Statics - TAM 210 & TAM 211

Lecture 24

March 14, 2018

Chap 7.3

Announcements

- ❑ Upcoming deadlines:
- Quiz 5 (3/14-16)
  - Sign up at CBTF
  - Up thru and including Lecture 22 (Shear Force & Bending Moment Diagrams), although review from Lectures 23-24 will be helpful.
- Monday (3/26)
  - Mastering Engineering Tutorial 11
- Thursday (3/29)
  - WA 4 due
- Tuesday (3/13)
  - PL HW 8
- Friday (3/30)
  - Last lecture for TAM 210 students
- Written exam (Thursday 4/5, 7-9pm in 1 Noyes Lab)
  - Conflict exam (Monday 4/2, 7-9pm)
    - **Must make arrangements with Prof. H-W by Friday 3/16**
  - DRES accommodation exam. Make arrangements at DRES. Must tell Prof. H-W

# How to orient positive V and M on a FBD?

"Positive" sign convention:

"Positive shear will create a clockwise rotation"   ⟹ Draw V arrows to create CW rotation

Therefore the direction of the arrow for the bending moment M on the same side of the segment follows the same sense as the shear force V pointing in the direction of the positive coordinate axis (the y-axes in these diagrams); thus both V and M create a clockwise rotation.
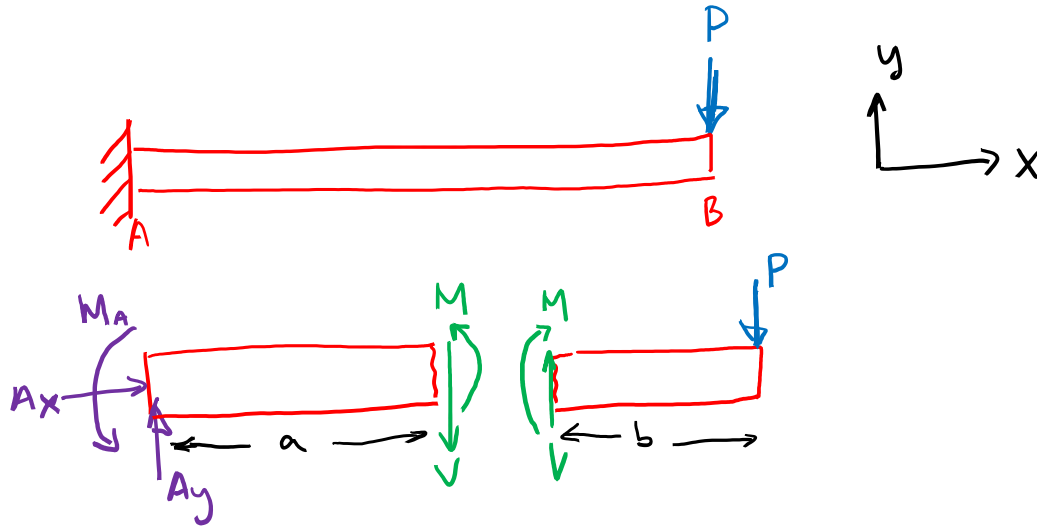
Horizontal Beam

Vertical Beam

Direction of the arrow for M depends on which coordinate system orientation chosen.
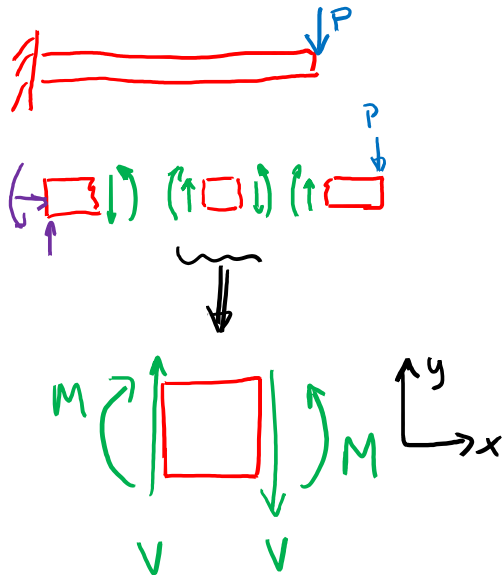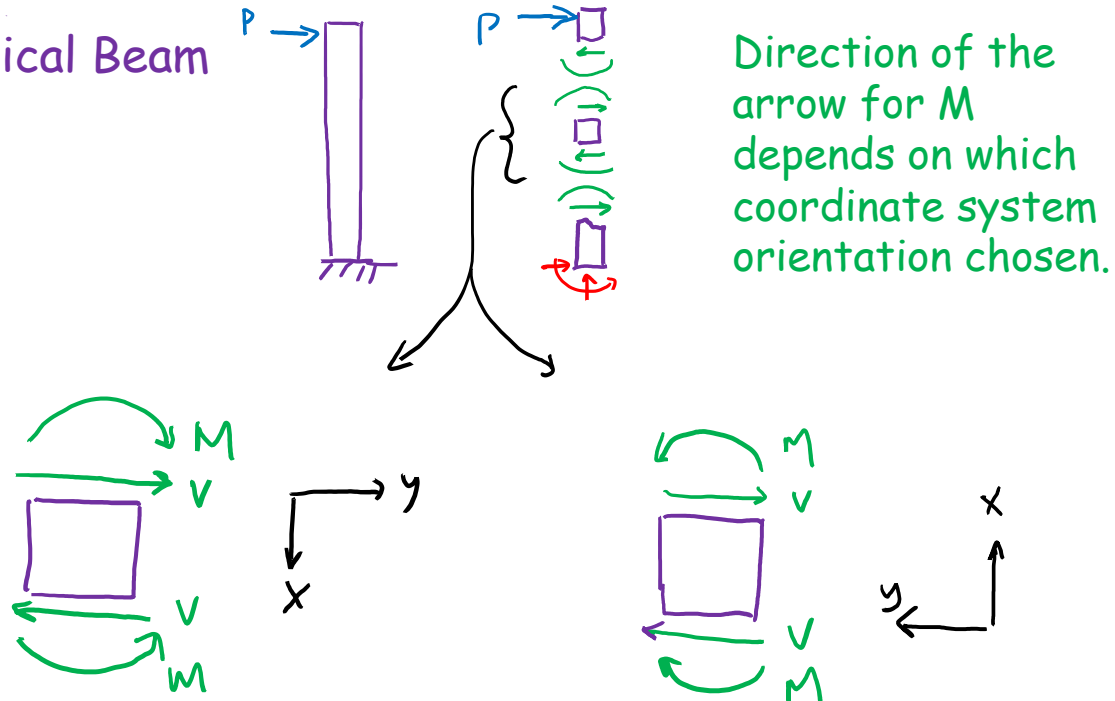
What sign to give V and M terms in equations of equilbrium?

Follow the positive orientations of the coordinate system.

For left side.

$+\uparrow \Sigma F_y: A_y - V = 0$

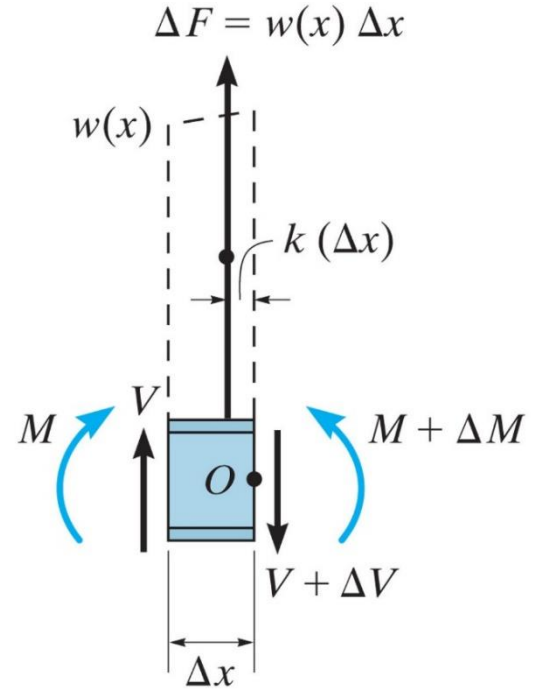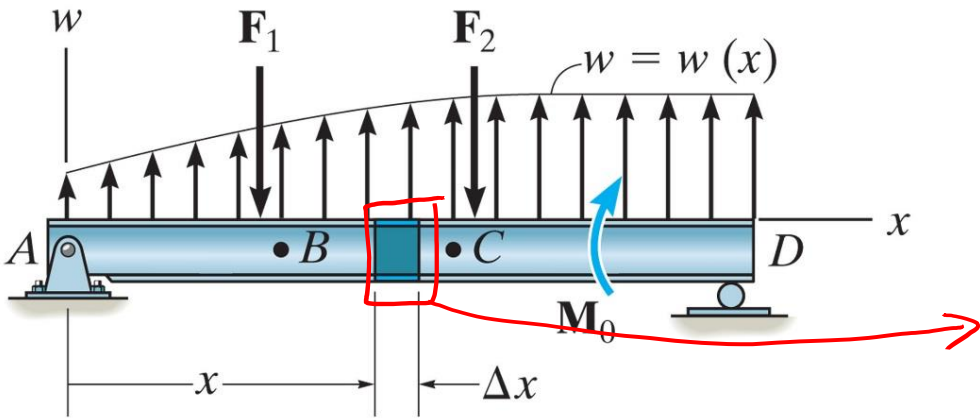$+\circlearrowleft \Sigma M_A:$

$M_A + M - a \cdot V = 0$

For right side:

$+\uparrow \Sigma F_y: V - P = 0$

$+\circlearrowleft \Sigma M_B:$

$-M - b \cdot V = 0$

Recap: Relations Among Distributed Load, Shear Force and Bending Moments

Relationship between <u>distributed load</u> and <u>shear</u>:

$$\frac{dV}{dx} = w$$

Slope of shear force = distributed load intensity

$$\Delta V = V_2 - V_1$$
$$= \int w \, dx$$

Change in shear force = area under loading curve

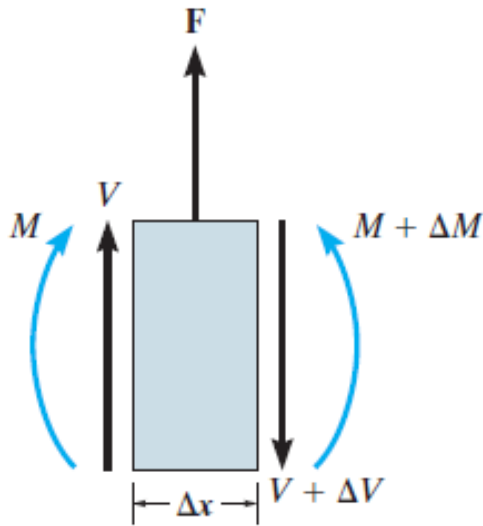Relationship between <u>shear</u> and <u>bending moment</u>:

$$\frac{dM}{dx} = V$$

Slope of bending moment = shear force

$$\Delta M = M_2 - M_1$$
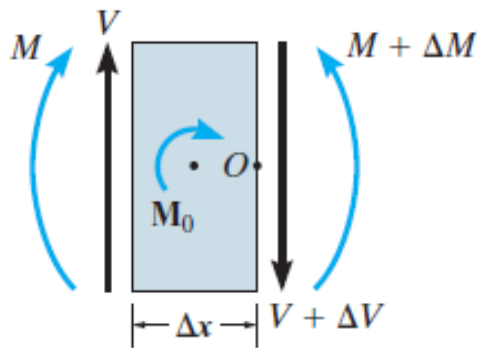$$= \int V \, dx$$

Change in moment= area under shear curve

Recap: Wherever there is an external concentrated force or a concentrated moment, there will be a change (jump) in shear or moment, respectively.

$\Sigma F_y:$

$V + F - (V + \Delta V) = 0$

$\Delta V = F$ — Jump in shear force due to concentrated load F
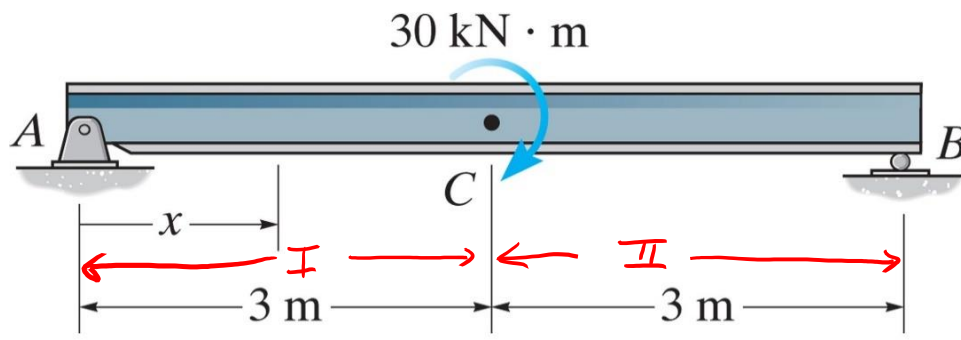
$+\!\!\!\curvearrowright \Sigma M_O:$

$(M + \Delta M) - M - M_O - V(\Delta x) = 0$

$\Delta M = M_O + V(\Delta x)$

$\Delta M = M_O$, when $\Delta x \to 0$

Jump in bending moment due to concentrated couple moment M_O

Note: the text, these notes, and convention assume that an applied concentrated moment $\underline{M_O}$ **in clockwise direction** results in a **positive change in $\bar{M}(x)$**

Draw the shear force and moment diagrams for the beam.
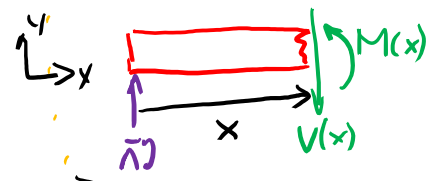
(1) Reaction supports:

$\Sigma F_y: \quad A_y = -B_y$

$+\circlearrowleft \Sigma M_A: \quad -30 kN \cdot m + (6m) B_y = 0$

$$\boxed{B_y = 5 kN \cdot m}$$

$$\boxed{A_y = -5 kN \cdot m}$$

$M_c = 30 kN \cdot m$

Region I: $0 < x < 3$

A) using FBD & EoE to create $V(x)$ & $M(x)$:

$\Sigma F_y: \quad \boxed{V(x) = A_y = -5 kN}$

constant, negative

$+\circlearrowleft \Sigma M_A: \quad M(x) - x \cdot V(x) = 0$

$$\boxed{M(x) = x \cdot V(x)}$$

linear w/ slope $V(x)$

slope $\dfrac{dM}{dx} = V(x) = -5 kN$

use BC's to find end points for $M(x)$

$x = 0: \quad M(0) = 0$

$x = 3m: \quad M(3m^{(-)}) = -15 kN \cdot m$

Region II: $3 < x < 6$

$M_c = 30 kN \cdot m$
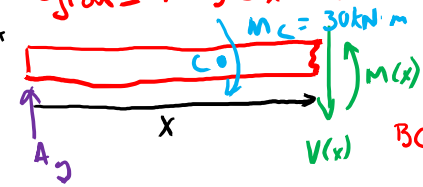
$\Sigma F_y: \quad \boxed{V(x) = A_y = -5 kN}$

$+\circlearrowleft \Sigma M_A: \quad M(x) - x \cdot V(x) - M_c = 0$

$M(x) = x \cdot V(x) + M_c$

BC's: $M(3m^{(+)}) = -15 + 30 = 15 kN \cdot m$

$M(6m) = -30 + 30 = 0$

$V(x)$

$-5 kN$

$M(x)$

$15 kN \cdot m$

$M_c = 30 kN \cdot m$

$-15 kN \cdot m$

30 kN·m

A

C

B

x

I   II

3 m   3 m
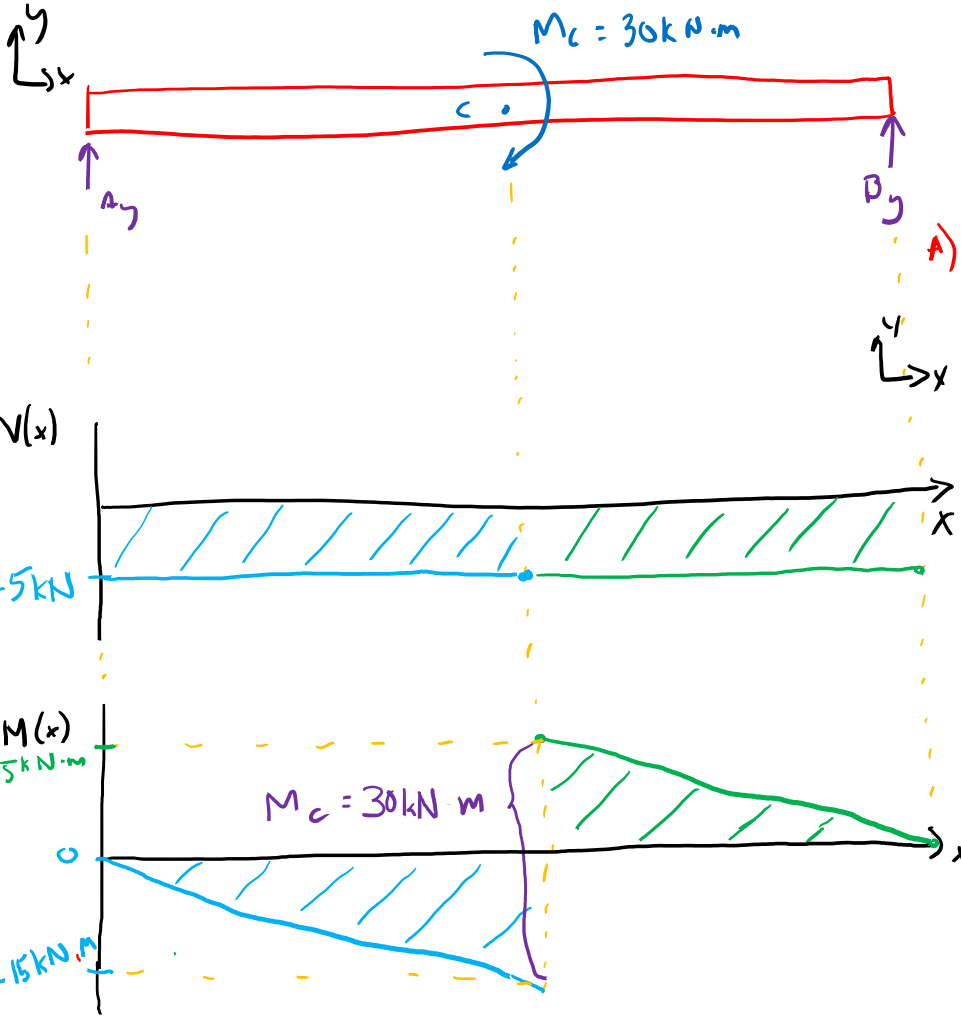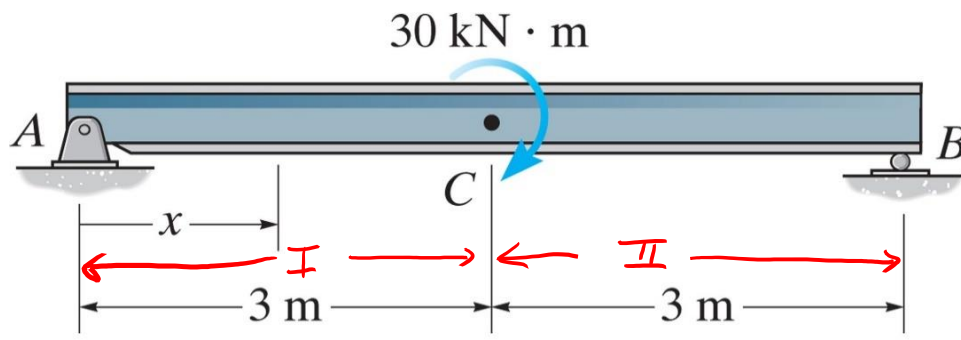
Draw the shear force and moment diagrams for the beam.

B) Alternative method to quickly draw V & M diagrams

use $\dfrac{dV}{dx} = W(x)$ to define slope of V(x)

$\Delta V = V_2 - V_1 = \int W(x)\,dx$ — change in shear = area under loading curve

$\dfrac{dM}{dx} = V(x)$ to define slope of M(x)

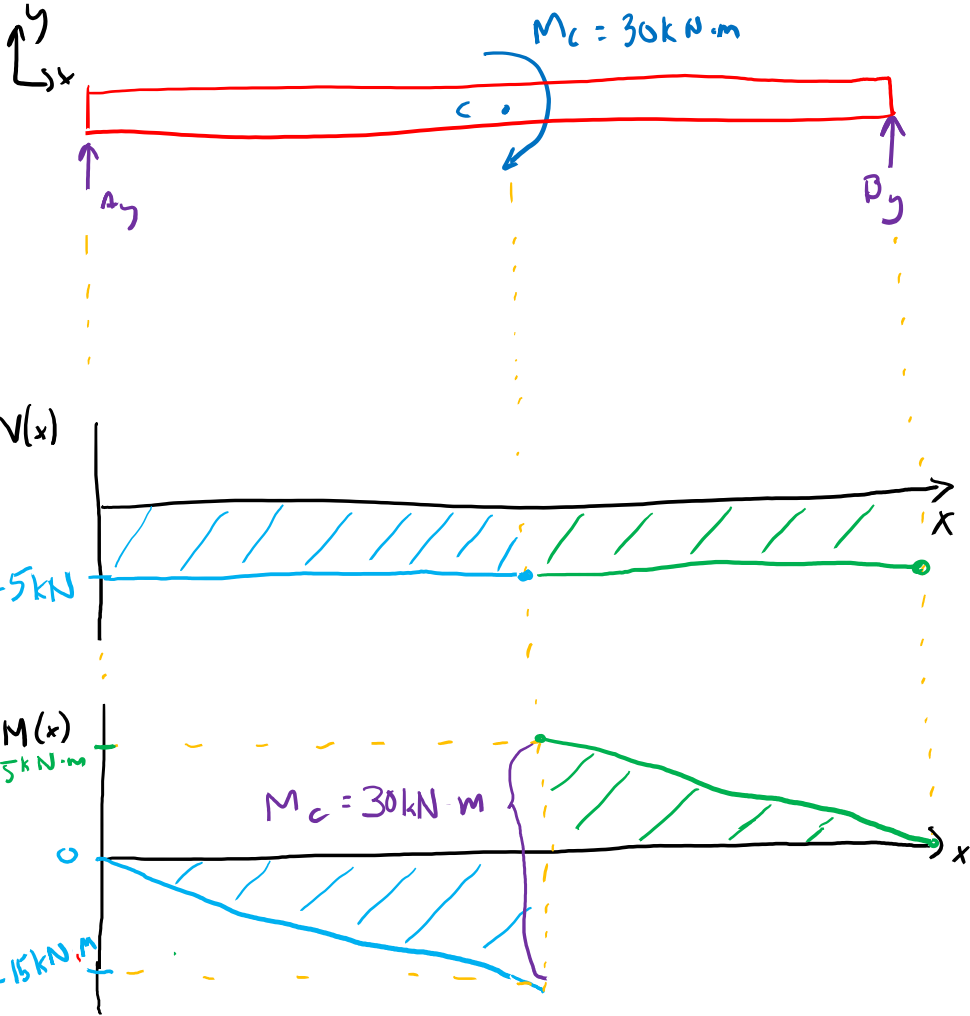$\Delta M = M_2 - M_1 = \int V(x)\,dx$ = area under shear curve — change in moment

For concentrated moment:
$W(x) = 0$
→ $\dfrac{dV}{dx} = 0$ (no slope)
$\Delta V = \int W\,dx = 0$ (no change in V)

$\dfrac{dM}{dx} = V(x) = neg.\,const = A_y = -5kN$

$\Delta M = M_c$ (from knowledge of applied moment M_c)

Since $M_c \circlearrowright$, then ΔM is in positive direction ↑ (see notes 2 slides prior)

$M_c = 30 kN\cdot m$

A_y B_y

$V(x)$

-5kN

$M(x)$

15 kN·m

$M_c = 30 kN\cdot m$

0

-15 kN·m

x

6 kN **12 kN**

A ... B

I   II   III

2 m   2 m   2 m

$W_I(x) = 0$ $W_{II}(x) = 0$ $W_{III}(x) = 0$

$\frac{dV}{dx} = W = 0$ $\frac{dV}{dx} = V = 0$ $\frac{dV}{dx} = W = 0$

$\Delta V_I = \int w\,dx = 0$ $\Delta V_{II} = 0$ $\Delta V_{III} = 0$

$\Delta V = F_1$ $\Delta V = F_2$

$V(x)$

8 kN

} 6 kN

2 kN

} 12 kN

−10 kN

$\frac{dM}{dx} = V = 8kN$ $\frac{dM}{dx} = V = 2kN$ $\frac{dM}{dx} = V = -10kN$

M(x=0) = 0 because no applied moment at x = 0

$M(x)$

16   2 kN   20   −10

8 kN   0

$\Delta M(2) = 2 \cdot 8 = 16 kN \cdot m$ $\Delta M(2) = 2 \cdot 2 = 4$ $\Delta M(2) \ 2 \cdot (-10) = -20$

Draw the shear force and moment diagrams for the beam.

[solution written outside of class]

1) Find support reactions

$\uparrow \Sigma F_y : A_y + B_y - 6kN - 12kN = 0$

$+ \circlearrowleft \Sigma M_A : (2m)6kN - (4m)(12kN) + (6m)B_y = 0$
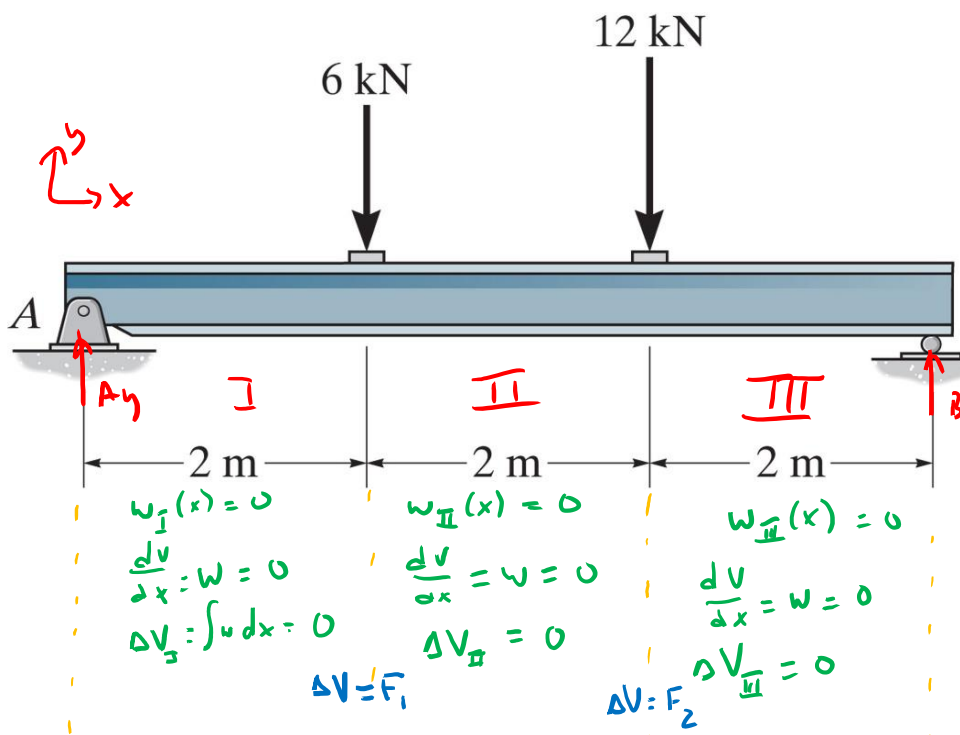
$\boxed{B_y = 10kN} \Rightarrow \boxed{A_y = 8kN}$

2) Quickly draw $V(x)$ & $M(x)$

within a region use:

$\frac{dV}{dx} = W$, $\Delta V = V_2 - V_1 = \int w\,dx$

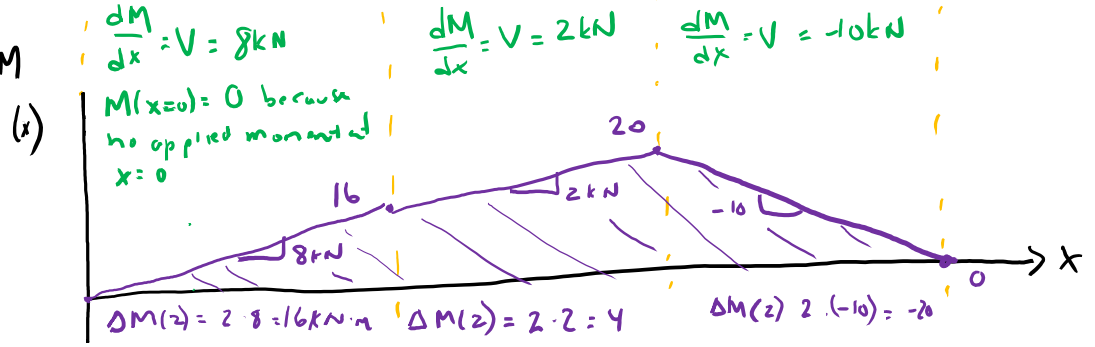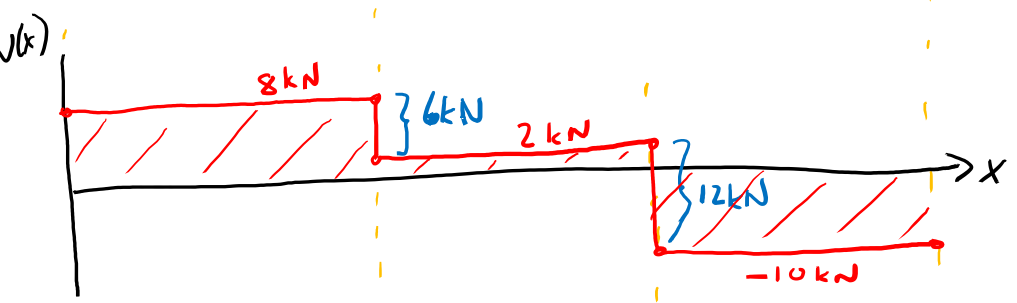$\frac{dM}{dx} = V$, $\Delta M = M_2 - M_1 = \int V\,dx$
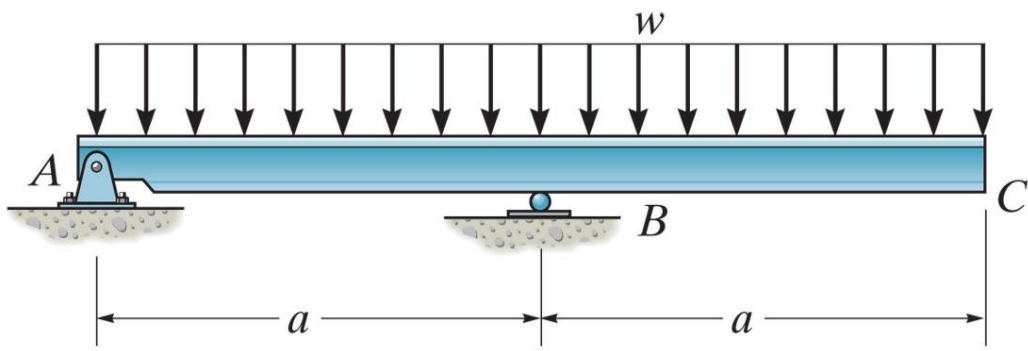
At locations of applied loads,

use $\Delta V = F$

Draw the shear force and moment diagrams for the beam.