# Statics - TAM 210 & TAM 211

**Spring 2018**

**Lecture 28**

**Exam Review**

# Announcements

- Concept  Inventory: Ungraded assessment of course knowledge
  - Extra credit: Complete #1 or #2 for 0.5 out of 100 pt of final grade each, or both for 1.5 out of 100 pt of final grade
  - #2: Sign up at CBTF (4/2-4 M-Th)
  - 50 min appointment, should take < 30 min

- Upcoming deadlines:
- Friday (3/30)
  - Last lecture for TAM 210 students
- Monday (4/2)
  - PL HW 9/11
  - ME Tutorial due for TAM 211 students
  - Lecture 29 for TAM 211 students
- Discussion section for (4/3-4/4)
  - Attendance taken for TAM 211, no worksheet
  - TA/CA answer questions about course material
  - TAM 210 students may attend

# Written exam

- Comprehensive from Lecture 1 through Lecture 27 (Chapters 1 -8)
- Thursday 4/5, 7-9pm
- TAM 210 students: 100 Material Science & Engineering Building (MSEB)
  - Do not confuse this building with Mechanical Engineering Building (MEB)
- TAM 211 students: 100 Noyes Lab
- Bring i-Card and **pencil**
- **No calculators**
- **Closed book, closed notes**
- Conflict exam & DRES accommodation exam: Prof. H-W is not taking anymore requests

- Composition of exam:
  A. (60 points) Short-answer using Scantron (must use pencil!)
  B. (40 points) Written answers on exam paper (NO need for blue exam booklet)
  C. Multiple variations of the exam

## Course Overview

**Description:** In this course, we will cover fundamental concepts that are used in every engineering discipline. We will begin with forces, moments and move towards structural analyses of frames, devices, and machines.  By the end, you will be able to solve rigid body mechanics problems that will inform the design of everything from bridges to biomedical devices.

**Big Idea:** Clear knowledge of external forces (boundary conditions) is required to determine what constraints are necessary for the safe (static equilibrium) development and design of any widget. Free body diagrams are an essential tool for understanding the forces and moments on a body.

**Resources:**

**PrairieLearn**: All past HW questions can be reviewed and reworked

**Mastering Engineering**: Extra practice and study material available under Menu>"Study Area", then "Additional Problems" under the menu of the new window

- Problems organized by book chapters
- Chose topic of interest from drop down menu near top

Chapter 1: General Principles
Chapter 2: Force Vectors
Chapter 3: Equilibrium of a particle
Chapter 4: Force System Resultants
Chapter 5: Equilibrium of Rigid Bodies
Chapter 6: Structural Analysis
Chapter 7: Internal Forces
Chapter 8: Friction

# Chapter 1: General Principles

# Chapter 1: General Principles
# Main goals and learning objectives

- Introduce the basic ideas of *Mechanics*
- Give a concise statement of Newton’s laws of motion and gravitation
- Review the principles for applying the SI system of units
- Examine standard procedures for performing numerical calculations
- Outline a general guide for solving problems

# General procedure for analysis

1. Read the problem carefully; write it down carefully.
2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.
3. Apply principles needed.
4. Solve problem symbolically. Make sure equations are dimensionally homogeneous
5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).
6. See if answer is reasonable.

*Most effective way to learn engineering mechanics is to **solve problems!***

# Chapter 2: Force Vectors

# Chapter 2: Force vectors
# Main goals and learning objectives

Define scalars, vectors and vector operations and use them to analyze forces acting on objects

- Add forces and resolve them into components
- Express force and position in Cartesian vector form
- Determine a vector’s magnitude and direction
- Introduce the unit vector and position vector
- Introduce the dot product and use it to find the angle between two vectors or the projection of one vector onto another

# Force vector

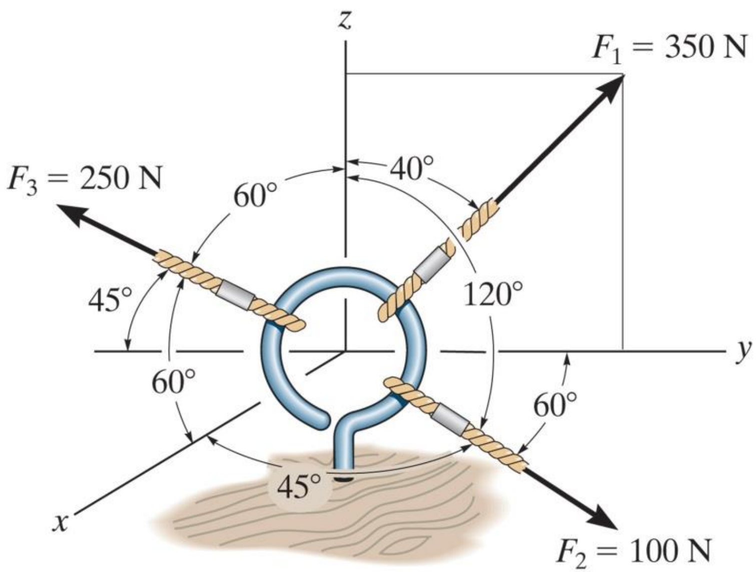
- A force can be treated as a vector, since forces obey all the rules that vectors do.

$$\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} \qquad \vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}} \qquad \vec{\boldsymbol{R'}} = \vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} = \vec{\boldsymbol{A}} + (-\vec{\boldsymbol{B}})$$

- Vector representations
  - Rectangular components $\vec{\boldsymbol{A}} = \overrightarrow{\boldsymbol{A_x}} + \overrightarrow{\boldsymbol{A_y}} + \overrightarrow{\boldsymbol{A_z}}$
  - Cartesian vectors $\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} + A_z\widehat{\boldsymbol{k}}$
  - Unit vector $\overrightarrow{\boldsymbol{u_A}} = \frac{\vec{\boldsymbol{A}}}{|\vec{\boldsymbol{A}}|} = \frac{A_x}{|\vec{\boldsymbol{A}}|}\hat{\boldsymbol{\imath}} + \frac{A_y}{|\vec{\boldsymbol{A}}|}\hat{\boldsymbol{\jmath}} + \frac{A_z}{|\vec{\boldsymbol{A}}|}\widehat{\boldsymbol{k}}$
  - Direction cosines $cos(\alpha) = \frac{A_x}{A},\ cos(\beta) = \frac{A_y}{A}, cos(\gamma) = \frac{A_z}{A}$

Recall: Magnitude of a vector (which is a scalar quantity) can be shown as a term with no font modification (*A*) or vector with norm bars ($|\vec{\boldsymbol{A}}|$), such that $A = |\vec{\boldsymbol{A}}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

# Example




The cables attached to the screw eye are subjected to the three forces shown.

(a) Express each force vector using the Cartesian vector form (components form).

(b) Determine the magnitude of the resultant force vector

(c) Determine the direction cosines of the resultant

$$\mathbf{F}_1 = 350\{\sin 40^\circ\mathbf{j} + \cos 40^\circ\mathbf{k}\}\text{ N}$$
$$= \{224.98\mathbf{j} + 268.12\mathbf{k}\}\text{ N}$$
$$= \{225\mathbf{j} + 268\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 100\{\cos 45^\circ\mathbf{i} + \cos 60^\circ\mathbf{j} + \cos 120^\circ\mathbf{k}\}\text{ N}$$
$$= \{70.71\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}\}\text{ N}$$
$$= \{70.7\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_3 = 250\{\cos 60^\circ\mathbf{i} + \cos 135^\circ\mathbf{j} + \cos 60^\circ\mathbf{k}\}\text{ N}$$
$$= \{125.0\mathbf{i} - 176.78\mathbf{j} + 125.0\mathbf{k}\}\text{ N}$$
$$= \{125\mathbf{i} - 177\mathbf{j} + 125\mathbf{k}\}\text{ N}$$

***Resultant Force:***

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$= \{(70.71 + 125.0)\mathbf{i} + (224.98 + 50.0 - 176.78)\mathbf{j} + (268.12 - 50.0 + 125.0)\mathbf{k}\}\text{ N}$$
$$= \{195.71\mathbf{i} + 98.20\mathbf{j} + 343.12\mathbf{k}\}\text{ N}$$

***Magnitude of the resultant force vector***

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$$
$$= \sqrt{195.71^2 + 98.20^2 + 343.12^2}$$
$$= 407.03\text{ N} = 407\text{ N}$$

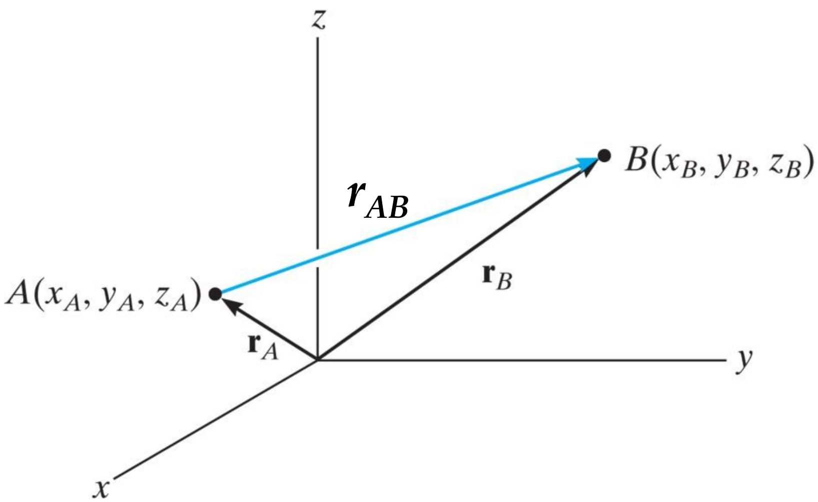
***Direction cosines of the resultant force vector***

$$\cos\alpha = \frac{F_{R_x}}{F_R} = \frac{195.71}{407.03}$$

$$\cos\beta = \frac{F_{R_y}}{F_R} = \frac{98.20}{407.03}$$

$$\cos\gamma = \frac{F_{R_z}}{F_R} = \frac{343.12}{407.03}$$

# Position vector



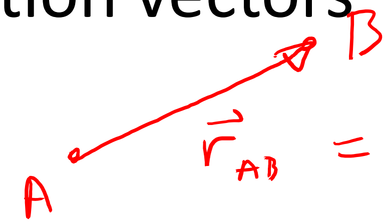

$$\overrightarrow{\boldsymbol{r_{AB}}} = \overrightarrow{\boldsymbol{r_B}} - \overrightarrow{\boldsymbol{r_A}}$$
$$= \left(x_B\hat{\boldsymbol{\imath}} + y_B\hat{\boldsymbol{\jmath}} + z_B\widehat{\boldsymbol{k}}\right) + \left(x_A\hat{\boldsymbol{\imath}} + y_A\hat{\boldsymbol{\jmath}} + z_A\widehat{\boldsymbol{k}}\right)$$

$$\overrightarrow{\boldsymbol{r_{AB}}} = (x_B - x_A)\hat{\boldsymbol{\imath}} + (y_B - y_A)\hat{\boldsymbol{\jmath}} + (z_B - z_A)\widehat{\boldsymbol{k}}$$

Thus, the $(\boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k})$ components of the positon vector $\boldsymbol{r}$ may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).

# Project of force vector

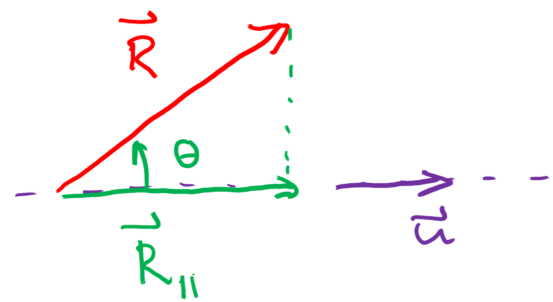
- Position vectors




$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = \vec{r}_{HEAD} - \vec{r}_{TAIL}$$

- Force vector directed along a line

$$\vec{F} = F\vec{u}, \quad \vec{u} = \frac{\vec{r}}{|\vec{r}|}$$




- Dot (scalar) product

$$\vec{A}\cdot\vec{B} = C = |\vec{A}||\vec{B}|\cos\theta = \sum_{i=x,y,z} A_i B_i$$

$$\vec{R}_{\parallel} = (\vec{R}\cdot\vec{u})\,\vec{u}$$

$$|\vec{R}_{\parallel}| = |\vec{R}|\cos\theta$$

- Cross (vector) product

$$\vec{A}\times\vec{B} = \vec{C} = (|\vec{A}||\vec{B}|\sin\theta)\,\vec{u}_c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

← determinant

# Example

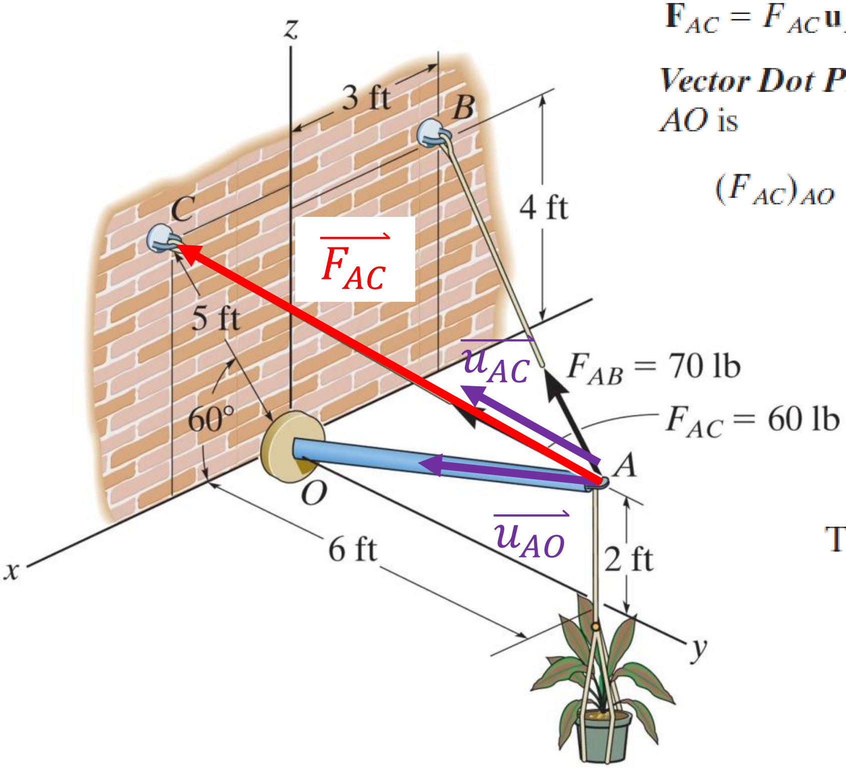
Determine the projected component of the force vector $F_{AC}$ along the axis of strut AO. Express your result as a Cartesian vector

***Unit Vectors:*** The unit vectors $\mathbf{u}_{AC}$ and $\mathbf{u}_{AO}$ must be determined first.

$$\mathbf{u}_{AC} = \frac{(5\cos 60^\circ - 0)\mathbf{i} + (0-6)\mathbf{j} + (5\sin 60^\circ - 2)\mathbf{k}}{\sqrt{(5\cos 60^\circ - 0)^2 + (0-6)^2 + (0-2)^2}} = 0.3621\mathbf{i} - 0.8689\mathbf{j} + 0.3375\mathbf{k}$$

$5\sin 60^o$

$$\mathbf{u}_{AO} = \frac{(0-0)\mathbf{i} + (0-6)\mathbf{j} + (0-2)\mathbf{k}}{\sqrt{(0-0)^2 + (0-6)^2 + (0-2)^2}} = -0.9487\mathbf{j} - 0.3162\,\mathbf{k}$$




Thus, the force vectors $\mathbf{F}_{AC}$ is given by

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 60(0.3621\mathbf{i} - 0.8689\mathbf{j} + 0.3375\mathbf{k}) = \{21.72\mathbf{i} - 52.14\mathbf{j} + 20.25\mathbf{k}\}\text{ lb}$$

***Vector Dot Product:*** The magnitude of the projected component of $\mathbf{F}_{AC}$ along strut $AO$ is

$$\begin{aligned}(F_{AC})_{AO} = \mathbf{F}_{AC}\cdot\mathbf{u}_{AO} &= (21.72\mathbf{i} - 52.14\mathbf{j} + 20.25\mathbf{k})\cdot(-0.9487\mathbf{j} - 0.3162\mathbf{k}) \\ &= (21.72)(0) + (-52.14)(-0.9487) + (20.25)(-0.3162) \\ &= 43.057\text{ lb}\end{aligned}$$

Thus, $(\mathbf{F}_{AC})_{AO}$ expressed in Cartesian vector form can be written as

$$(\mathbf{F}_{AC})_{AO} = (F_{AC})_{AO}\mathbf{u}_{AO} = 43.057(-0.9487\mathbf{j} - 0.3162\mathbf{k})$$

$$= \{-40.8\mathbf{j} - 13.6\mathbf{k}\}\text{ lb}$$

# Chapter 3: Equilibrium of a particle

# Goals and Objectives

- Practice following general procedure for analysis.
- Introduce the concept of a free-body diagram for an object modeled as a particle.
- Solve equilibrium problems using the equations of equilibrium.
    - 3D, 2D planar, idealizations (smooth surfaces, pulleys, springs)

# Free body diagram

Drawing of a body, or part of a body, on which all forces acting on the body are shown.

- Key to writing the equations of equilibrium.
- Can draw for any object/subsystem of system. Pick the most appropriate object. (Equal & opposite forces on interacting bodies.)

- ❑ Draw Outlined Shape: image object free of its surroundings
  - ❑ Sometimes may collapse large object into point mass
- ❑ Establish x, y, z axes in any suitable orientation
  - ❑ Show positive directions for translation and rotation
- ❑ Show all forces acting on the object at points of application
- ❑ Label all known and unknown forces
- ❑ Sense (“direction”) of unknown force can be assumed. If solution is negative, then the sense is reverse of that shown on FBD

# Equations of equilibrium

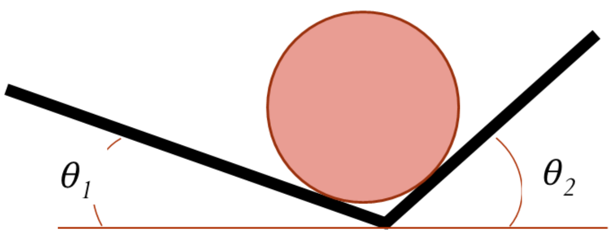
- Use FBD to write equilibrium equations in x, y, z directions
  - $\sum \overrightarrow{\boldsymbol{F_x}} = 0, \sum \overrightarrow{\boldsymbol{F_y}} = 0,$ and if 3D $\sum \overrightarrow{\boldsymbol{F_z}} = 0,$
  - If # equations ≥ # unknown forces, **statically determinate** (can solve for unknowns)
  - If # equations < # unknown forces, **indeterminate** (can **NOT** solve for unknowns), need more equations
- Get more equations from FBD of other bodies in the problem

# Idealizations

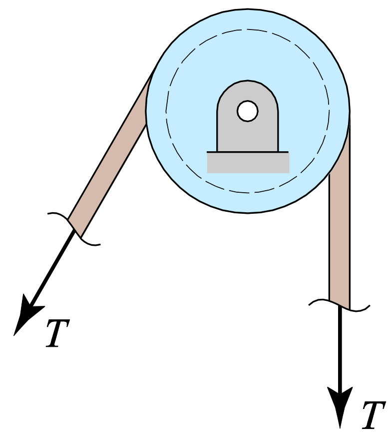
**Smooth surfaces:** regarded as frictionless; force is perpendicular to surface

**Pulleys:** (usually) regarded as frictionless; tension around pulley is same on either side.

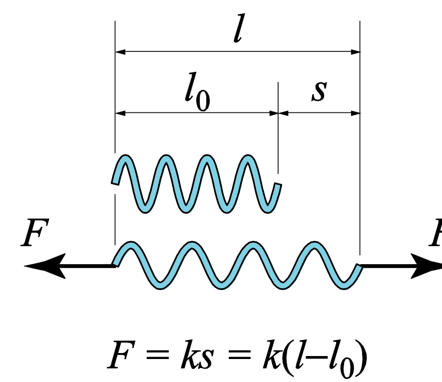
**Springs**: (usually) regarded as linearly elastic; tension is proportional to *change* in length *s*.



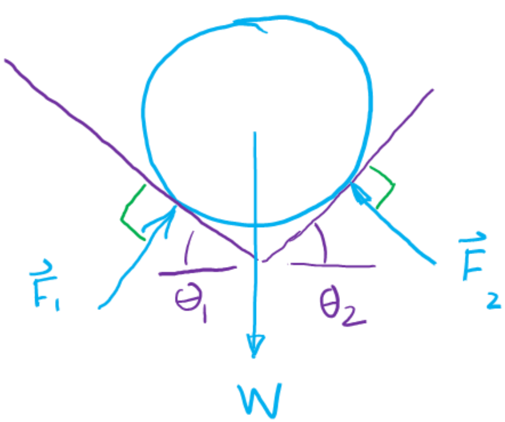

Smooth surface

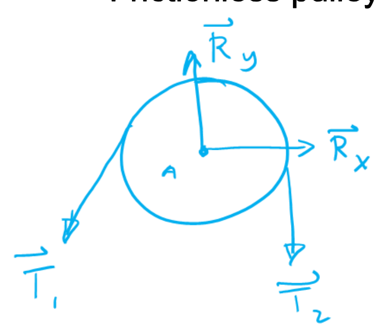



Frictionless pulley




Linearly elastic spring




$|\vec{T}_1| = |\vec{T}_2|$

Assuming cable is massless & rigid

Magnitudes are same

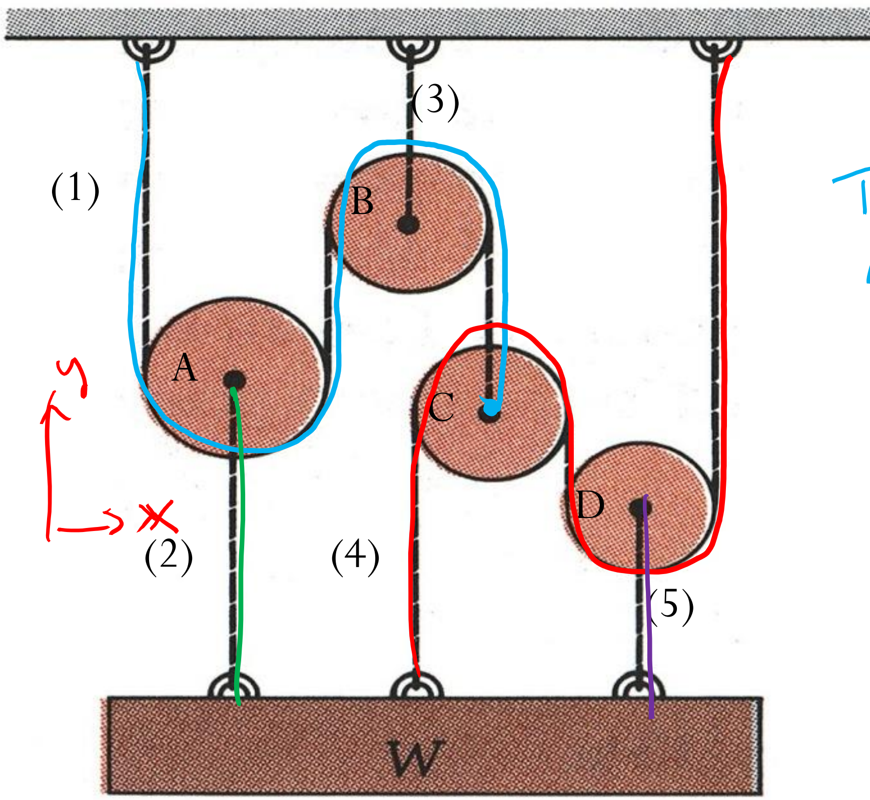
Directions do not need to be the same

$s = l_f - l_0$

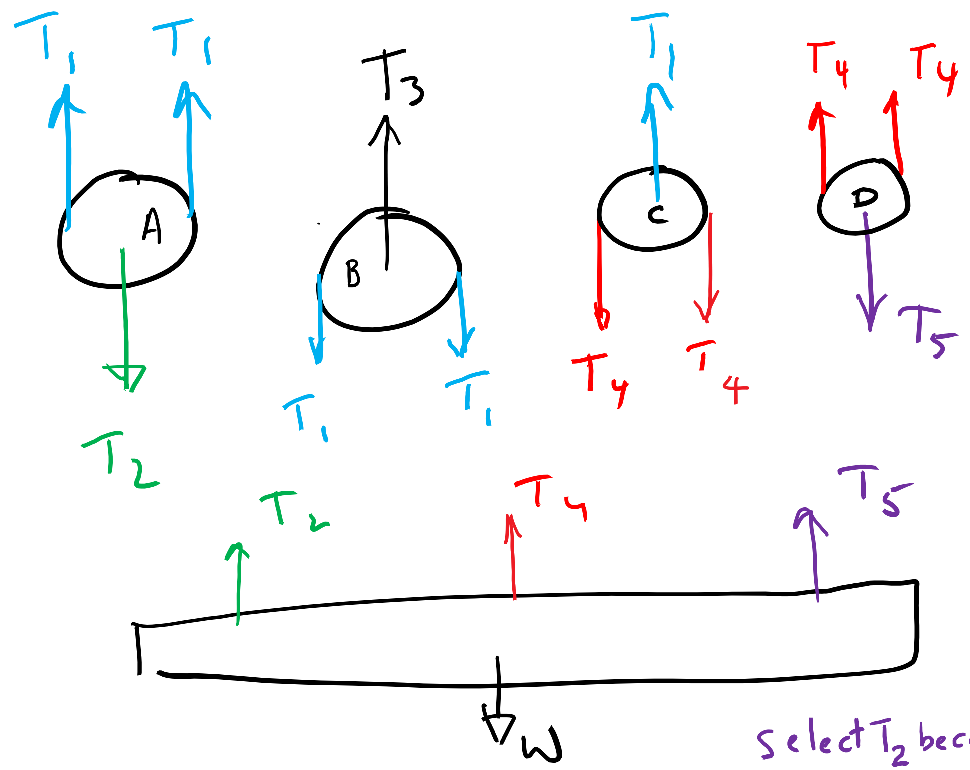
if $s > 0$ → elongation

if $s < 0$ → compression

The five ropes can each take 1500 N without breaking. How heavy can *W* be without breaking any?




Note: No pin jt reaction forces at center of pulleys because these pulleys are not secured to a fixed (or grounded) pin jt.




$\Sigma F_y = 0$

write eqns of equilibrium from each FBD

A: $2T_1 - T_2 = 0$

B: $-2T_1 + T_3 = 0$

C: $T_1 - 2T_4 = 0$

D: $2T_4 - T_5 = 0$

W: $T_2 + T_4 + T_5 - W = 0$

$T_2 = 2T_1$

$T_3 = 2T_1 = T_2$

$T_4 = \frac{1}{2} T_1$

$T_5 = 2T_4 = T_1$

$W = 2T_1 + \frac{1}{2} T_1 + T_1 \Rightarrow W = 3.5 T_1 = \frac{3.5}{2} T_2$

If $T_2 = 1500N$

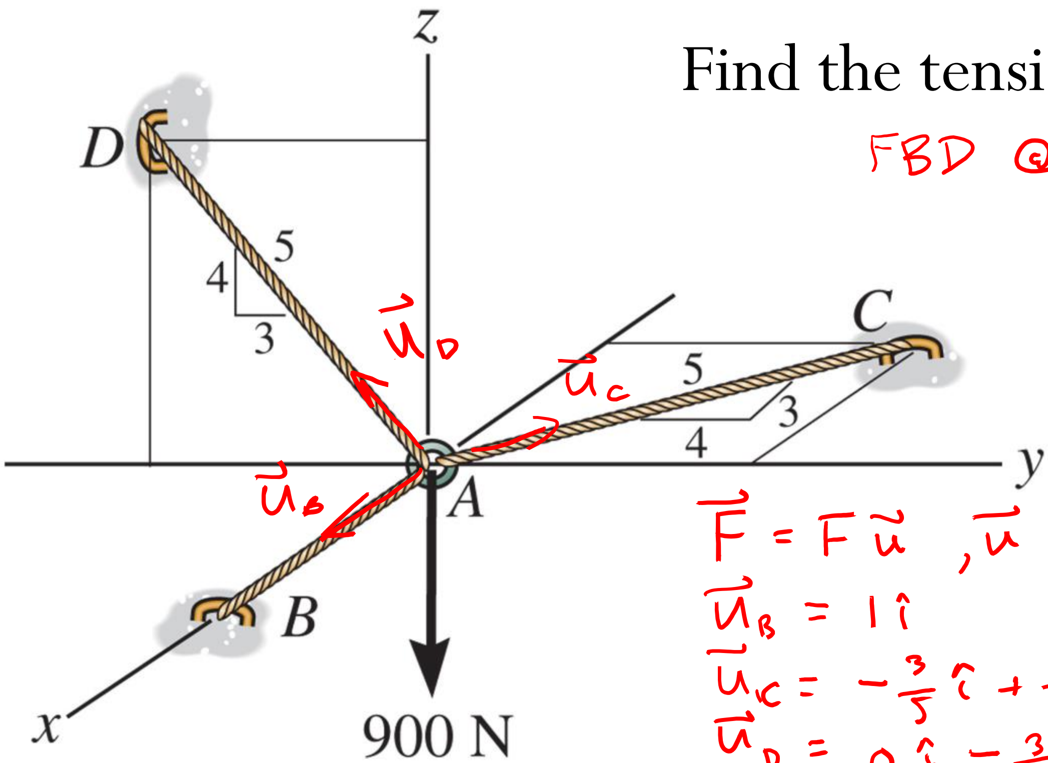
$W = 263N$

Select $T_2$ because want a rope that will support the largest load. Then set it's load to the breakage limit. $T_3$ could also be possible choice

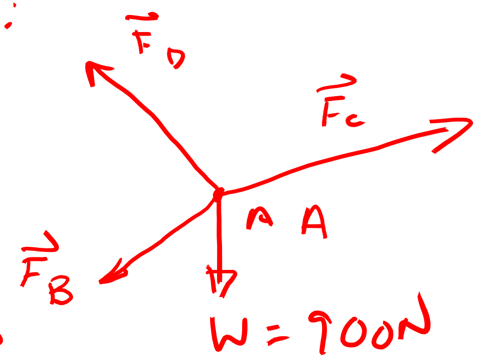
# 3D force systems

Use $\sum \overrightarrow{F_x} = 0, \sum \overrightarrow{F_y} = 0, \sum \overrightarrow{F_z} = 0$

Find the tension developed in each cable




FBD @ A:




$\vec{F} = F\vec{u}, \quad \vec{u} = \frac{\vec{r}}{|\vec{r}|}$

$\vec{u}_B = 1\hat{i}$

$\vec{u}_C = -\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$

$\vec{u}_D = 0\hat{i} - \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}$

$\Sigma F_x: \; F_B - F_C\left(\frac{3}{5}\right) = 0$

$\Sigma F_y: \; -F_D\left(\frac{3}{5}\right) + F_C\left(\frac{4}{5}\right) = 0$

$\Sigma F_z: \; F_D\left(\frac{4}{5}\right) - 900 = 0$

$\Rightarrow F_B, F_C, F_D$

Solve for the magnitudes (tensions) of the 3 cables

If wanted the forces, then compute the vectors. $\vec{F}_B = F_B\vec{u}_B$, etc.

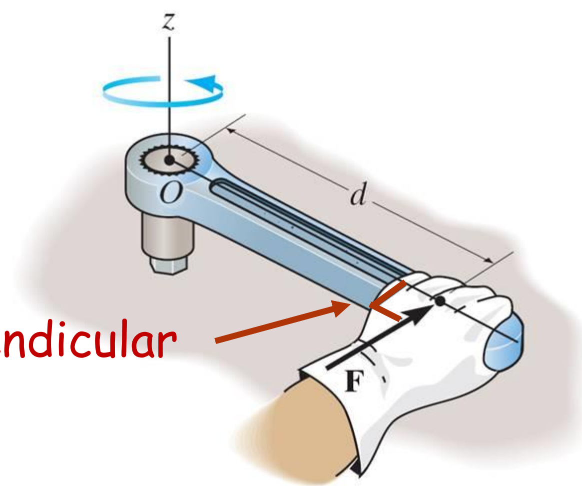
# Chapter 4: Force System Resultants

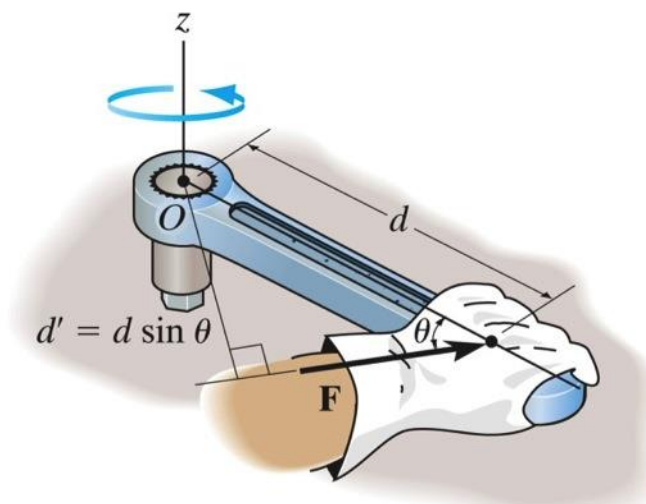
# Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- How to find the moment about a specified axis (Scalar Triple Product)
- Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading

# Moment of a force

perpendicular



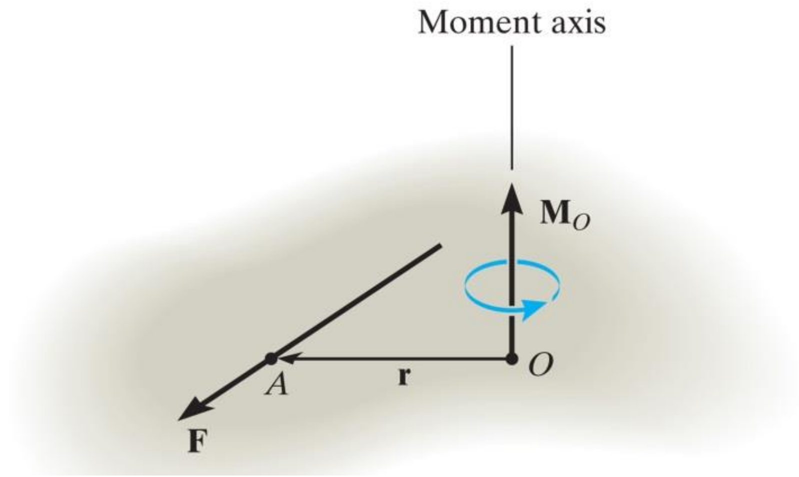

**Scalar Formulation:** $M_O = F\,d$

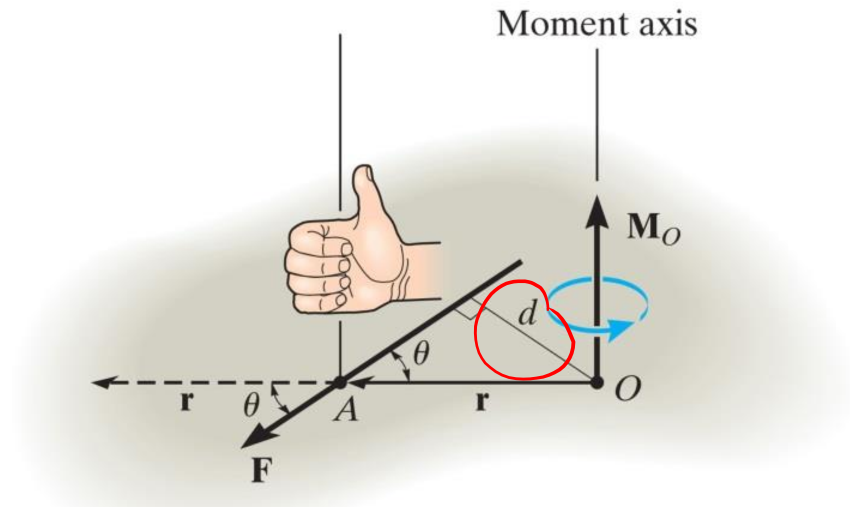
**Scalar Formulation:** $M_O = F\,d'$

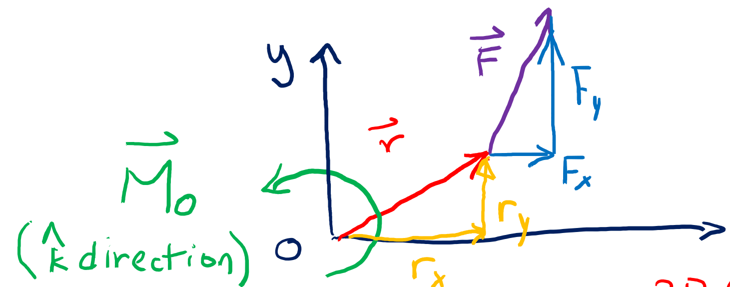
**Direction**: Moment about point O $\overrightarrow{M_O}$ is perpendicular to the plane that contains the force $\vec{F}$ and its moment arm $\vec{d}$. The right-hand rule is used to define the sense.

**Magnitude**: In a 2D case (where $\vec{F}$ is perpendicular to $\vec{d}$), the magnitude of the moment about point O is $M_O = F\,d$

# Moment of a force




## Vector Formulation

Use cross product: $\overrightarrow{M_O} = \vec{r} \times \vec{F}$

**Direction**: Defined by right hand rule.

2D (x,y) case

$$\overrightarrow{M_O} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = (r_yF_z - r_zF_y)\hat{\imath} - (r_xF_z - r_zF_x)\hat{\jmath} + (r_xF_y - r_yF_x)\hat{k}$$

$= \vec{M}_o$ (x,y case)

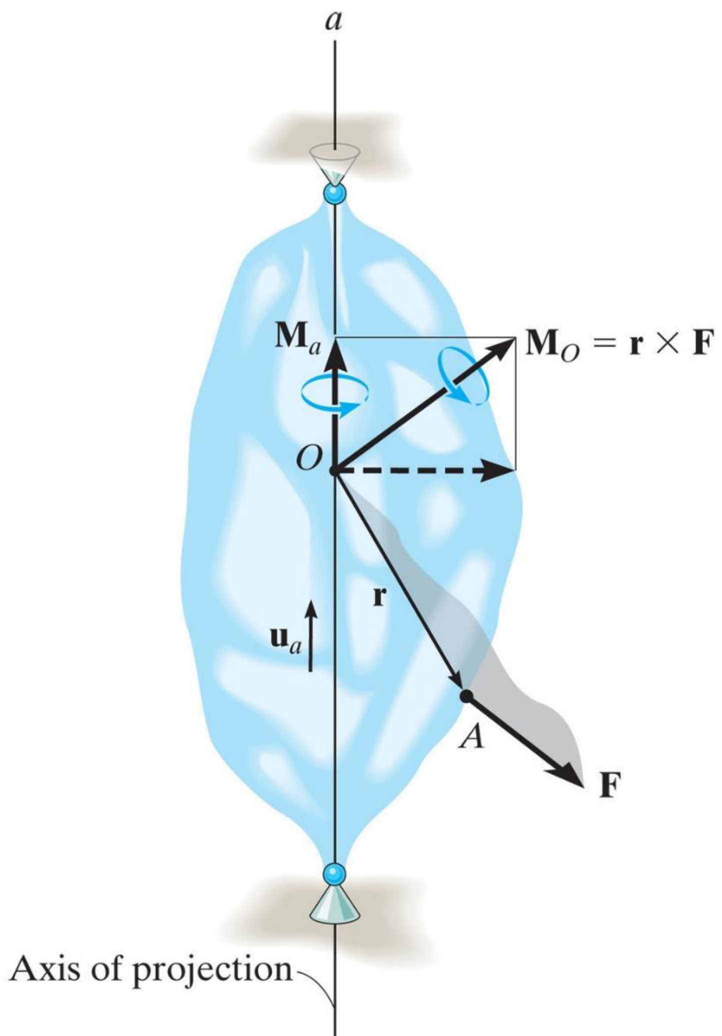
**Magnitude:**

$$M_O = \left|\overrightarrow{M_O}\right| = |\vec{r}|\left|\vec{F}\right| sin\theta = F(rsin\theta) = Fd$$

d is the perpendicular distance from O to $\vec{F}$

# Moment of a force about a specified axis (Scalar Triple Product)

The <u>magnitude</u> of the projected moment about any generic axis *a* can be computed using the scalar triple product:




$$\left|\overrightarrow{\boldsymbol{M}_a}\right| = \overrightarrow{\boldsymbol{u}_a} \cdot \left(\vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}\right)$$

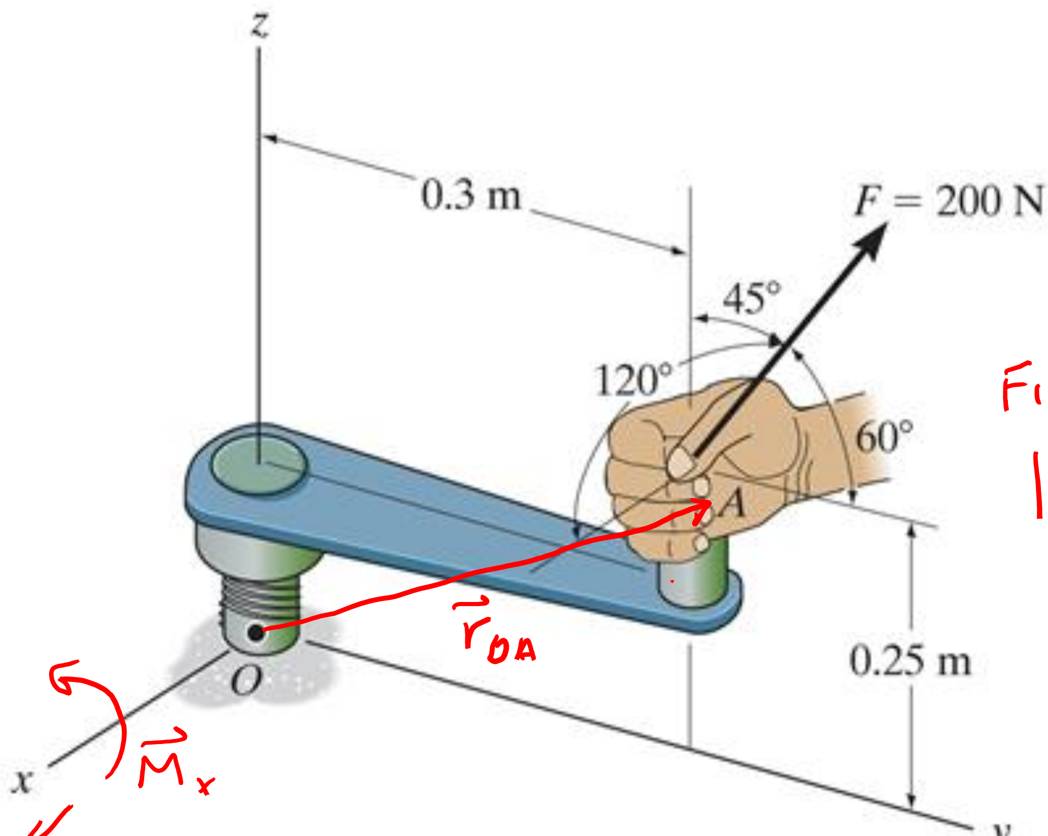
$$= \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The <u>direction</u> of the projected moment about any generic axis *a* can be defined using :

$$\overrightarrow{\boldsymbol{M}_a} = \left|\overrightarrow{\boldsymbol{M}_a}\right| \overrightarrow{\boldsymbol{u}_a}$$

where $\overrightarrow{\boldsymbol{u}_a}$ is the unit vector along axis *a*

A force is applied to the tool as shown. Find the magnitude of the moment of this force about the x axis.




$\vec{u}_x = \hat{i}$

Find: $\vec{M}_x$

$$|\vec{M}_x| = \underbrace{\vec{u}_x}_{\hat{i}} \cdot \underbrace{(\vec{r}_{OA} \times \vec{F})}_{\vec{M}_O}$$

$$\vec{r}_{OA} = 0\hat{i} + 0.3m\,\hat{j} + 0.25m\,\hat{k}$$

$$\vec{F} = 200(\cos 120\,\hat{i} + \cos 60\,\hat{j} + \cos 45\,\hat{k})\ N$$

$$= -100\hat{i} + 100\hat{j} + 141.4\hat{k}\ N$$

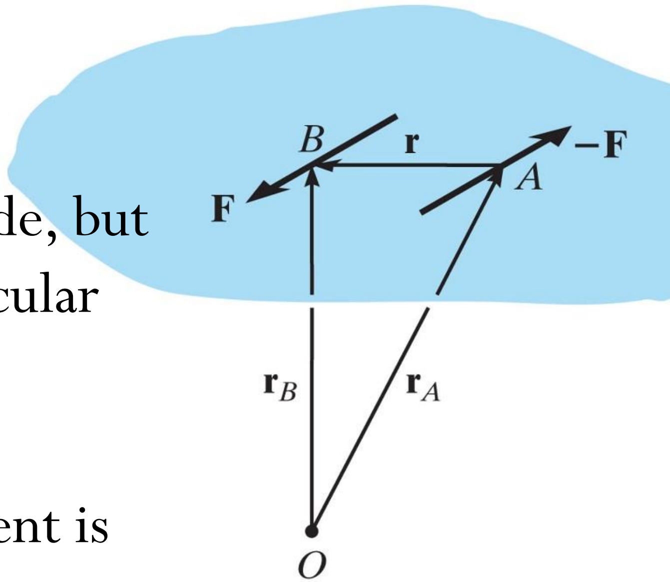
$$M_x = \hat{i} \cdot (\vec{r}_{OA} \times \vec{F})$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.3 & 0.25 \\ -100 & 100 & 141.4 \end{vmatrix}$$

$$= 1\left[(0.3m \cdot 141.4N) - (0.25m \cdot 100N)\right] = 17.4\ Nm$$

$$\therefore \vec{M}_x = 17.4\ Nm\ \hat{i}$$

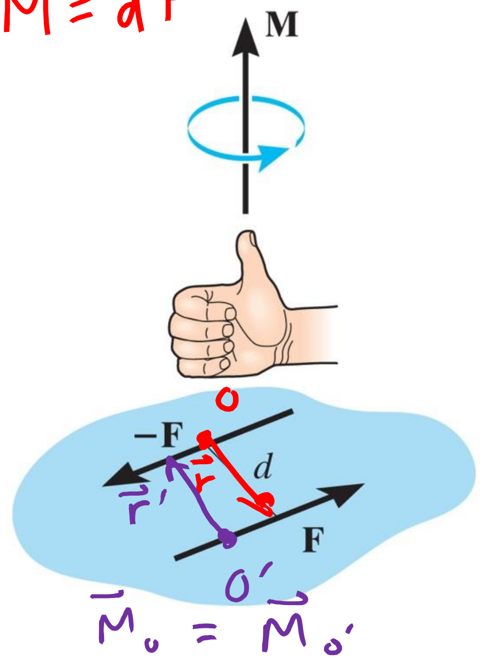
# Moment of a couple




**Couple:** two parallel forces that have same magnitude, but opposite directions, and are separated by a perpendicular distance *d*.

- Resultant force is zero. $\vec{F}_R = \vec{F} + (-\vec{F}) = 0$
- Couple produces actual rotation, or if no movement is possible, tendency of rotation in a specified direction.

Moment produced by a couple is called **couple moment**. $M = dF$




Sum of moments of both couple forces about **any** arbitrary point:

$$\vec{M} = (\vec{r_B} \times \vec{F}) + \left(\vec{r_A} \times (-\vec{F})\right)$$

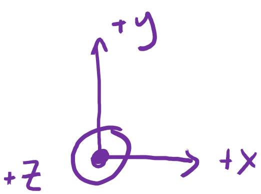
$$= (\vec{r_B} - \vec{r_A}) \times \vec{F}$$

$$= \vec{r} \times \vec{F}$$

Couple moment is a **free vector**, i.e. is **independent** of the choice of O!

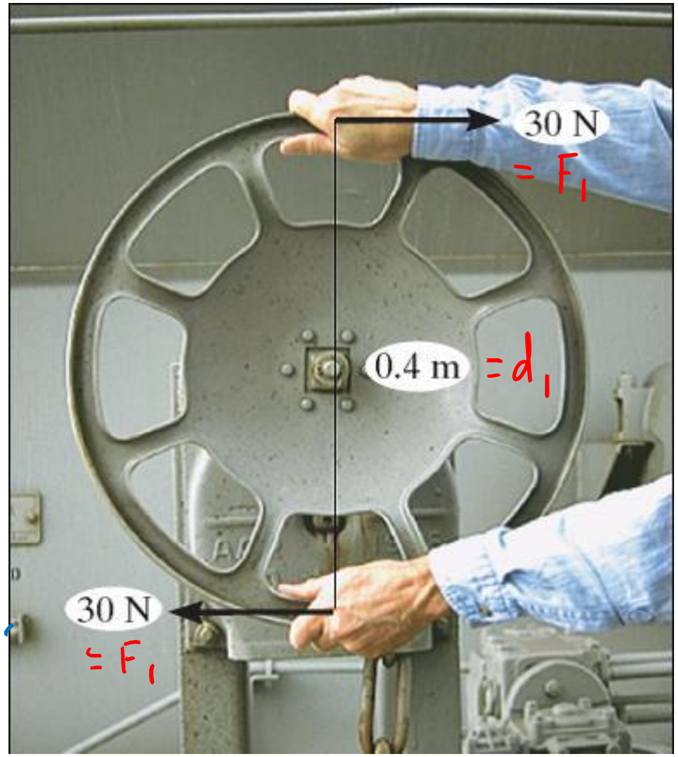
# Equivalent couples

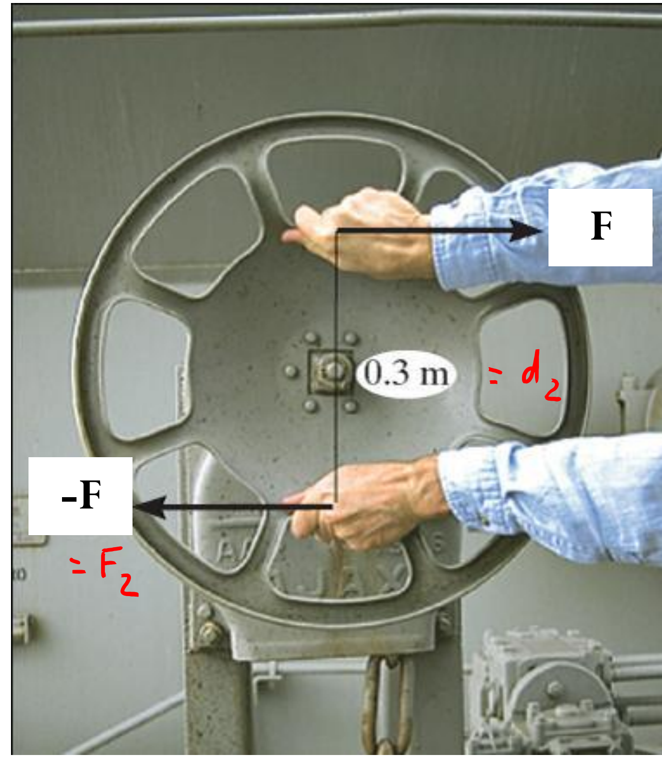
Define coordinate frame




Notation for representing a vector pointing perpendicular (in or out of screen)

⊙ OUT tip   ⊗ IN tail   ↖ Like arrow




A torque or moment of 12 N·m is required to rotate the wheel.

Would F be greater or less than 30 N?

$M_1 = d_1 F_1 = (0.4\,m)(30\,N)$ CW moment $(-\hat{k})$

$\vec{M}_1 = -12\,Nm\,\hat{k}$

$\vec{M}_2 = -12\,Nm\,\hat{k}$

$M_2 = d_2 F$

$12\,Nm = 0.3\,m\,F$

$F = 40\,N$

$\vec{F} = 40\,N\,\hat{i}$

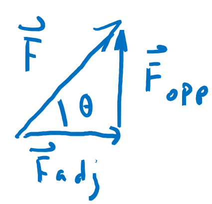
# Equipollent (or equivalent) force systems

A force **system** is a collection of **forces** and **couples** applied to a body.

Two force systems are said to be **equipollent** (or equivalent) if they have the **same resultant force** AND the **same resultant moment** with respect to any point $O$.

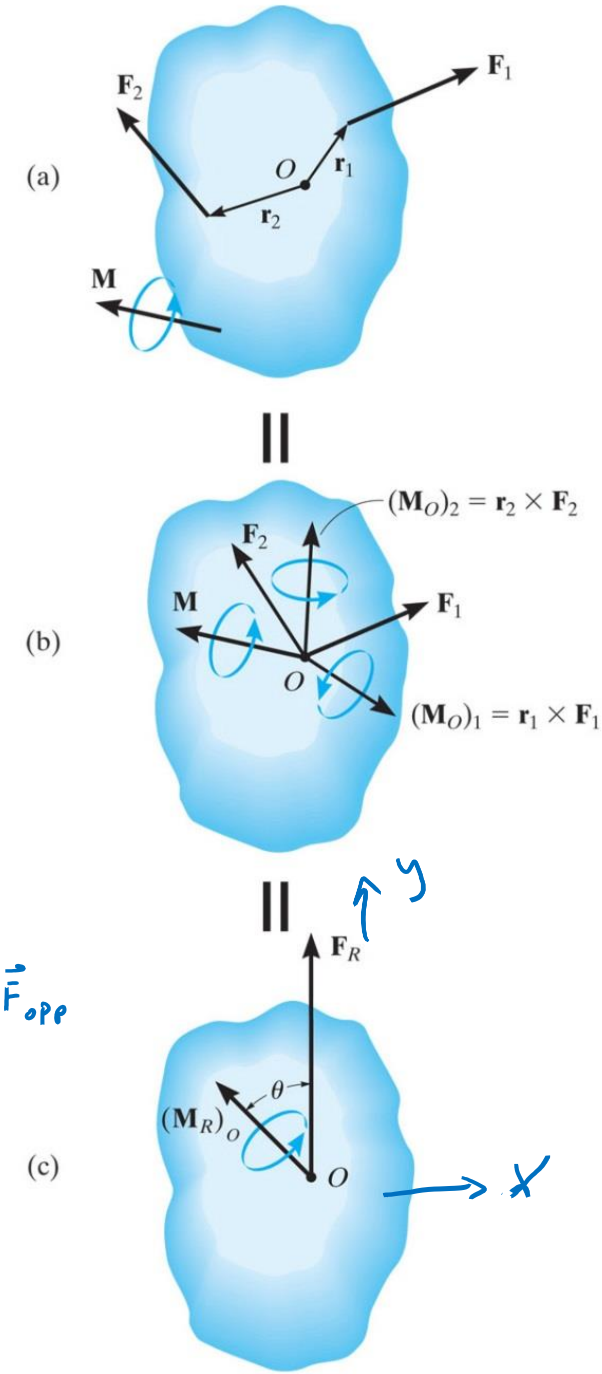
Reducing a force system to a single resultant force $\boldsymbol{F}_R$ and a single resultant couple moment $(\boldsymbol{M}_R)_o$:

$$\overrightarrow{\boldsymbol{F}_R} = \Sigma F_x \hat{\boldsymbol{\imath}} + \Sigma F_y \hat{\boldsymbol{\jmath}} + \Sigma F_z \widehat{\boldsymbol{k}}$$




$$|\overrightarrow{\boldsymbol{F}_R}| = \sqrt{F_x^2 + F_y^2 + F_z^2} \qquad \theta = \tan^{-1} \frac{F_{opp}}{F_{adj}}$$

$$(\boldsymbol{M}_R)_o = \sum \boldsymbol{M}_o + \sum \boldsymbol{M}$$



# Distributed loads

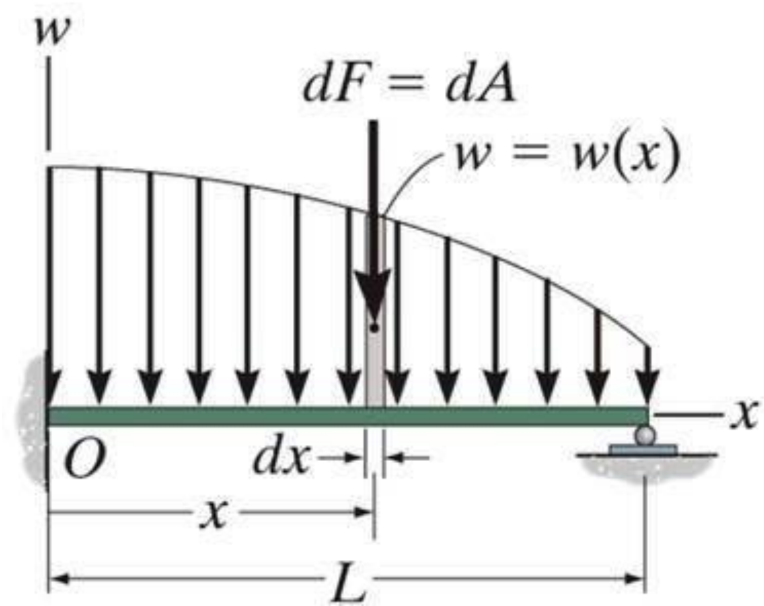
- Equivalent force system for distributed loading function $w(x)$ with units of $\frac{\text{force}}{\text{length}}$.
- Find magnitude $F_R$ and location $\bar{x}$ of the equivalent resultant force for $\overrightarrow{\boldsymbol{F_R}}$

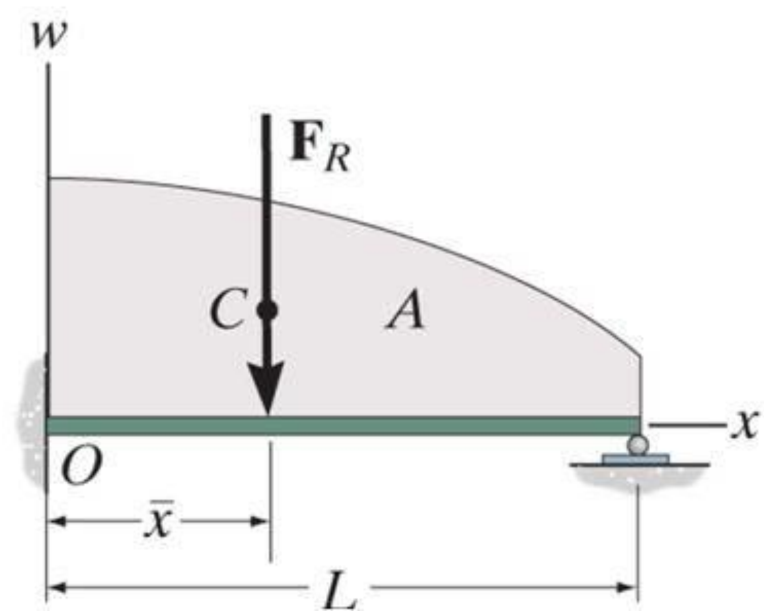
$$|\overrightarrow{\boldsymbol{F_R}}| = F_R = \int_0^L dF = \int_0^L w(x)\,dx = A$$

$$M_o = \int_0^L x\ w(x)\,dx = \bar{x}\,F_R$$

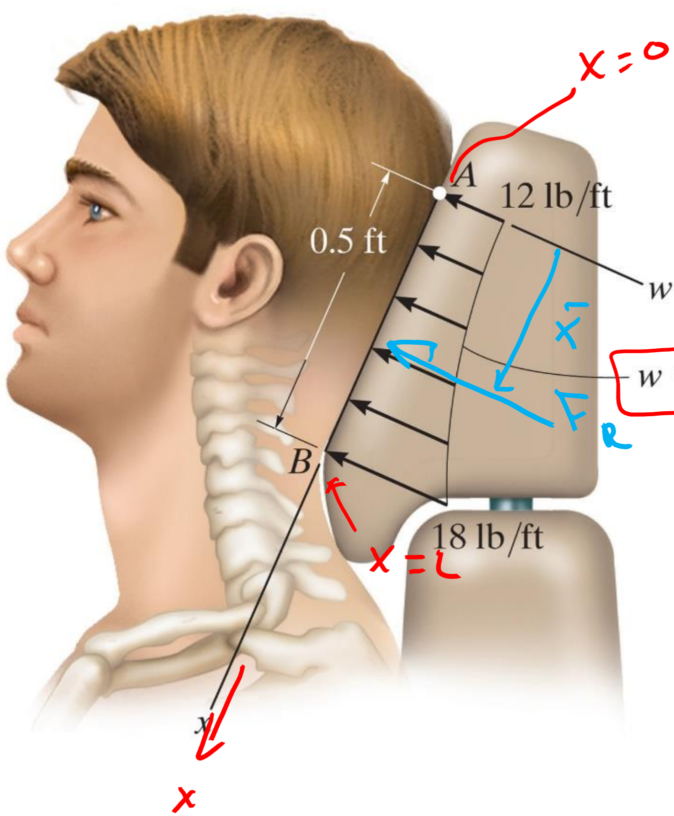
$$\bar{x} = \frac{M_o}{F_R} = \frac{\int_0^L x\ w(x)\,dx}{\int_0^L w(x)\,dx}$$

$\bar{x}$ = **geometric center or centroid** of area $A$ under loading curve $w(x)$.



Find equivalent force and its location from point A for loading on headrest.




FIND: $F_R$ & $\bar{x}$ wrt A

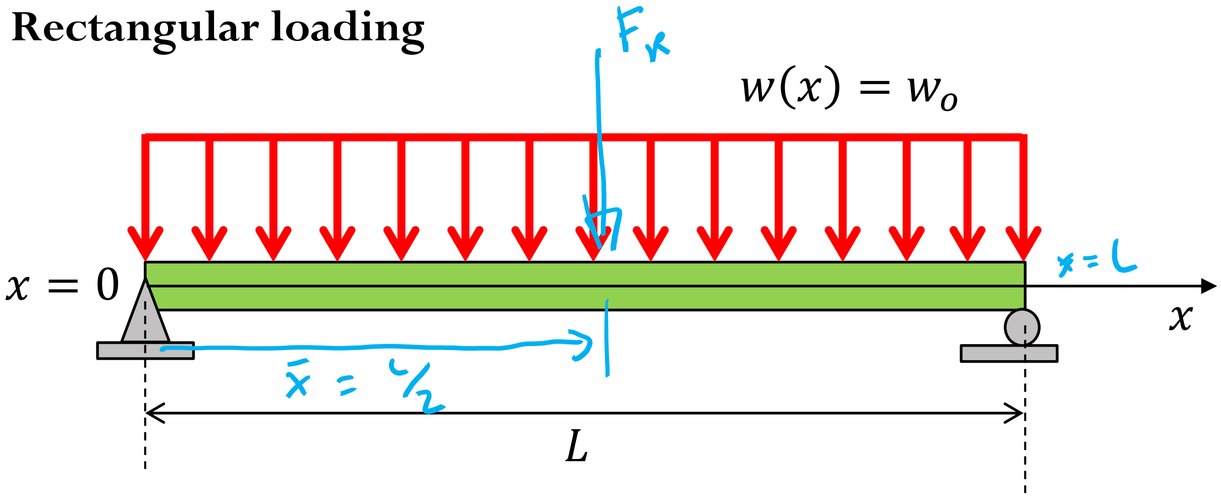
$$F_R = \int_0^L w(x)\,dx$$

$$F_R = \int_0^{0.5} 12(1 + 2x^2)\,dx$$

$$\bar{x} = \frac{M_A}{F_R} = \frac{\int_0^L x\,w(x)\,dx}{\int_0^L w(x)\,dx}$$

$$\bar{x} = \frac{\int_0^{0.5} x\left[12(1 + 2x^2)\right]dx}{\int_0^{0.5} 12(1 + 2x^2)\,dx}$$

## Rectangular loading

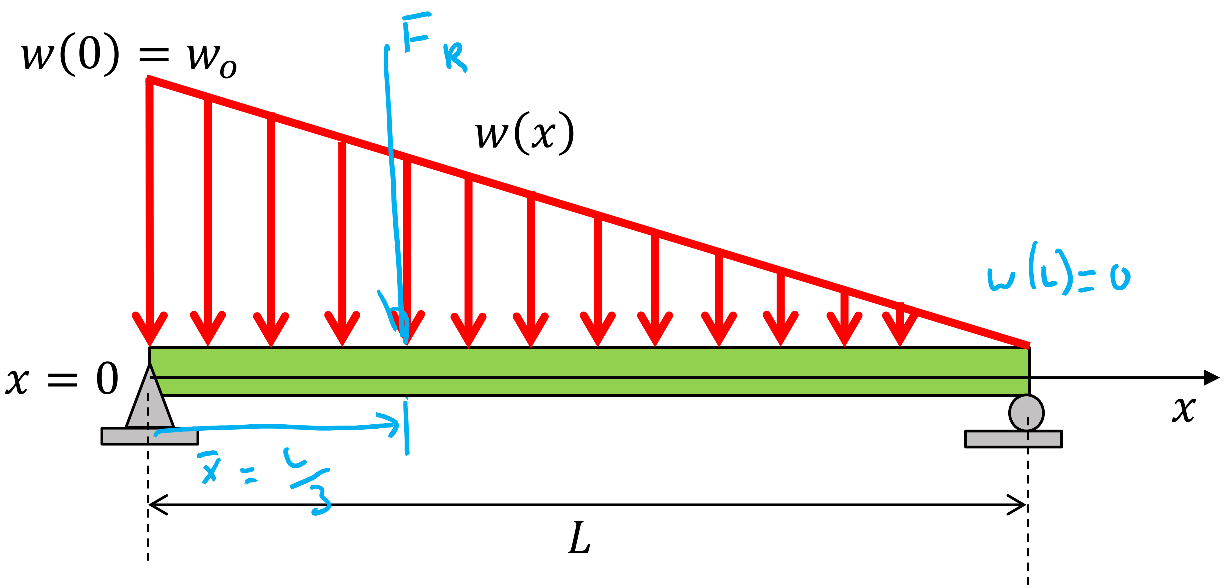



$$w(x) = w_o$$

$$|\vec{\boldsymbol{F}}| = F = w_o L$$

$$\bar{x} = \frac{L}{2}$$

## Triangular loading




$$w(x) = w_o - \frac{w_o x}{L}$$

$$F = w_o \frac{L}{2}$$

$$\bar{x} = \frac{L}{3}$$

# Chapter 5: Equilibrium of Rigid Bodies

Focus on 2D problems

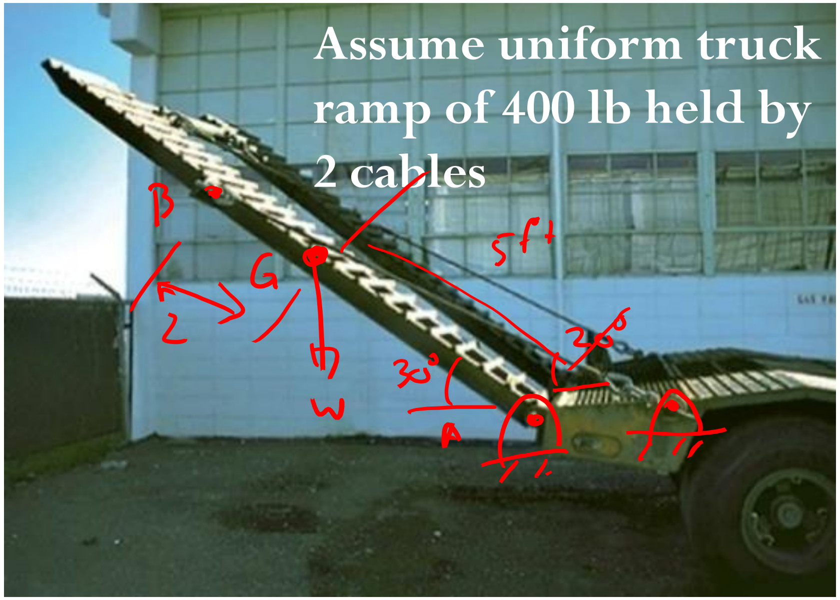
Sections 5.1-5.4, 5.7

TAM 211 students will cover 3D problems (sections 5.5-5.6) in week 13

# Goals and Objectives

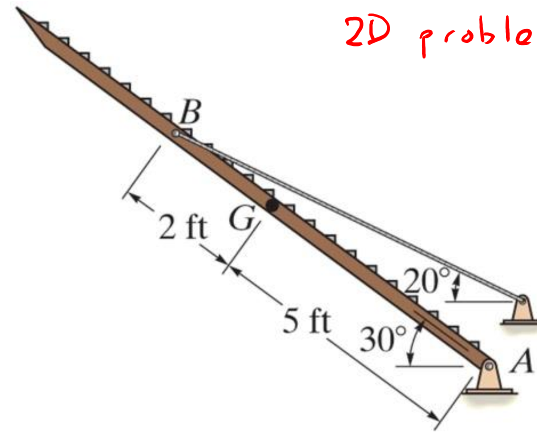
- Introduce the free-body diagram for a 2D rigid body
- Develop the equations of equilibrium for a 2D rigid body
- Solve 2D rigid body equilibrium problems using the equations of equilibrium
- Introduce concepts of
  - Support reactions
  - Two- and three-force members
  - Constraints and statical determinacy

# Process of solving rigid body equilibrium problems



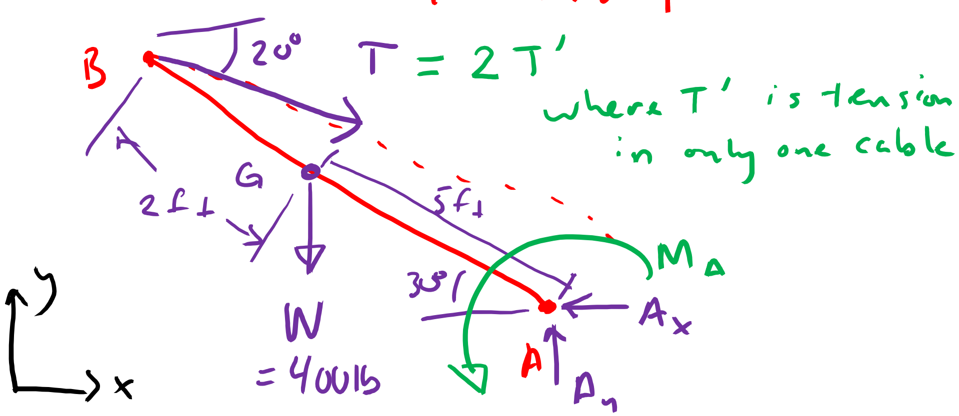

**1. Create idealized model (modeling and assumptions)**

Idealize ramp as 2D problem




**2. Draw free body diagram showing ALL the external (applied loads and supports)**

FBD : ramp




**3. Apply equations of equilibrium**

$$\overrightarrow{F_R} = \sum \vec{F} = 0$$

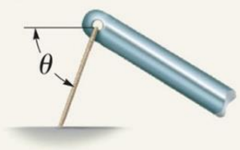
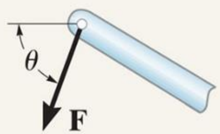
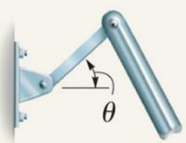
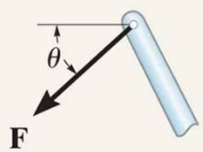
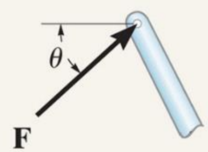
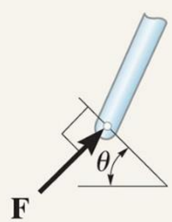
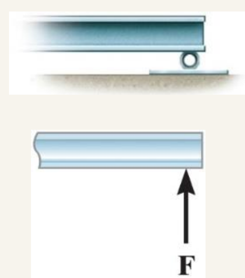
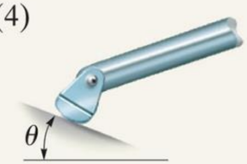
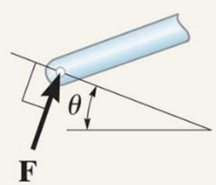
$$(\overrightarrow{M_R})_A = \sum \overrightarrow{M_A} = 0 = f(T, W)$$

In this case, let's sum moments about pt A

See Example 5.11 in text for full derivation

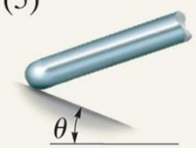
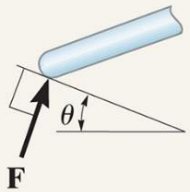
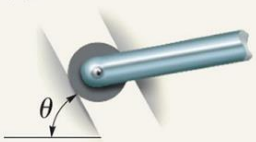
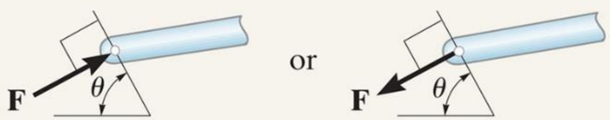
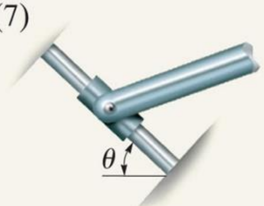
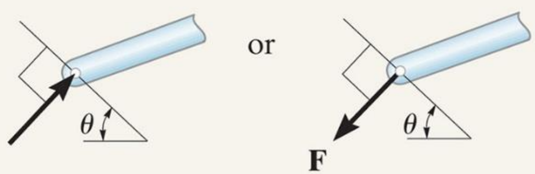
# Types of connectors/supports

**TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems**

| Types of Connection | Reaction | Number of Unknowns |
|---|---|---|
| (1)  cable |  | One unknown. The reaction is a tension force which acts away from the member in the direction of the cable. |
| (2)  weightless link |  or  | One unknown. The reaction is a force which acts along the axis of the link. |
| (3)  roller |   | One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact. |
| (4)  rocker |  | One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact. |

# Types of connectors/supports

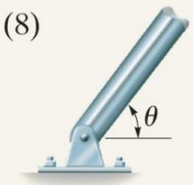
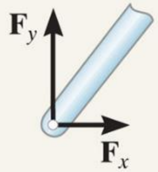
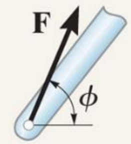
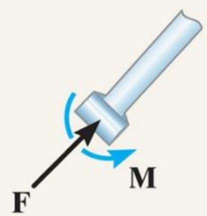
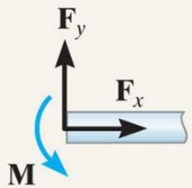
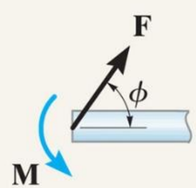
**TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems**

| Types of Connection | Reaction | Number of Unknowns |
|---|---|---|
| (5)   smooth contacting surface |   | One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact. |
| (6)   roller or pin in confined smooth slot |   | One unknown. The reaction is a force which acts perpendicular to the slot. |
| (7)   member pin connected to collar on smooth rod |   | One unknown. The reaction is a force which acts perpendicular to the rod. |

*continued*

# Types of connectors/supports

**TABLE 5–1 Continued**

| Types of Connection | Reaction | Number of Unknowns |
|---|---|---|
| (8)   smooth pin or hinge |   or   | Two unknowns. The reactions are two components of force, or the magnitude and direction $\phi$ of the resultant force. Note that $\phi$ and $\theta$ are not necessarily equal [usually not, unless the rod shown is a link as in (2)]. |
| (9)  member fixed connected to collar on smooth rod |   | Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod. |
| (10)  fixed support |   or   | Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction $\phi$ of the resultant force. |

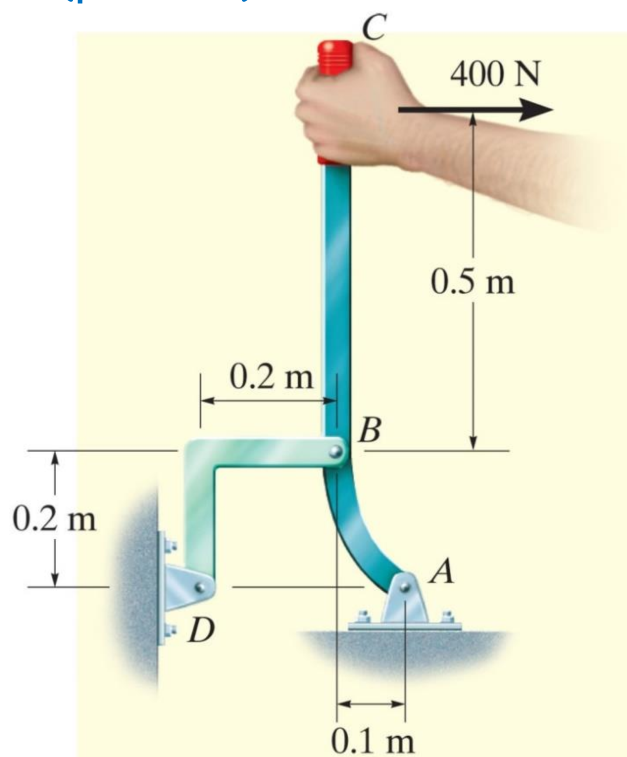
# Two-force and three-force members

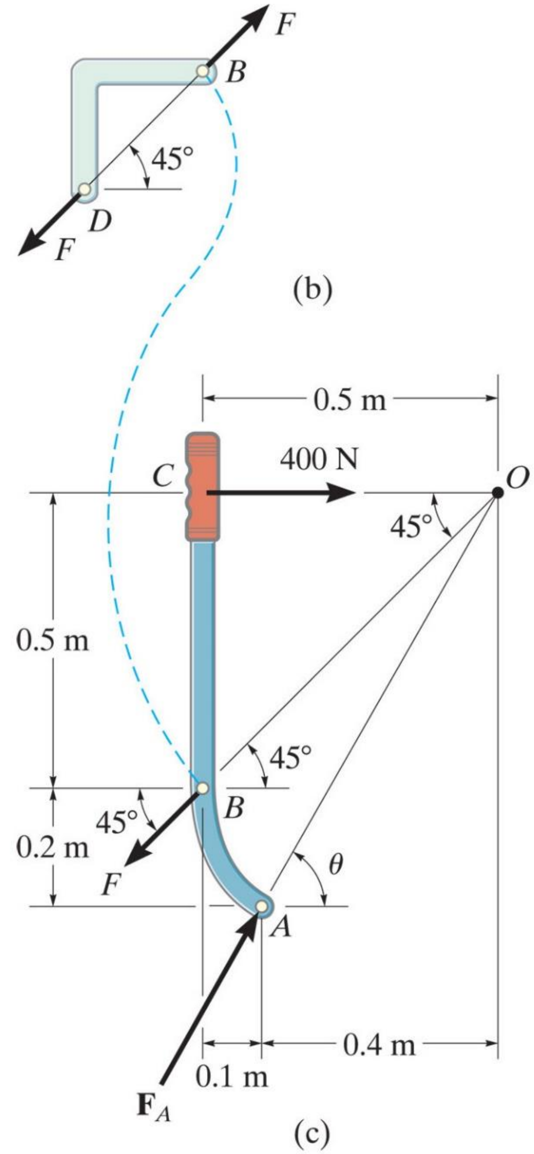
One can use these concepts to quickly identify the direction of an unknown force.

Two-force member:
the two forces at ends are equal, opposite, collinear

Three-force member: a force system where the three forces

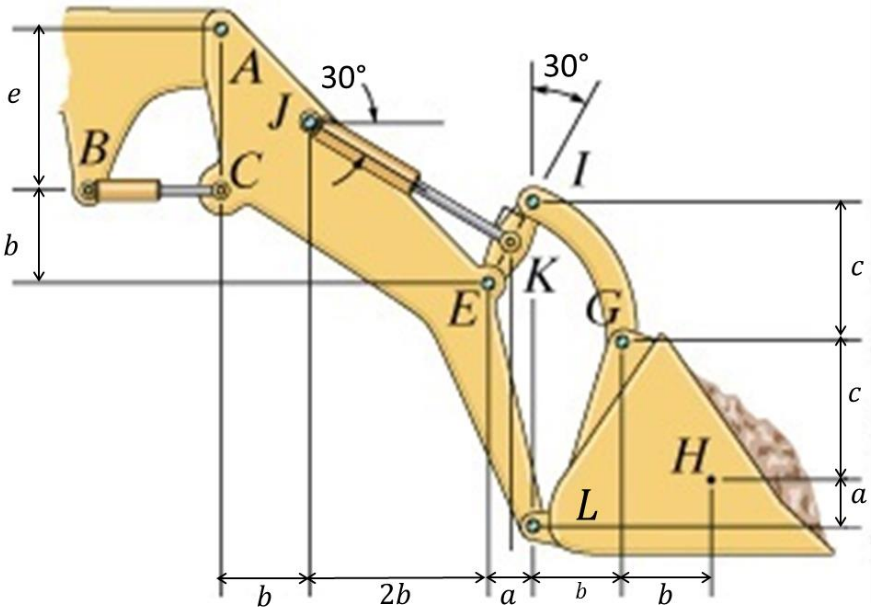
1. meet at the same point (point O), or
2. are parallel



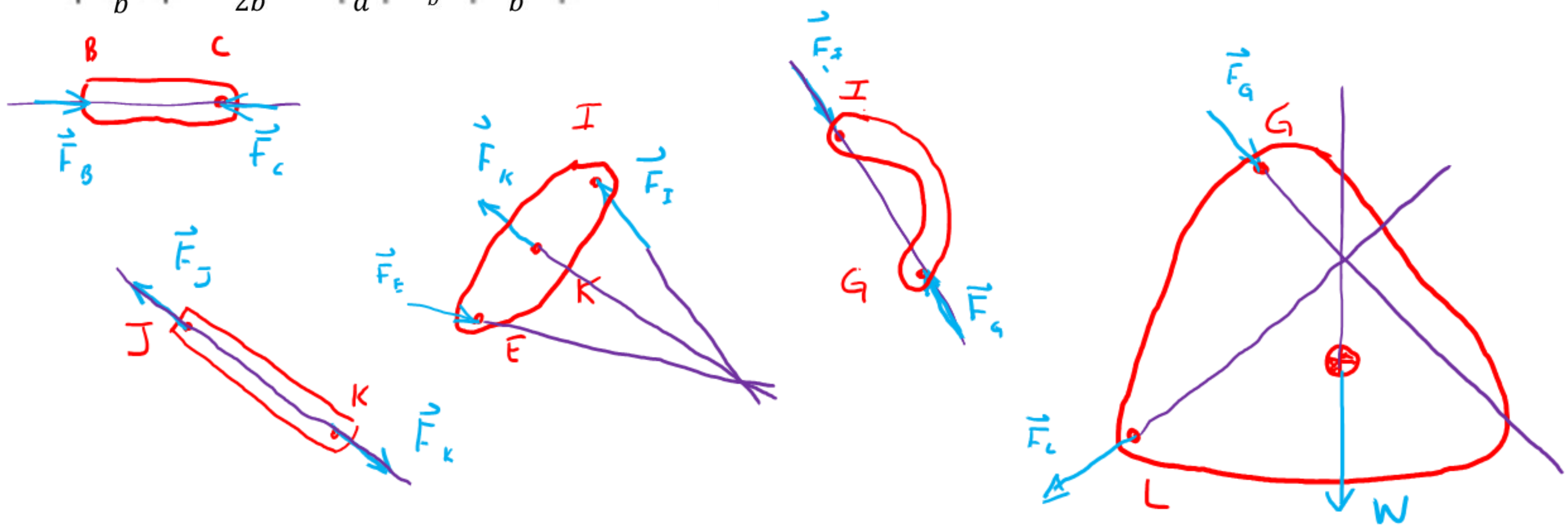




See Example 5.13 in text

Draw FBDs for each two or three force member (BC, JK, IE, IG, Bucket). Ignore weight of each link. Include dirt weight in bucket.




Directions of arrows of unknown forces/moments are arbitrary on FBD. Actual direction will be determined after solving for unknown values




Line of action of an unknown force can be determined from 2- or 3-force members

2-force member: The 2 forces at ends are equal, opposite, collinear

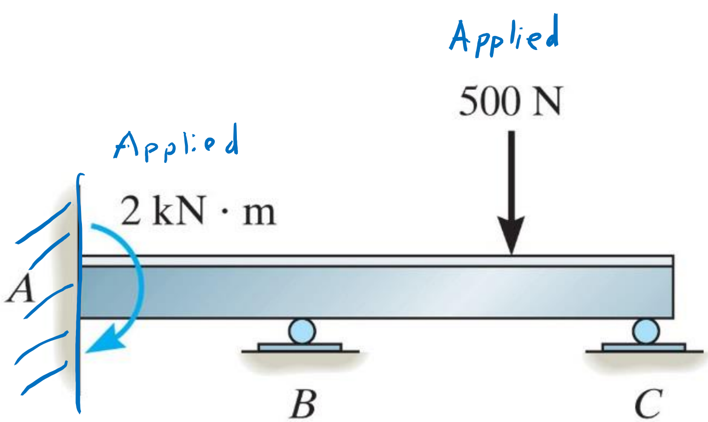
3-force member: force system where the 3 forces

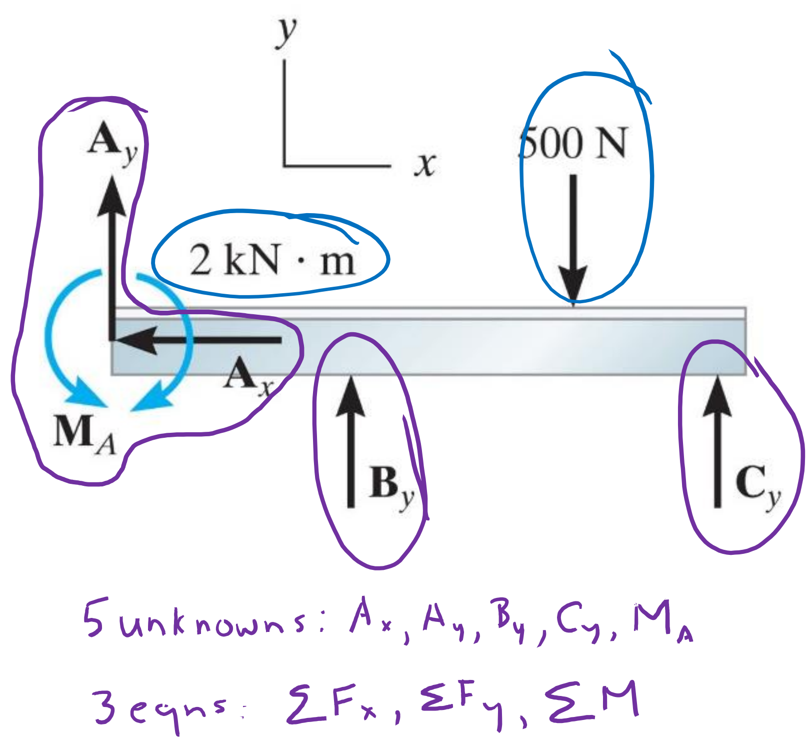
1. meet at the same point, or
2. are parallel

# Constraints

To ensure equilibrium of a rigid body, it is not only necessary to satisfy equations of equilibrium, but the body must also be properly constrained by its supports

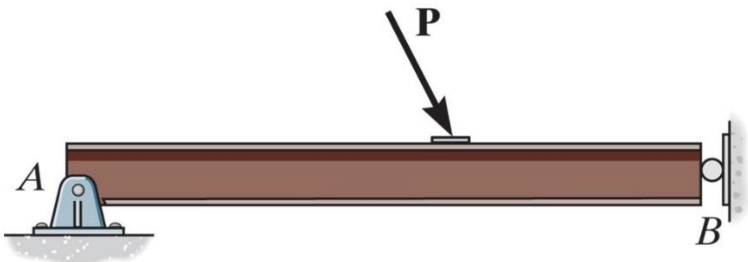
- **Redundant constraints**: the body has more supports than necessary to hold it in equilibrium; the problem is STATICALLY INDERTERMINATE and cannot be solved with statics alone. Too many unknowns, not enough equations

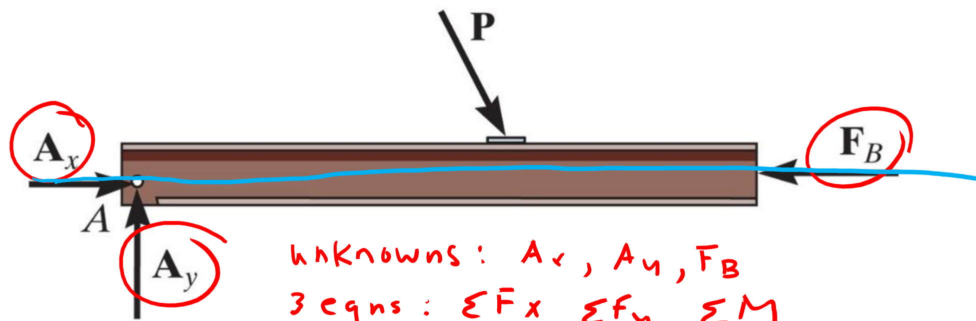


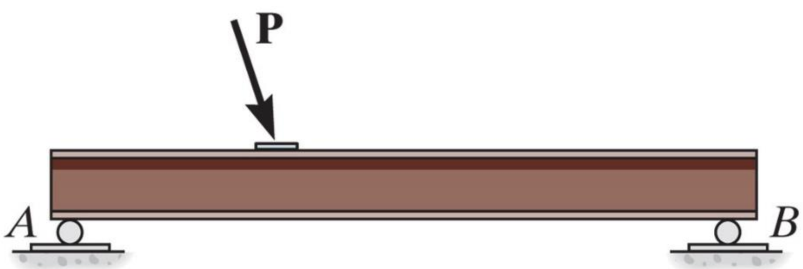
# Constraints

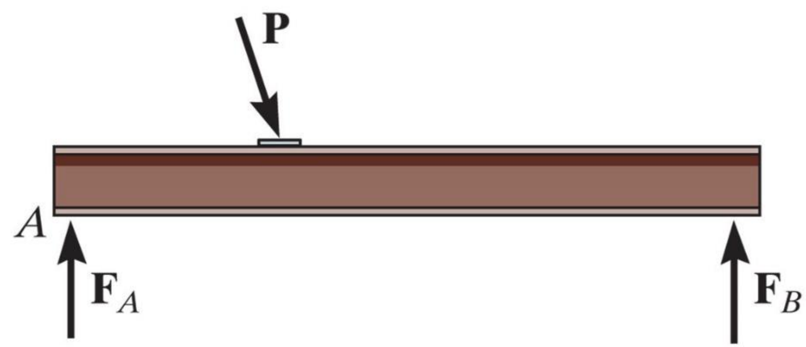
- **Improper constraints:** In some cases, there may be as many unknown reactions as there are equations of equilibrium (statically determinate). However, if the supports are not properly constrained, the body may become unstable for some loading cases.
    - BAD: Reactive forces are concurrent at same point (point A) or line of action




- BAD: Reactive forces are parallel




Stable body: lines of action of reactive forces
do not intersect at common axis, and are not parallel

# Chapter 6: Structural Analysis

# Goals and Objectives

- Determine the forces in members of a truss using the method of joints
- Determine zero-force members
- Determine the forces in members of a truss using the method of sections
- Determine the forces and moments in members of a frame or machine

# Simple trusses

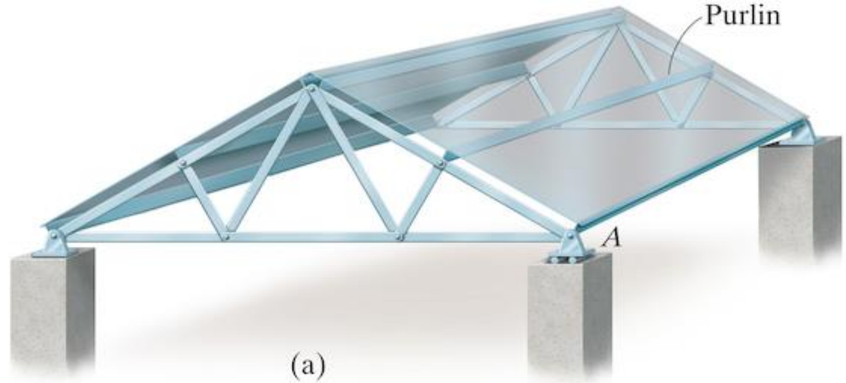
**Truss:**

- Structure composed of slender members joined together at end points
- Transmit loads to supports

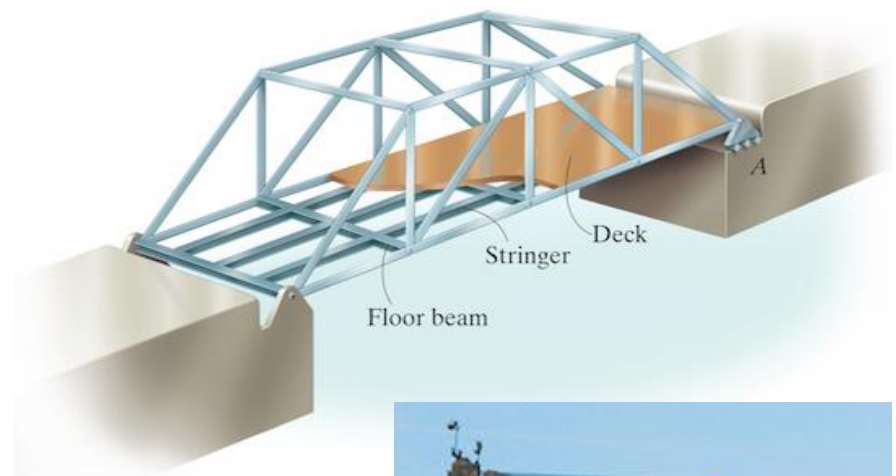
**Assumption of trusses**

- Loading applied at joints, with negligible weight (If weight included, vertical and split at joints)
- Members joined by smooth pins

**Result:** **all truss members are two-force members,** and therefore the force acting at the end of each member will be directed along the axis of the member

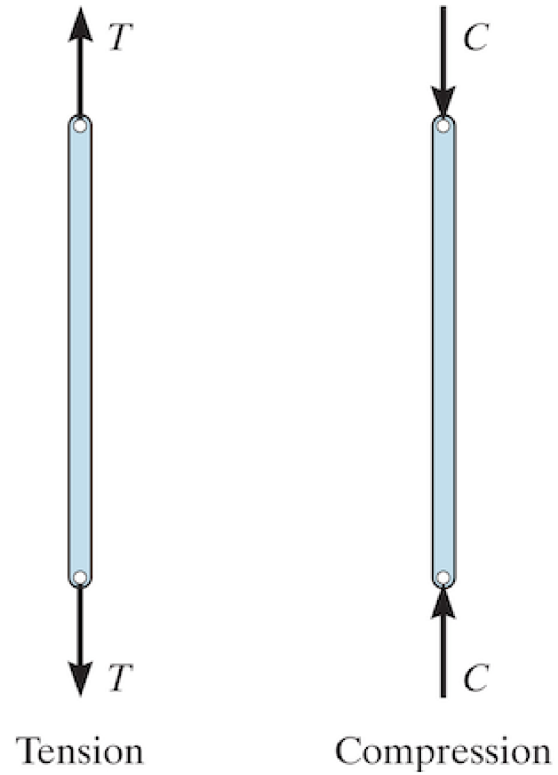


(a)




# Method of joints

- Entire truss is in equilibrium if and only if all individual pieces (truss members and connecting pins) are in equilibrium.
- Truss members are two-force members: equilibrium satisfied by equal, opposite, collinear forces.
    - Tension: member has forces elongating.
    - Compression: member has forces shortening.
- Pins in equilibrium: $\sum F_x = 0$ and $\sum F_y = 0$




**Procedure for analysis:**

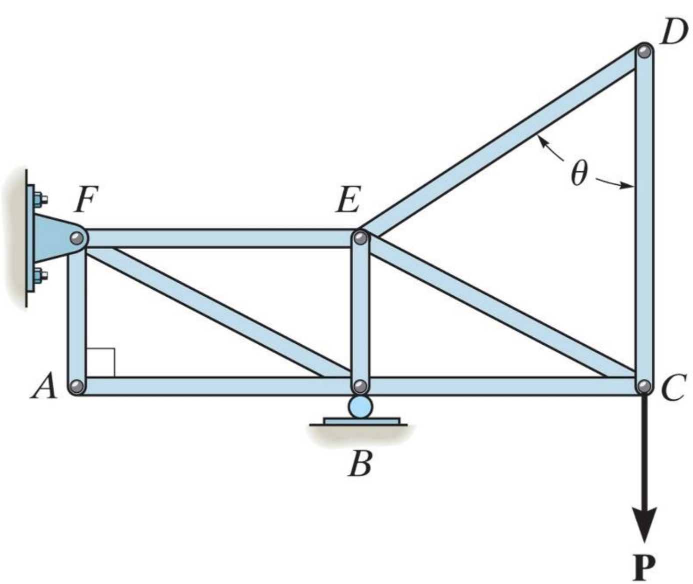
- Free-body diagram for each joint
- Start with joints with at least 1 known force and 1-2 unknown forces.
- Generates two equations, 1-2 unknowns for each joint.
- Assume the unknown force members to be in *tension; i.e. the forces "pull" on the pin*. Numerical solutions will yield positive scalars for members in tension and negative scalar for members in compression.

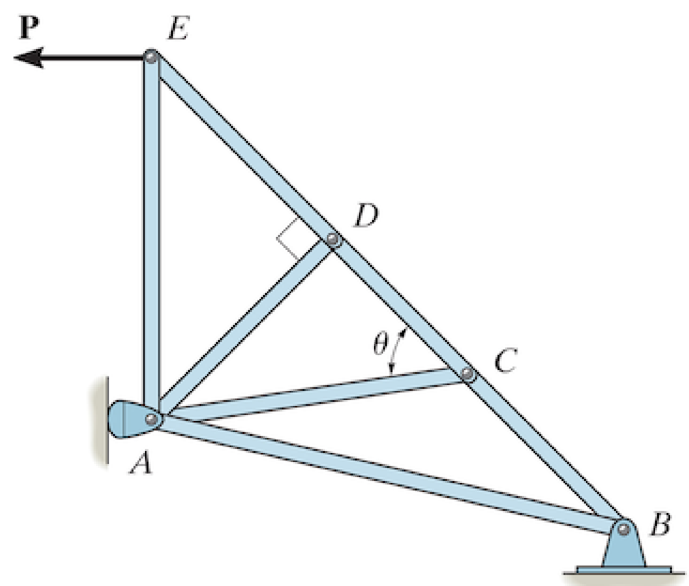
# Zero-force members

- Particular members in a structure may experience no force for certain loads.
- Zero-force members are used to increase stability
- Identifying members with zero-force can expedite analysis. ⇒ can quickly say $F_{member} = 0$

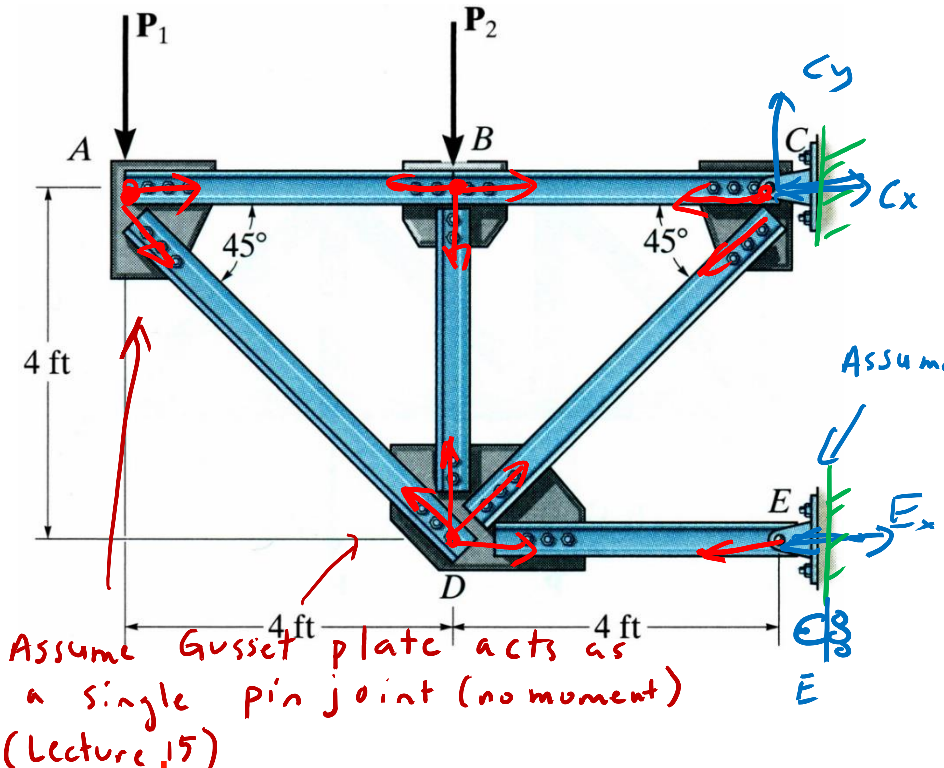
Two situations:

- Joint with two non-collinear members, no external or support reaction applied to the joint → **Both members are zero-force members**.
- Joint with two collinear member, plus third non-collinear, no loads applied to the joint → **Non-collinear member is a zero-force member**.



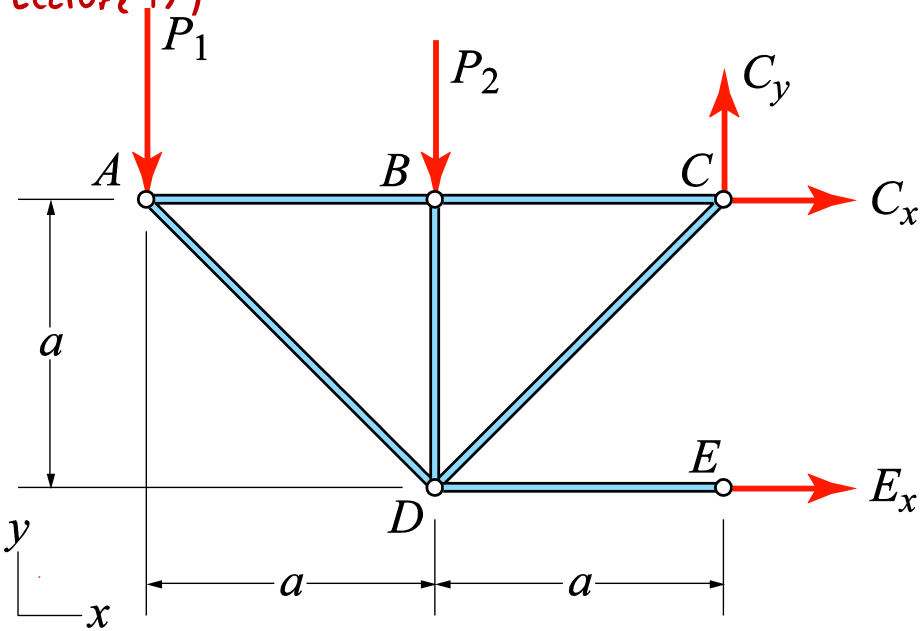
The truss, used to support a balcony, is subjected to the loading shown. Approximate each joint as a pin and determine the force in each member. State whether the members are in tension or compression.




Assume E is supported by Roller

**Solution**:

Start by setting the entire structure into **external** equilibrium. Draw the FBD.




Equilibrium requires $\sum \boldsymbol{F} = \mathbf{0}$ and $(\sum \boldsymbol{M})_C = \mathbf{0}$

$$\Sigma F_x = 0: \quad C_x + E_x = 0\,,$$

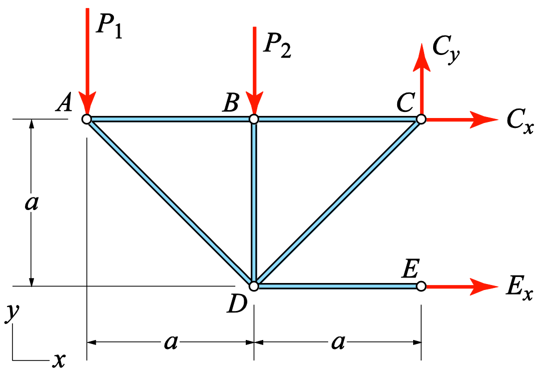
$$\Sigma F_y = 0: \quad C_y - P_1 - P_2 = 0\,,$$

$$\Sigma M_C = 0: \quad 2aP_1 + aP_2 + aE_x = 0\,.$$

Solving these equations gives the *external* reactions

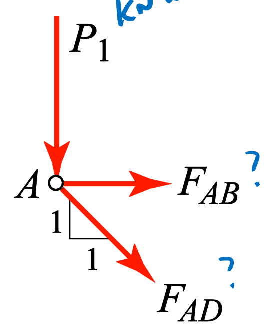
$$\boxed{C_x = 2P_1 + P_2, \quad C_y = P_1 + P_2, \quad E_x = -(2P_1 + P_2)\,.}$$

Next, start with a joint, draw the FBD, set it into *force* equilibrium *only*, and move to the next joint. Start with joints with at least 1 known force and 1-2 unknown forces.




$C_x = 2P_1 + P_2, \quad C_y = P_1 + P_2, \quad E_x = -(2P_1 + P_2)\,.$

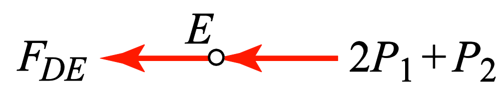
For example, start with **joint *A***:



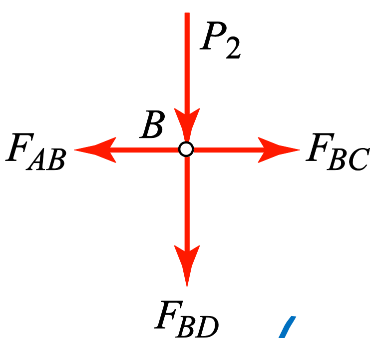

$$\Sigma F_x = 0: \quad F_{AB} + \tfrac{1}{\sqrt{2}} F_{AD} = 0\,,$$
$$\Sigma F_y = 0: \quad -P_1 - \tfrac{1}{\sqrt{2}} F_{AD} = 0\,.$$

$$F_{AD} = -\sqrt{2}P_1, \quad F_{AB} = -\tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) = +P_1\,.$$

## Joint E:



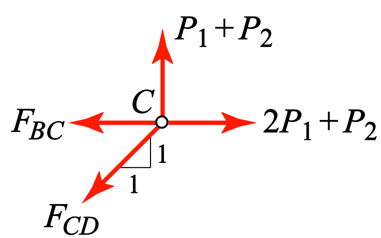

## Joint B:




$$\Sigma F_x = 0: \quad -F_{AB} + F_{BC} = 0\,,$$
$$\Sigma F_y = 0: \quad -P_2 - F_{BD} = 0\,.$$

$$F_{BC} = F_{AB} = +P_1, \quad F_{BD} = -P_2\,.$$

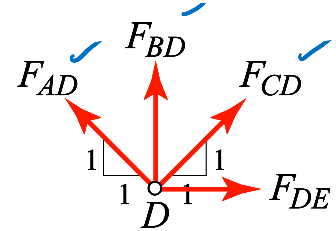
## Joint C:




$$\Sigma F_x = 0: \quad -F_{BC} - \tfrac{1}{\sqrt{2}} F_{CD} + 2P_1 + P_2 = 0\,,$$
$$\Sigma F_y = 0: \quad -\tfrac{1}{\sqrt{2}} F_{CD} + P_1 + P_2 = 0\,.$$

$$F_{CD} = \sqrt{2}(2P_1 + P_2 - P_1) = \sqrt{2}(P_1 + P_2)\,,$$
$$F_{CD} = \sqrt{2}(P_1 + P_2) \quad \text{(check)}\,.$$

## Joint D: *only needed for check*




$$\Sigma F_x = 0: \quad -\tfrac{1}{\sqrt{2}} F_{AD} + \tfrac{1}{\sqrt{2}} F_{CD} + F_{DE} = 0\,,$$
$$\Sigma F_y = 0: \quad \tfrac{1}{\sqrt{2}} F_{AD} + F_{BD} + \tfrac{1}{\sqrt{2}} F_{CD} = 0\,.$$

$$F_{DE} = \tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) - \tfrac{1}{\sqrt{2}}\sqrt{2}(P_1 + P_2) = -(2P_1 + P_2)\,,$$
$$\tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) - P_2 + \tfrac{1}{\sqrt{2}}\sqrt{2}(P_1 + P_2) = 0 \quad \text{(check)}\,.$$

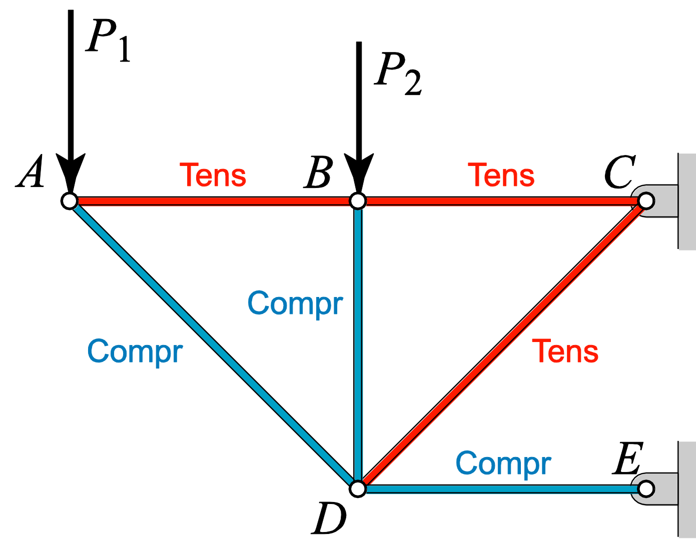
**Note:** The checks would not have been satisfied if the external reactions had been calculated incorrectly.

**Note:** The order in which the joints are set in equilibrium is usually arbitrary. Sometimes not all member loads are requested.

If provided numerical values:

$P_1 = 800 \text{ lb}$

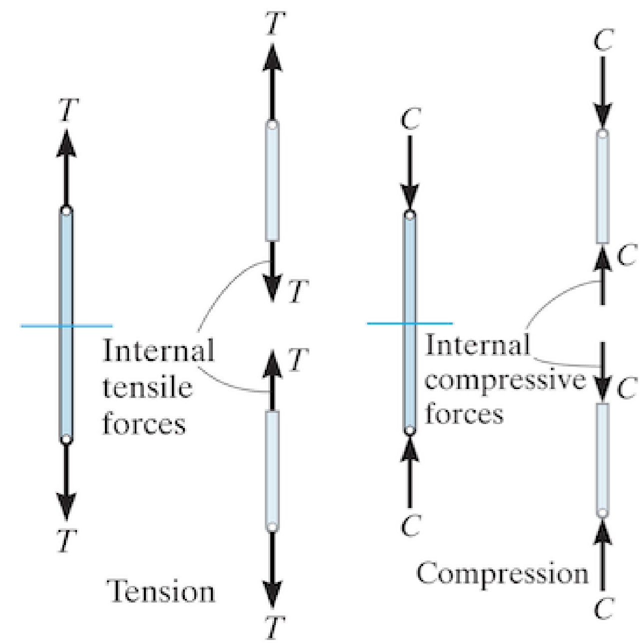
$P_2 = 0$



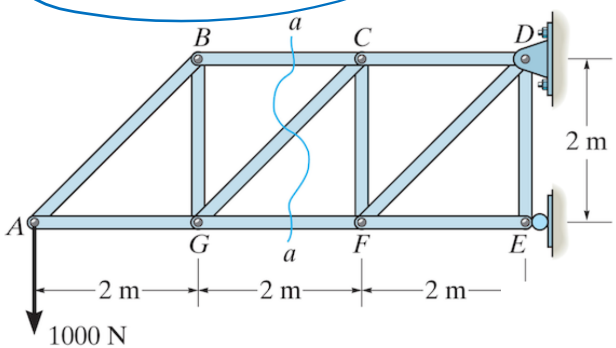

$$F_{AB} = P_1 = 800 \text{ lb (T)}$$
$$F_{BC} = P_1 = 800 \text{ lb (T)}$$
$$F_{AD} = -\sqrt{2}P_1 = -1130 \text{ lb (C)}$$
$$F_{BD} = -P_2 = 0$$
$$F_{CD} = \sqrt{2}(P_1 + P_2) = 1130 \text{ lb (T)}$$
$$F_{DE} = -(2P_1 + P_2) = -1600 \text{ lb (C)}$$

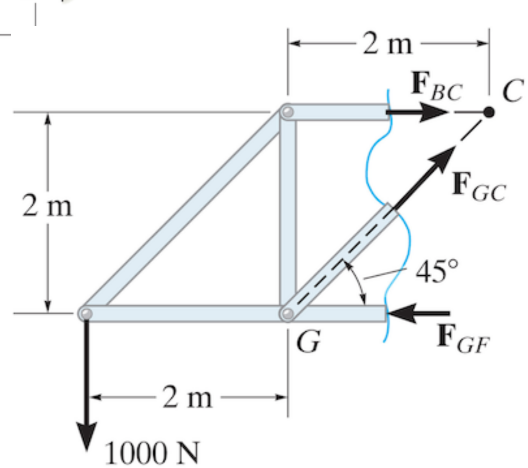
Note that, in the absence of $P_2$, member BD is a zero-force member

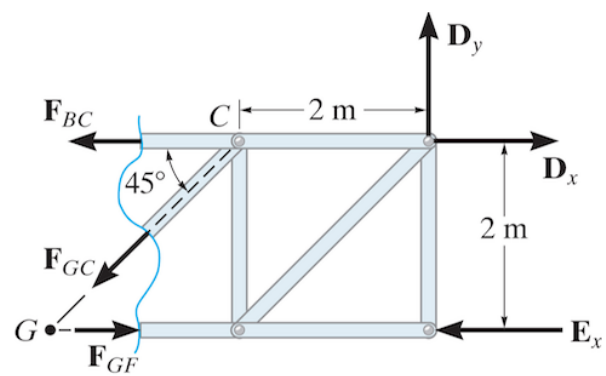
# Method of sections




- Determine external support reactions
- "Cut" the structure at a section of interest into two separate pieces and set either part into force and moment equilibrium (your cut should be such that you have **no more than** three unknowns)



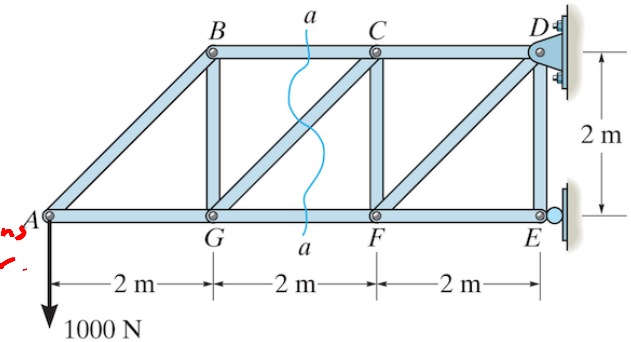



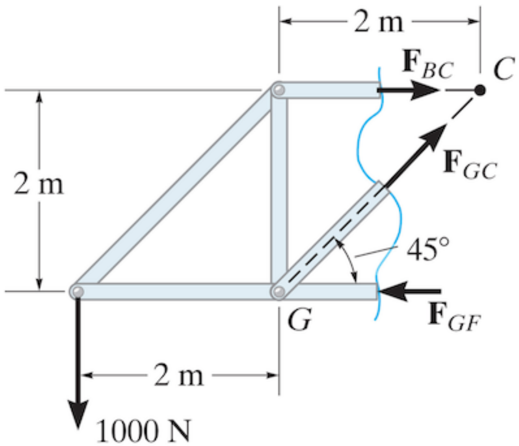




- Determine equilibrium equations (e.g., moment around point of intersection of two lines)
- Assume all internal loads are tensile.

# Method of sections

- Determine equilibrium equations (e.g., moment around point of intersection of two lines) ⇐ Reduces # unknowns in eqn. Solve faster.
- Assume all internal loads are tensile.

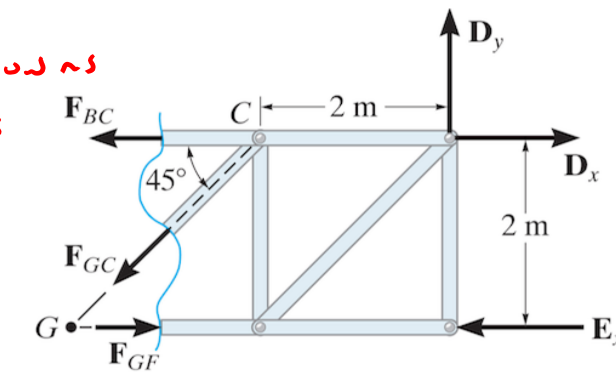



Left section : 3 unknowns

Right section : 6 unkns

⇒ solve left section first!




$+\circlearrowleft \Sigma M_C : -2m(F_{GF}) + 4m(1000N) = 0$

$\Sigma F_x$

$\Sigma F_y$ ⇒ $F_{BC}$, $F_{GC}$

$+\circlearrowleft \Sigma M_G$ :

$2m(F_{BC}) - 2m(D_x) + 2m(D_y) = 0$

$E_x$ passes thru G, so no effect on moment.

$\Sigma F_x$

$\Sigma F_y$ ⇒ $D_x, D_y, E_x$

# Frames and machines

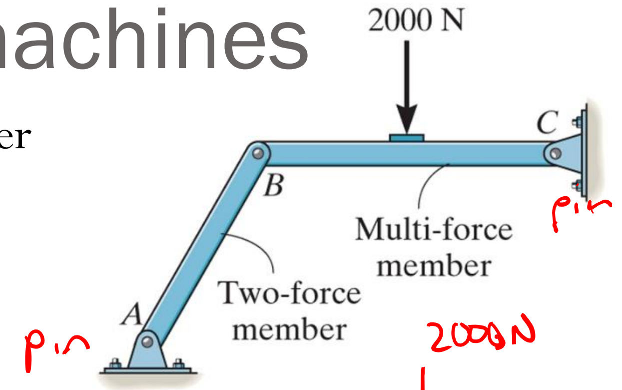
Frames and machines are two common types of structures that have at least **one multi-force member.** (Recall that trusses have <u>**only**</u> two-force members.) Therefore, it is not appropriate to use Method of Joints or Method of Sections for frames and machines.

**Frames** are generally **stationary** and used to support various external loads.

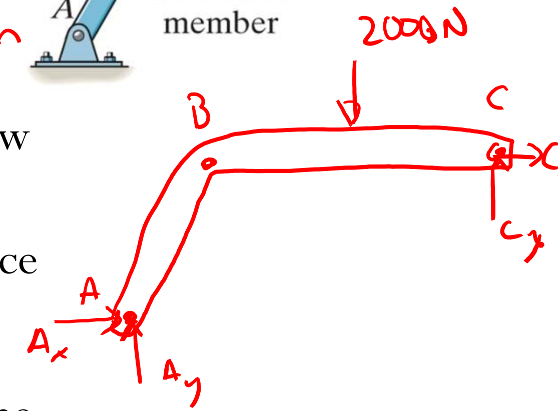

**Machines** contain **moving parts** and are designed to alter the effect of forces

# Forces/Moment in frames and machines

The members can be truss elements, beams, pulleys, cables, and other components. The general solution method is the same:

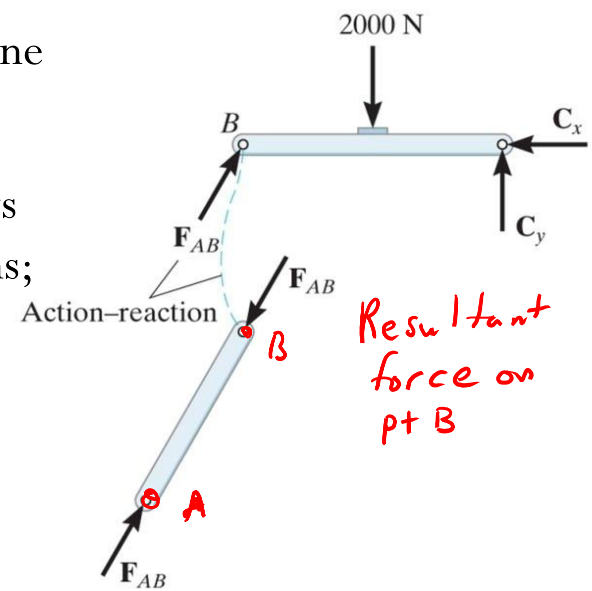



1. Identify two-force member(s) to simplify direction of unknown force(s).




2. Identify external support reactions on entire frame or machine. (Draw FDB of entire structure. Set the structure into external equilibrium: $\sum F_x = 0, \sum F_y = 0, \sum M_{most\ efficient\ pt} = 0$. This step will generally produce more unknowns than there are relevant equations of equilibrium.)

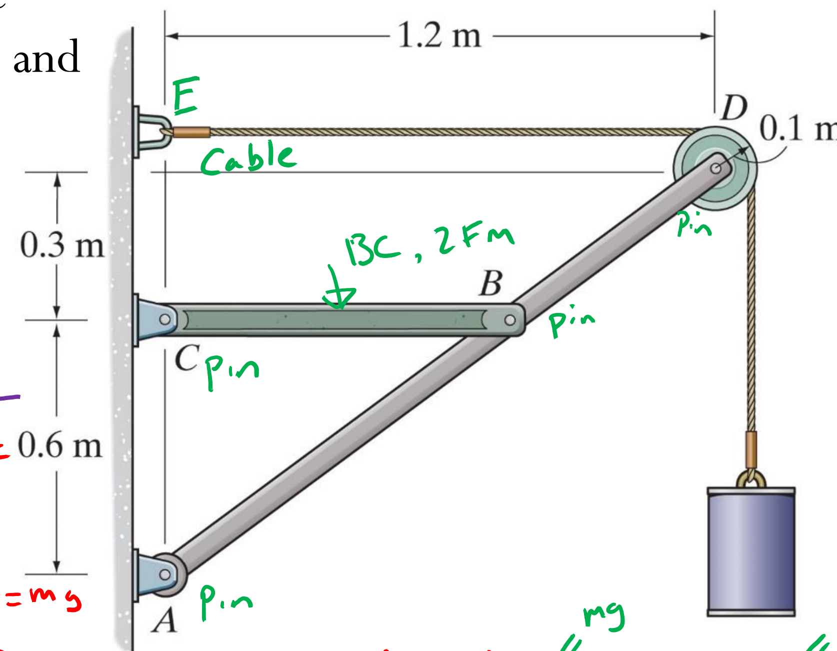
3. Draw FDBs of individual subsystems (members). (Isolate part(s) of the structure, setting each part into equilibrium $\sum F_x = 0, \sum F_y = 0$, $\sum M_{most\ efficient\ pt} = 0$. The sought forces or couples must appear in one or more free-body diagrams.)




4. Solve for the requested unknown forces or moments. (Look for ways to solve efficiently and quickly: single equations and single unknowns; equations with least # unknowns.)

Problems are going to be **challenging** since there are usually several unknowns (and several solution steps). A lot of practice is needed to develop good strategies and ease of solving these problems.

The frame supports a 50kg cylinder. Determine the horizontal and vertical components of reaction at A and the force at C



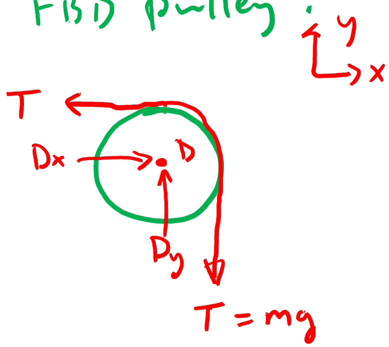

Find: $A_x$, $A_y$, $F_{BC}$

ID: 2FM ?

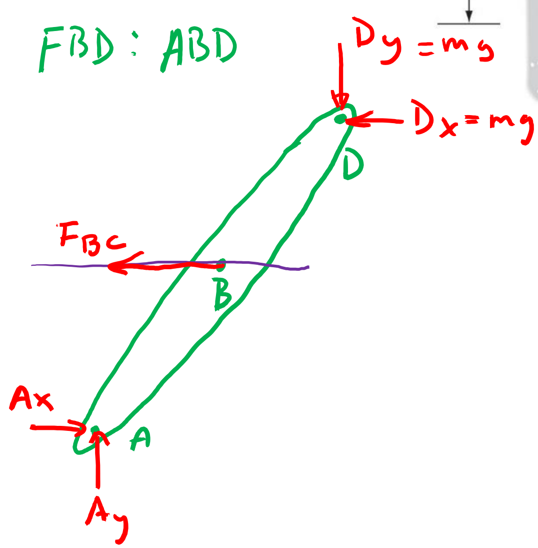
ID: supports

FBD: BC (2FM)


FBD pulley:




FBD: ABD




$\sum F_x: D_x - T = 0$

$D_x = T = mg$

$\sum F_y: D_y - T = 0$

$D_y = mg$

$+\circlearrowleft \sum M_A: (0.9m) D_x - (1.2m) D_y + (0.6m) F_{BC} = 0$ ($D_x = mg$, $D_y = mg$)

$F_{BC} = 245N$ $\quad m = 50kg$

$\sum F_x: A_x - F_{BC} - D_x = 0$

$A_x = 736N$

$\sum F_y: A_y - D_y = 0$

$A_y = 490N$

# Chapter 7: Internal Forces

# Goals and Objectives

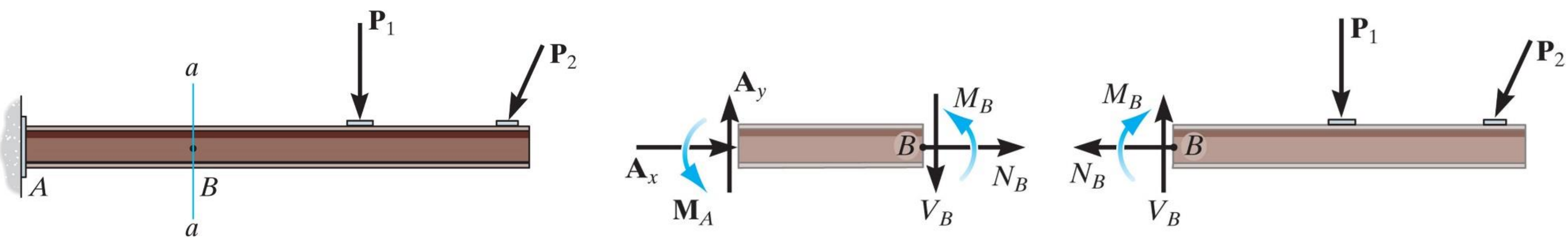
- Determine the internal loadings in members using the method of sections
- Generalize this procedure and formulate equations that describe the internal shear force and bending moment throughout a member
- Be able to construct or identify shear a force nd bending moment diagrams for beams when distributed loads, concentrated forces, and/or concentrated couple moments are applied

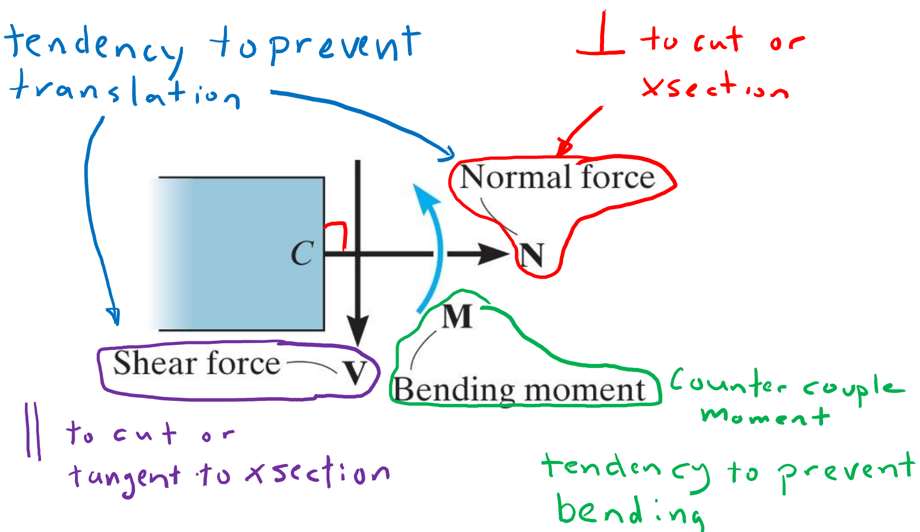
# Internal loadings developed in structural members

N, V, M ← key labels to learn

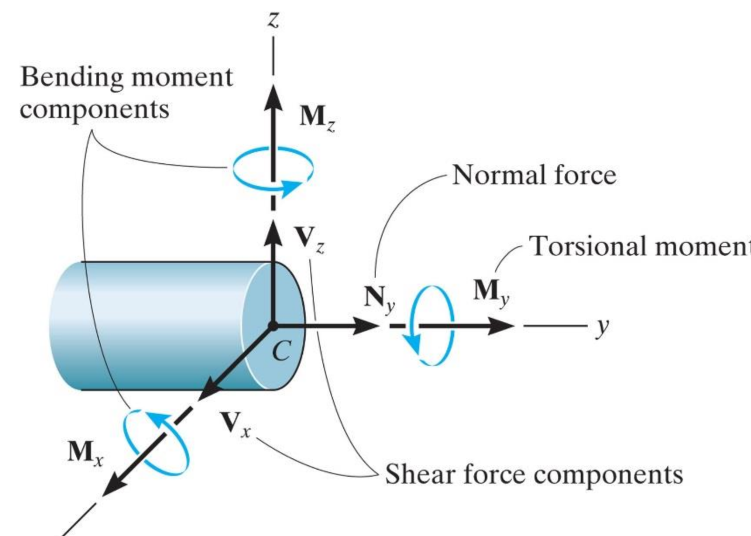
Structural Design: need to know the loading acting within the member in order to be sure the material can resist this loading

**Cutting** members at internal points reveal **internal forces and moments** (Method of Sections)



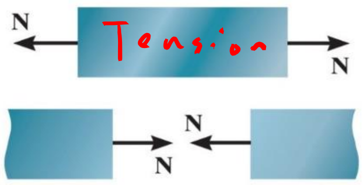

**2D**



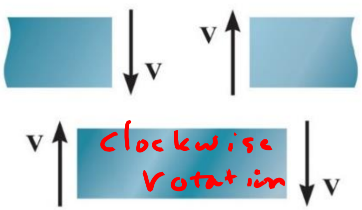

(b)

**3D**

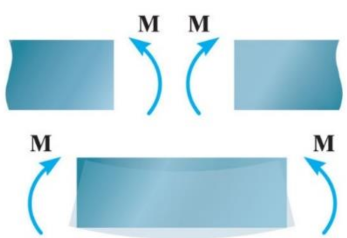
# Sign conventions:




Positive normal force

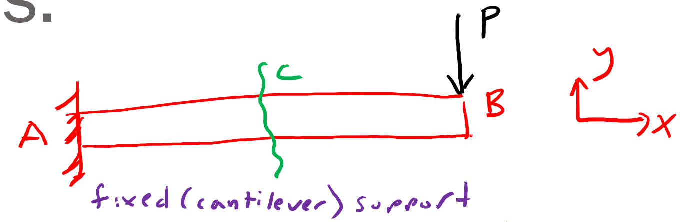



Positive shear



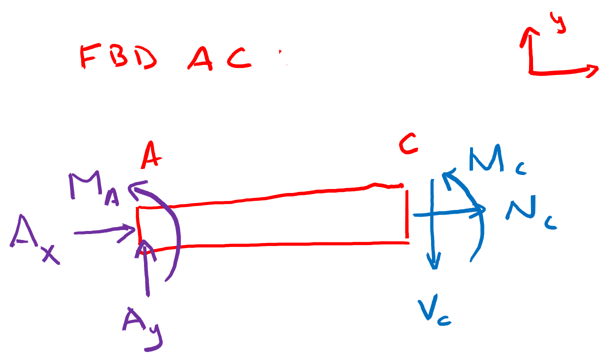

Positive moment

Concave up
"happy moment"

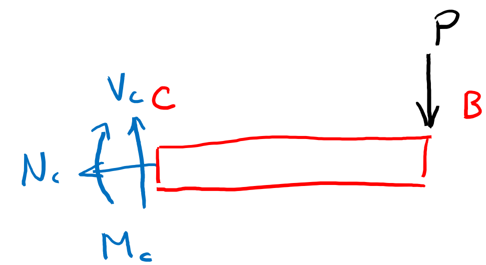



If beam AB is cut at C, draw FBDs of sections AC, CB illustrating assumptions of N, V, M drawn in positive directions.

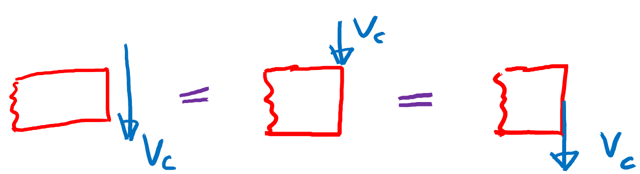
FBD AC:




FBD CB:




Note: although draw V off the side of the cut section, V is actually applied at the cut.



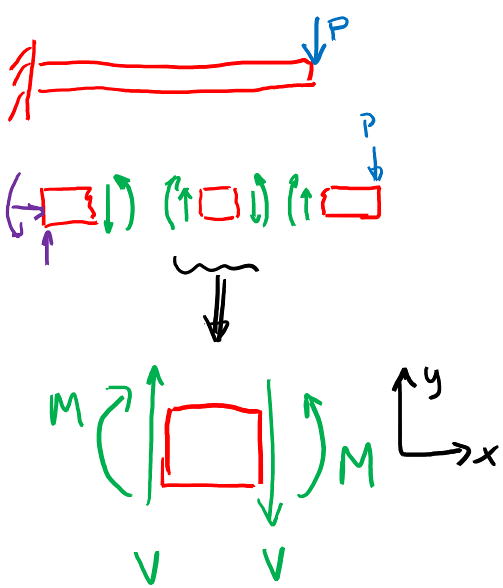
# How to orient positive V and M on a FBD?

"Positive" sign convention:

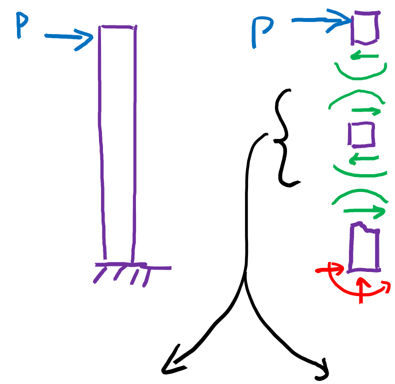
"Positive shear will create a clockwise rotation" $\Rightarrow$ Draw V arrows to create CW rotation

Therefore the direction of the arrow for the bending moment M on the same side of the segment follows the same sense as the shear force V pointing in the direction of the positive coordinate axis (the y-axes in these diagrams); thus both V and M create a clockwise rotation.

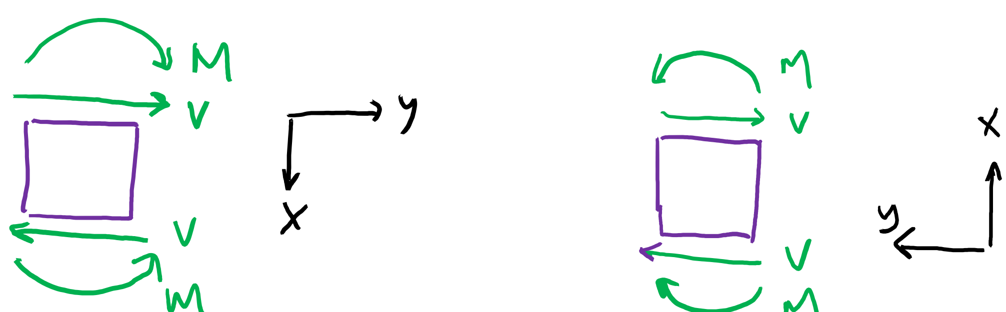
Horizontal Beam




Vertical Beam

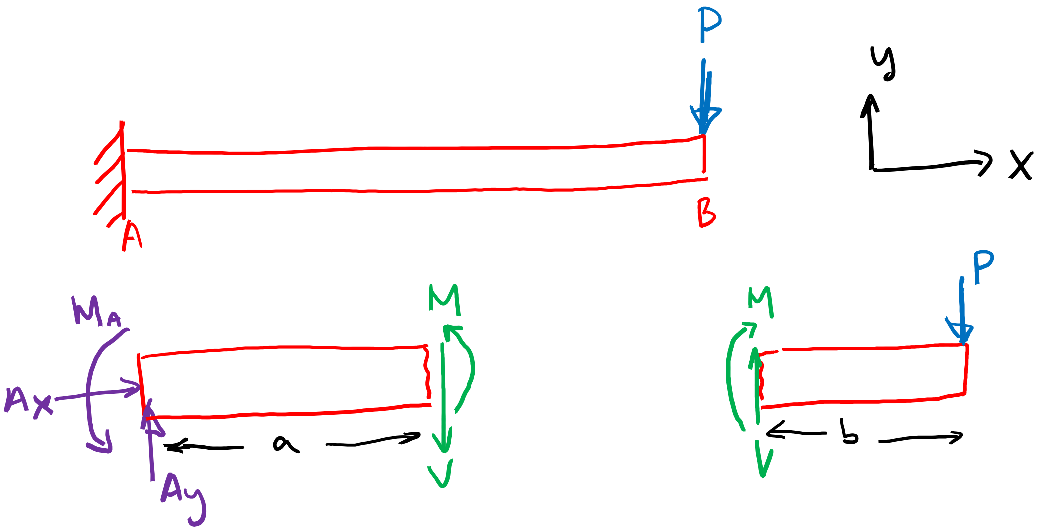



Direction of the arrow for M depends on which coordinate system orientation chosen.



# What sign to give V and M terms in equations of equilbrium?

Follow the positive orientations of the coordinate system.




For left side:

$+\uparrow \Sigma F_y: A_y - V = 0$

$+\circlearrowleft \Sigma M_A:$

$M_A + M - a \cdot V = 0$

For right side:

$+\uparrow \Sigma F_y: V - P = 0$

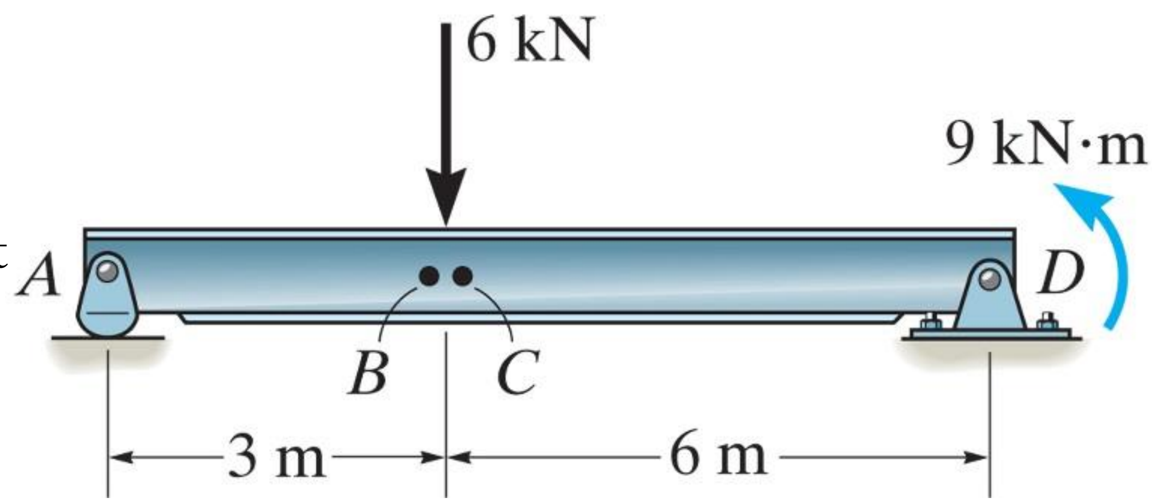
$+\circlearrowleft \Sigma M_B:$

$-M - b \cdot V = 0$

# Procedure for analysis:

1. Find support reactions (free-body diagram of entire structure)
2. Pass an imaginary section through the member
3. Draw a free-body diagram of the segment that has the least number of loads on it
4. Apply the equations of equilibrium

Find the internal forces and moments at B (just to the left of load P) and at C (just to the right of load P)



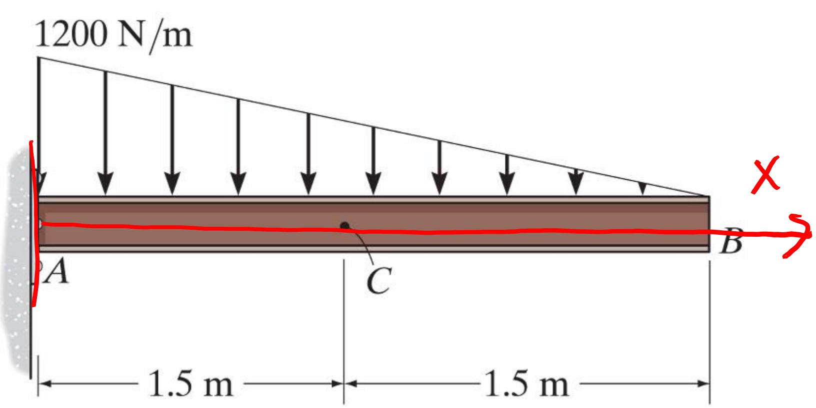
# Shear Force and Bending Moment Diagrams

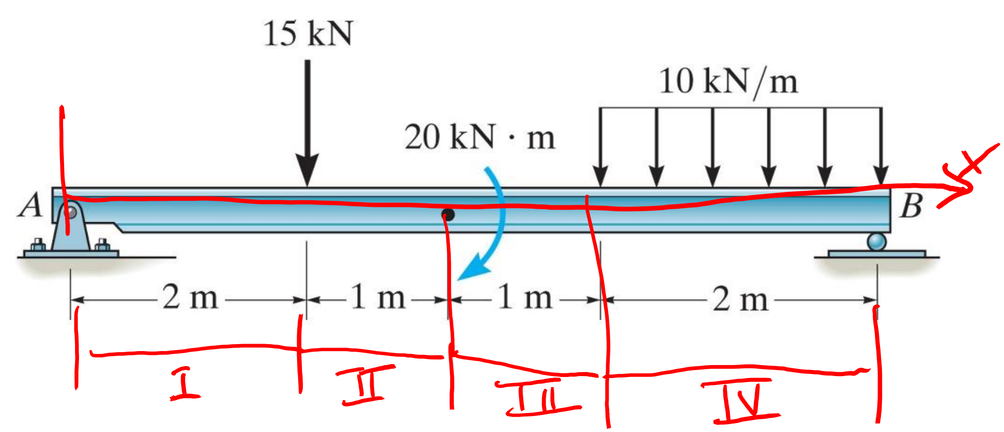
Goal: provide detailed knowledge of the variations of internal shear force and bending moments (V and M) throughout a beam when perpendicular distributed loads, concentrated forces, and/or concentrated couple moments are applied.

Normal forces (N) in such beams are zero, so we will not consider normal force diagrams.

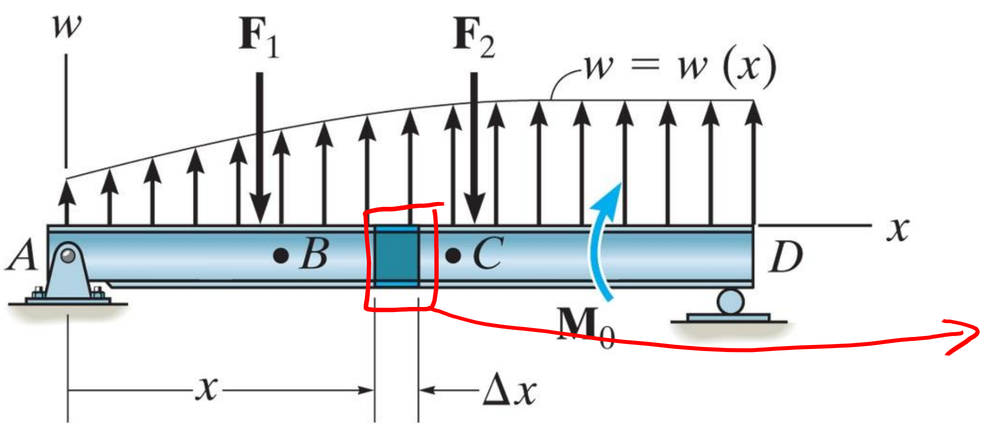
Procedure

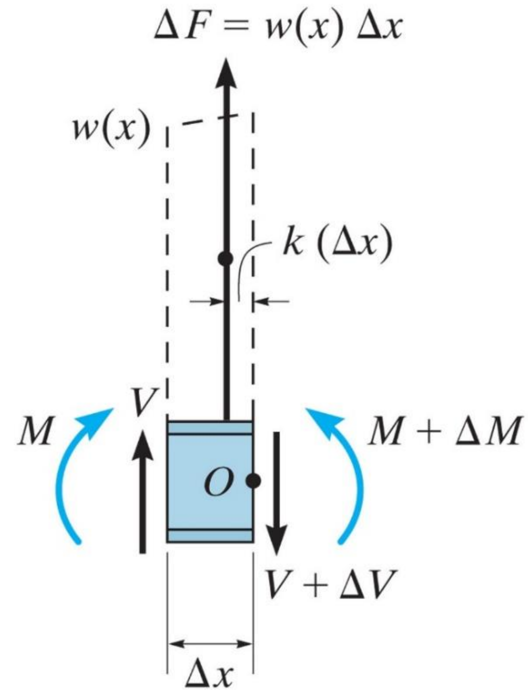
1. Find support reactions (free-body diagram of entire structure)
2. Specify coordinate *x* (start from left)
3. Divide the beam into sections according to loadings
4. Draw FBD of a section
5. Apply equations of equilibrium to derive V and M as functions of *x*: V(*x*), M(*x*)



# Relations Among Distributed Load, Shear Force and Bending Moments




Relationship between distributed load and shear:

$$\frac{dV}{dx} = w$$

Slope of shear force = distributed load intensity

$$\Delta V = V_2 - V_1 = \int w\, dx$$

Change in shear force = area under loading curve

Relationship between shear and bending moment:

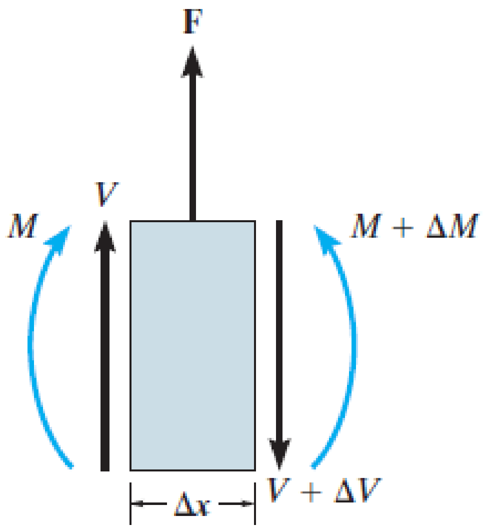
$$\frac{dM}{dx} = V$$

Slope of bending moment = shear force

$$\Delta M = M_2 - M_1 = \int V\, dx$$

Change in moment= area under shear curve

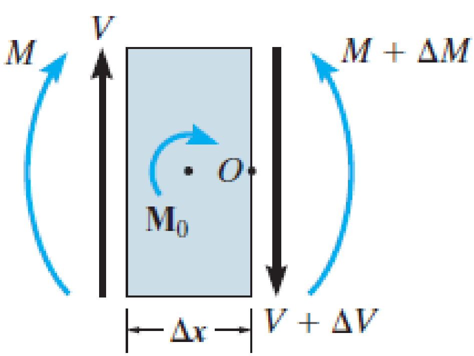
Wherever there is an external concentrated force or a concentrated moment, there will be a change (jump) in shear or moment, respectively.




$\Sigma F_y$:

$V + F - (V + \Delta V) = 0$

$\Delta V = F$ Jump in shear force due to concentrated load *F*




+↺ $\Sigma M_O$:

$(M + \Delta M) - M - M_O - V(\Delta x) = 0$

$\Delta M = M_O + V(\Delta x)$

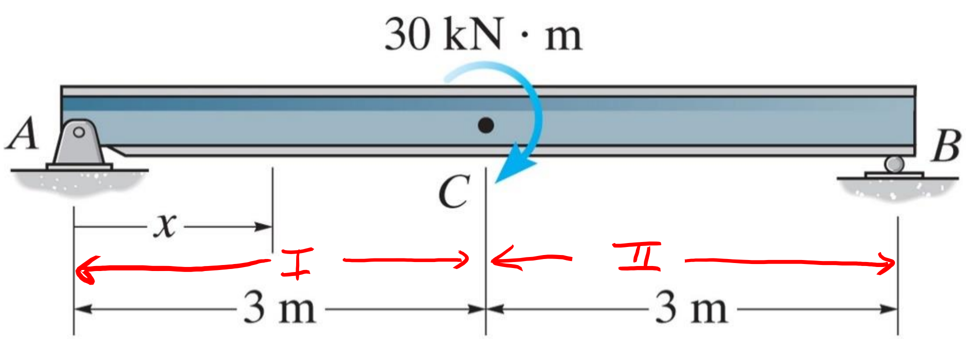
$\Delta M = M_O$, when $\Delta x \rightarrow 0$

Jump in bending moment due to concentrated couple moment $M_O$

* Note: the text, these notes, and convention assume that an applied concentrated moment **$M_O$ in clockwise direction**

* results in a **positive change in $M(x)$**

Draw the shear force and moment diagrams for the beam.



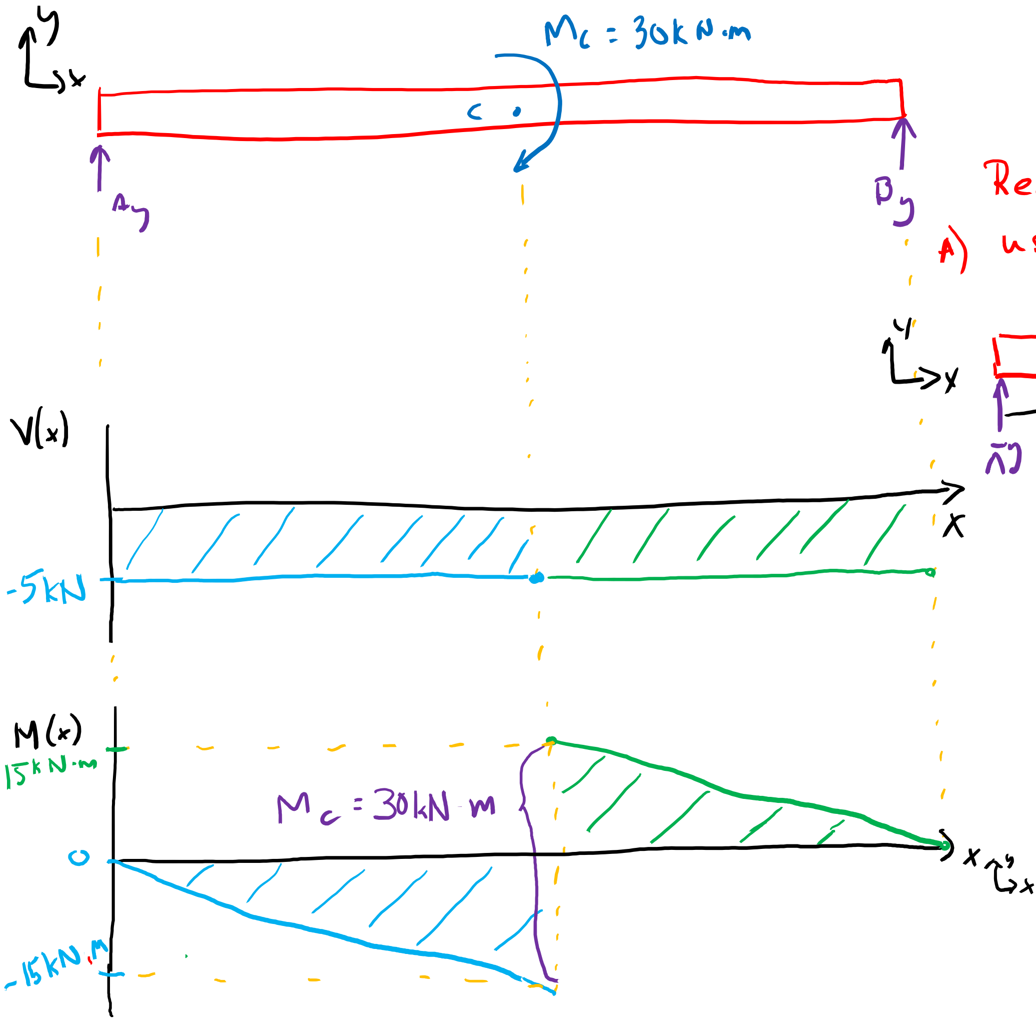

(1) Reaction supports:

$\Sigma F_y$: $A_y = -B_y$

$+\circlearrowleft \Sigma M_A$: $-30\text{ kN m} + (6\text{m})B_y = 0$

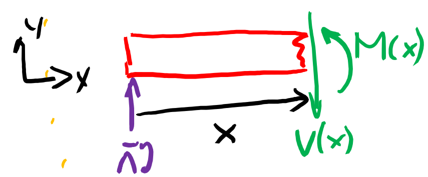
$B_y = 5\text{ kN.m}$

$A_y = -5\text{ kN.m}$




Region I: $0 < x < 3$

A) using FBD & EoE to create V(x) & M(x):




$\Sigma F_y$: $V(x) = A_y = -5\text{kN}$

constant, negative

$+\circlearrowleft \Sigma M_A$: $M(x) - x \cdot V(x) = 0$

$M(x) = x\ V(x)$

linear w/ slope V(x)

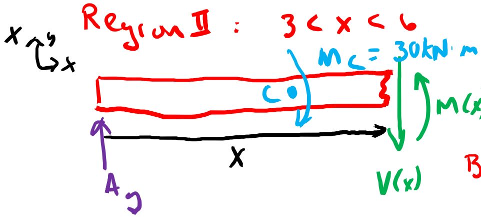
slope $\frac{dM}{dx} = V(x) = -5\text{kN}$

use BC's to find end points for M(x)

$x = 0$: $M(0) = 0$

$x = 3\text{m}$: $M(3m^{(-)}) = -15\text{ kN m}$

Region II: $3 < x < 6$




$\Sigma F_y$: $V(x) = A_y = -5\text{kN}$

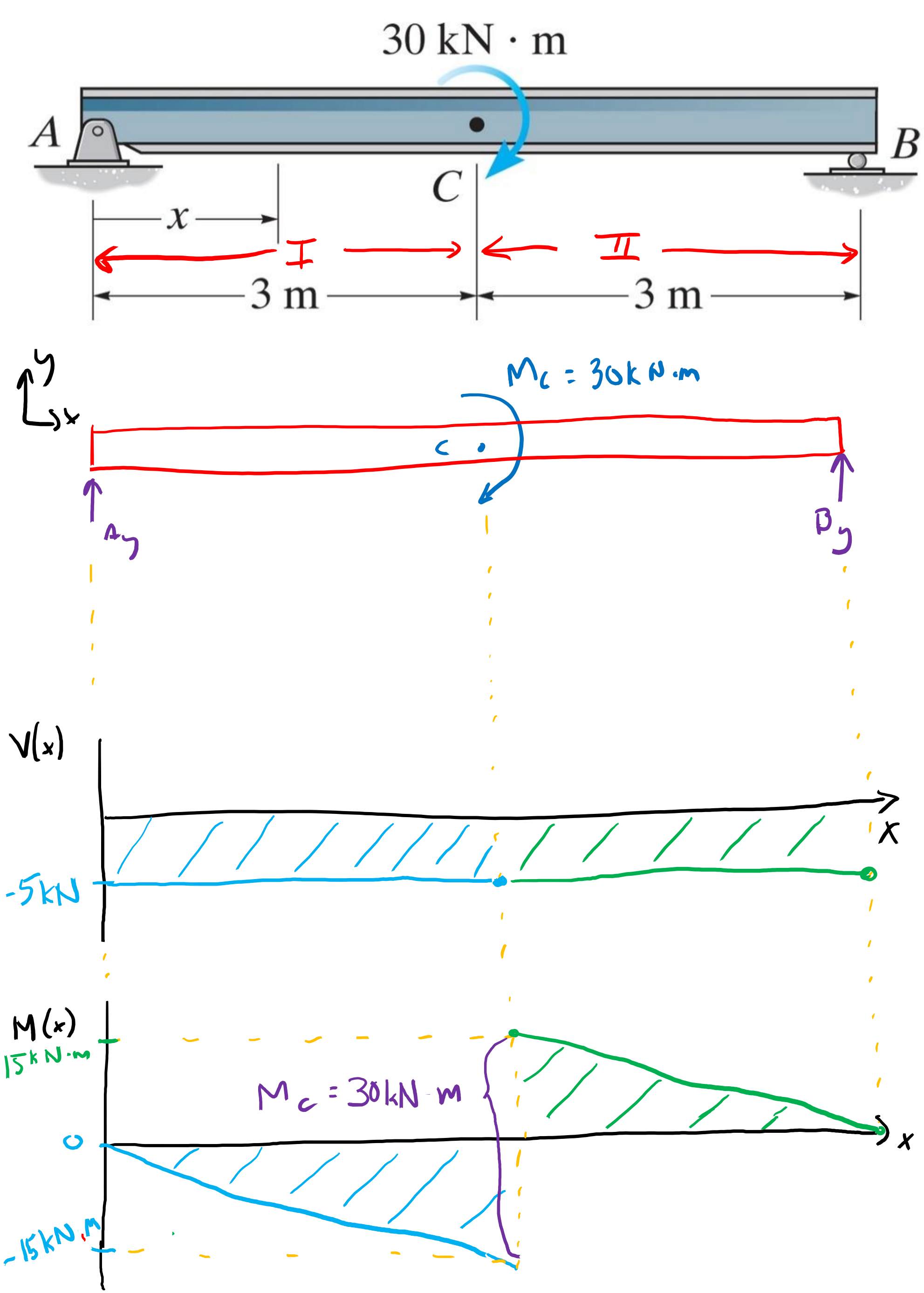
$+\circlearrowleft \Sigma M_A$: $M(x) - x \cdot V(x) - M_c = 0$

$M(x) = x \cdot V(x) + M_c$

BC's: $M(3m^{(+)}) = -15 + 30 = 15\text{kN m}$

$M(6m) = -30 + 30 = 0$

Draw the shear force and moment diagrams for the beam.




B) Alternative method to quickly draw V & M diagrams

use $\frac{dV}{dx} = w(x)$ to define slope of $V(x)$

$\Delta V = V_2 - V_1 = \int w(x)\,dx$ change in shear = area under loading curve

$\frac{dM}{dx} = V(x)$ to define slope of $M(x)$

$\Delta M = M_2 - M_1 = \int V(x)\,dx$ change in moment = area under shear curve

For concentrated moment:

$w(x) = 0$

$\rightarrow \frac{dV}{dx} = 0$ (no slope)

$\Delta V = \int w\,dx = 0$ (no change in V)

$\frac{dM}{dx} = V(x) =$ neg. const $= A_y = -5\text{kN}$

$\Delta M = M_c$ (from knowledge of applied moment $M_c$)

Since $M_c$ ↻ cw, then $\Delta M$ is in positive direction ↑

(see notes 2 slides prior)

# Chapter 8: Friction

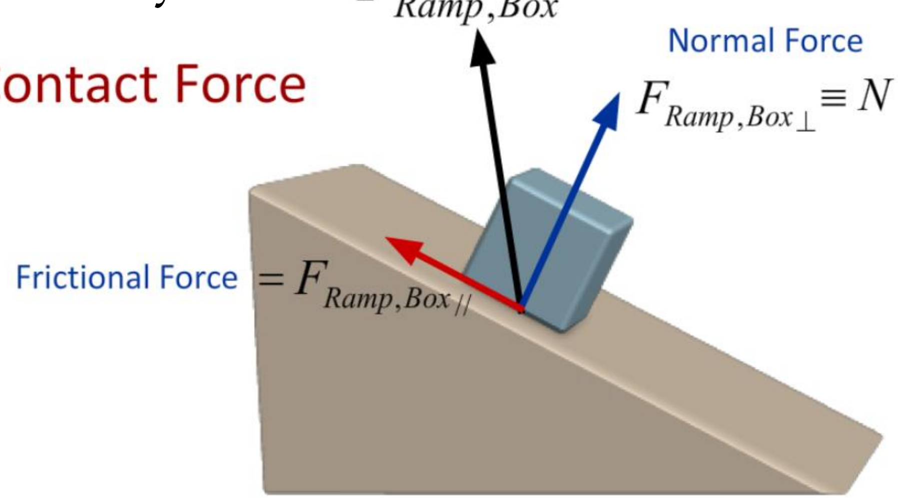
# Goals and Objectives

- Sections 8.1-8.2
- Introduce the concept of dry friction
- Analyze the equilibrium of rigid bodies subjected to this force

# Dry friction (or Coulomb friction)

- Friction acts tangent to contacting surfaces and in a direction opposed to motion of one surface relative to another
- Friction force $F$ is related to the coefficient of friction and normal force $N$
  - Static friction (no motion): $F_s \leq \mu_s N$
  - Kinetic friction (moving): $F_k = \mu_k N$
- Magnitude of coefficient of friction depends on the two contacting materials
- Maximum static frictional force occurs when motion is impending
- Kinetic friction is the tangent force between two bodies after motion begins. Less than static friction by ~25%.

## Components of a Contact Force

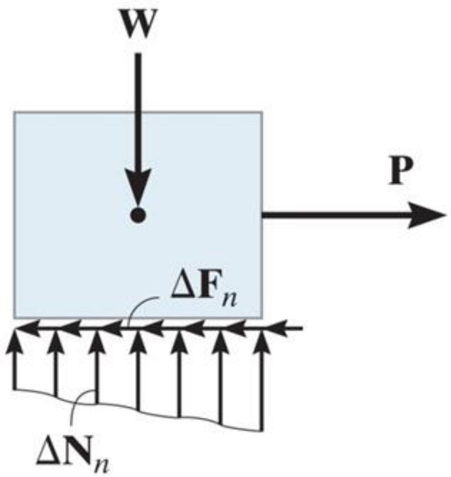



| Contact Materials | Coefficient of Static Friction ($\mu_s$) |
|---|---|
| Metal on ice | 0.03–0.05 |
| Wood on wood | 0.30–0.70 |
| Leather on wood | 0.20–0.50 |
| Leather on metal | 0.30–0.60 |
| Aluminum on aluminum | 1.10–1.70 |

Table 8–1 Typical Values for $\mu_s$

Physics 211: Flipitphysics.com

# Dry Friction Problems




- 3 types of static problems with dry friction

1. No apparent impending motion
2. Impending motion at all points of contact
3. Impending motion at some points of contact

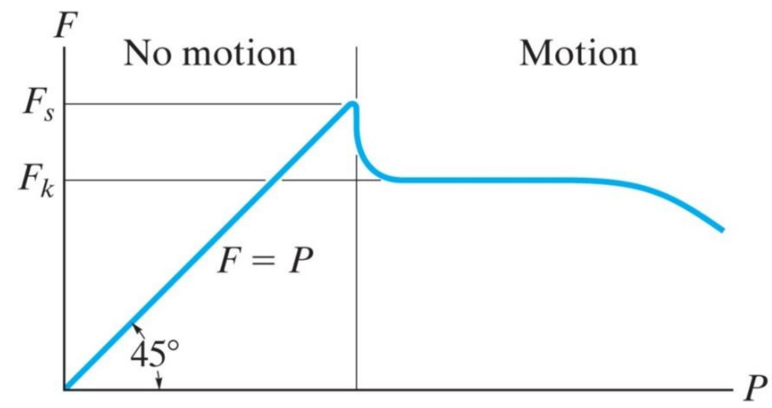
Note that all of these cases are for IMPENDING motion (since static case). Therefore, in tipping problems, the entire bottom surface is still in contact with ground.

# Slipping and Tipping

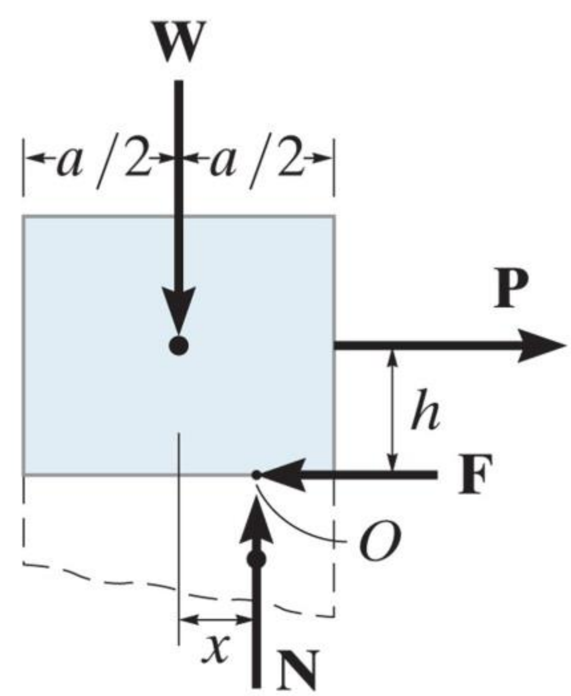
- **Impending slipping motion**: the maximum force $F_s$ before slipping begins is given by

$$F_s = \mu_s N$$

Slipping starts when $P$ just exceeds $\mu_s N$



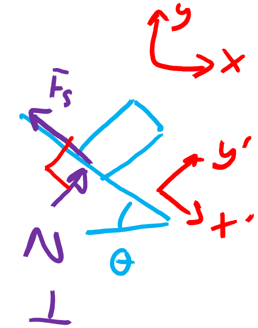

- **Tipping condition**: to avoid tipping of the block, the following equilibrium should be satisfied:




$$\sum M_O = -Ph + Wx = 0 \rightarrow x = \frac{P\,h}{W}$$

Compute value for $x$ based on the applied loads:
If $x > a/2$, then these loads would cause tipping.
Otherwise $x < a/2$, will only slip

# Dry Friction Problems




- Procedure

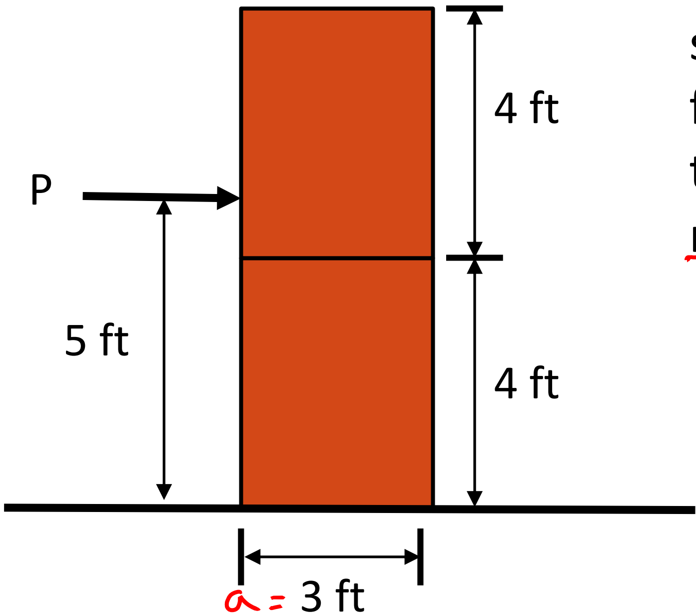
A. Draw FBD for each body
   - Friction force points opposite direction of impending motion

B. Determine # unknowns

C. Apply eqns of equilibrium and necessary frictional eqns (or conditional eqns if tipping is possible)

$\Sigma F_x \quad \Sigma F_y$ → Slip, $F_s = \mu_s N$

$\Sigma M_o$ → Tip




Two uniform boxes, each with weight 200 lb, are simply stacked as shown. If the coefficient of static friction between the boxes is $\mu_s = 0.8$ and between the box and the floor is $\mu_s = 0.5$, determine the minimum force $P$ to cause motion.

*How many possible motions?*

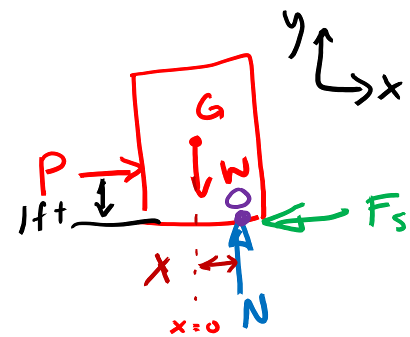
I) 1 slips

II) 1 tips

III) 1 + 2 slip

IV) 1 + 2 tip

FBD for box 1 :




Case I : Assume 1 slip

$+\uparrow \Sigma F_y : \; N - W = 0 \;, \; N = W$

$+\rightarrow \Sigma F_x : \; P - F_s = 0$

assume slipping $F_s = \mu_s N$

$P = \mu_s N = \mu_s W$

$P = (0.8)(200\,lb)$

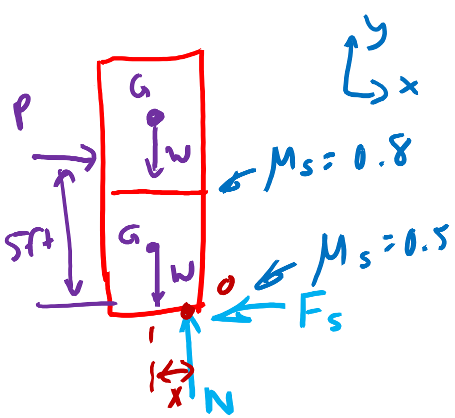
$P_I = 160\,lb$

Case II : Assume 1 tips $\Rightarrow \therefore x = \frac{a}{2} = 1.5\,ft$

$+\circlearrowleft \Sigma M_o : \; W(1.5\,ft) - P(1\,ft) = 0$

$P = 1.5\,W = 1.5(200\,lb)$

$P_{II} = 300\,lb$




Case III : Assume 1+2 combo slips

$+\uparrow \sum F_y : N - 2W = 0 \qquad N = 2W$

$\xrightarrow{+} F_x : P - F_s = 0 \quad , F_s = \mu_s N$

$P = \mu_s (2W) = (0.5)(2)(200\,\text{lb})$

$P_{III} = 200\,\text{lb}$

Case IV : Assume 1+2 combo tip , $x = \frac{a}{2}$

$+\circlearrowleft \sum M_o : (2W)(1.5\,\text{ft}) - P(5\,\text{ft}) = 0$

$P = \frac{3}{5} W$

$P_{IV} = 120\,\text{lb}$

Case IV will happen first since $P_{IV}$ is minimum.