Statics - TAM 210 & TAM 211

Lecture 29

April 2, 2018

Chap 9.1

Announcements

- **No class Wednesday April 4 or Wednesday April 11**
- Concept Inventory: Ungraded assessment of course knowledge
  - Extra credit: Complete #1 or #2 for 0.5 out of 100 pt of final grade each, or both for 1.5 out of 100 pt of final grade
  - #2: Sign up at CBTF (4/2-4 M-Th)
  - 50 min appointment, should take < 30 min

- Upcoming deadlines:
- Discussion section for (4/3-4/4)
  - Attendance taken for TAM 211, no worksheet
  - TA/CA answer questions about course material
  - TAM 210 students may attend
- Written exam
  - Comprehensive from Lecture 1 through Lecture 27 (Chapters 1 -8)
  - Thursday 4/5, 7-9pm
  - TAM 210 students: 100 Material Science & Engineering Building (MSEB)
  - TAM 211 students: 100 Noyes Lab
  - Bring i-Card. No calculators
  - Closed book, closed notes

Chapter 9: Center of Gravity and Centroid

Goals and Objectives

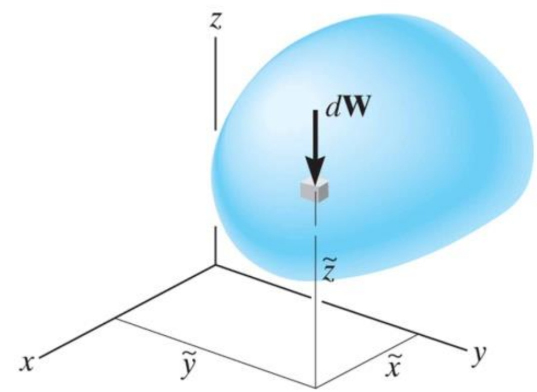
- Understand the concepts of center of gravity, center of mass, and centroid.
- Determine the location of the center of gravity and centroid for a system of discrete particles and a body of arbitrary shape.

Center of gravity

To design the structure for supporting a water tank, we will need to know the weight of the tank and water as well as the locations where the resultant forces representing these distributed loads act.

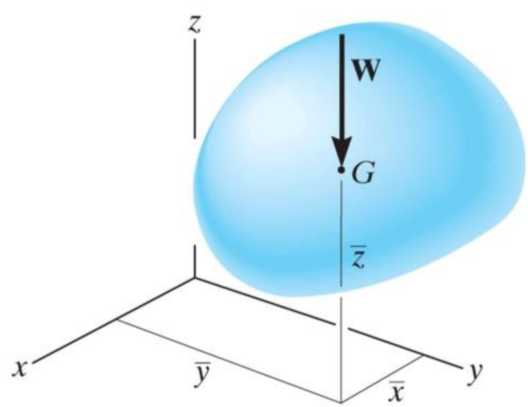
How can we determine these resultant weights and their lines of action?

Center of gravity



A body is composed of an infinite number of particles, and so if the body is located within a gravitational field, then each of these particles will have a weight dW.

The **center of gravity (CG)** is a point, often shown as G, which locates the resultant weight of a system of particles or a solid body.



From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G.

If dW is located at point $(\tilde{x}, \tilde{y}, \tilde{z})$ then

$$\bar{x}\,W = \int \tilde{x}\,\mathrm{dW}$$
$$\bar{y}\,W = \int \tilde{y}\,\mathrm{dW}$$
$$\bar{z}\,W = \int \tilde{z}\,\mathrm{dW}$$

$$\bar{x} = \frac{\int \tilde{x}\,dW}{\int dW}$$
$$\bar{y} = \frac{\int \tilde{y}\,dW}{\int dW}$$
$$\bar{z} = \frac{\int \tilde{z}\,dW}{\int dW}$$

Center of Mass

Given: $dW = g\,dm$

Provided that $g =$ constant:

$$\bar{x} = \frac{\int \tilde{x}\,dm}{\int dm}$$

$$\bar{y} = \frac{\int \tilde{y}\,dm}{\int dm}$$

$$\bar{z} = \frac{\int \tilde{z}\,dm}{\int dm}$$

Center of Volume

For homogeneous material,

$\rho =$ constant.

Therefore, $dm = \rho\,dV$

$$\bar{x} = \frac{\int \tilde{x}\,dV}{\int dV}$$

$$\bar{y} = \frac{\int \tilde{y}\,dV}{\int dV}$$

$$\bar{z} = \frac{\int \tilde{z}\,dV}{\int dV}$$

Center of Area

$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA}$$

$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA}$$

$$\bar{z} = \frac{\int \tilde{z}\,dA}{\int dA}$$

If use rectangular strip,

simplify to $dA = y\,dx$ and $\tilde{x} = x,\ \tilde{y} = y/2$.

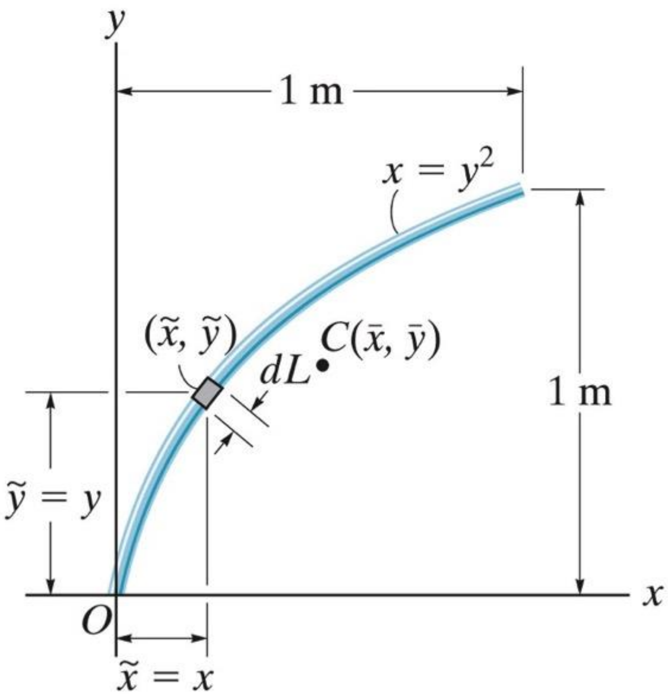
and $dA = x\,dy$ and $\tilde{x} = x/2,\ \tilde{y} = y$.

Center of Line

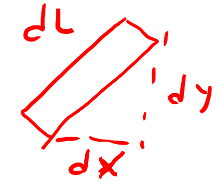
$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL}$$

$$\bar{y} = \frac{\int \tilde{y}\, dL}{\int dL}$$

$$\bar{z} = \frac{\int \tilde{z}\, dL}{\int dL}$$



$y = f(x)$ or $x = f(y)$



Use Pythagorean Theorem:

$$dL = \sqrt{(dx)^2 + (dy)^2}$$

$$dL = \sqrt{\left(\frac{dx}{dx}\right)^2 (dx)^2 + \left(\frac{dy}{dx}\right)^2 (dx)^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right) dx$$

Or

$$dL = \sqrt{\left(\frac{dx}{dy}\right)^2 (dy)^2 + \left(\frac{dy}{dy}\right)^2 (dy)^2} = \left(\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\right) dy$$

Centroid

The centroid, C, is a point defining the geometric center of an object.

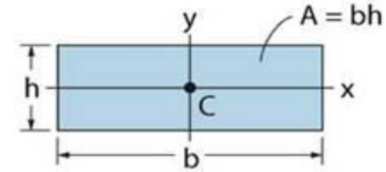
The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogeneous (density or specific weight is constant throughout the body).

If an object has an axis of symmetry, then the centroid of object lies on that axis.

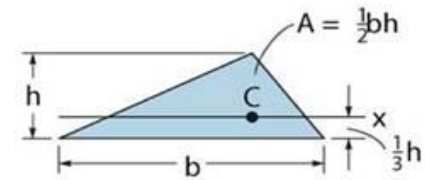
$\bar{z}' = \frac{h}{2}$

$(0, 0, \frac{h}{2})$

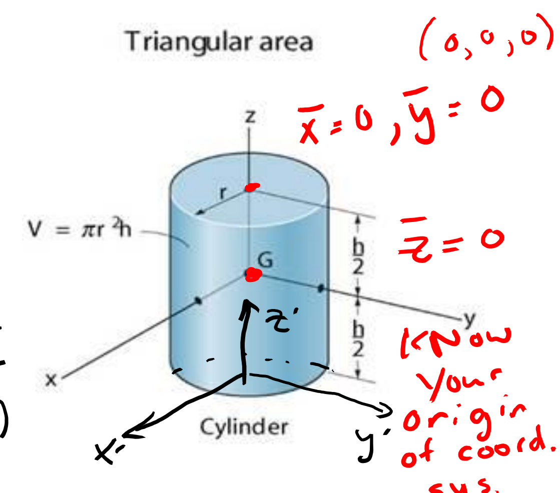
In some cases, the centroid may not be located on the object.



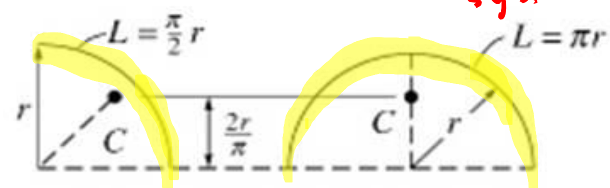
Rectangular area



Triangular area

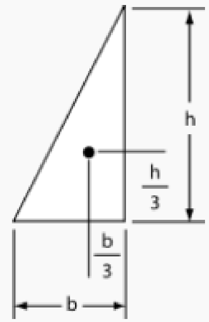
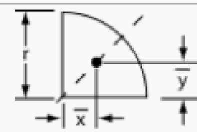
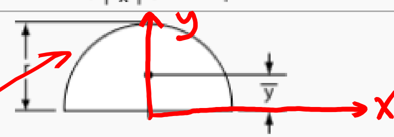
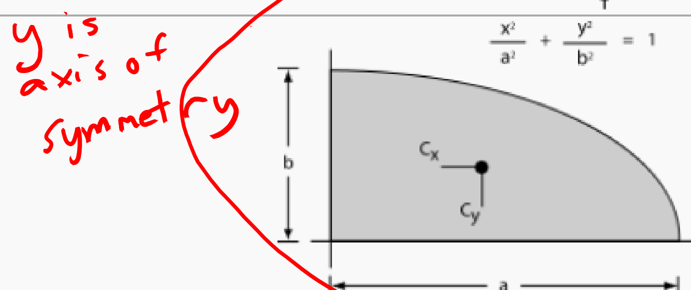
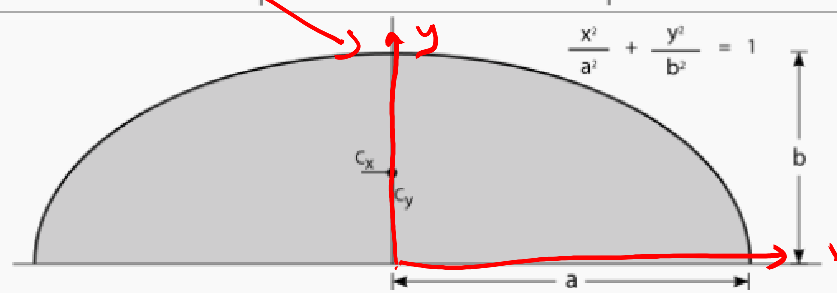


Cylinder



Quarter and semicircle arcs

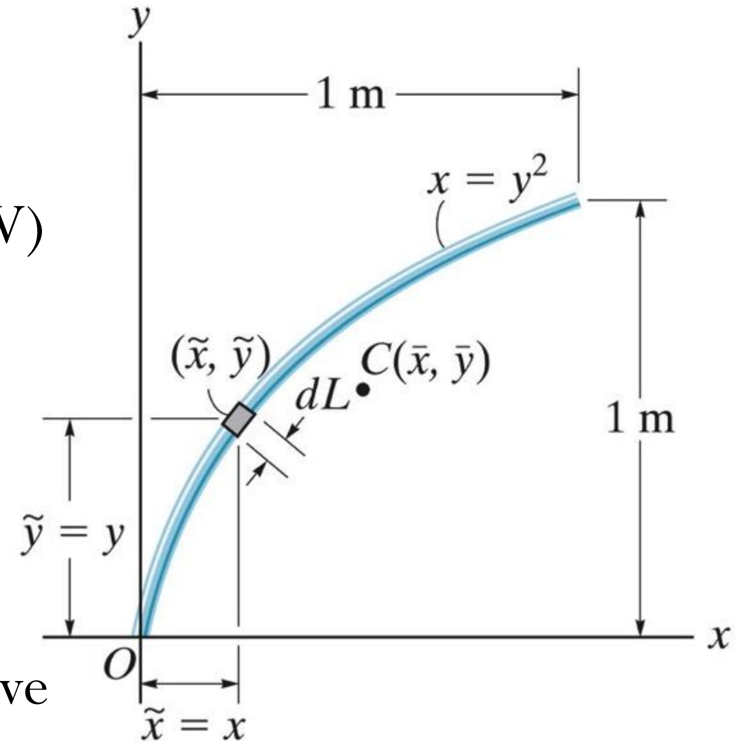
Centroid of typical 2D shapes

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Right-triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

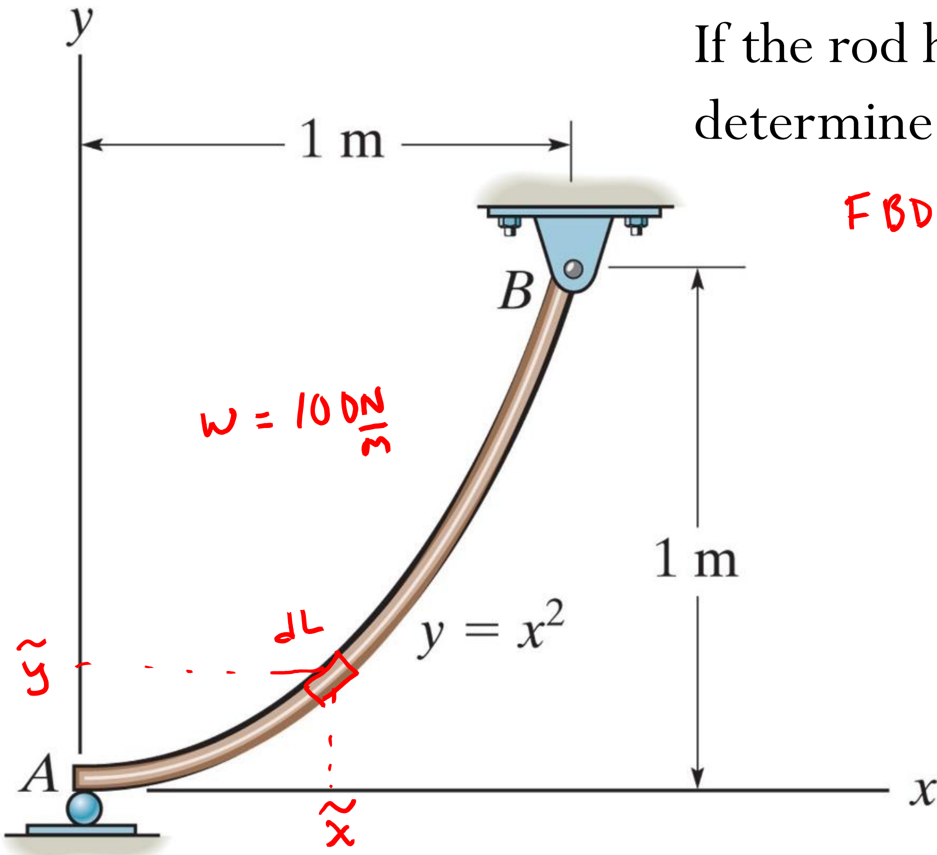
http://en.wikipedia.org/wiki/List_of_centroids

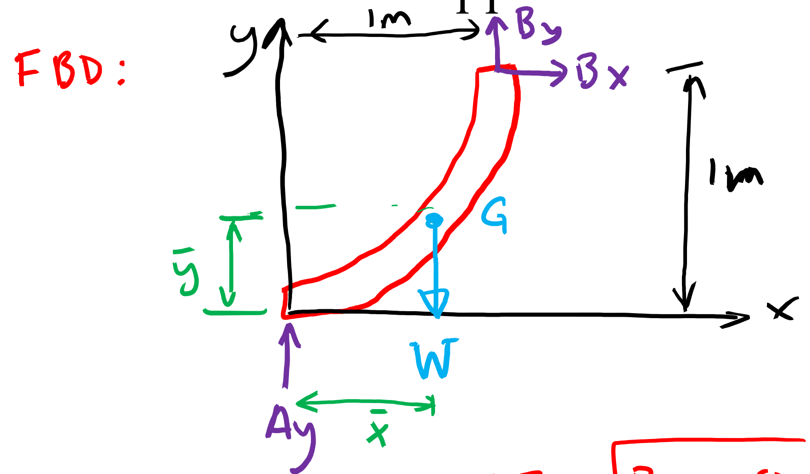
Centroid – Analysis Procedure

1. Select an appropriate coordinate system
2. Define the appropriate element (dL, dA, or dV)
3. Express (2) in terms of the coordinate system
4. Identify any symmetry
5. Express the moment arms (centroid) of (2)
6. Substitute (3) and (4) into the integral and solve



If the rod has a weight per unit length of 100 N/m, determine the reaction supports at A and B.





E.E.:

$\xrightarrow{+} \Sigma F_x: \boxed{B_x = 0}$

$+\uparrow \Sigma F_y: A_y + B_y - W = 0$

$\curvearrowleft + \Sigma M_B: -(1\text{m})A_y + (1-\bar{x})W = 0$

5 unknowns: $A_y, B_y, B_x, W, \bar{x}$ ✓

Solve for $\bar{x}$

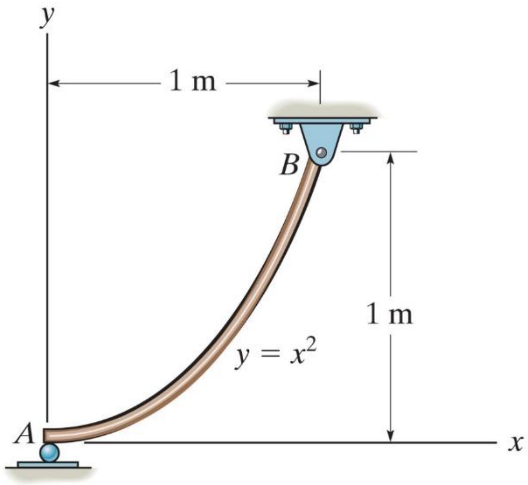
$y = x^2 \rightarrow dy = 2x\,dx$

$\tilde{x} = x$

$\tilde{y} = y$

$dL = \sqrt{(dx)^2 + (dy)^2}$

$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \sqrt{1 + (2x)^2}\,dx = \sqrt{4x^2 + 1}\,dx$



$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL} = \frac{\int_0^1 x\left(\sqrt{4x^2+1}\right) dx}{\int_0^1 \sqrt{4x^2+1}\, dx}$$

} everything all in one term (x)

$$= \frac{2\int_0^1 x\sqrt{x^2+\frac{1}{4}}\, dx}{2\int_0^1 \sqrt{x^2+\frac{1}{4}}\, dx}$$

$$= \frac{0.8484\ m^2}{1.478\ m} \Rightarrow \boxed{\bar{x} = 0.574\,m}$$

Solve for W:

$$W = w\int_0^1 dL$$

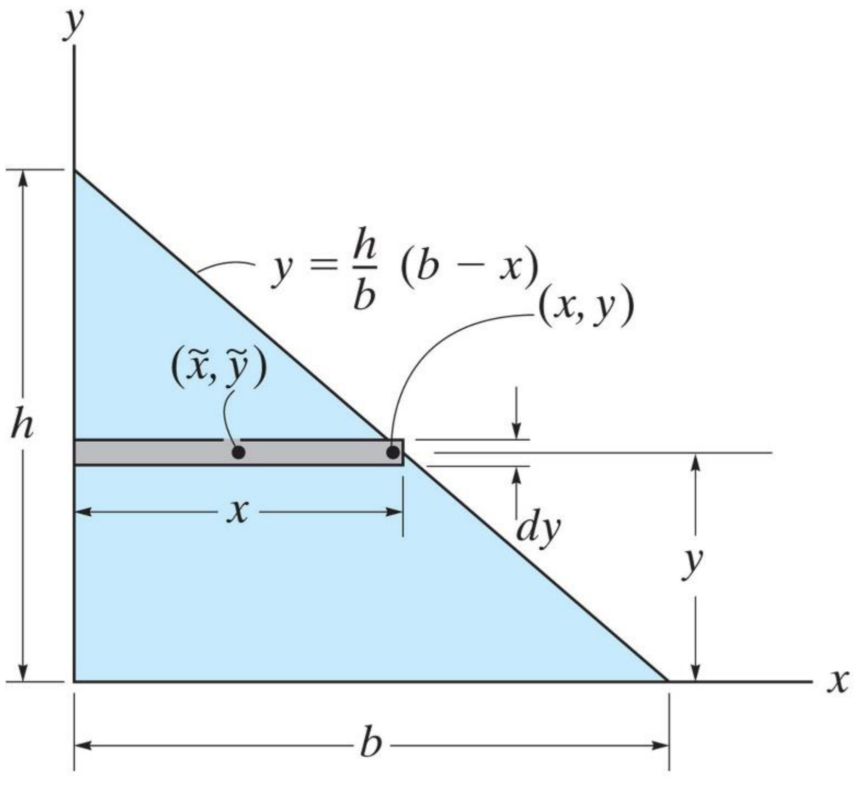
$$W = 100\frac{N}{m}\int_0^1 \sqrt{4x^2+1}\, dx$$

$$= 100\frac{N}{m}(1.478\,m)$$

$$\boxed{W = 147.8\,N}$$

Solve for A_y & B_y:

Will get: $\boxed{A_y = 63.1\,N}$ $\boxed{B_y = 84.8\,N}$



Determine the distance y measured from the x axis to the centroid of the area of the triangle.

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA} \qquad \bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$$(\tilde{x}, \tilde{y}) = \left(\frac{x}{2}, y\right)$$

$$y = \frac{h}{b}(b-x) \quad \text{or} \quad x = f(y)$$

$$x = \frac{b}{h}(h-y)$$

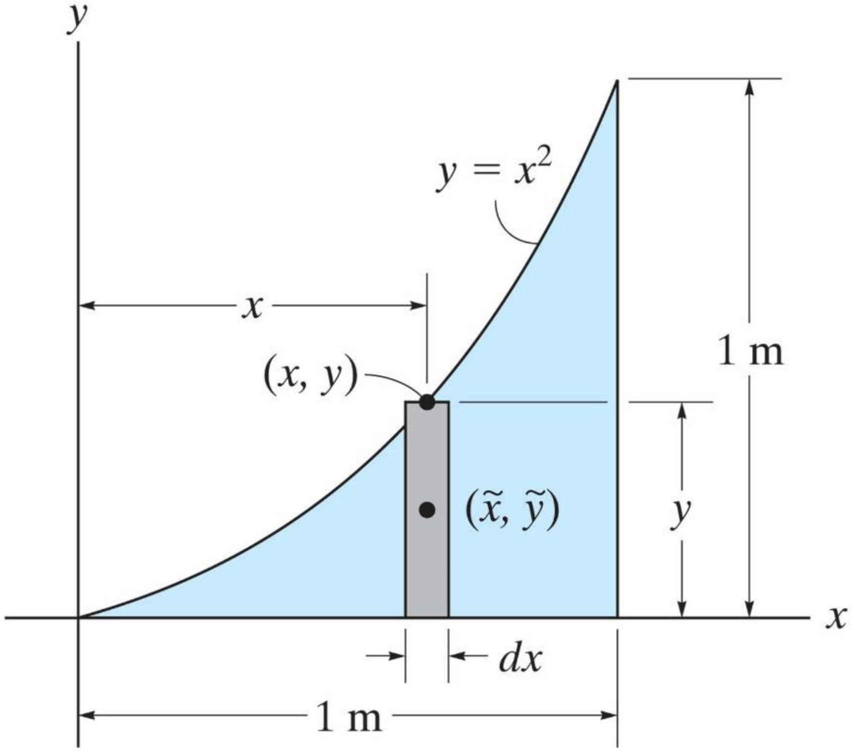
$$\bar{y} = \frac{\int y\, dA}{\int dA} \qquad dA = x\, dy$$

$$\therefore \bar{y} = \frac{\int_0^h y\left(\frac{b}{h}(h-y)\right) dy}{\int_0^h \frac{b}{h}(h-y)\, dy}$$

$$= \frac{\frac{b}{h}\int_0^h (yh - y^2)\, dy}{\frac{b}{h}\int_0^h (h-y)\, dy} = \frac{hy^2 - \frac{y^3}{3}\Big|_0^h}{hy - \frac{y^2}{2}\Big|_0^h} = \frac{h}{3}$$

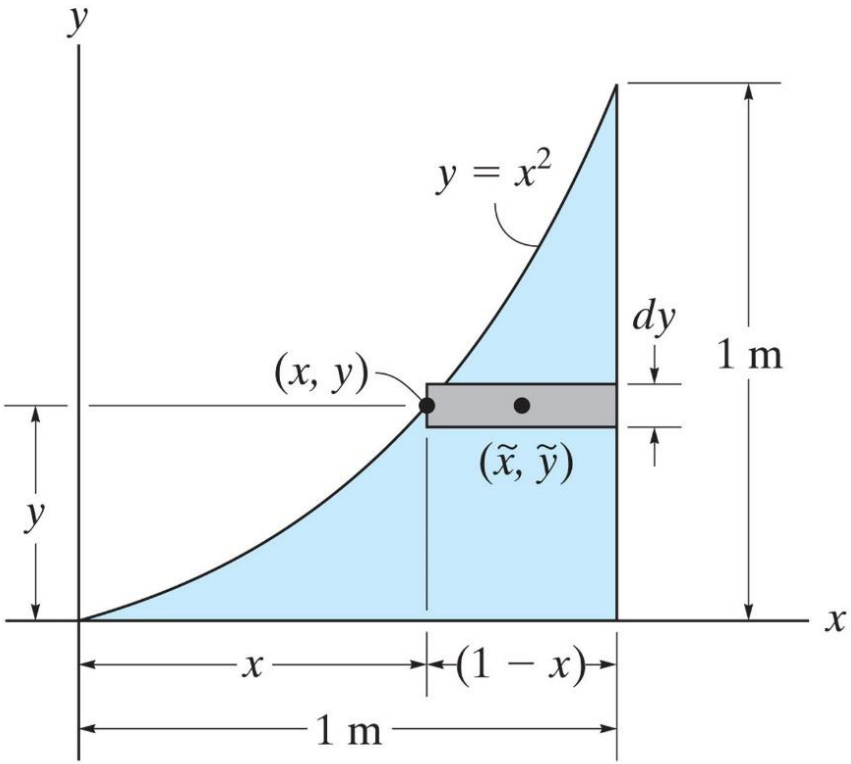
$\Rightarrow$ $\boxed{\bar{y} = \frac{h}{3}}$ Centroid for triangle cf. Chap. 4

See Text Example: 9.3



Locate the centroid of the area.

See Text Example: 9.5



Locate the centroid of the area.

See Text Example: 9.5