# Statics - TAM 210 & TAM 211

**Lecture 31**

(no lecture 30)

**April 6, 2018**

**Chap 9.2**

# Announcements

- **No class Wednesday April 11**
- **No office hours for Prof. H-W on Wednesday April 11**

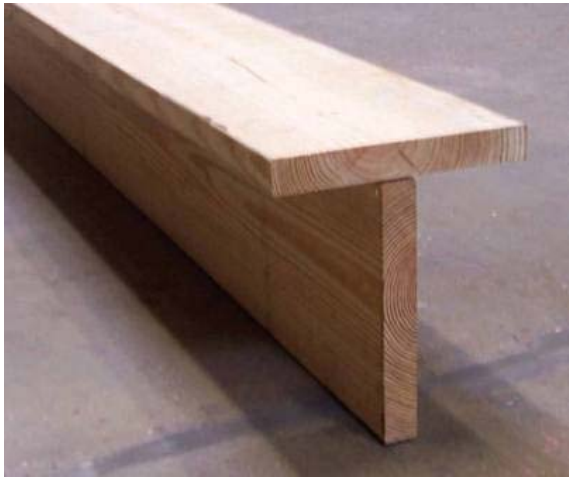
- Upcoming deadlines:
  - Monday (4/9)
    - Mastering Engineering Tutorial 13
  - Tuesday (4/10)
    - PL HW 12
  - Thursday (4/12)
    - WA 5 due

# Chapter 9: Center of Gravity and Centroid

# Goals and Objectives

- Understand the concepts of center of gravity, center of mass, and centroid.
- Determine the location of the center of gravity and centroid for a system of discrete particles and a body of arbitrary shape.

# Composite bodies



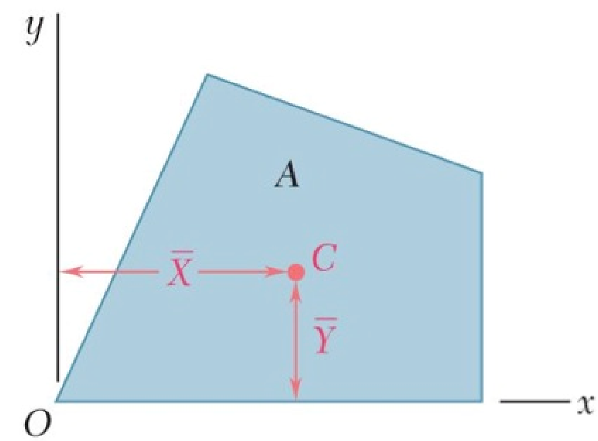
The I-beam (top) or T-beam (bottom) shown are commonly used in building various types of structures.

How can we easily determine the location of the centroid for different beam shapes?

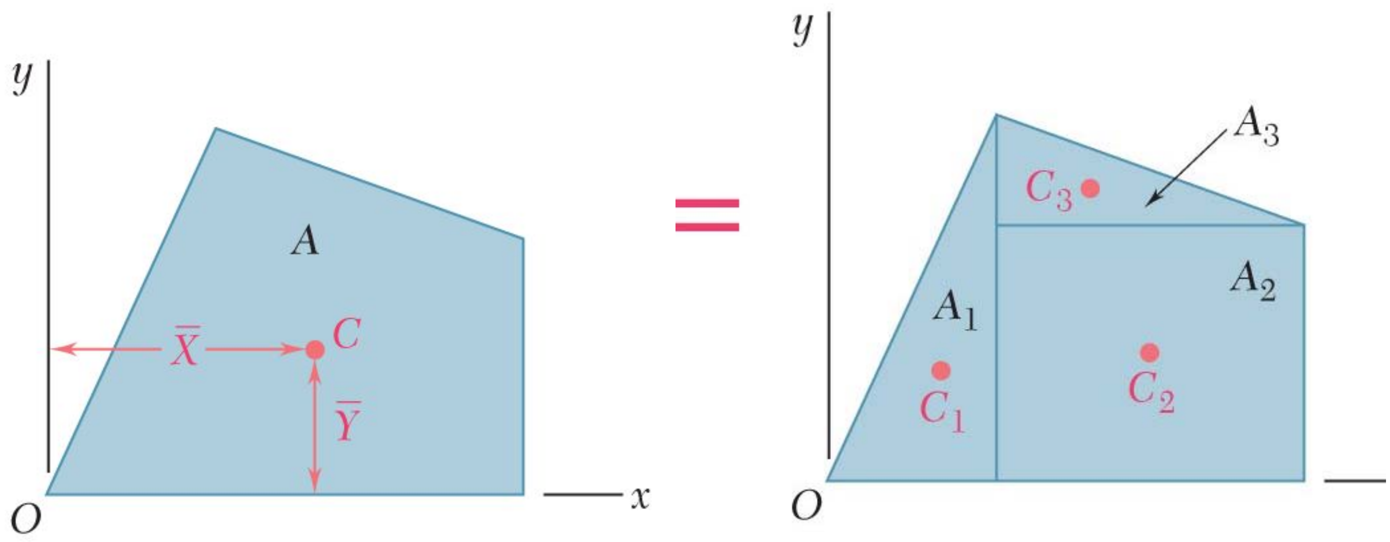
# Composite bodies

A composite body consists of a series of connected simpler shaped bodies.

Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.

For example, the centroid of the area A is located at point C of coordinates $\bar{x}$ and $\bar{y}$. In the case of a composite area, we divide the area A into parts $A_1, A_2, A_3$




$$\bar{x}\ A_{total} = \sum_{i=1}^{n} \widetilde{x_i}\ A_i$$

$$\bar{y}\ A_{total} = \sum_{i=1}^{n} \widetilde{y_i}\ A_i$$

Where: $A_{total} = \sum_{i=1}^{n} A_i$

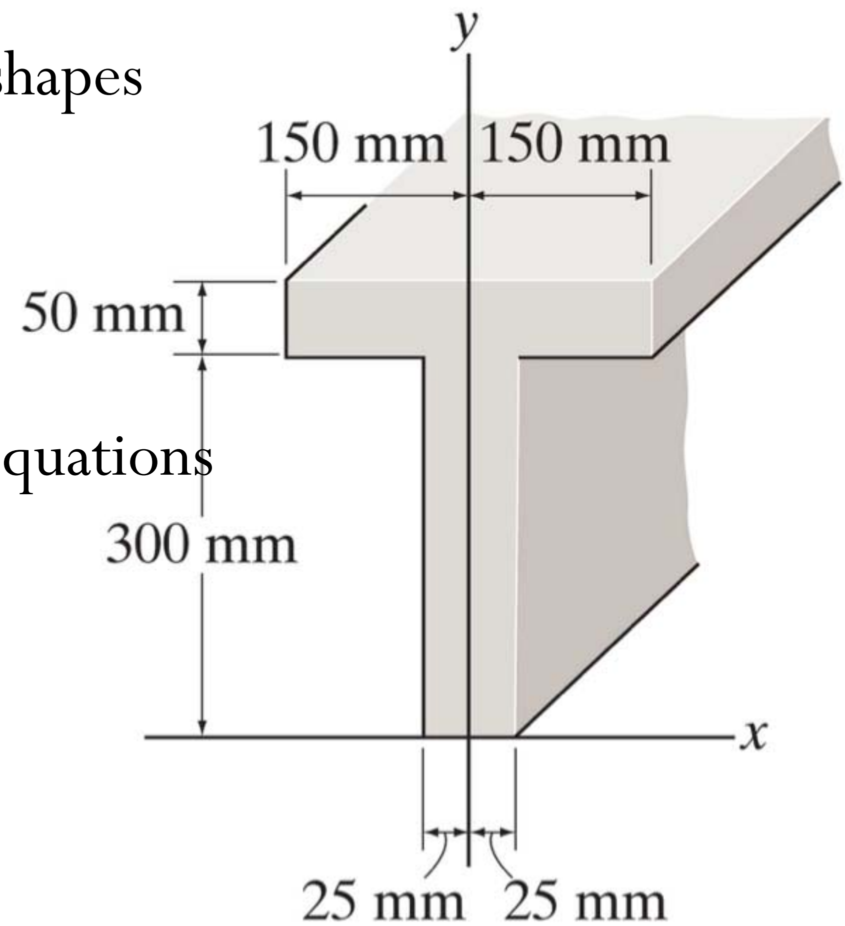
Therefore:

$\bar{x} = \dfrac{\sum_{i=1}^{n} \widetilde{x_i}\ A_i}{\sum_{i=1}^{n} A_i}$, shorthand: $\bar{x} = \dfrac{\sum \tilde{x} A}{\sum A}$

$\bar{y} = \dfrac{\sum_{i=1}^{n} \widetilde{y_i}\ A_i}{\sum_{i=1}^{n} A_i}$ or $\bar{y} = \dfrac{\sum \tilde{y} A}{\sum A}$

# Composite bodies – Analysis Procedure

1. Divide the body into finite number of simple shapes
2. Consider "holes" as "negative" parts
3. Establish coordinate axes
4. Determine centroid location by applying the equations



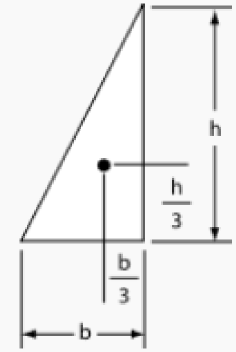
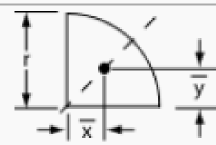
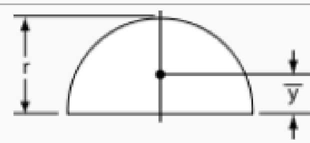
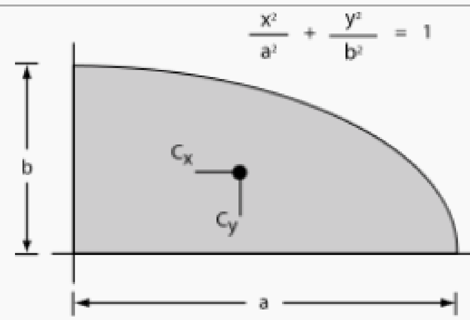
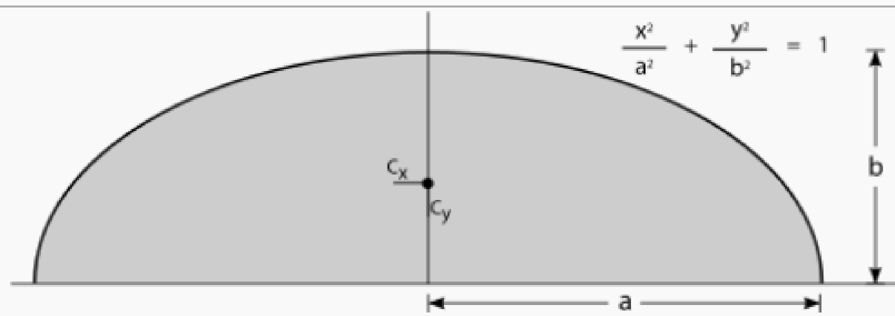

$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W} \qquad \bar{x} = \frac{\sum \tilde{x}A}{\sum A}$$

$$\bar{y} = \frac{\sum \tilde{y}W}{\sum W} \qquad \bar{y} = \frac{\sum \tilde{y}A}{\sum A}$$

$$\bar{z} = \frac{\sum \tilde{z}W}{\sum W} \qquad \bar{z} = \frac{\sum \tilde{z}A}{\sum A}$$

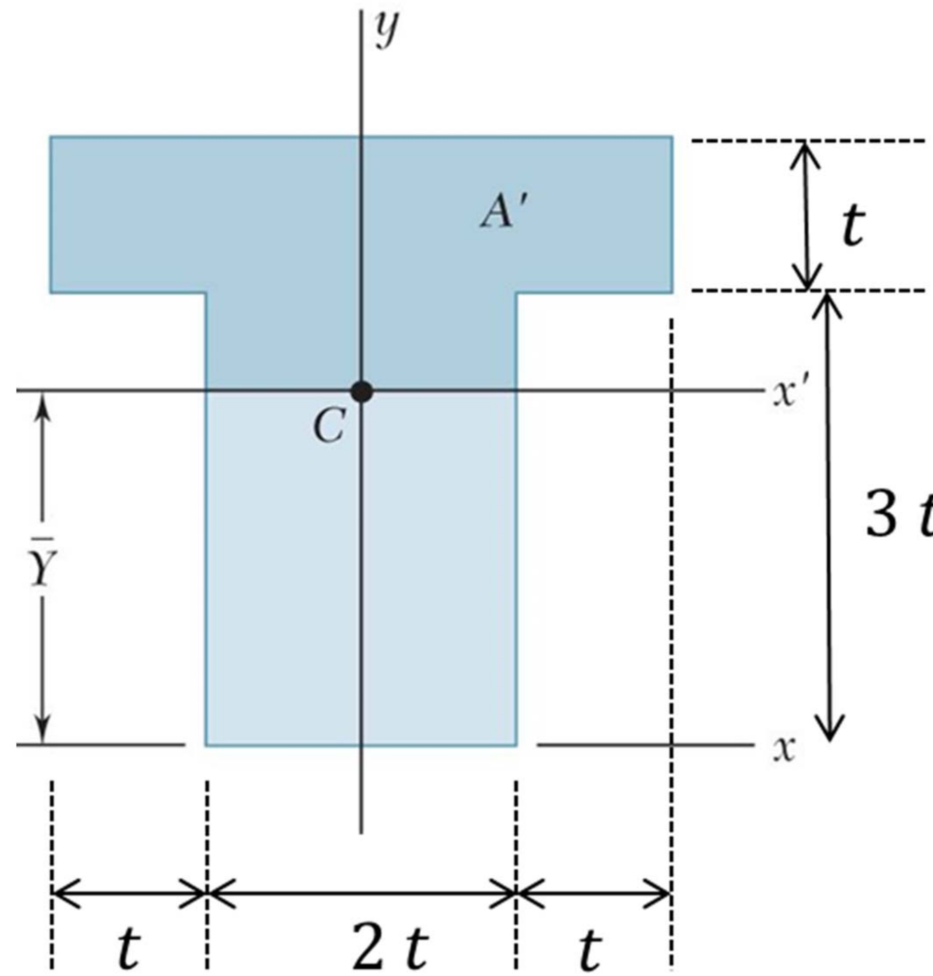
Similarly for mass (m), volume (V), or line (L)

# Centroid of typical 2D shapes

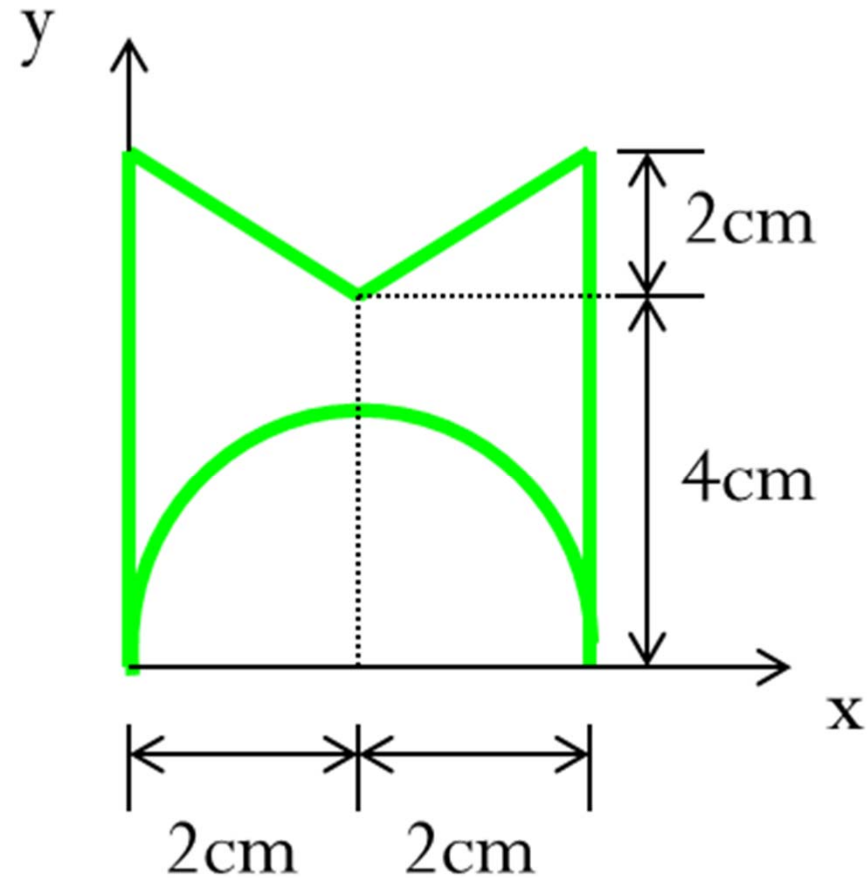
| Shape | Figure | $\bar{x}$ | $\bar{y}$ | Area |
|---|---|---|---|---|
| Right-triangular area |  | $\frac{b}{3}$ | $\frac{h}{3}$ | $\frac{bh}{2}$ |
| Quarter-circular area |  | $\frac{4r}{3\pi}$ | $\frac{4r}{3\pi}$ | $\frac{\pi r^2}{4}$ |
| Semicircular area |  | 0 | $\frac{4r}{3\pi}$ | $\frac{\pi r^2}{2}$ |
| Quarter-elliptical area |  | $\frac{4a}{3\pi}$ | $\frac{4b}{3\pi}$ | $\frac{\pi ab}{4}$ |
| Semielliptical area |  | 0 | $\frac{4b}{3\pi}$ | $\frac{\pi ab}{2}$ |

http://en.wikipedia.org/wiki/List_of_centroids

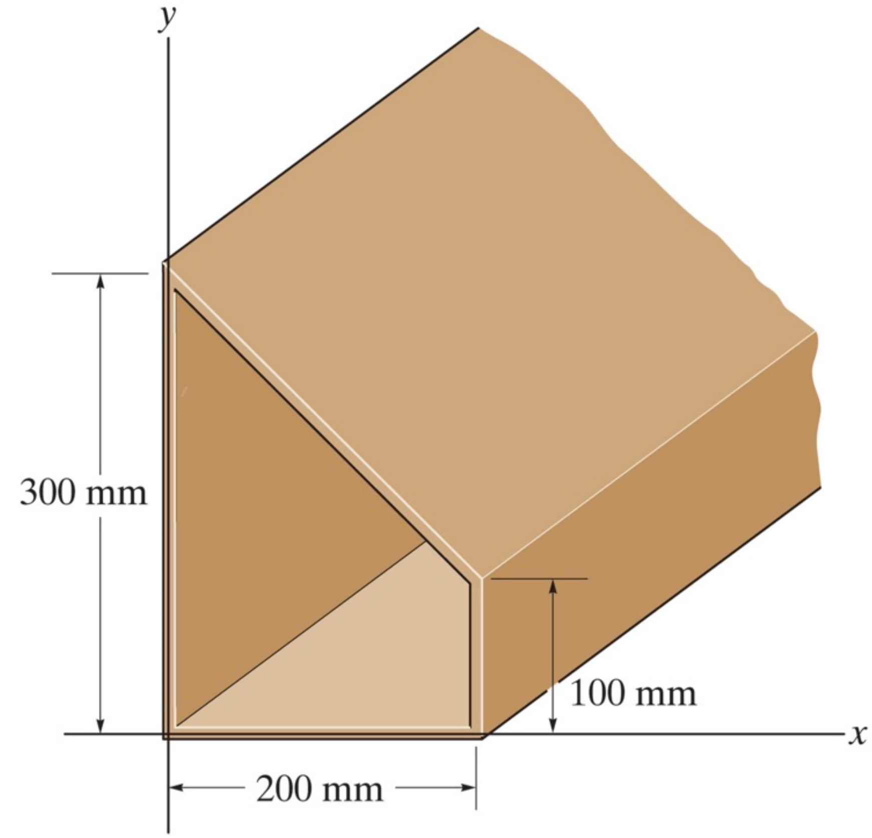
Find the centroid of the area.



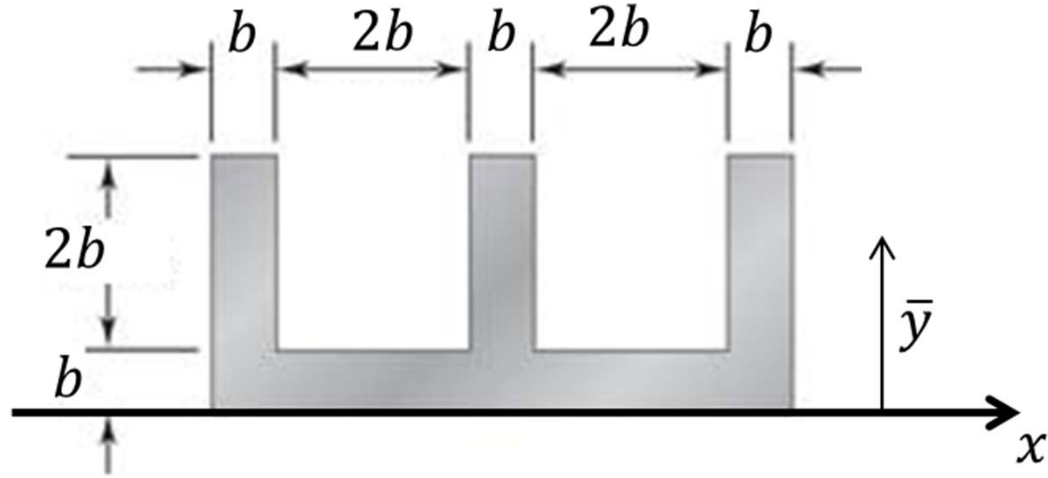
A rectangular area has semicircular and triangular cuts as shown. What is the centroid of the resultant area?



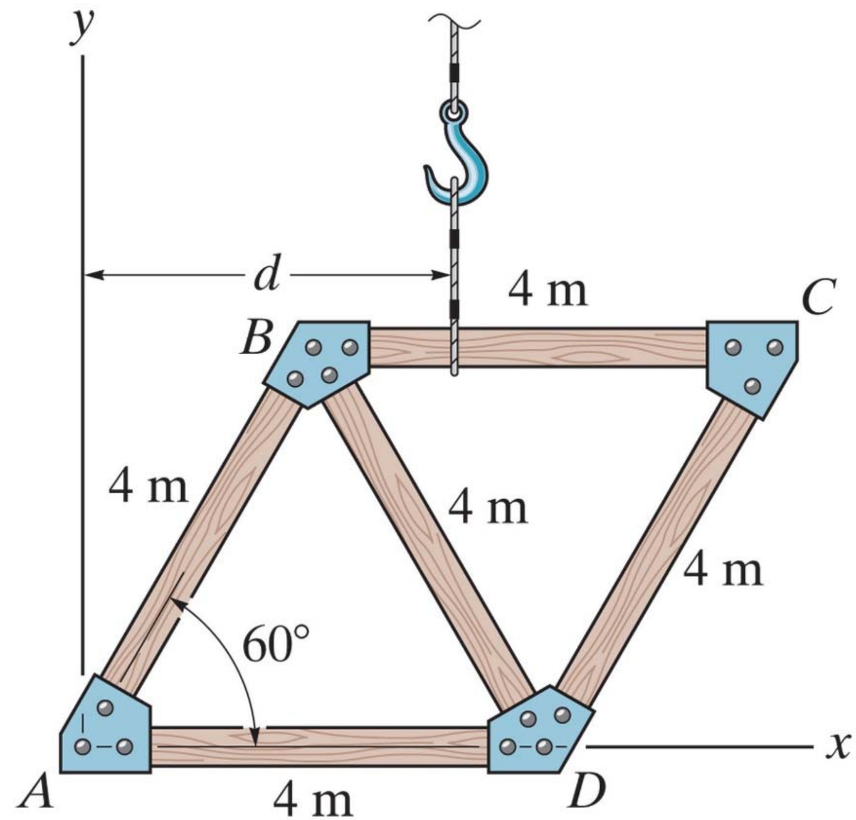
Locate the centroid of the cross section area.



Find the centroid of the area.



A truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. Determine the distance *d* to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.



Determine the location of the center of gravity of the three-wheeler. If the three-wheeler is symmetrical with respect to the x-y plane, determine the normal reaction each of its wheels exerts on the ground.