Statics - TAM 211

Lecture 31

(no lecture 30)

April 6, 2018

Chap 9.2

Announcements

- **No class Wednesday April 11**
- **No office hours for Prof. H-W on Wednesday April 11**

- Upcoming deadlines:
  - Monday (4/9)
    - Mastering Engineering Tutorial 13
  - Tuesday (4/10)
    - PL HW 12
  - Thursday (4/12)
    - WA 5 due

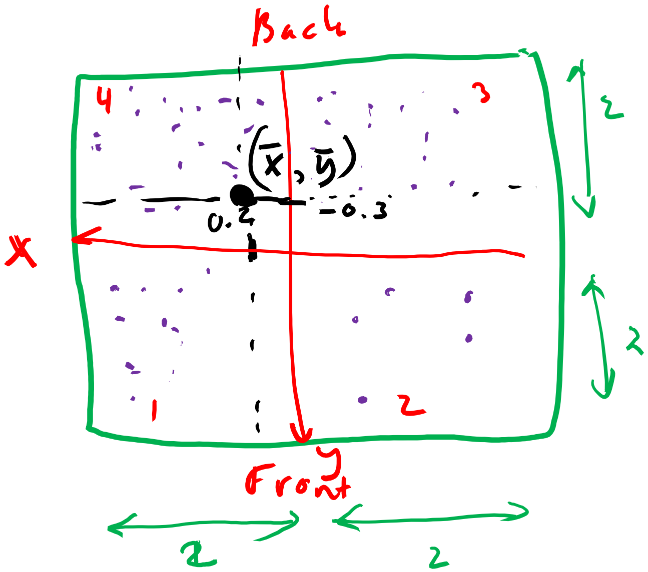
Chapter 9: Center of Gravity and Centroid

Goals and Objectives

- Understand the concepts of center of gravity, center of mass, and centroid.

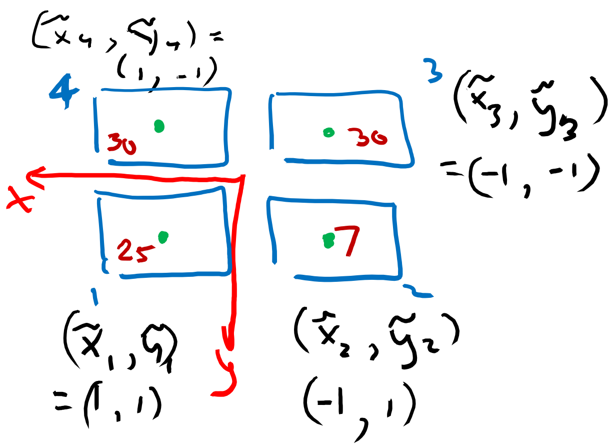
- Determine the location of the center of gravity and centroid for a system of discrete particles and a body of arbitrary shape.

Notes on class discussion about finding the center of distribution of the students in the 112 Gregory Hall lecture room, as an example of how to use composite bodies (4 quadrants) to find the center for the entire room.



$\bar{x}, \bar{y}$ Entire = ?

$\tilde{x}_i, \tilde{y}_i$ Segments



$$\bar{x} = \frac{\sum_{i=1}^{4} \tilde{x}_i (\text{bodies}_i)}{\sum (\text{bodies})}$$

$$\bar{x} = \frac{1(25) + -1(7) + (-1)30 + (1)30}{25 + 7 + 30 + 30}$$

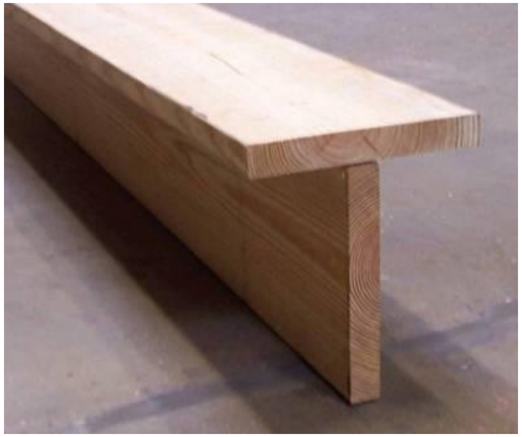
$$\boxed{\bar{x} = 0.2}$$

$$\bar{y} = \frac{1(25) + 1(7) + (-1)30 + (-1)30}{92}$$

$$\boxed{\bar{y} = -0.3}$$

I assumed that, within each quadrant, the distribution of bodies is uniform such that the center of bodies is located in the centroid of the area $(\tilde{x}_i, \tilde{y}_i)$

Composite bodies



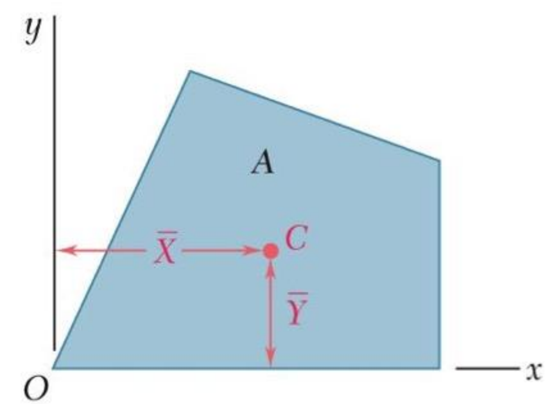
The I-beam (top) or T-beam (bottom) shown are commonly used in building various types of structures.

How can we easily determine the location of the centroid for different beam shapes?

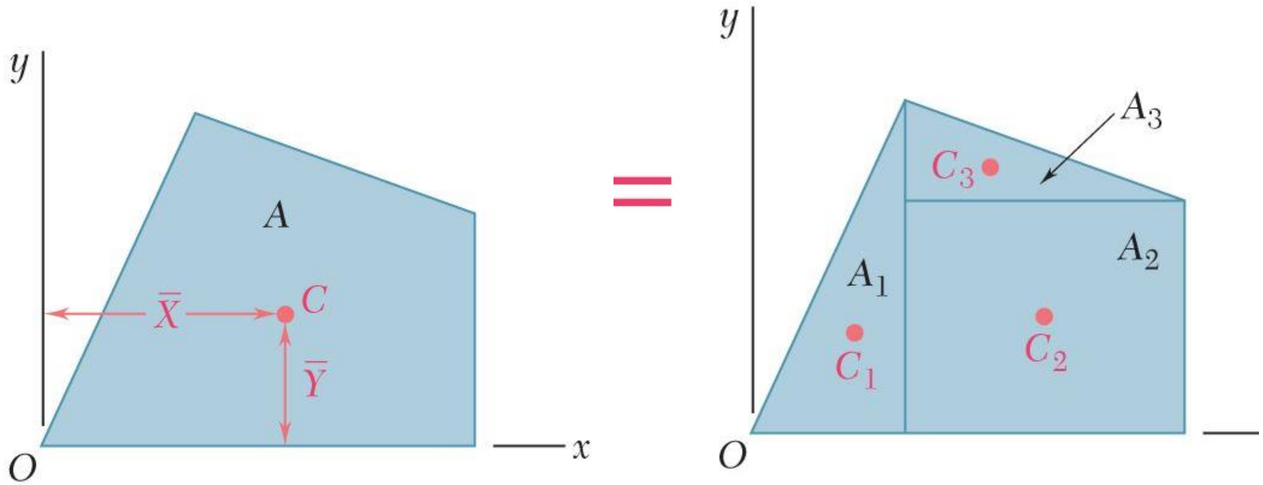
Composite bodies

A composite body consists of a series of connected simpler shaped bodies.

Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.



For example, the centroid of the area A is located at point C of coordinates $\bar{x}$ and $\bar{y}$. In the case of a composite area, we divide the area A into parts A_1, A_2, A_3



$\sum \tilde{x} A = \sum_{i=1}^{n} \tilde{x}_i A_i = ?$ Products, multiplication

$$\bar{x}\ A_{total} = \sum_{i=1}^{n} \widetilde{x_i}\ A_i$$

$$\bar{y}\ A_{total} = \sum_{i=1}^{n} \widetilde{y_i}\ A_i$$

Where: $A_{total} = \sum_{i=1}^{n} A_i$ Summation

Therefore:

$$\bar{x} = \frac{\sum_{i=1}^{n} \widetilde{x_i}\ A_i}{\sum_{i=1}^{n} A_i}\text{, shorthand: } \bar{x} = \frac{\sum \tilde{x} A}{\sum A}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} \widetilde{y_i}\ A_i}{\sum_{i=1}^{n} A_i} \text{ or } \bar{y} = \frac{\sum \tilde{y} A}{\sum A}$$

Composite bodies

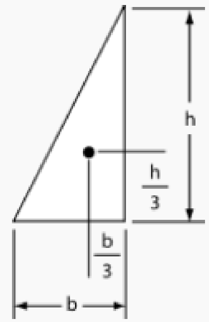
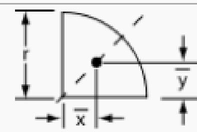
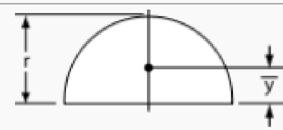
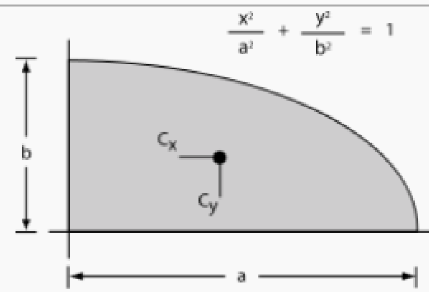
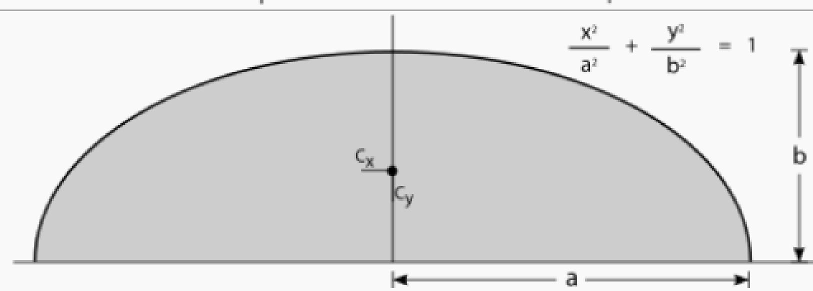
$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W} \qquad \bar{x} = \frac{\sum \tilde{x}A}{\sum A}$$

$$\bar{y} = \frac{\sum \tilde{y}W}{\sum W} \qquad \bar{y} = \frac{\sum \tilde{y}A}{\sum A}$$

$$\bar{z} = \frac{\sum \tilde{z}W}{\sum W} \qquad \bar{z} = \frac{\sum \tilde{z}A}{\sum A}$$

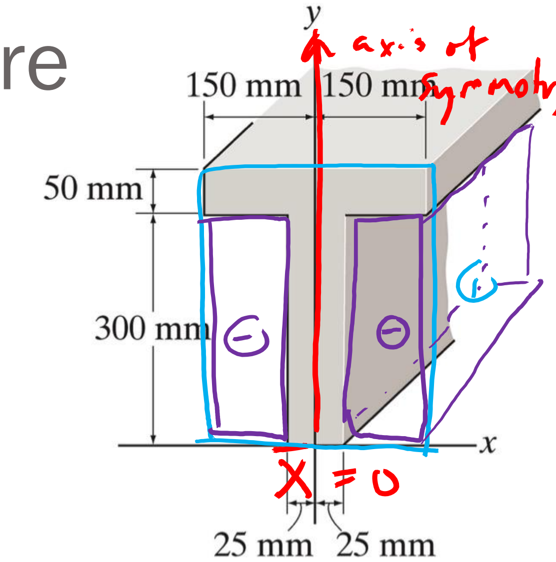
Similarly for mass (m), volume (V), or line (L)

Centroid of typical 2D shapes

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Right-triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

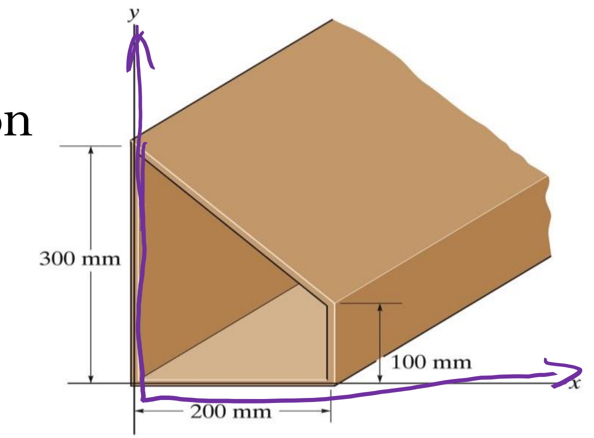
http://en.wikipedia.org/wiki/List_of_centroids

Composite bodies – Analysis Procedure



1. Divide the body into finite number of simple shapes
2. Identify possible axis (axes) of symmetry
3. Consider "holes" as "negative" parts
4. Establish coordinate axes
5. Make a table to help with bookkeeping
6. Determine total centroid location by applying equation

$$\bar{x} = \frac{\sum \tilde{x} A}{\sum A} \qquad \bar{y} = \frac{\sum \tilde{y} A}{\sum A}$$

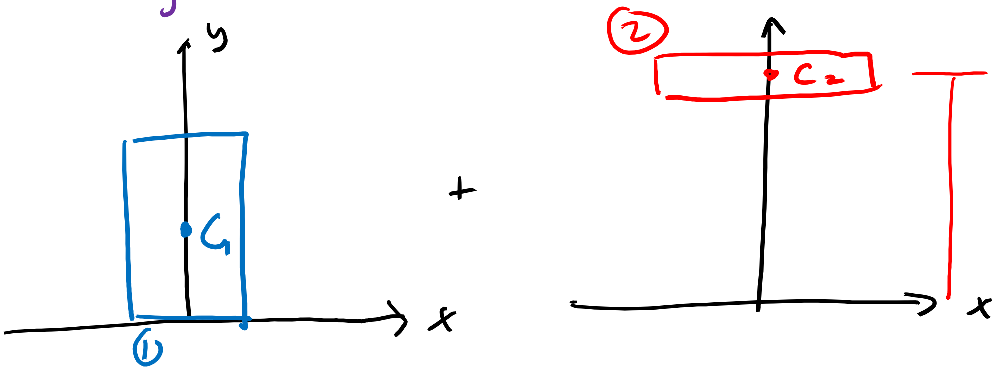


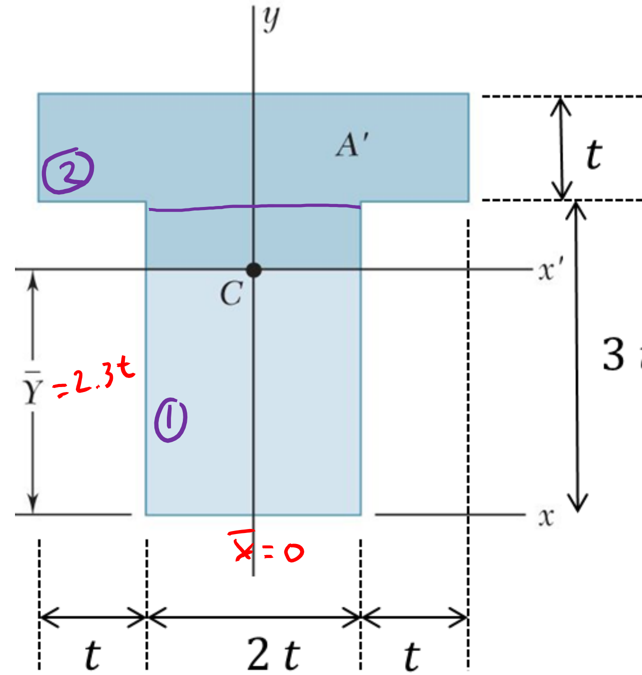
Segment #	W, m, A, V, or L (units)	Moment arm [Coord of part] (units)			Summations (units)		
		$\tilde{x}$	$\tilde{y}$	$\tilde{z}$	$\tilde{x}_i A_i$	$\tilde{y}_i A_i$	$\tilde{z}_i A_i$
	$\Sigma A=$				$\Sigma\tilde{x}A=$	$\Sigma\tilde{y}A=$	$\Sigma\tilde{z}A=$

Find the centroid of the area.

Any symmetry? y-axis ⇒ $\bar{x} = 0$

How segments?



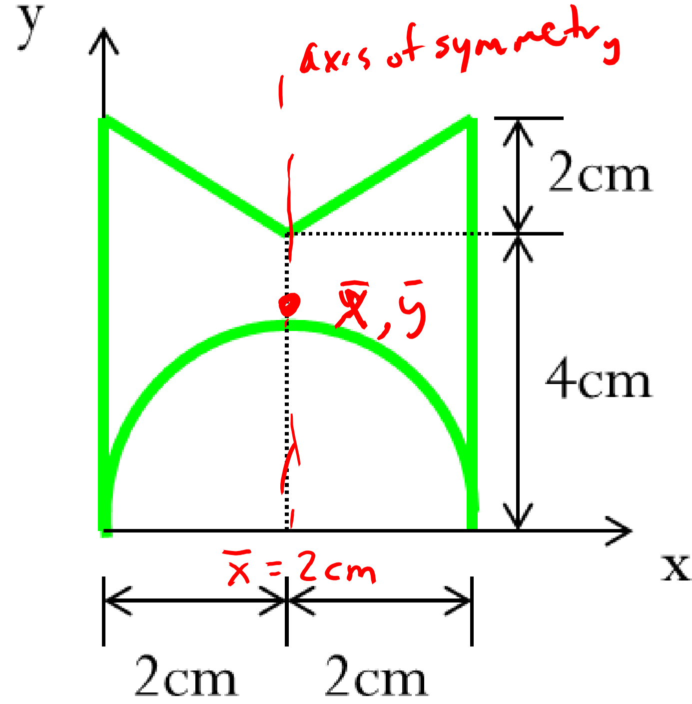


Seg #	Area	$\tilde{x}$	$\tilde{y}$	$\tilde{x}_i\, A_i$	$\tilde{y}_i\, A_i$
1	$6t^2$	0	$1.5t$	0	$9t^3$
2	$4t^2$	0	$3.5t$	0	$14t^3$
	$\Sigma A = 10t^2$			$\Sigma \tilde{x} A = 0$	$\Sigma \tilde{y} A = 23t^3$

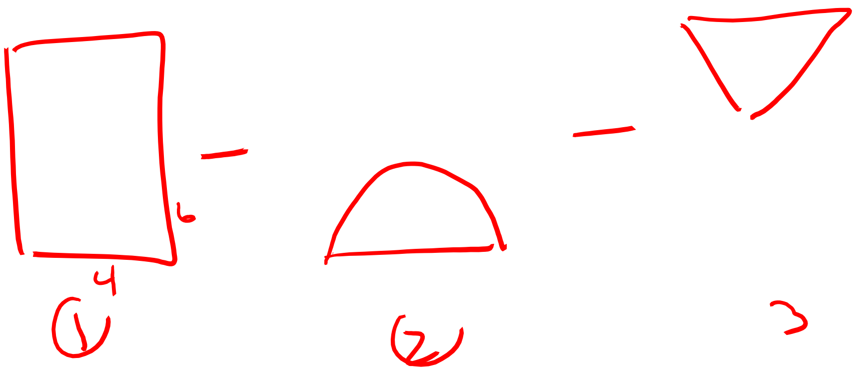
$$\bar{x} = \frac{\sum \tilde{x} A}{\sum A} = 0$$

$$\bar{y} = \frac{\sum \tilde{y} A}{\sum A} = \frac{23t^3}{10t} = \boxed{2.3t = \bar{y}}$$

A rectangular area has semicircular and triangular cuts as shown. What is the centroid of the resultant area?



Any axis of symmetry? Yes (parallel to y-axis, but at x = 2 cm).
How many segments? People in class presented multiple options for segmentation. Select an approach that needs to **the least number** of segments.



Seg #	Area (cm^2)	$\tilde{x}$	$\tilde{y}$	$\tilde{x}_i A_i$	$\tilde{y}_i A_i$
1	24 cm^2	2	3		72
2	−4	2	$\frac{8}{3}\pi$		$-16/3$
3	-2π	2	$\frac{16}{3}$		$-64/3$
	$\Sigma A =$ 13.7			$\Sigma\tilde{x}A =$	$\Sigma\tilde{y}A =$ 45.3 cm^3

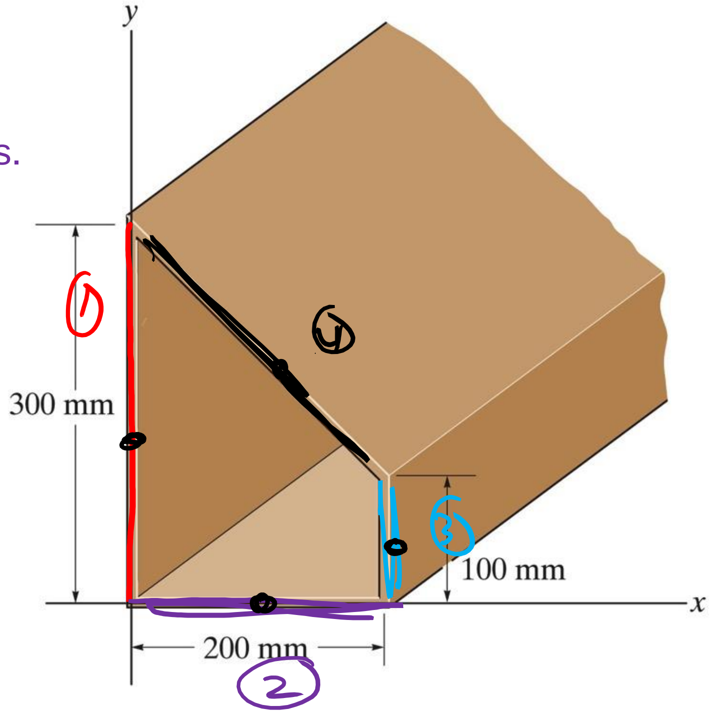
$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} = 2$$

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} = \frac{45.3}{13.7} = 3.3 = \bar{y}$$

Locate the centroid of the **cross section area**.

Any axis of symmetry? No
How many segments? The following approach used 4 line segments.
Complete the table to verify the proposed solution for the centroid.

A classmate suggested a triangle and rectangle. See if that approach would also produce the same solution.

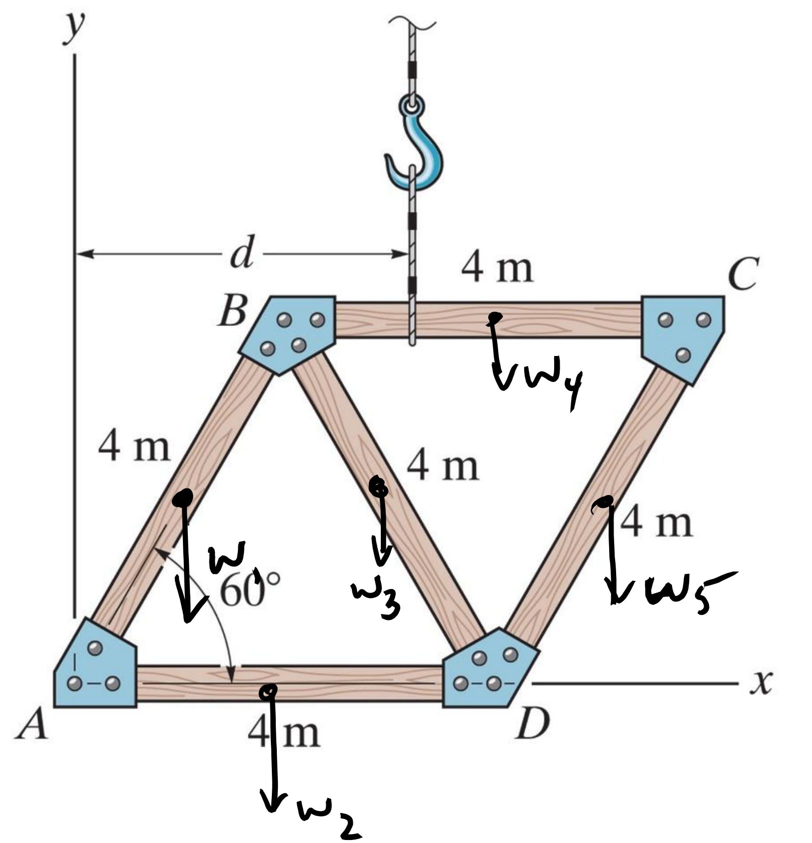


	L	$\tilde{x}$	$\tilde{y}$	$\tilde{x}L$	$\tilde{y}L$
1	300				
2	200				
3	100				
4	$200\sqrt{2}$				

$\bar{x} = 77.3$ mm $= \dfrac{\Sigma \tilde{x} L}{\Sigma L}$

$\bar{y} = 120.7$ mm

A truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. Determine the distance *d* to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.



$$W = \left(7\,\frac{kg}{m}\right)(4\,m)\,g = 28\,kg\,(g)$$

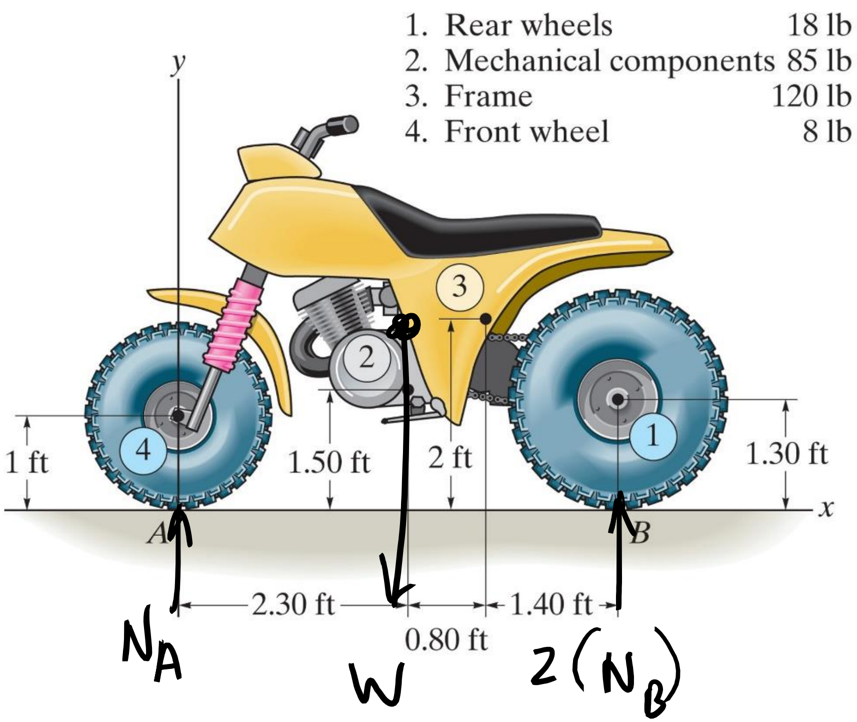
$$m = 28\,kg$$

$$\bar{x} = d = \frac{\Sigma \tilde{x}_i m_i}{\Sigma m}$$

For this problem, since distance "d" is the only needed value, then only $\bar{x}$ is needed.

As an aside, after class, a student noted that this structure has two axes of symmetry: one between AC, another between BD; like the supports for a diamond-shaped kite. Fun fact: If one can identify two axes of symmetry, the intersection is the centroid.
http://mathdemos.org/mathdemos/centroids/centroid.html

Determine the location of the center of gravity of the three-wheeler. If the three-wheeler is symmetrical with respect to the x-y plane, determine the normal reaction each of its wheels exerts on the ground.



To solve this problem,

1. Determine the location of the center of gravity (CoG) of the vehicle.
2. Draw the FBD of the vehicle in the x-y plane.
3. To solve for the unknown normal reaction forces at the points of contact of the wheels, use the equations of equilibrium. Note that the total weight (W) would be located at the CoG found in step 1, and that there are two normal forces at B due to two back wheels of the "three-wheeler".