x) - x A_y + (x - \frac{a}{2}) \cdot aw + (x - b) \cdot P = 0$

$$M(x) = x(A_y - aw - P) + bP + \frac{1}{2}a^2 w$$

$V(x)$

↖ linear w/ slope $V(x)$

BC's:

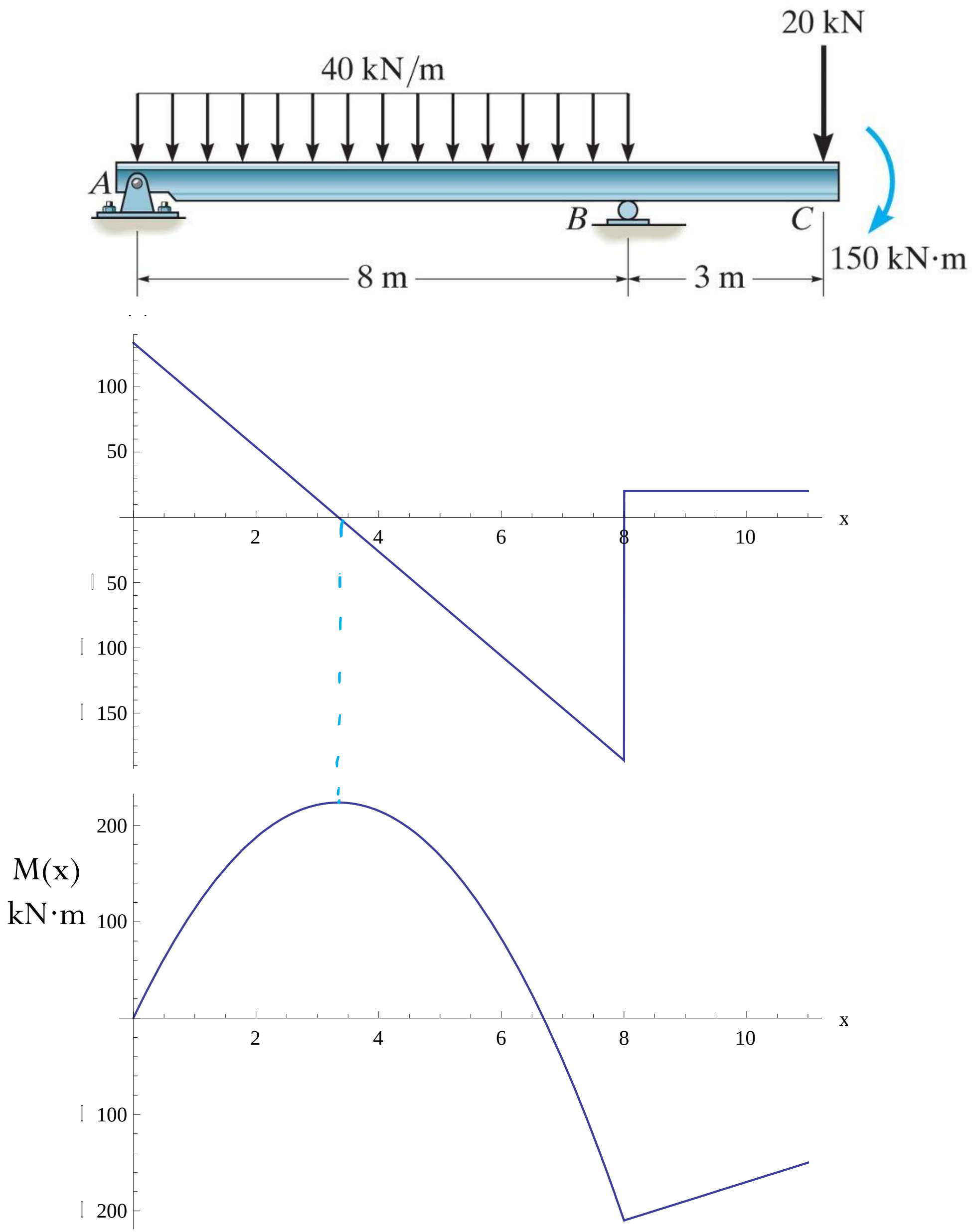
$x = b^{(+)}$: $V(b) = A_y - aw - P$, $M(b) = b(A_y - aw) + \frac{1}{2}a^2 w$

$x = L$: $V(L) = A_y - aw - P$, $M(L) = 0$

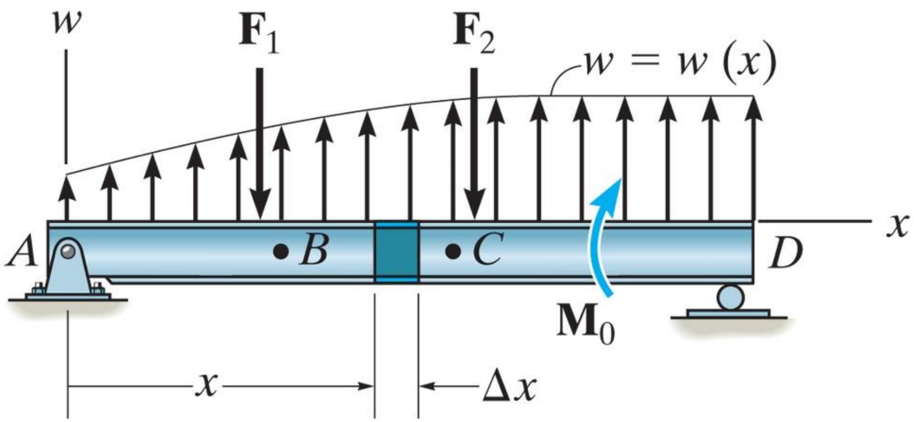
Note step change of $-P$ in $V(x)$ at $x = b$ due to concentrated load of P

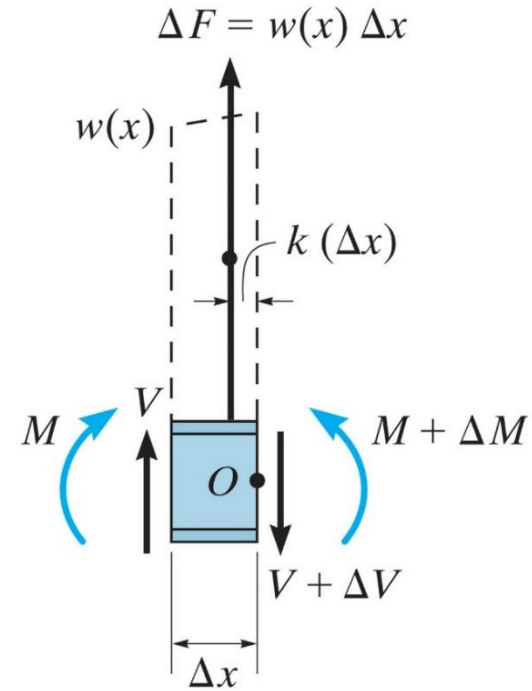
Explore and re-create the shear force and bending moment diagrams for the beam.

Example: concentrated load, rectangular distributed load, concentrated couple moment



Relations Among Distributed Load, Shear Force and Bending Moments





Relationship between <u>distributed load</u> and <u>shear</u>:

$$\sum F_y = 0: \quad V - (V + \Delta V) + w\,\Delta x = 0$$

$$\Delta V = w\,\Delta x$$

Dividing by Δx and letting $\Delta x \to 0$, we get:

$$\frac{dV}{dx} = w \qquad \Delta V = \int w\;dx$$

Relationship between <u>shear</u> and <u>bending moment</u>:

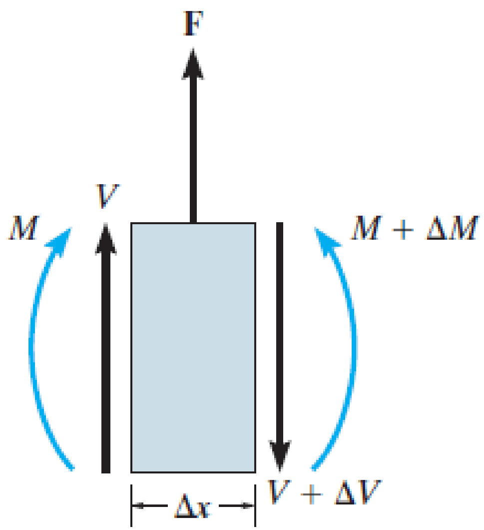
$$\sum M_O = 0: \quad (M + \Delta M) - M - V\,\Delta x - w\,\Delta x\,(k\,\Delta x) = 0$$

$$\Delta M = V\,\Delta x + w\,k\,(\Delta x)^2$$

Dividing by Δx and letting $\Delta x \to 0$, we get:

$$\frac{dM}{dx} = V \qquad \Delta M = \int V\;dx$$

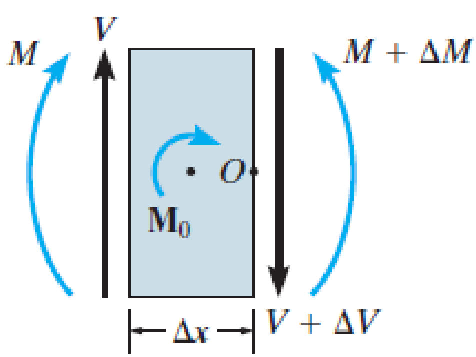
Wherever there is an external concentrated force, or a concentrated moment, there will be a change (jump) in shear or moment, respectively.



ΣF_y:

$V + F - (V + \Delta V) = 0$

$\Delta V = \mathrm{F}$

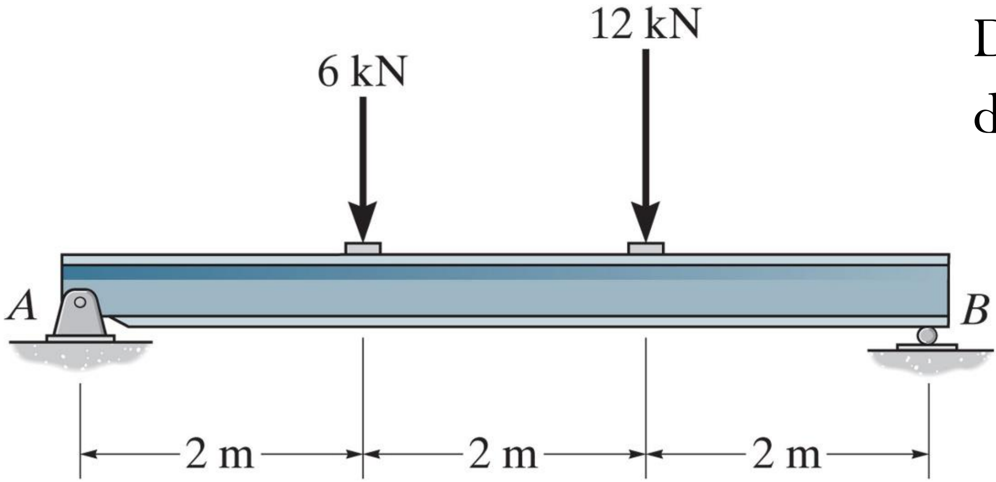


ΣM_O:

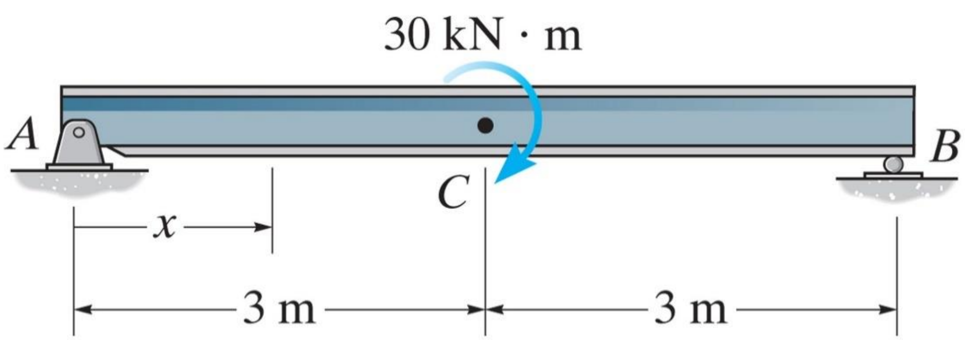
$(M + \Delta M) - M - M_O - V(\Delta x) = 0$

$\Delta M = M_O + V(\Delta x)$

$\Delta M = M_O$, when $\Delta x \to 0$



Draw the shear force and moment diagrams for the beam.



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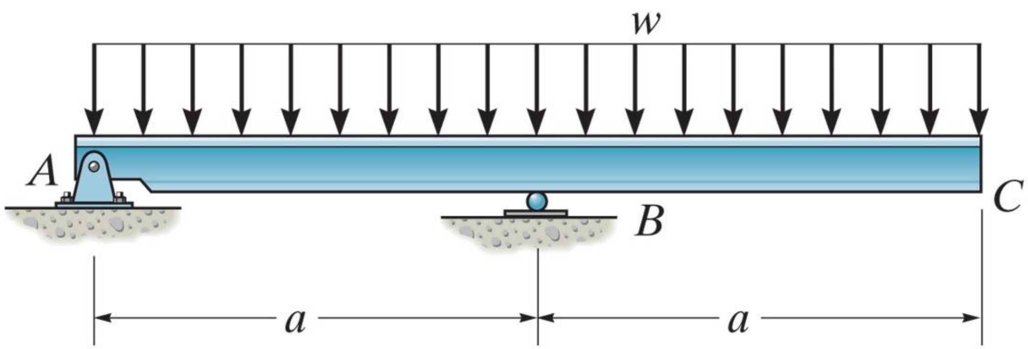

Draw the shear force and moment diagrams for the beam.

Draw the shear force and bending moment diagrams for the beam.

Example: concentrated load, rectangular distributed load, concentrated couple moment