Statics - TAM 210 & TAM 211

Lecture 24

March 14, 2018

Chap 7.3

Announcements

- Upcoming deadlines:
- Quiz 5 (3/14-16)
  - Sign up at CBTF
  - Up thru and including Lecture 22 (Shear Force & Bending Moment Diagrams), although review from Lectures 23-24 will be helpful.
- Monday (3/26)
  - Mastering Engineering Tutorial 11
- Thursday (3/29)
  - WA 4 due
- ~~Tuesday (3/13)~~ Monday April 2
  - PL HW ~~8~~ 9/11
- Friday (3/30)
  - Last lecture for TAM 210 students
- Written exam (Thursday 4/5, 7-9pm in 1 Noyes Lab)
  - Conflict exam (Monday 4/2, 7-9pm)
    - **Must make arrangements with Prof. H-W by Friday 3/16**
  - DRES accommodation exam. Make arrangements at DRES. Must tell Prof. H-W

How to orient positive V and M on a FBD?

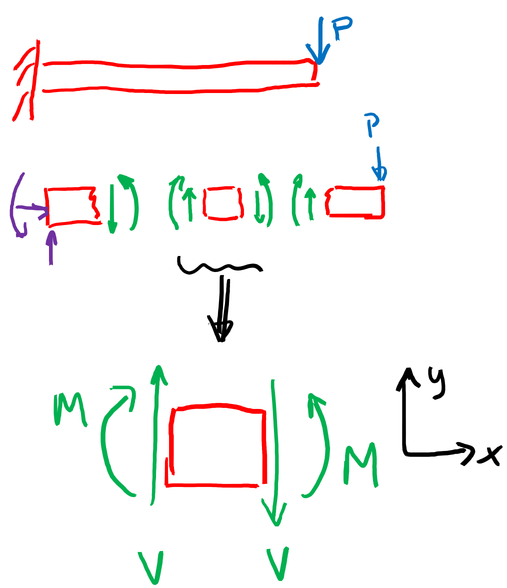
"Positive" sign convention:

"Positive shear will create a clockwise rotation"

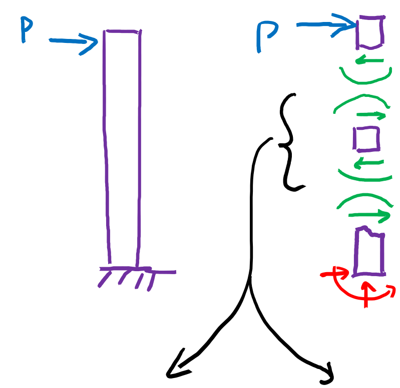
⟹ Draw V arrows to create CW rotation

Therefore the direction of the arrow for the bending moment M on the same side of the segment follows the same sense as the shear force V pointing in the direction of the positive coordinate axis (the y-axes in these diagrams); thus both V and M create a clockwise rotation.

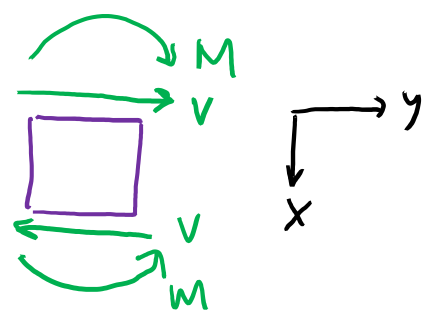
Horizontal Beam

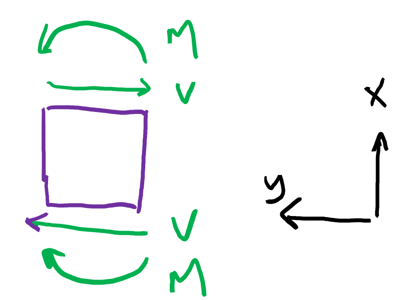


Vertical Beam



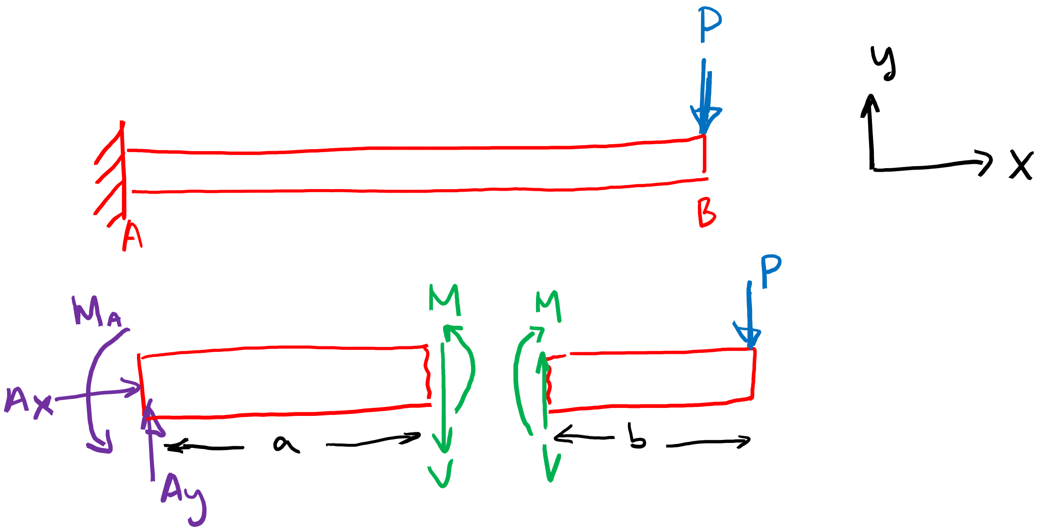
Direction of the arrow for M depends on which coordinate system orientation chosen.





What sign to give V and M terms in equations of equilbrium?

Follow the positive orientations of the coordinate system.



For left side:

$+\uparrow \Sigma F_y: \ A_y - V = 0$

$+\circlearrowleft \Sigma M_A:$

$M_A + M - a \cdot V = 0$

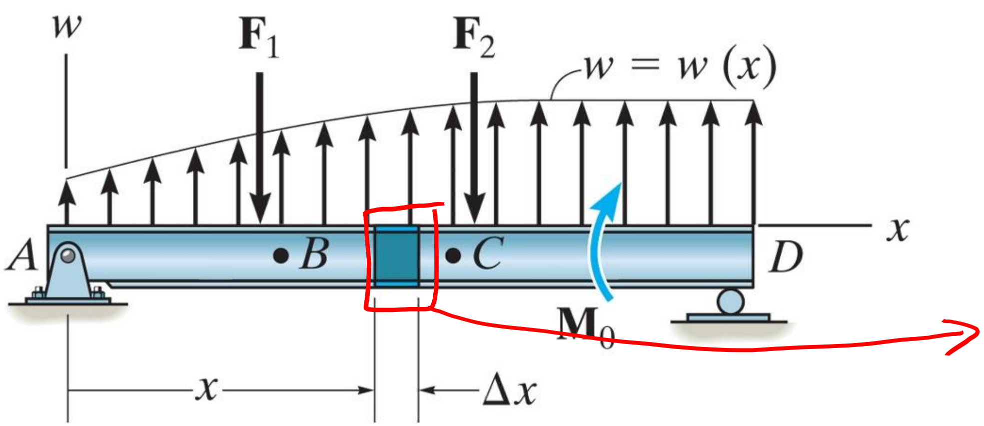
For right side:

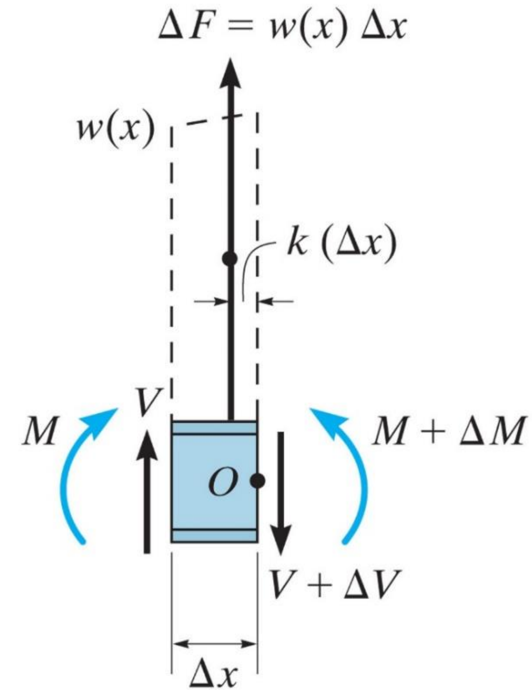
$+\uparrow \Sigma F_y: \ V - P = 0$

$+\circlearrowleft \Sigma M_B:$

$-M - b \cdot V = 0$

Recap: Relations Among Distributed Load, Shear Force and Bending Moments





Relationship between <u>distributed load</u> and <u>shear</u>:

$$\frac{dV}{dx} = w$$

Slope of shear force = distributed load intensity

$$\Delta V = V_2 - V_1 = \int w\, dx$$

Change in shear force = area under loading curve

Relationship between <u>shear</u> and <u>bending moment</u>:

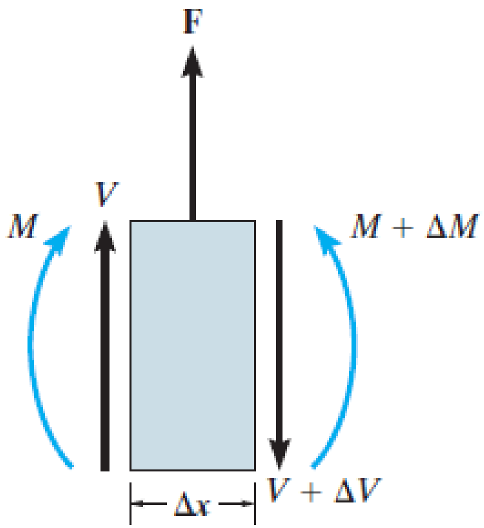
$$\frac{dM}{dx} = V$$

Slope of bending moment = shear force

$$\Delta M = M_2 - M_1 = \int V\, dx$$

Change in moment= area under shear curve

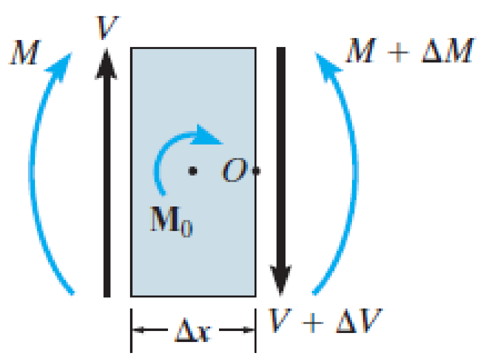
Recap: Wherever there is an external concentrated force or a concentrated moment, there will be a change (jump) in shear or moment, respectively.



ΣF_y:

$$V + F - (V + \Delta V) = 0$$

$\Delta V = \mathrm{F}$ Jump in shear force due to concentrated load *F*



$+\circlearrowright\ \Sigma M_O$:

$$(M + \Delta M) - M - M_O - V(\Delta x) = 0$$

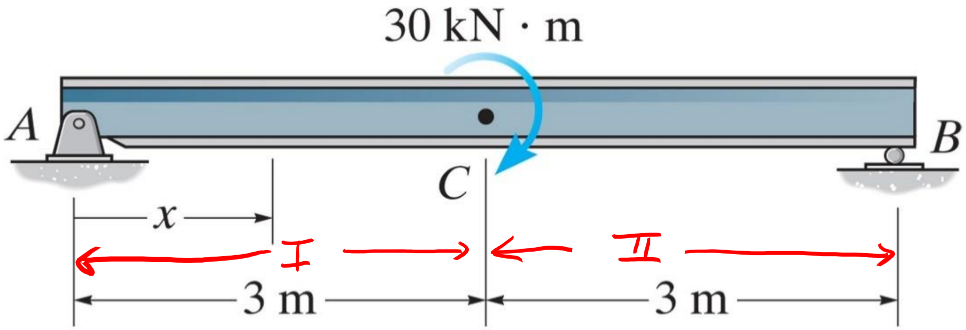
$$\Delta M = M_O + V(\Delta x)$$

$$\Delta M = M_O, \text{ when } \Delta x \to 0$$

Jump in bending moment due to concentrated couple moment M_O

Note: the text, these notes, and convention assume that an applied concentrated moment **M_O in clockwise direction** results in a **positive change in $M(x)$**

Draw the shear force and moment diagrams for the beam.



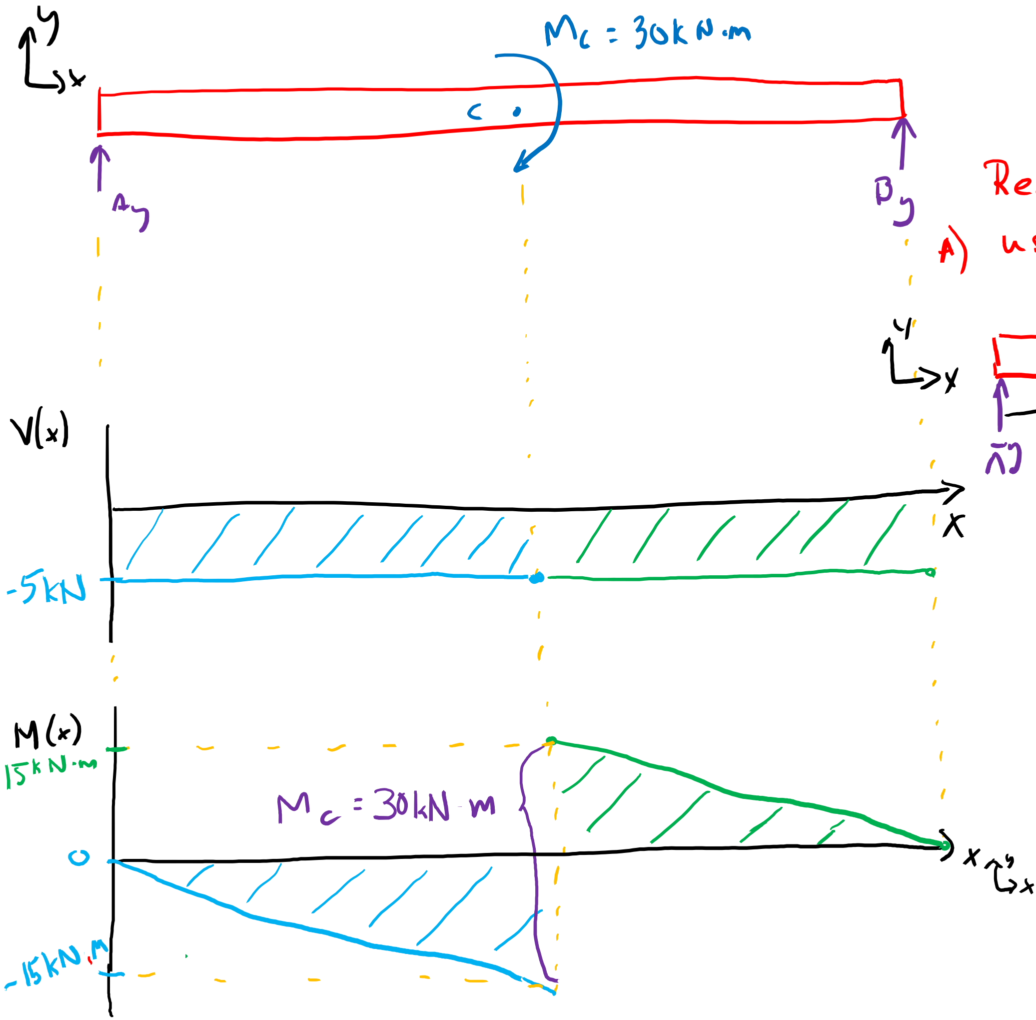
(1) Reaction supports :

$\Sigma F_y : \quad A_y = -B_y$

$+\circlearrowleft \Sigma M_A : \quad -30 \text{ kN m} + (6m) B_y = 0$

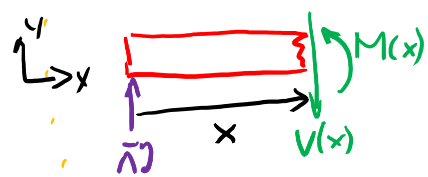
$\boxed{B_y = 5 \text{ kN.m}}$

$\boxed{A_y = -5 \text{ kN.m}}$



Region I : $0 < x < 3$

A) using FBD & EoE to create $V(x)$ & $M(x)$:



$\Sigma F_y : \boxed{V(x) = A_y = -5 \text{kN}}$

constant, negative

$+\circlearrowleft \Sigma M_A : \quad M(x) - x \cdot V(x) = 0$

$\boxed{M(x) = x \; V(x)}$

linear w/ slope V(x)

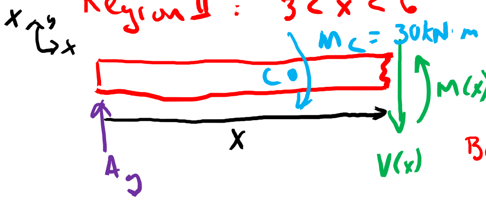
slope $\frac{dM}{dx} = V(x) = -5 \text{kN}$

use BC's to find end points for M(x)

$x = 0 : \; M(0) = 0$

$x = 3m : \; M(3m^{(-)}) = -15 \text{ kN} \cdot m$

Region II : $3 < x < 6$



$\Sigma F_y : \boxed{V(x) = A_y = -5 \text{kN}}$

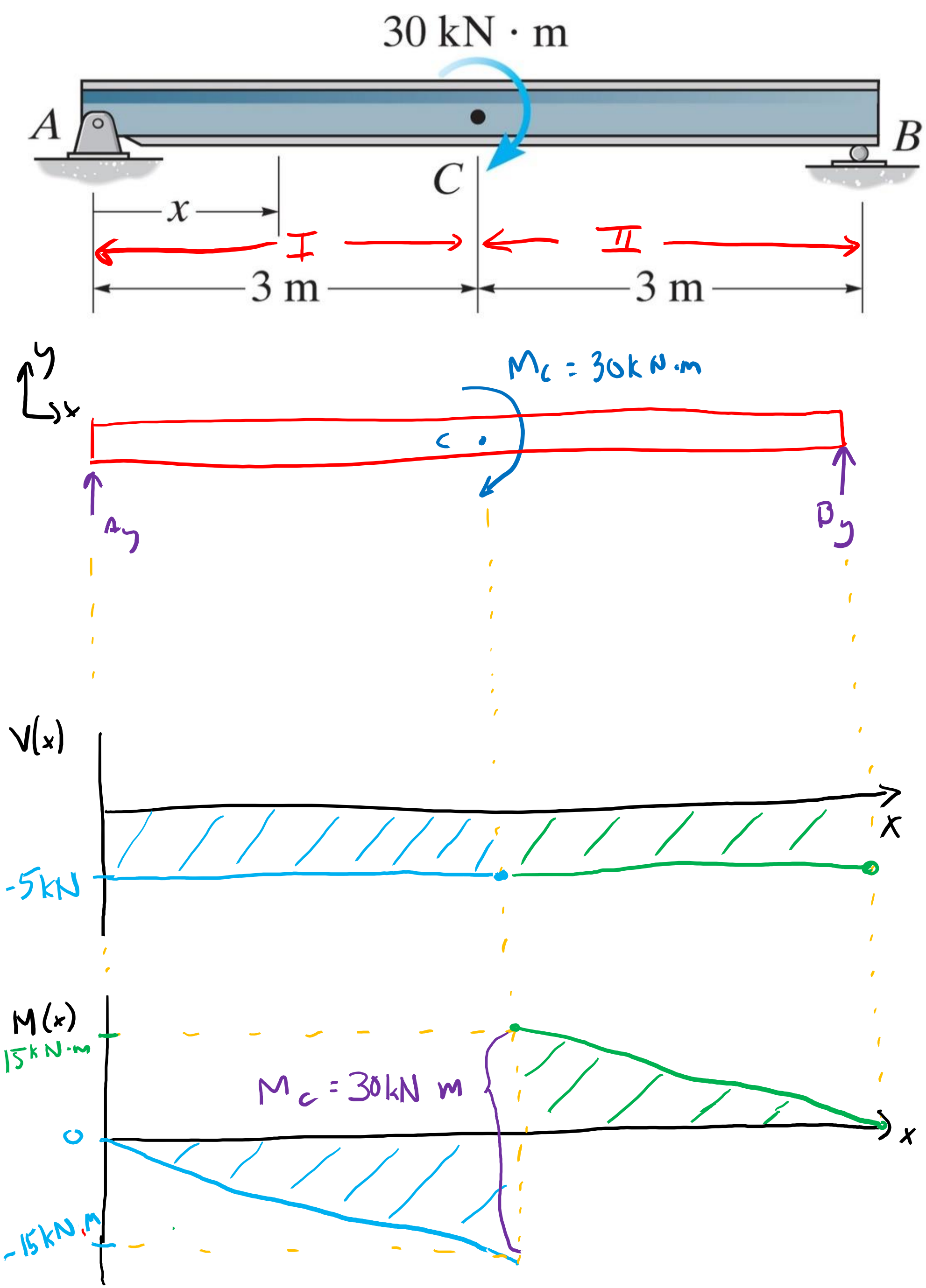
$+\circlearrowleft \Sigma M_A : \quad M(x) - x \cdot V(x) - M_c = 0$

$M(x) = x \cdot V(x) + M_c$

BC's : $M(3m^{(+)}) : -15 + 30 = 15 \text{kN} \cdot m$

$M(6m) = -30 + 30 = 0$

Draw the shear force and moment diagrams for the beam.



B) Alternative method to quickly draw V & M diagrams

use $\frac{dV}{dx} = w(x)$ to define slope of V(x)

$\Delta V = V_2 - V_1 = \int w(x)\,dx$ change in shear = area under loading curve

$\frac{dM}{dx} = V(x)$ to define slope of M(x)

$\Delta M = M_2 - M_1 = \int V(x)\,dx$ change in moment = area under shear curve

For concentrated moment:

$w(x) = 0$

$\Rightarrow \frac{dV}{dx} = 0$ (no slope)

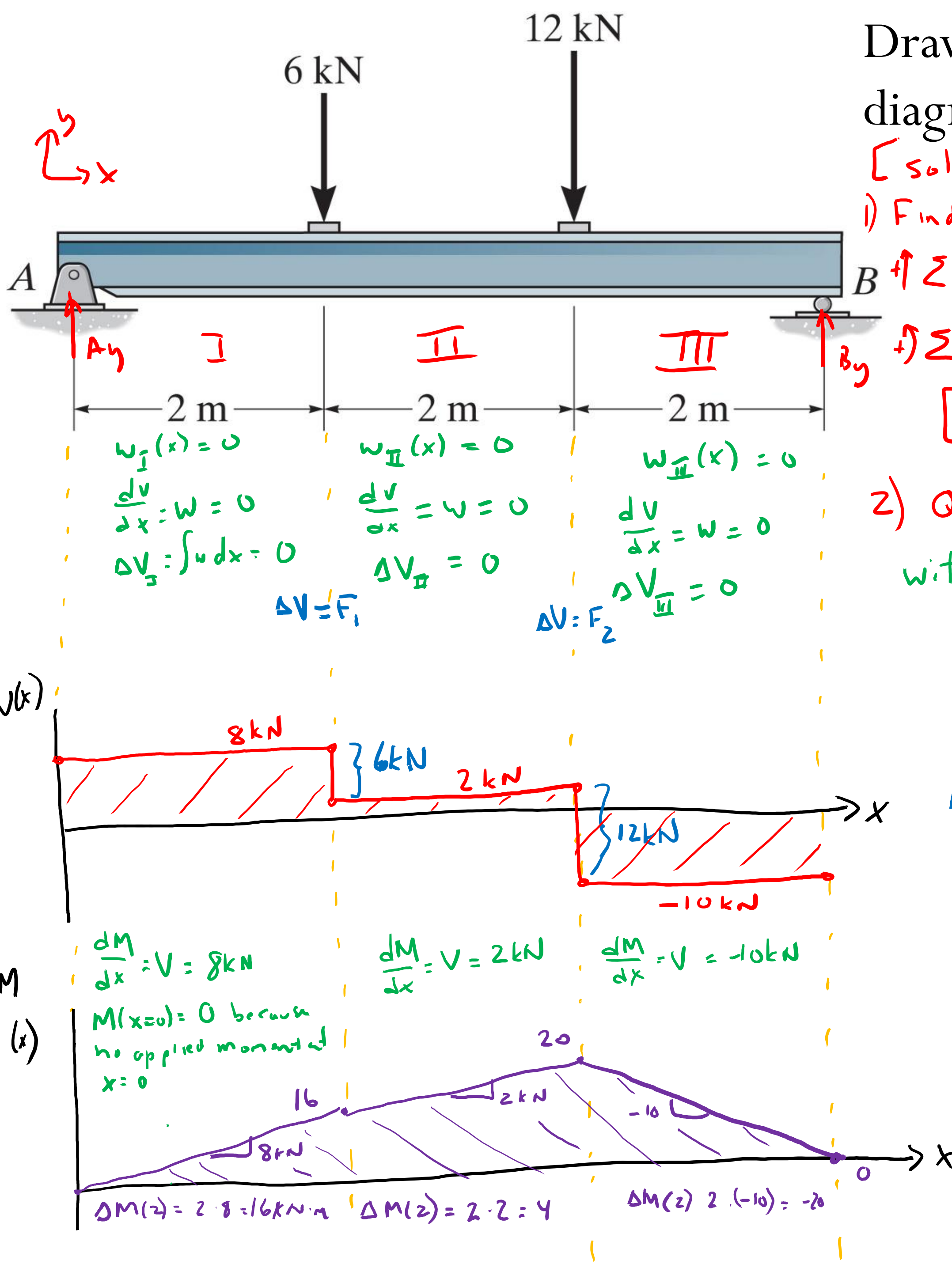
$\Delta V = \int w\,dx = 0$ (no change in V)

$\frac{dM}{dx} = V(x) = \text{neg. const} = A_y = -5kN$

$\Delta M = M_c$ (from knowledge of applied moment M_c)

Since M_c ↻ cw, then ΔM is in positive direction ↑

(see notes 2 slides prior)



Draw the shear force and moment diagrams for the beam.

[solution written outside of class]

1) Find support reactions

$+\uparrow \Sigma F_y: A_y + B_y - 6kN - 12kN = 0$

$+\circlearrowleft \Sigma M_A: (2m)6kN - (4m)(12kN) + (6m)B_y = 0$

$\boxed{B_y = 10kN} \Rightarrow \boxed{A_y = 8kN}$

2) Quickly draw V(x) & M(x)

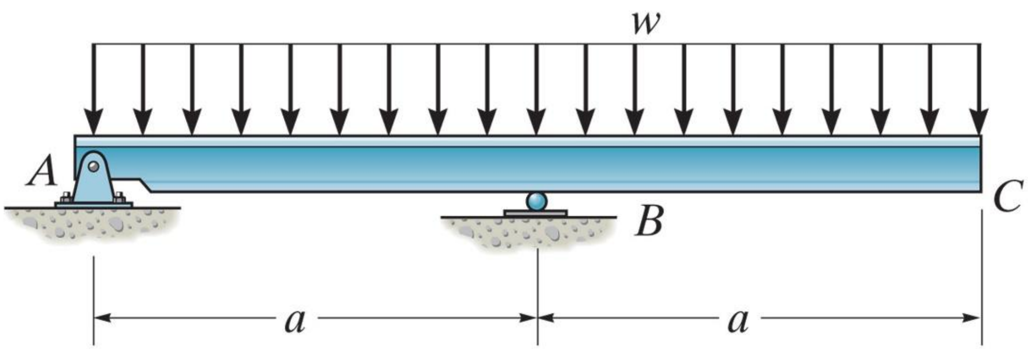
within a region use:

$\frac{dV}{dx} = w$, $\Delta V = V_2 - V_1 = \int w\,dx$

$\frac{dM}{dx} = V$, $\Delta M = M_2 - M_1 = \int V\,dx$

At locations of applied loads,

use $\Delta V = F$

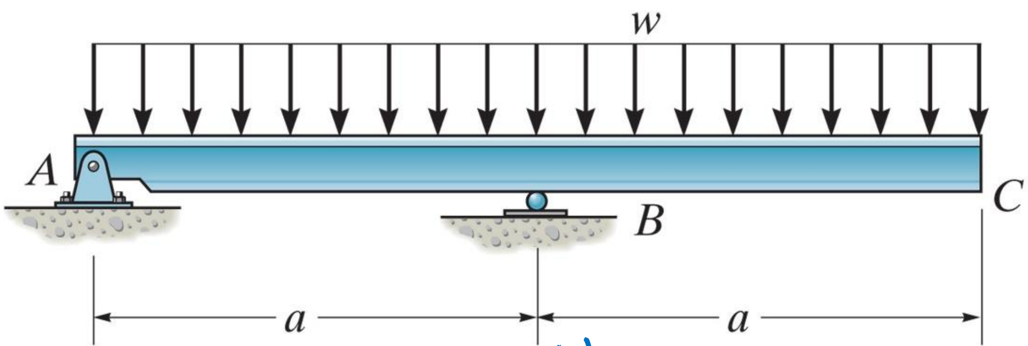


Draw the shear force and moment diagrams for the beam.

What's different about this beam?

Ans: Distributed load *w* is also over support B!

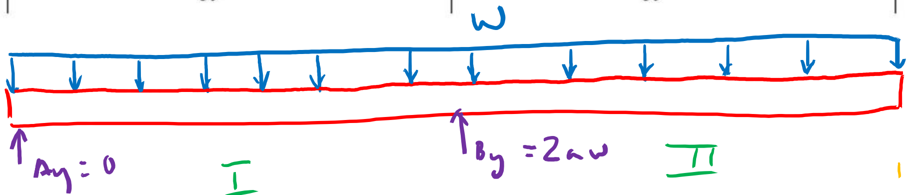
→ Can't automatically draw V & M diagram for a rectangular distributed load



Draw the shear force and moment diagrams for the beam.

Prove to yourself that

$A_y = 0$, $B_y = 2aw$



I

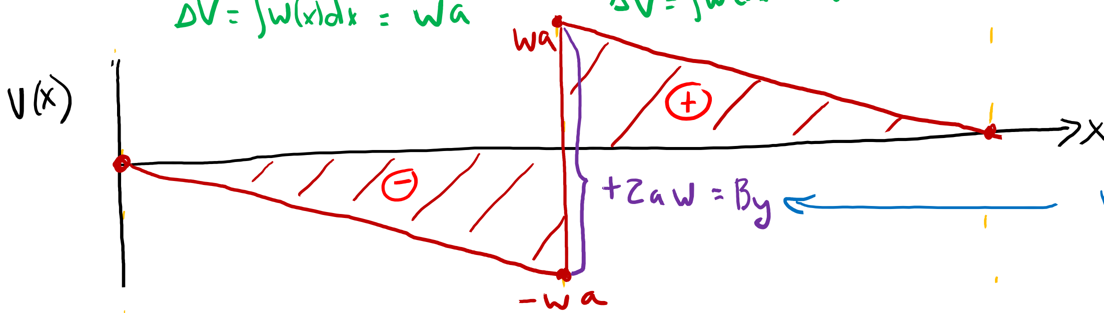
$\frac{dV}{dx} = w(x) \equiv$ Constant, negative

$\Delta V = \int w(x)dx = wa$

II

$\frac{dV}{dx} = w(x) \equiv$ const, neg

$\Delta V = \int w\,dx = wa$



In this scenario, reaction support force B_y acts like an upward concentrated load!

$\frac{dM}{dx} = V \equiv$ decr, neg

$\Delta M = \int V(x)dx = \frac{1}{2}wa^2$

$\frac{dM}{dx} = V \equiv$ decr, pos

$\Delta M = \int V dx = \frac{1}{2}wa^2$