Statics - TAM 210 & TAM 211

Lecture 23

March 12, 2018

Chap 7.3

Announcements

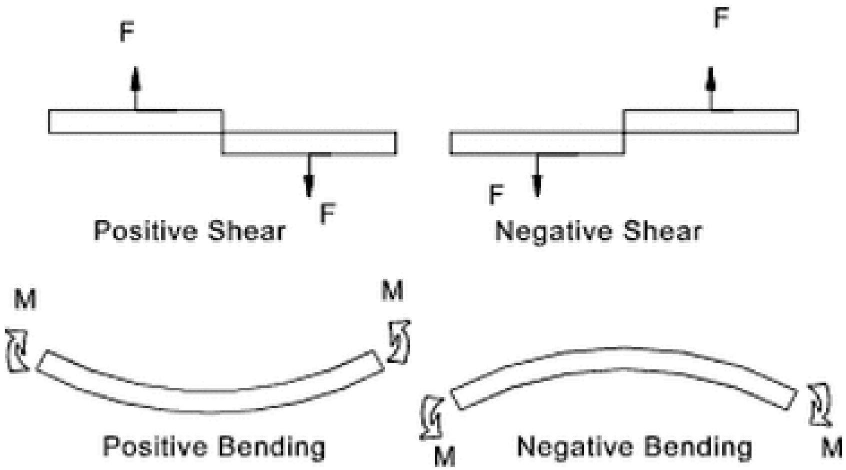
- Upcoming deadlines:
- Monday (3/12)
  - Mastering Engineering Tutorial 9
- Tuesday (3/13)
  - PL HW 8
- Quiz 5 (3/14-16)
  - Sign up at CBTF
  - Up thru and including Lecture 22 (Shear Force & Bending Moment Diagrams), although review/new material from today's lecture will be helpful.
- Last lecture for TAM 210 students (3/30)
- Written exam (Thursday 4/5, 7-9pm in 1 Noyes Lab)
  - Conflict exam (Monday 4/2, 7-9pm)
    - **Must make arrangements with Prof. H-W by Friday 3/16**
  - DRES accommodation exam. Make arrangements at DRES. Must tell Prof. H-W

Chapter 7: Internal Forces

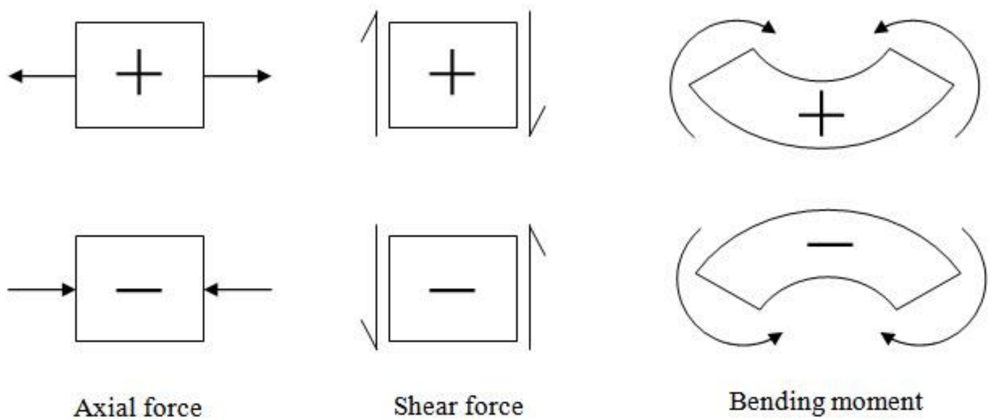
Goals and Objectives

- Determine the internal loadings in members using the method of sections
- Generalize this procedure and formulate equations that describe the internal shear force and bending moment throughout a member
- Be able to construct or identify shear a force nd bending moment diagrams for beams when distributed loads, concentrated forces, and/or concentrated couple moments are applied

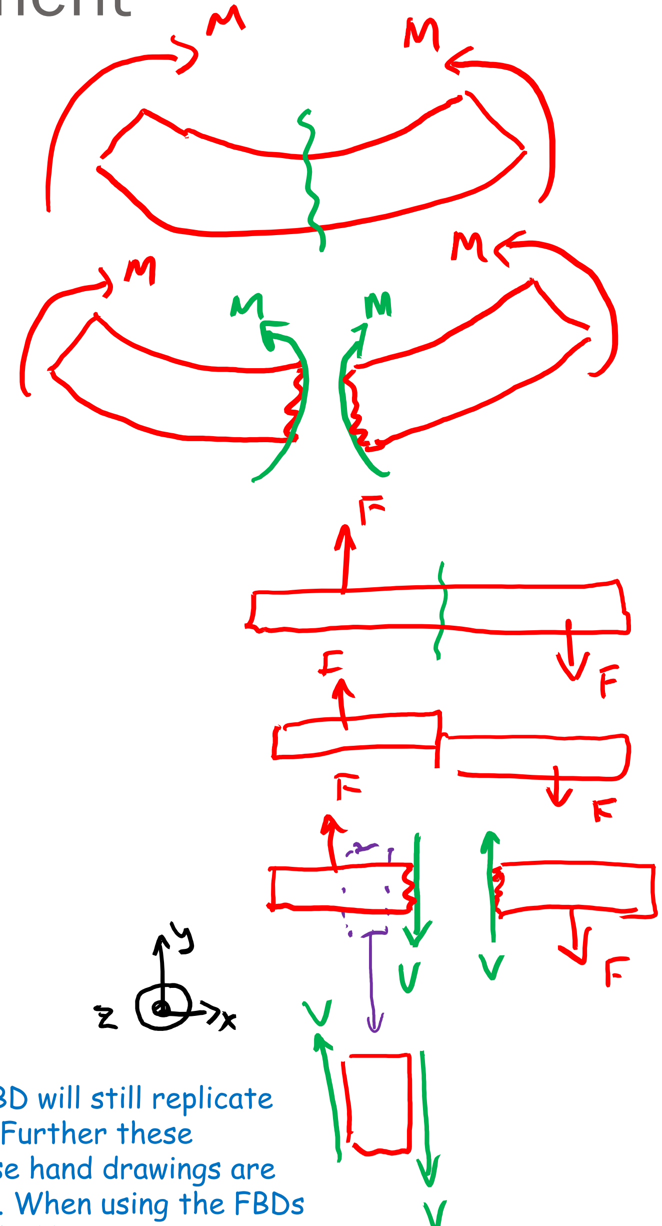
Recap: Shear Force and Bending Moment



http://structureanalysis.weebly.com/bending-moment--shear-force.html



https://ecoursesonline.icar.gov.in/mod/page/view.php?id=125191



Notes about hand-drawn material: even when cutting a beam (green), the new smaller FBD will still replicate the bending moments or shear forces on the cut surfaces on the left or right segment. Further these replicated moments/forces should be drawn to be equal and in opposite directions. These hand drawings are for when the bending moments and shear forces are drawn to be in the "positive" sense. When using the FBDs to write out the eqns of equilibrium, use the axes of your coordinate system diagram (black) to define whether the vectors are pointing in a positive or negative direction – see any example problem to see if a particular force or moment is + or – in the eqn.

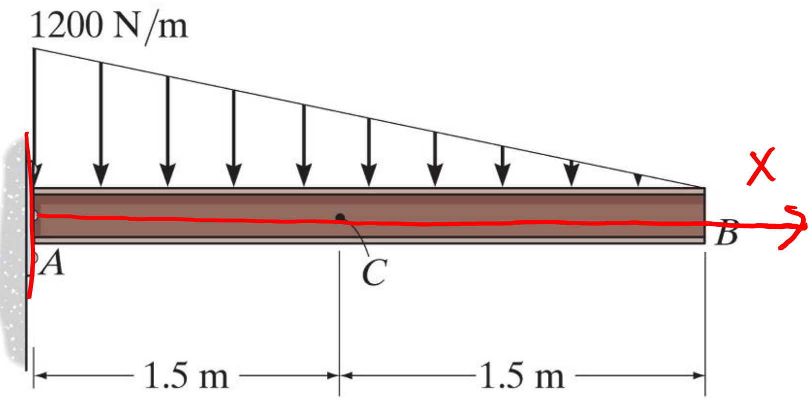
Recap: Shear Force and Bending Moment Diagrams

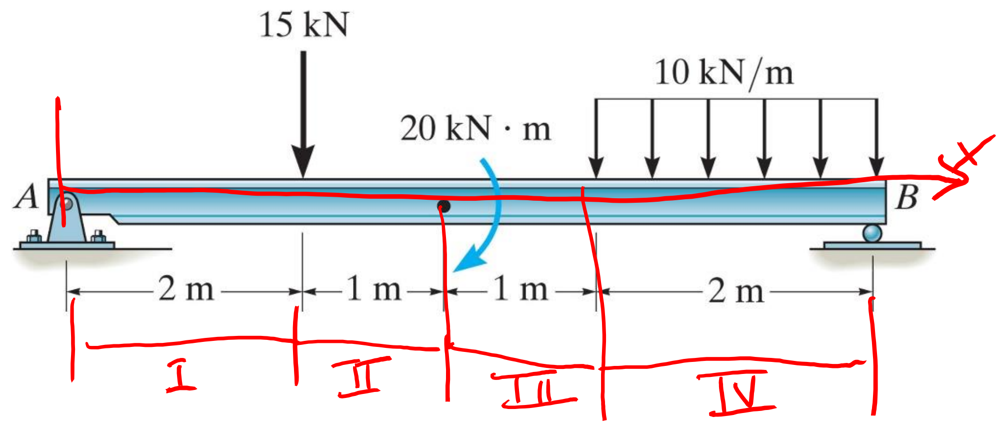
Goal: provide detailed knowledge of the variations of internal shear force and bending moments (V and M) throughout a beam when perpendicular distributed loads, concentrated forces, and/or concentrated couple moments are applied.

Normal forces (N) in such beams are zero, so we will not consider normal force diagrams.

Procedure

1. Find support reactions (free-body diagram of entire structure)
2. Specify coordinate x (start from left)
3. Divide the beam into sections according to loadings
4. Draw FBD of a section
5. Apply equations of equilibrium to derive V and M as functions of x: $V(x)$, $M(x)$





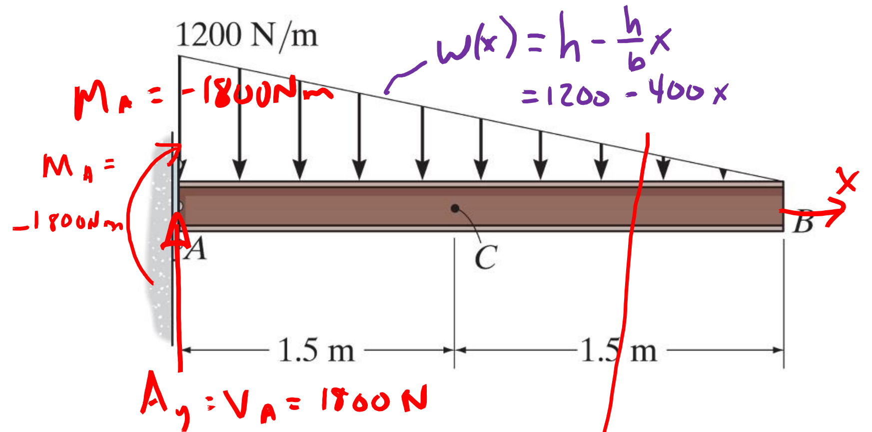
Recap: Draw the shear and bending moment diagrams for the beam.

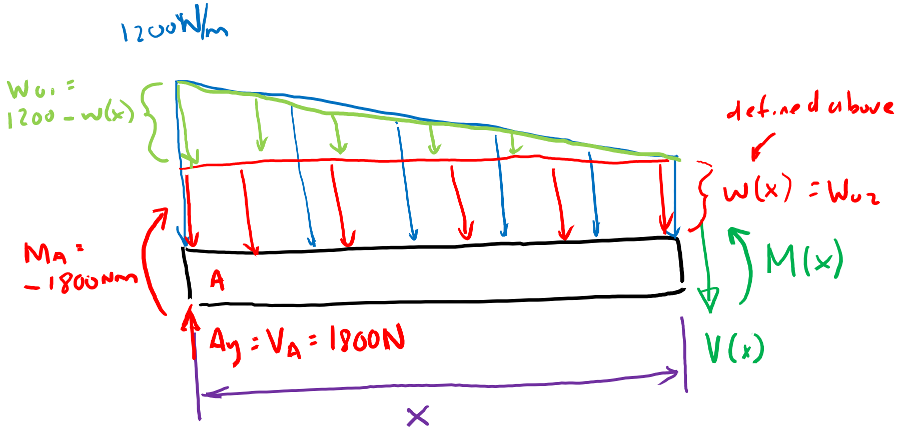
Detailed notes added to post-lecture version of Lecture 21

From previous example, we know that the support reactions are: $A_x = 0$, $A_y = 1800N \uparrow$, $M_A = -1800\ Nm$

We are interested in finding $V(x)$ & $M(x)$ as these vary along the length of the beam.

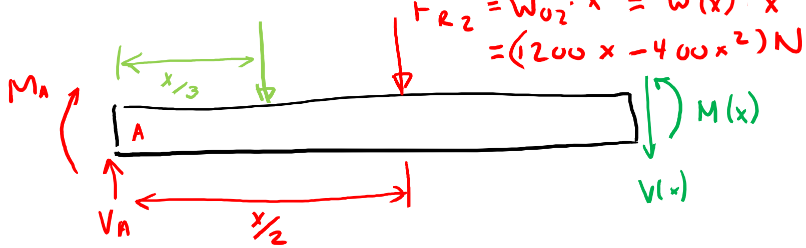
So for any length x of the beam, we get the following generic FBD as a function of x.





$F_{R_1} = w_{01} \frac{L}{2} = [1200 - w(x)]\frac{x}{2} = (200x^2)N$

$F_{R_2} = w_{02} \cdot x = w(x) \cdot x$
$= (1200x - 400x^2)N$



$\Sigma F_y: A_y - F_{R_1} - F_{R_2} - V(x) = 0$

$$\boxed{V(x) = (200x^2 - 1200x + 1800)\,N}$$

Quadratic

Boundary conditions:

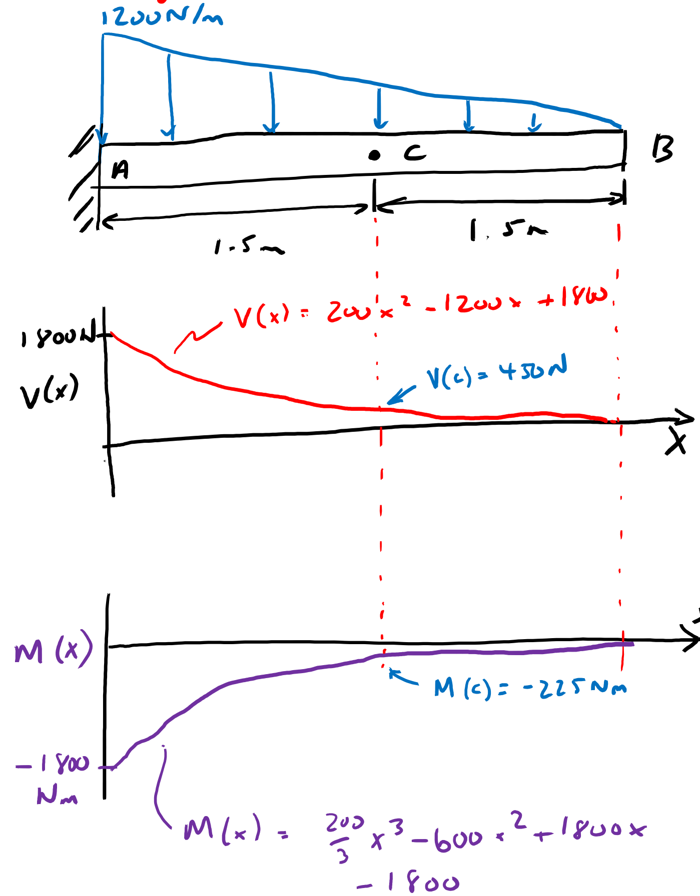
$V(x=0) = 1800N = A_y$

$V(x=L=3m) = 0N$

cf. $V(@C = 1.5m) = 450N$ ✓ w/ previous example

$V = \frac{dM}{dx}$

Draw Shear Force $V(x)$ & Bending Moment $M(x)$ diagrams



$+\circlearrowleft \Sigma M_A: -M_A - \left(\frac{x}{3}\right)F_{R_1} - \left(\frac{x}{2}\right)F_{R_2} - x \cdot V(x) + M(x) = 0$

$$\boxed{M(x) = \left(\frac{200}{3}x^3 - 600x^2 + 1800x - 1800\right) Nm}$$

3rd Order Polynomial

B.C.

$M(0) = -1800\,Nm = M_A$

$M(L) = 0$

cf. $M(@C = 1.5m) = -225\,Nm$ ✓ w/ previous

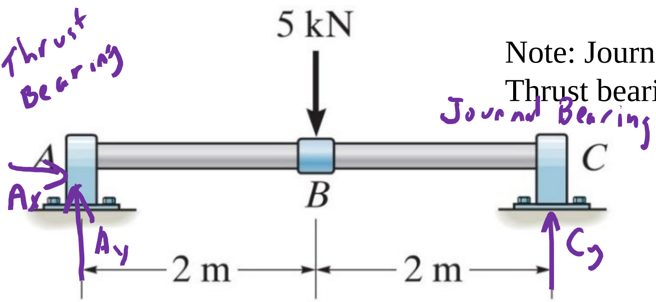
Note that since the applied load is a single distributed load along the entire length of the beam, then V(x) and M(x) are continuous functions. We will see (in Lecture 22) that V(x) and M(x) will be discontinuous functions when multiple loads are applied to a beam, and these discontinuities will happen at the transitions between loading regions.

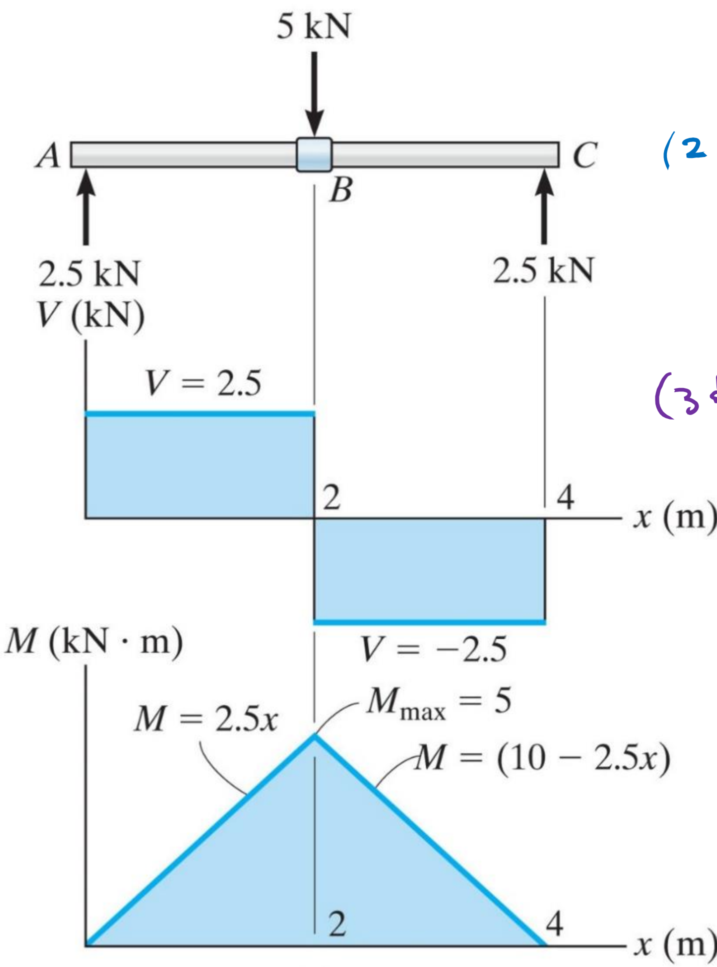
Recap: Explore and re-create the shear force and bending moment diagrams for the beam. A is thrust bearing & C is journal bearing.

Example: single concentrated load

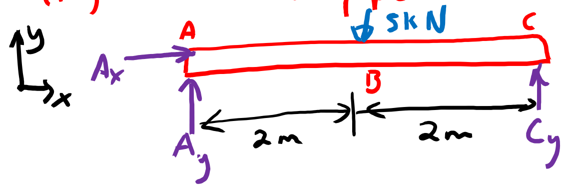
See Example 7.6 in text

Note: Journal bearings only have support reaction forces and moments on axes perpendicular to shaft. Thrust bearings are similar to journal bearings but with added support reaction force along axis of shaft





(1) Find support reactions:



ΣF_x : $\boxed{A_x = 0}$

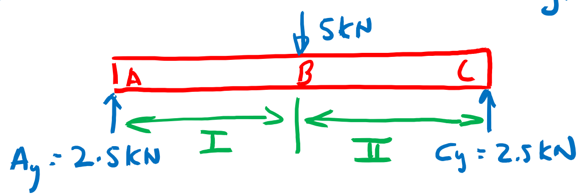
ΣF_y : $A_y + C_y - 5kN = 0$

$+\circlearrowleft\ \Sigma M_A$: $-(2m)5kN + (4m)C_y = 0$

$\rightarrow \boxed{C_y = 2.5kN}$

$\therefore \boxed{A_y = 2.5kN}$

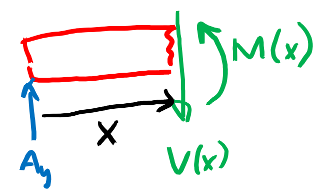
(2) Divide beam into regions according to loadings.



(3 & 4) Draw FBD of a region. Use Eq of Eq to derive V(x) & M(x):

Region I:

$0 < x < 2m$



ΣF_y : $A_y - V(x) = 0$

$\boxed{V(x) = A_y = 2.5kN}$

constant, positive

$+\circlearrowleft\ \Sigma M_A$: $-x \cdot V(x) + M(x) = 0 \quad \therefore M(x) = x \cdot V(x)$

$\boxed{M(x) = x A_y = 2.5x \ kN \cdot m}$

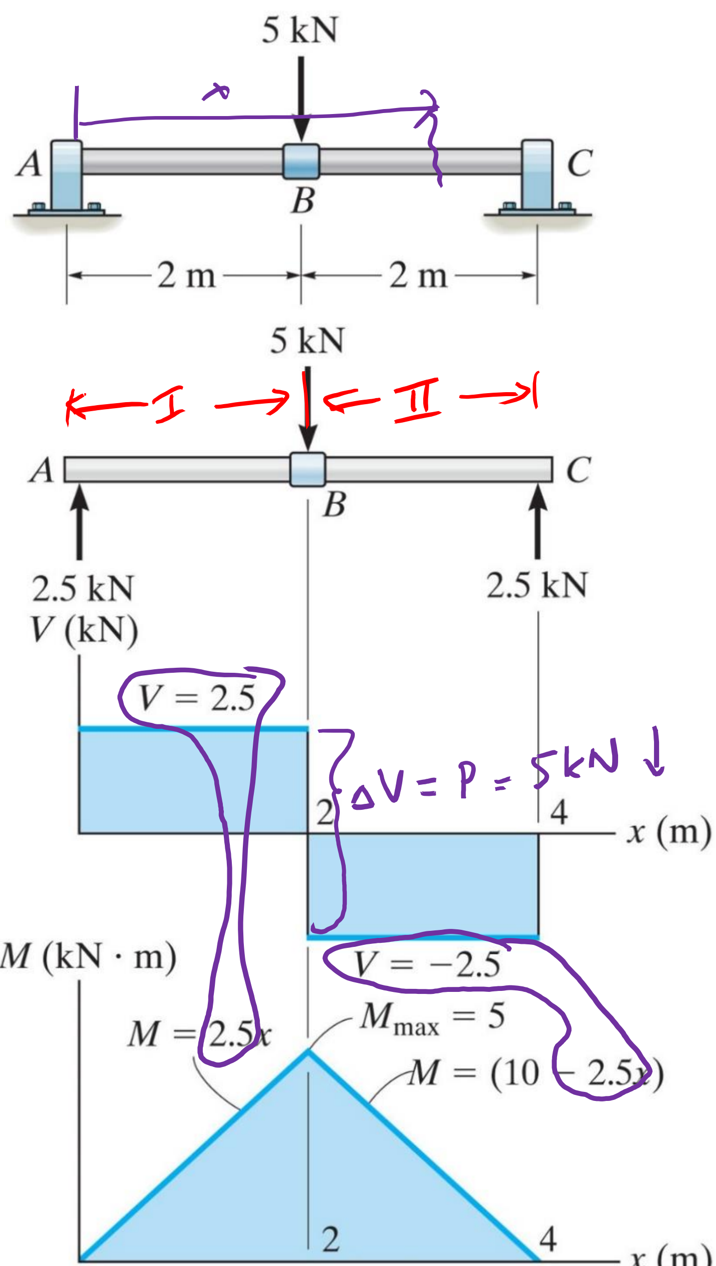
Note: could chose ΣM_x where x is at cut on right side

linear w/ slope A_y

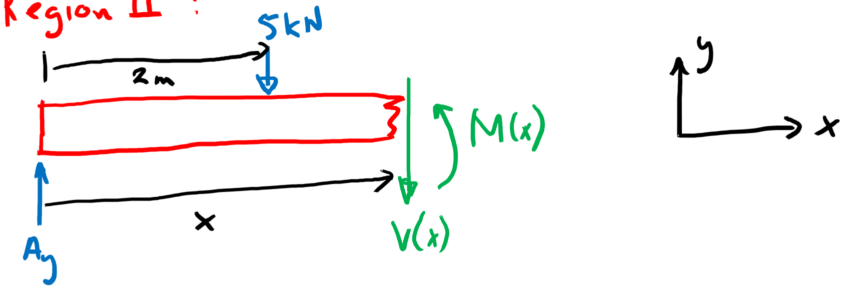
Boundary Conditions: Compare results to plots to left

$x = 0$: $V(0) = 2.5kN$, $M(0) = 0$

$x = 2m^{(-)}$ {use (−) as immediately to left of 2m} : $V(2m^{(-)}) = 2.5kN$, $M(2m^{(-)}) = 5kN \cdot m$



Region II : $2m < x < 4m$



ΣF_x: $A_y - V(x) - 5\,kN = 0$

$$\boxed{V(x) = A_y - 5\ kN = -2.5\,kN}$$

constant, negative

$+\circlearrowleft \Sigma M_A$: $-(2m)5kN - x \cdot V(x) + M(x) = 0$

$M(x) = 10kN\cdot m + x \cdot V(x)$

$= 10\ kN\cdot m + x\,(A_y - 5\ kN)$

$$\therefore \boxed{M(x) = (10 - 2.5x)\ kN\ m}$$

linear w/ negative slope

cf. BC's :

$x = 2m^{(+)}$: $V(2m^{(+)}) = -2.5kN$, $M(2m^{(+)}) = 5kN\cdot m$

$x = 4m$: $V(4) = -2.5\,kN$, $M(4) = 0$

compare results to plots to left

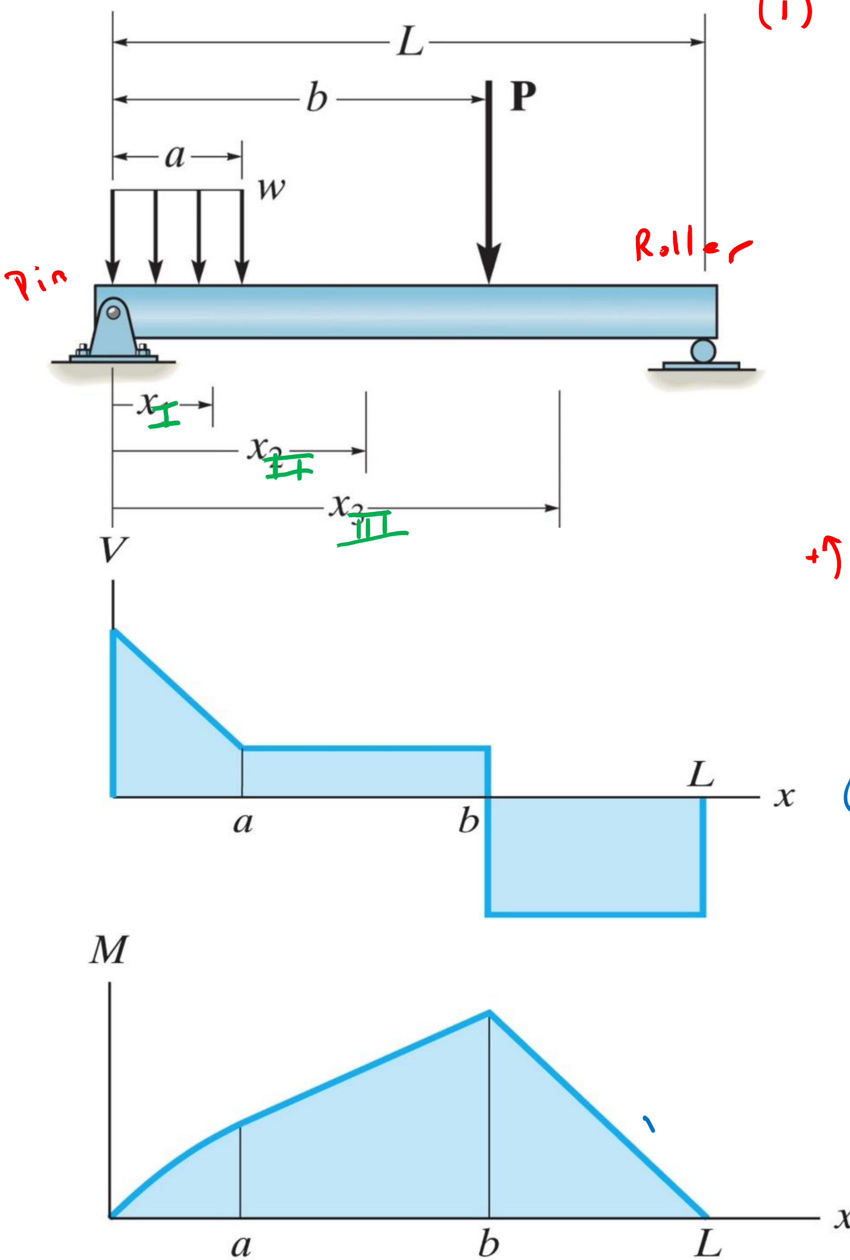
Note for <u>single concentrated load (P)</u>:

- $V(x)$ is <u>constant</u> within a region. $V(x)$ has a step change at location of load that is equivalent to magnitude and direction of applied load (e.g., -P$\hat{j}$ or -5 kN).
- $M(x)$ is <u>linear</u>.

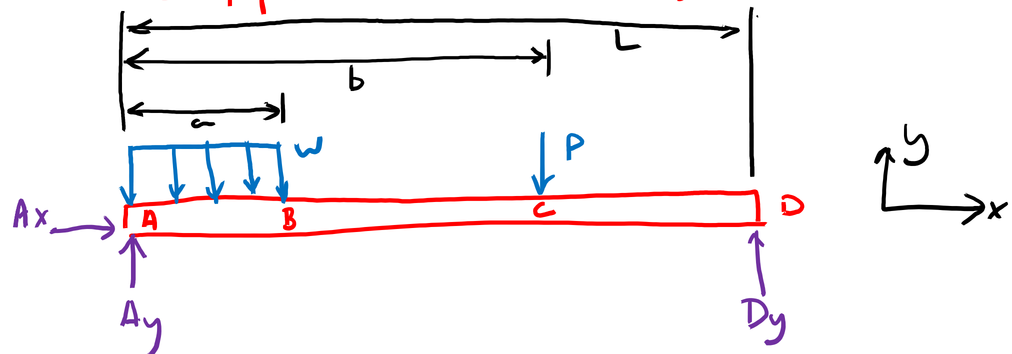
Also note that $V(x) = \frac{d}{dx} M(x)$, or slope of moment diagram

Recap: Explore and re-create the shear force and bending moment diagrams for the beam.

Example: single concentrated load, rectangular distributed load



(1) Find support reactions:



ΣF_x : $\boxed{A_x = 0}$

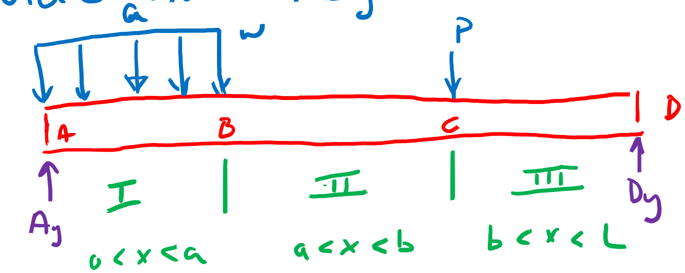
ΣF_y : $A_y + D_y - aw - P = 0$

$+\circlearrowleft \Sigma M_A$: $-(\frac{a}{2})aw - b \cdot P + L \cdot D_y = 0$

$D_y = \frac{1}{L}\left(\frac{a^2 w}{2} + bP\right) \equiv$ constant

$A_y = aw + P - D_y = aw\left(1 - \frac{a}{2L}\right) + P\left(1 - \frac{b}{L}\right)$
$\equiv$ constant

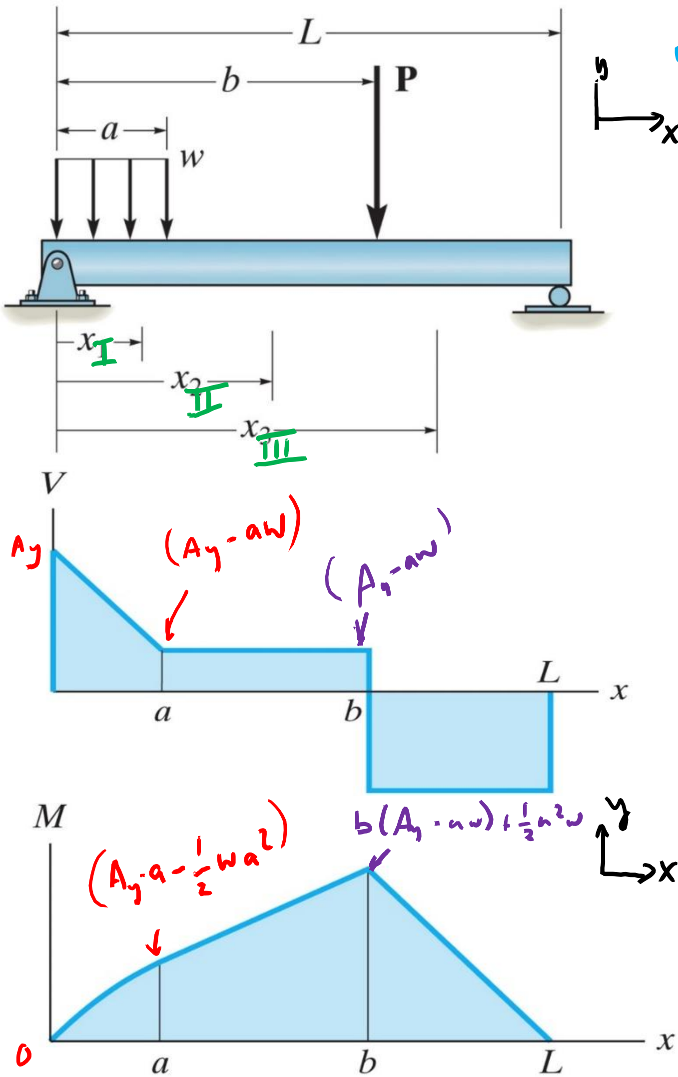
(2) Divide into regions



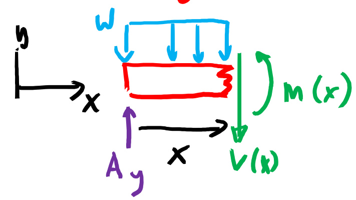
3 regions, 3 FBDs

Recap: Explore and re-create the shear force and bending moment diagrams for the beam.

Example: single concentrated load, rectangular distributed load



Region I : $0 < x < a$



$\Sigma F_y : \; A_y - V(x) - x \cdot w = 0$

$\boxed{V(x) = A_y - xw}$ linear w/ slope w

$+\circlearrowleft M_A : \; M(x) - \left(\frac{x}{2}\right)(xw) - x \cdot V(x) = 0$

$M(x) = \frac{x^2 w}{2} + x\,(A_y - xw)$

$\therefore \boxed{M(x) = A_y x - \frac{1}{2} w x^2}$ quadratic

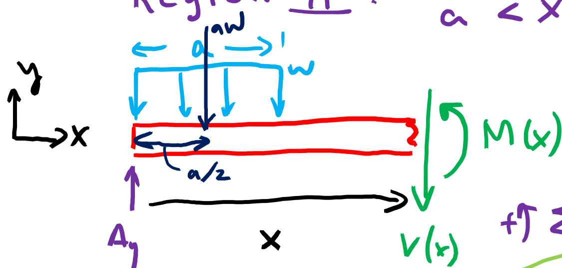
BC's:

$x = 0 : \quad V(0) = A_y , \quad M(0) = 0$

$x = a^{(-)} : \quad V(a) = A_y - aw , \quad M(a) = A_y \cdot a - \frac{1}{2} w a^2$

Compare to values labeled on V & M diagrams to left

Region II : $a < x < b$



$\Sigma F_y : \; A_y - aw - V(x) = 0$

$\boxed{V(x) = A_y - aw}$ constant

$+\circlearrowleft \Sigma M_x : \; M(x) - x \cdot A_y - \left(x - \frac{a}{2}\right) aw = 0$

$\boxed{M(x) = x\,(A_y - aw) + \frac{1}{2} a^2 w}$ linear w/ slope $(A_y - aw) = V(x)$

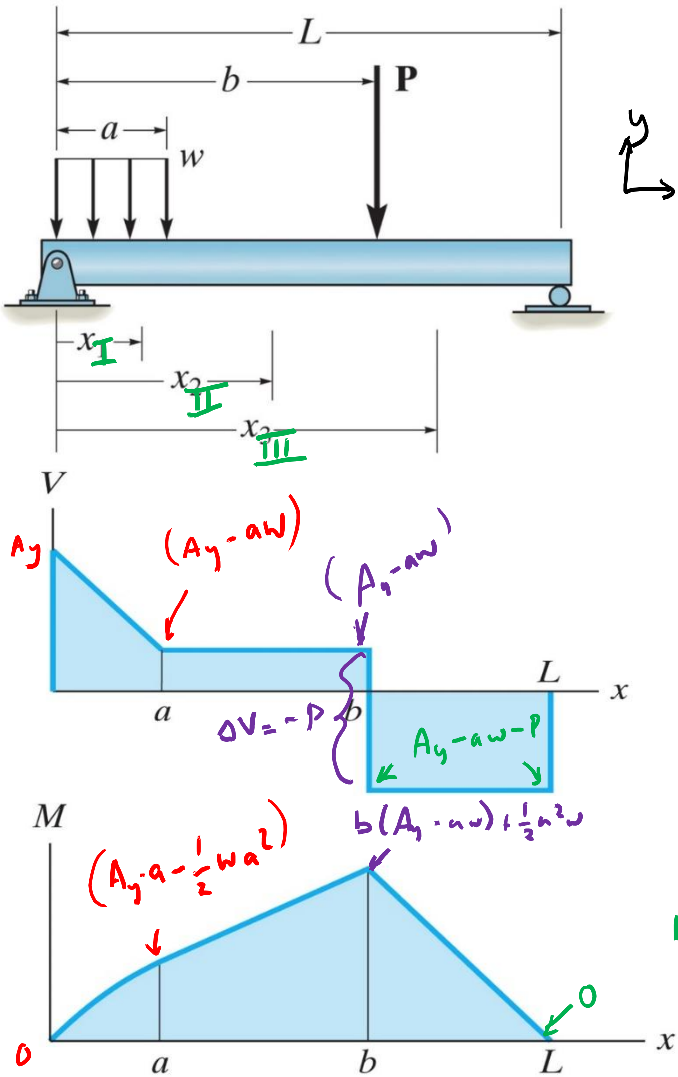
Note used pt x to sum moments

BC's: $x = a^{(+)} : \quad V(a^{(+)}) = A_y - aw, \quad M(a) = A_y a - \frac{1}{2} w a^2$

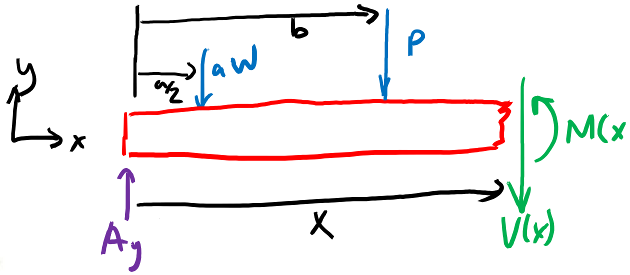
$x = b^{(-)} : \quad V(b^{(-)}) = A_y - aw , \quad M(b^{(-)}) = b(A_y - aw) + \frac{1}{2} a^2 w$

Recap: Explore and re-create the shear force and bending moment diagrams for the beam.

Example: single concentrated load, rectangular distributed load



Region III : $b < x < L$



$\Sigma F_y: A_y - aw - P - V(x) = 0$

$\boxed{V(x) = A_y - aw - P}$

constant

$+\circlearrowleft \Sigma M_x : M(x) - x A_y + (x - \frac{a}{2}) \cdot aw + (x - b) \cdot P = 0$

$$\boxed{M(x) = x \underbrace{(A_y - aw - P)}_{V(x)} + bP + \frac{1}{2} a^2 w}$$

← linear w/ slope V(x)

BC's:

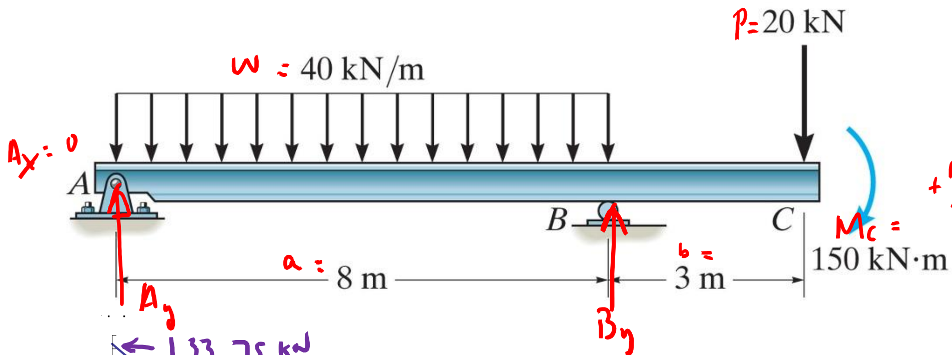
$x = b^{(+)}$: $V(b) = A_y - aw - P$, $M(b) = b(A_y - aw) + \frac{1}{2} a^2 w$

$x = L$: $V(L) = A_y - aw - P$, $M(L) = 0$

Note step change of $-P$ in $V(x)$ at $x = b$ due to concentrated load of P

Explore and re-create the shear force and bending moment diagrams for the beam.

Example: concentrated load, rectangular distributed load, concentrated couple moment

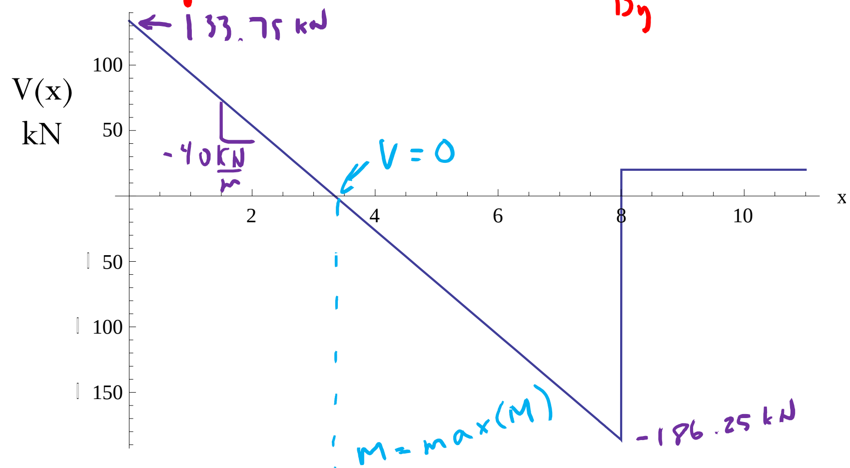


1) Find A_y & B_y:

$\Sigma F_y : A_y + B_y - aw - P = 0$

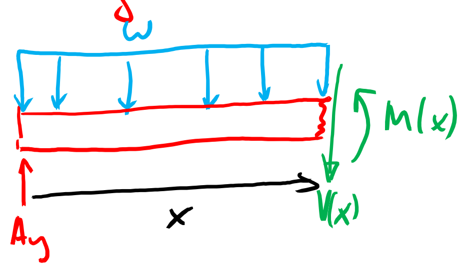
$+\circlearrowleft \; \Sigma M_A : -\left(\frac{a}{2}\right)(aw) + aB_y - (a+b)P - M_c = 0$

$$\boxed{B_y = \frac{M_c + (a+b)P + \frac{a^2 w}{2}}{a} = 206.25 \text{ kN} \qquad A_y = aw + P - B_y = 133.75 \text{ kN}}$$



(2-4) Solve for each region.

Region I $\quad 0 < x < a$

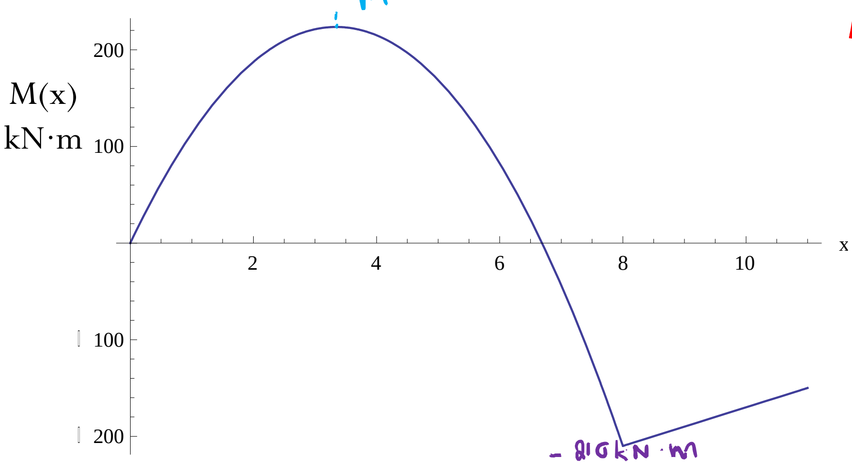


$\Sigma F_y : \boxed{V(x) = A_y - wx}$

$V(x) = (133.75 - 40x)$ kN

$+\circlearrowleft \; M_x : \boxed{M(x) = A_y \cdot x - \left(\frac{x}{2}\right)(x w)}$

$M(x) = (133.75x - 20x^2)$ kN·m



BC's: $x = 0$: $V(0) = A_y = 133.75$ kN, $M(0) = 0$

$x = a$: $V(a) = -186.25$ kN, $M(a) = -210$ kN·m

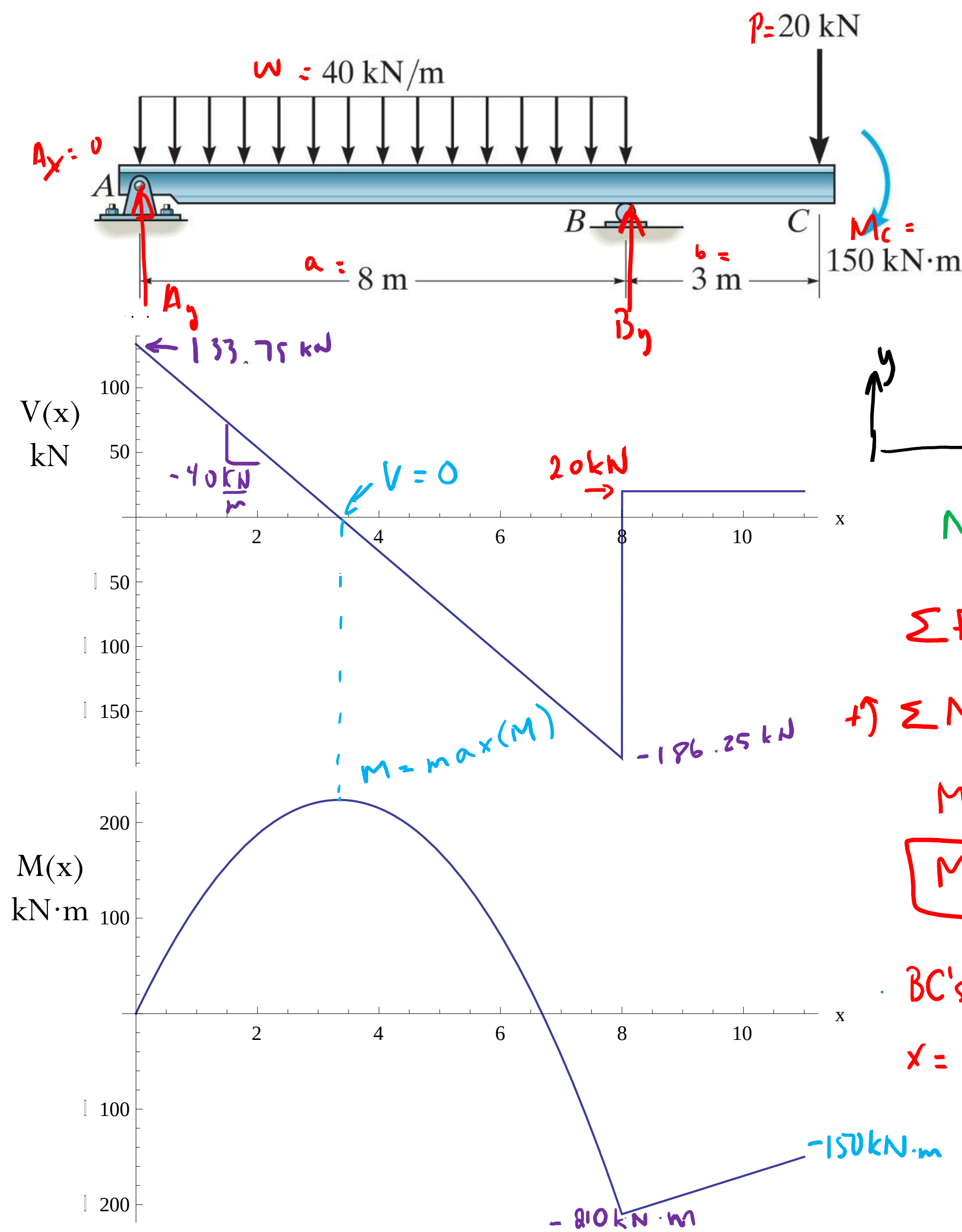
Note for rectangular distributed load:

$V(x)$ is linear with slope of w. In this case, slope is $-w$ since w is pointing in –y direction.

$M(x)$ is quadratic. When $V(x) = 0$, and $M(x) = \max(M)$.

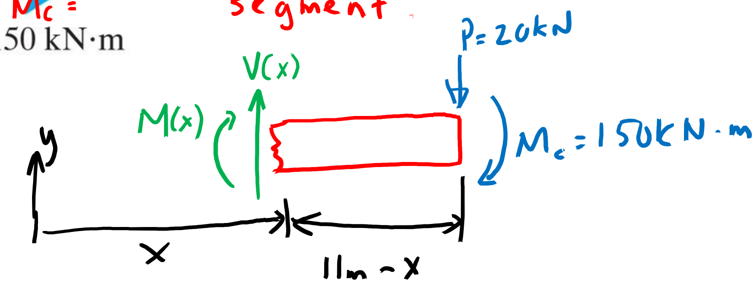
Explore and re-create the shear force and bending moment diagrams for the beam.

Example: concentrated load, rectangular distributed load, concentrated couple moment



Region II : $B < x < C$ or $a < x < (a+b)$

Due to concentrated load P & couple moment M_c at end C, draw FBD for right side segment.



Note directions of arrows for V(x) & M(x)

$\Sigma F_y : V(x) - P = 0 \quad \boxed{V(x) = P = 20\text{ kN}}$

$+\circlearrowleft\ \Sigma M_c : -M(x) - (11\text{m} - x)\cdot V(x) - M_c = 0$

$M(x) = -M_c - (11\text{m}(20\text{kN}) + x\,V(x) = -370 + x \cdot V(x)$

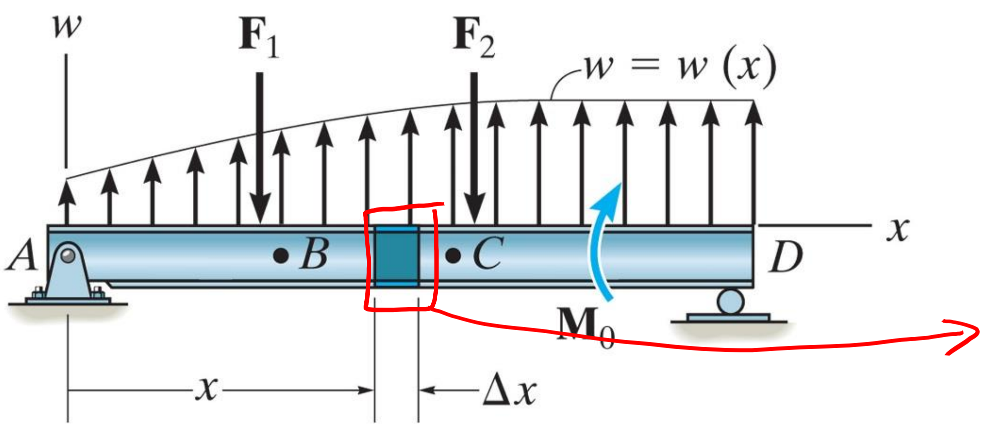
$\boxed{M(x) = (-370 + 20x)\text{ kN m}}$

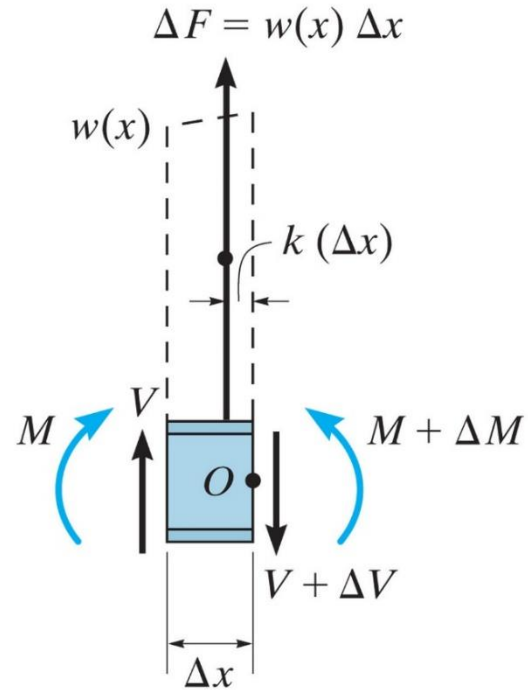
BC's: $\underset{= 8\text{m}}{x = a^{(+)}}$: $V(a^{(+)}) = 20\text{kN}$, $M(a^{(+)}) = -210\text{ kN}\cdot\text{m}$

$x = L = 11\text{m}$. $V(L) = 20\text{kN}$, $M(L) = -150\text{ kN}\cdot\text{m}$

As expected, since have applied $M_c = -150\text{ kN}\cdot\text{m}\ \hat{k}$

Relations Among Distributed Load, Shear Force and Bending Moments





Relationship between distributed load and shear:

$$\sum F_y = 0: \quad V - (V + \Delta V) + w\,\Delta x = 0$$

$$\Delta V = w\,\Delta x$$

Dividing by Δx and letting $\Delta x \to 0$, we get:

$$\boxed{\frac{dV}{dx} = w \qquad \Delta V = \int w\,dx}$$

slope of shear force = distributed load intensity

Relationship between shear and bending moment:

$$\sum M_O = 0: \quad (M + \Delta M) - M - V\,\Delta x - w\,\Delta x\,(k\,\Delta x) = 0$$

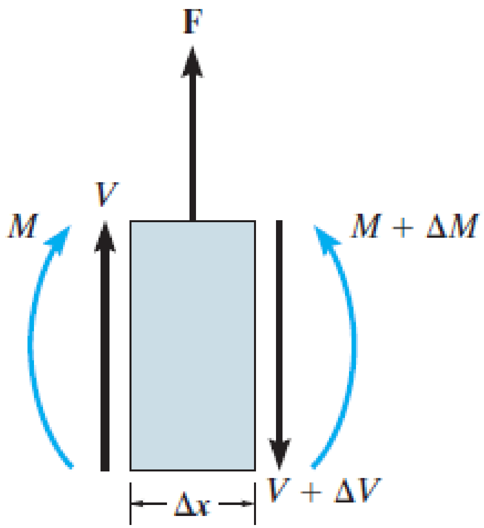
$$\Delta M = V\,\Delta x + w\,k\,(\Delta x)^2$$

Dividing by Δx and letting $\Delta x \to 0$, we get:

$$\boxed{\frac{dM}{dx} = V \qquad \Delta M = \int V\,dx}$$

slope of B.M = shear force

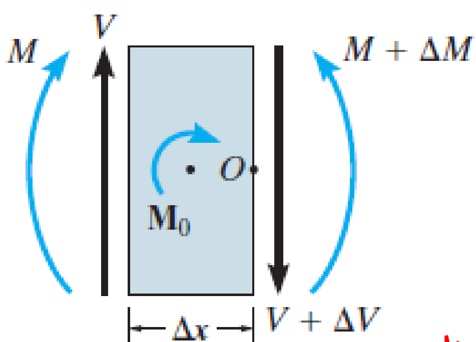
Wherever there is an external concentrated force or a concentrated moment, there will be a change (jump) in shear or moment, respectively.



ΣF_y:

$V + F - (V + \Delta V) = 0$

$\Delta V = \mathrm{F}$ jump in shear force due to concentrated load F



$+\circlearrowright$ ΣM_O:

$(M + \Delta M) - M - M_O - V(\Delta x) = 0$

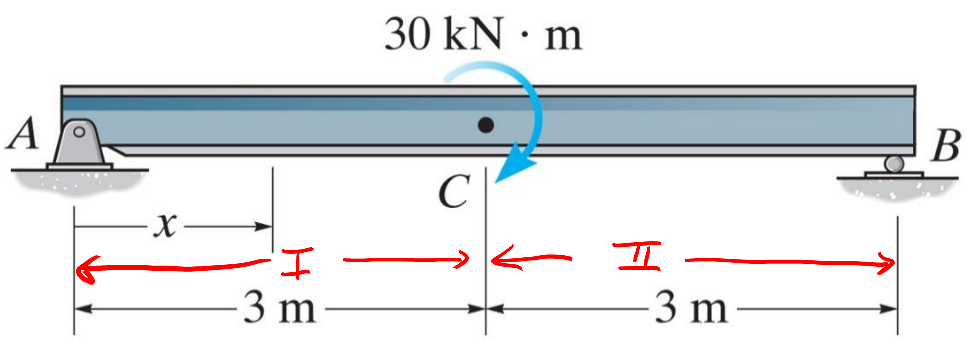
$\Delta M = M_O + V(\Delta x)$

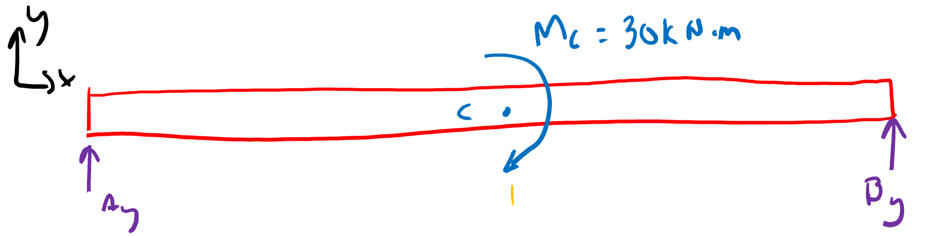
$\Delta M = M_O$, when $\Delta x \rightarrow 0$

jump in bending moment due to concentrated couple moment M_o

* Note: the text, these notes, and convention assume that an applied concentrated moment **M_O in clockwise direction** results in a **positive change in $M(x)$**

Draw the shear force and moment diagrams for the beam.





(1) Reaction supports:

ΣF_y: $A_y = -B_y$

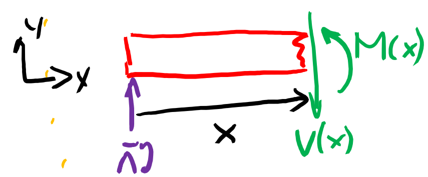
$+\circlearrowleft \Sigma M_A$: $-30\,kN\,m + (6m)B_y = 0$

$\boxed{B_y = 5\,kN.m}$

$\boxed{A_y = -5\,kN.m}$

Region I: $0 < x < 3$

A) using FBD & EoE to create $V(x)$ & $M(x)$:



ΣF_y: $\boxed{V(x) = A_y = -5kN}$

constant, negative

$+\circlearrowleft \Sigma M_A$: $M(x) - x \cdot V(x) = 0$

$\boxed{M(x) = x\ V(x)}$

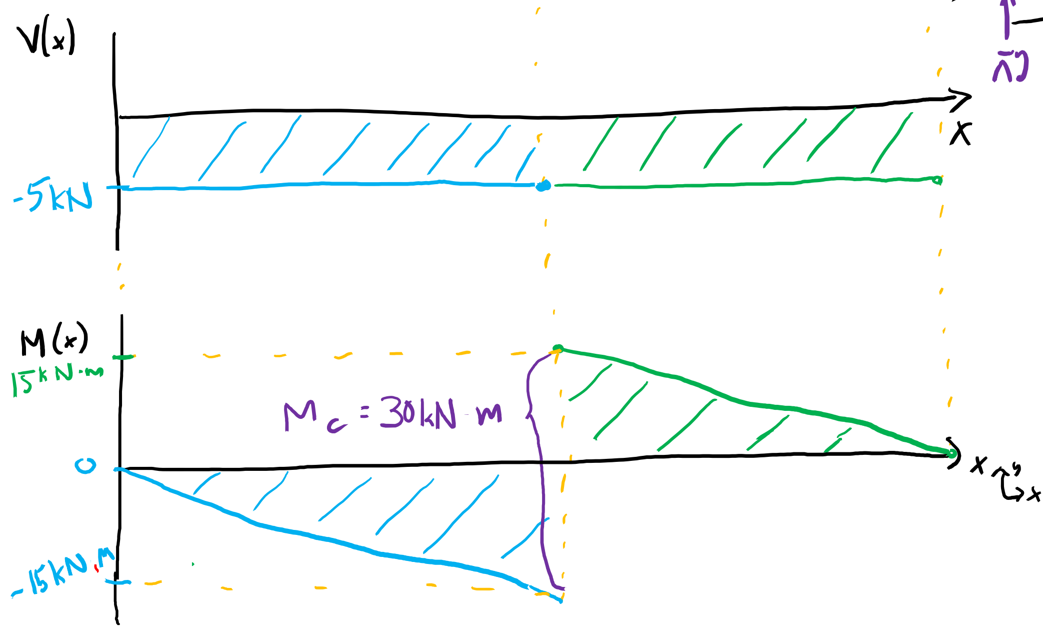
linear w/ slope $V(x)$

slope $\frac{dM}{dx} = V(x) = -5kN$

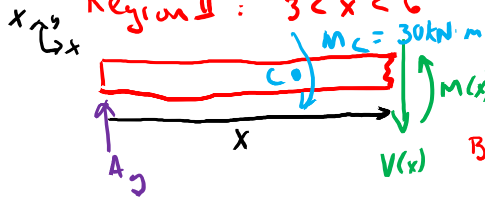
use BC's to find end points for $M(x)$

$x = 0$: $M(0) = 0$

$x = 3m$: $M(3m^{(-)}) = -15\,kN \cdot m$



Region II: $3 < x < 6$



ΣF_y: $\boxed{V(x) = A_y = -5kN}$

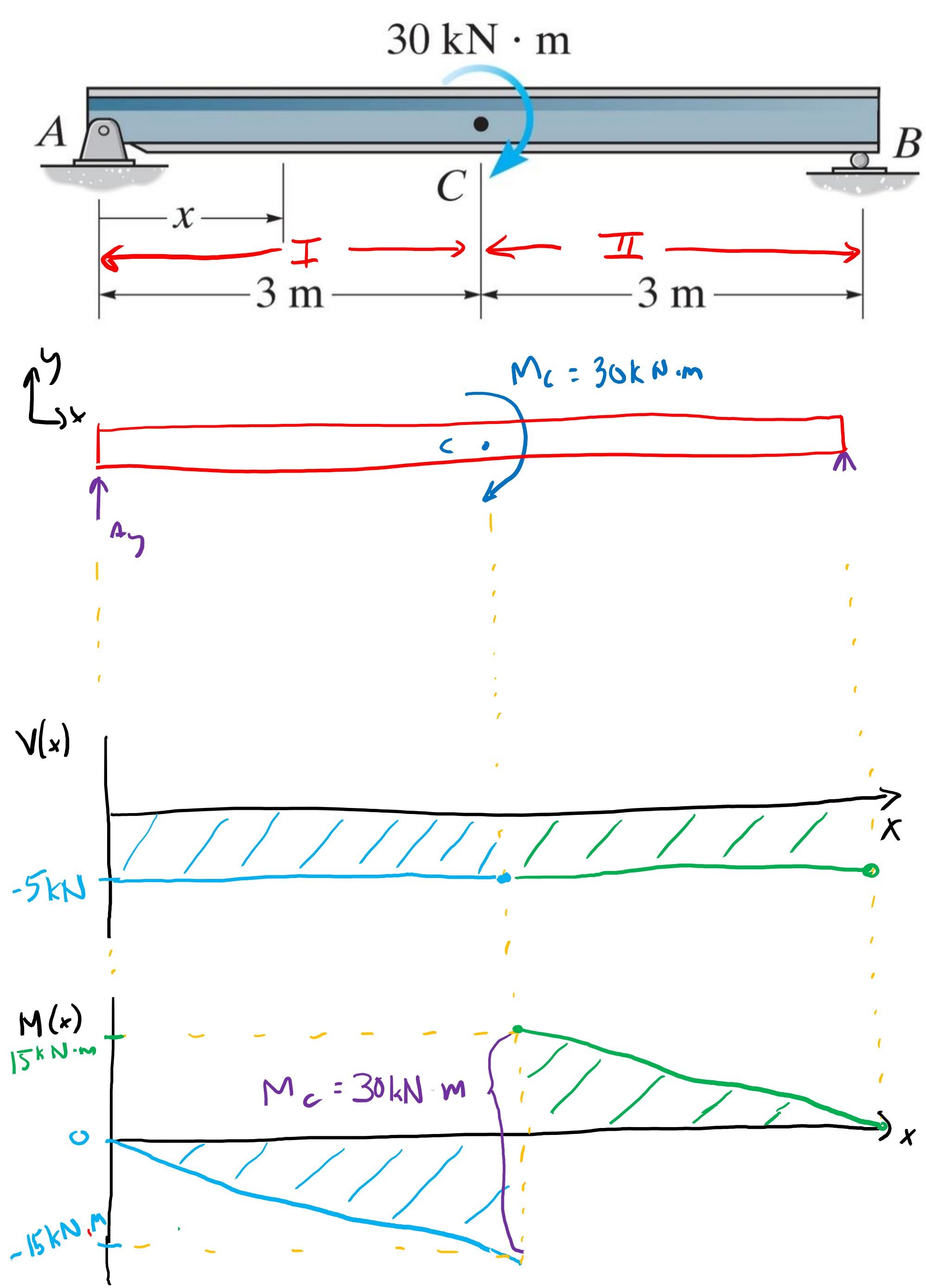
$+\circlearrowleft \Sigma M_A$: $M(x) - x \cdot V(x) - M_c = 0$

$M(x) = x \cdot V(x) + M_c$

BC's: $M(3m^{(+)}) = -15 + 30 = 15\,kN \cdot m$

$M(6m) = -30 + 30 = 0$

Draw the shear force and moment diagrams for the beam.



B) Alternative method to quickly draw V & M diagrams

use $\frac{dV}{dx} = w(x)$ to define slope of V(x)

$\Delta V = V_2 - V_1 = \int w(x)\,dx$ change in shear = area under loading curve

$\frac{dM}{dx} = V(x)$ to define slope of M(x)

$\Delta M = M_2 - M_1 = \int V(x)\,dx$ change in moment = area under shear curve

For concentrated moment:

$w(x) = 0$

$\rightarrow \frac{dV}{dx} = 0$ (no slope)

$\Delta V = \int w\,dx = 0$ (no change in V)

$\frac{dM}{dx} = V(x) = \text{neg. const} = A_y = -5\text{kN}$

$\Delta M = M_c$ (from knowledge of applied moment M_c)

Since M_c ↻ cw, then ΔM is in positive direction ↑

(see notes 2 slides prior)

Draw the shear force and moment diagrams for the beam.

[solution written outside of class]

1) Find support reactions

$+\uparrow \Sigma F_y: A_y + B_y - 6\,kN - 12\,kN = 0$

$+\circlearrowleft \Sigma M_A: (2m)\,6kN - (4m)(12kN) + (6m)B_y = 0$

$\boxed{B_y = 10\,kN} \Rightarrow \boxed{A_y = 8\,kN}$

2) Quickly draw $V(x)$ & $M(x)$

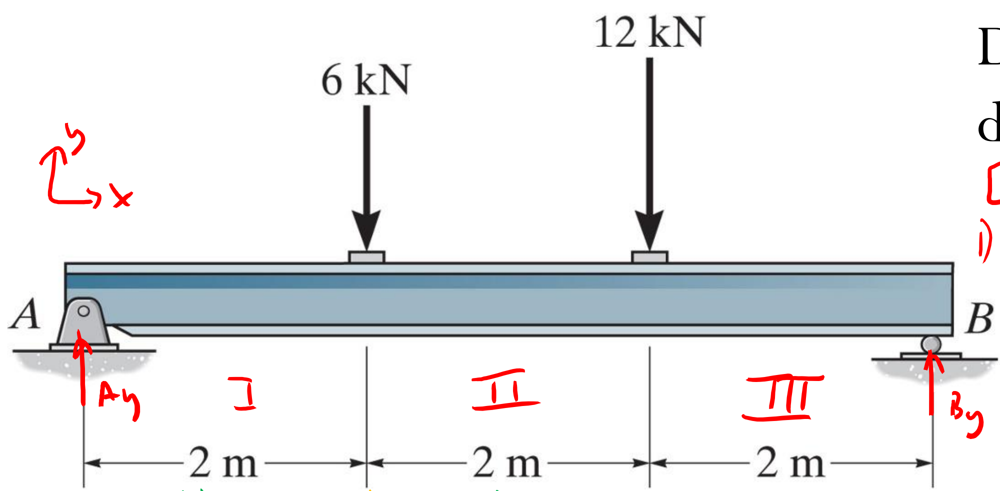
within a region use:

$\frac{dV}{dx} = w$, $\Delta V = V_2 - V_1 = \int w\,dx$

$\frac{dM}{dx} = V$, $\Delta M = M_2 - M_1 = \int V\,dx$

At locations of applied loads,

use $\Delta V = F$

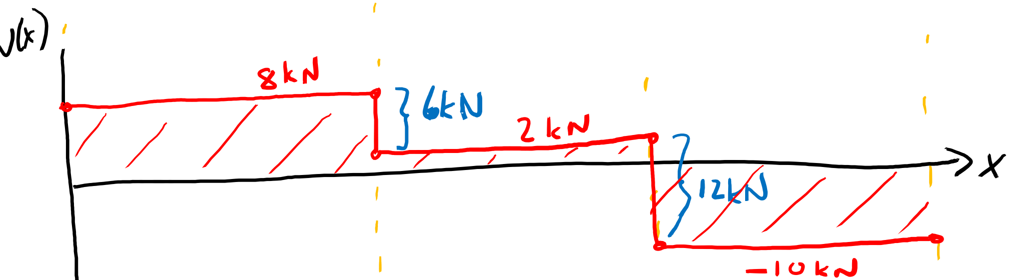


$w_I(x) = 0$ $\qquad$ $w_{II}(x) = 0$ $\qquad$ $w_{III}(x) = 0$

$\frac{dV}{dx} = w = 0$ $\qquad$ $\frac{dV}{dx} = w = 0$ $\qquad$ $\frac{dV}{dx} = w = 0$

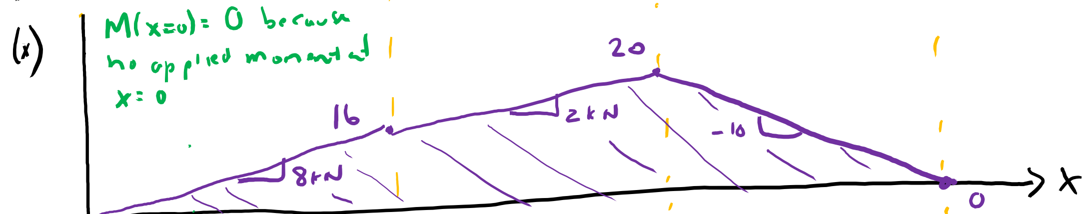
$\Delta V_I = \int w\,dx = 0$ $\qquad$ $\Delta V_{II} = 0$ $\qquad$ $\Delta V_{III} = 0$

$\Delta V = F_1$ $\qquad$ $\Delta V = F_2$

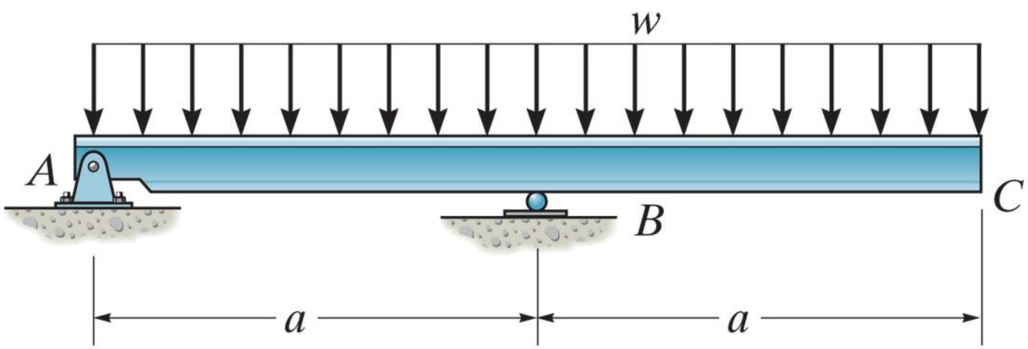


$\frac{dM}{dx} = V = 8kN$ $\qquad$ $\frac{dM}{dx} = V = 2kN$ $\qquad$ $\frac{dM}{dx} = V = -10kN$

$M(x=0) = 0$ because no applied moment at $x = 0$



$\Delta M(2) = 2 \cdot 8 = 16\,kN \cdot m$ $\qquad$ $\Delta M(2) = 2 \cdot 2 = 4$ $\qquad$ $\Delta M(2)\ 2 \cdot (-10) = -20$



Draw the shear force and moment diagrams for the beam.

Draw the shear force and bending moment diagrams for the beam.

Example: concentrated load, rectangular distributed load, concentrated couple moment