Statics - TAM 211

Lecture 34

(no lecture 33)

April 13, 2018

Chap 10.1, 10.2, 10.4, 10.8, Chap 5.5-5.6

Announcements

- Quiz 6 and Written Assignment 6 scheduling conflict
  - What Piazza for scheduling announcements

- Upcoming deadlines:
  - Monday (4/16)
    - Mastering Engineering Tutorial 14
  - Tuesday (4/17)
    - PL HW 13
  - Quiz 6
  - Written Assignment 6

Chapter 10: Moments of Inertia

Goals and Objectives

- Understand the term "moment" as used in this chapter

- Determine and know the differences between
  - First/second moment of area
  - Moment of inertia for an area
  - Polar moment of inertia
  - Mass moment of inertia

Inertia ≈ mass, distribution of mass, distribution of area

- Introduce the parallel-axis theorem.

- Be able to compute the moments of inertia of composite areas.

Second moment of area

First moment of area

$Q_x = \int_A \tilde{y}\, dA$

$Q_y = \int_A \tilde{x}\, dA$

"Second moment of area" ≈ "Area moment of inertia"; note differences in names, but they both represent the same concept.

Moment of inertia is the property of a deformable body that determines the moment needed to obtain a desired curvature about an axis.

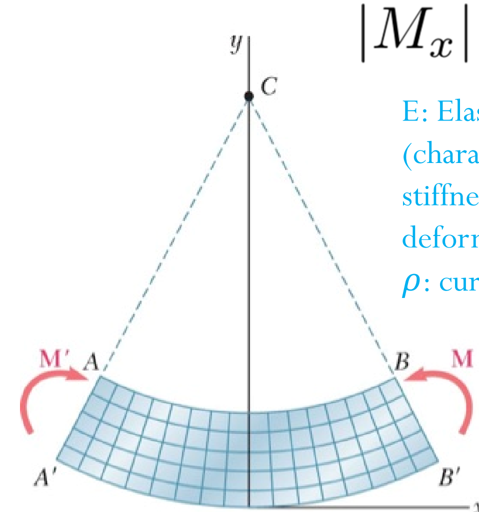
Moment of inertia depends on the shape of the body and may be different around different axes of rotation.

Moment-curvature relation:

$$|M_x| = \frac{E\, I_x}{\rho}$$

E: Elasticity modulus (characterizes stiffness of the deformable body)

ρ: curvature



a.k.a: "Area moment of inertia"

- The moment of inertia of the area A with respect to the x-axis is given by

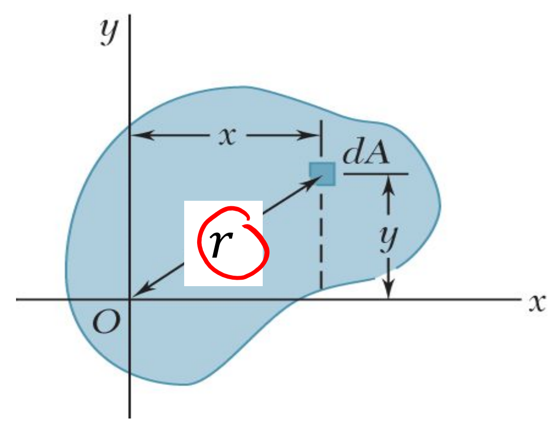
$$I_x = \int_A y^2\, dA$$

- The moment of inertia of the area A with respect to the y-axis is given by

$$I_y = \int_A x^2\, dA$$

- The moment of inertia of the area A with respect to the origin O is given by (Polar moment of inertia)

$$J_O = \int_A r^2\, dA = \int_A (x^2 + y^2)\, dA = I_y + I_x$$



Parallel axis theorem

- Often, the **moment of inertia** of an area is known for an axis passing through the **centroid**; e.g., *x'* and *y'*:
- The moments around other axes can be computed from the known $I_{x'}$ and $I_{y'}$:

$$I_x = \int_{\text{area}} (y' + d_y)^2 \, dA$$

$$= \int_{\text{area}} (y')^2 \, dA + 2d_y \int_{\text{area}} y' \, dA$$

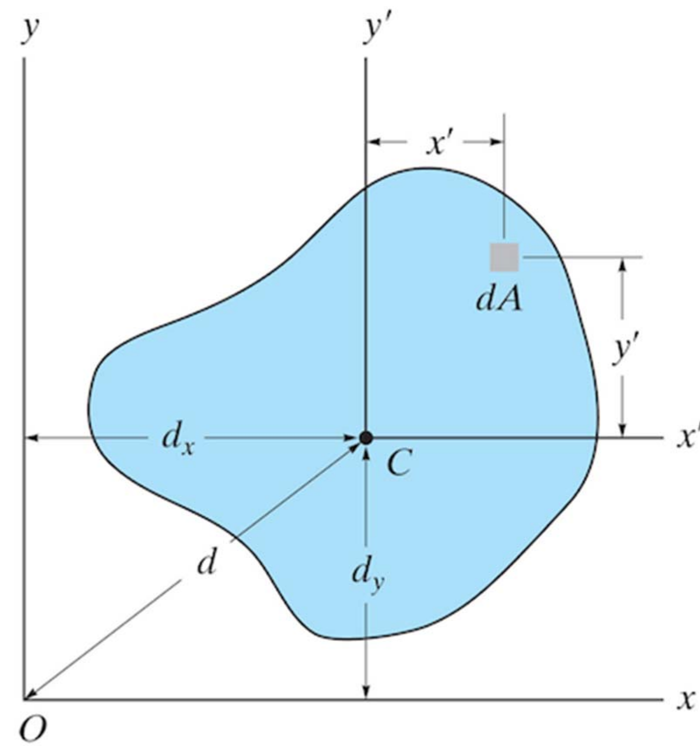
$$+ d_y^2 \int_{\text{area}} dA$$

$I_{x'}$ $= 0$

$$I_x = I_{x'} + Ad_y^2$$

$$I_y = I_{y'} + Ad_x^2$$

$$J_O = J_C + A(d_x^2 + d_y^2) = J_C + Ad^2$$

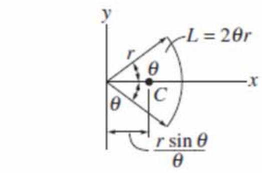


Note: the integral over y' gives zero *when done through the centroid axis*.

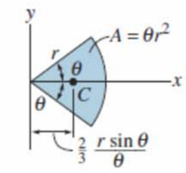
From inside back cover of Hibbler textbook

Geometric Properties of Line and Area Elements

Centroid Location | **Centroid Location** | **Area Moment of Inertia**



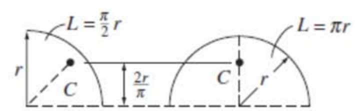
Circular arc segment



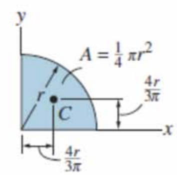
Circular sector area

$I_x = \frac{1}{4} r^4 (\theta - \frac{1}{2}\sin 2\theta)$

$I_y = \frac{1}{4} r^4 (\theta + \frac{1}{2}\sin 2\theta)$



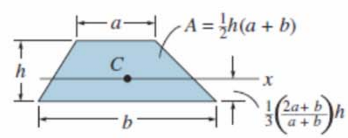
Quarter and semicircle arcs



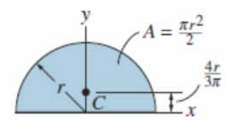
Quarter circle area

$I_x = \frac{1}{16}\pi r^4$

$I_y = \frac{1}{16}\pi r^4$



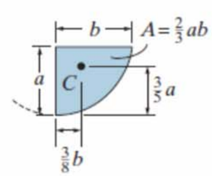
Trapezoidal area



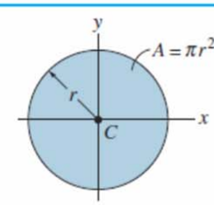
Semicircular area

$I_x = \frac{1}{8}\pi r^4$

$I_y = \frac{1}{8}\pi r^4$



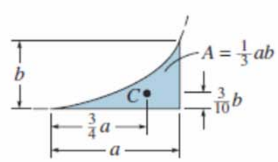
Semiparabolic area



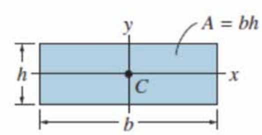
Circular area

$I_x = \frac{1}{4}\pi r^4$

$I_y = \frac{1}{4}\pi r^4$



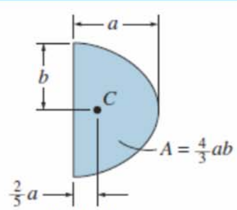
Exparabolic area



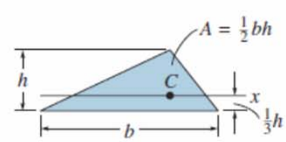
Rectangular area

$I_x = \frac{1}{12}bh^3$

$I_y = \frac{1}{12}hb^3$



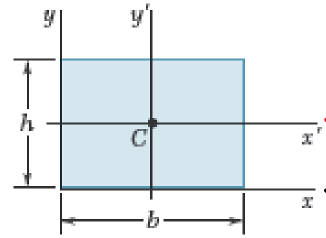
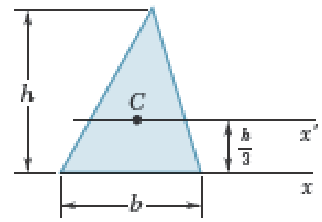
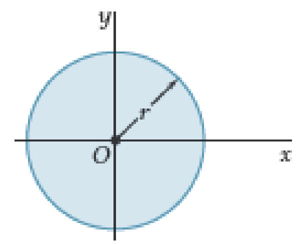
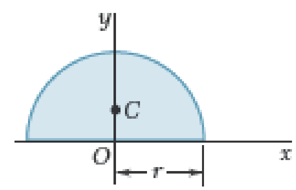
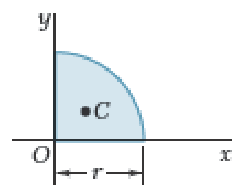
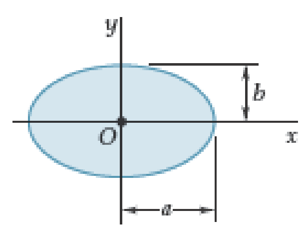
Parabolic area



Triangular area

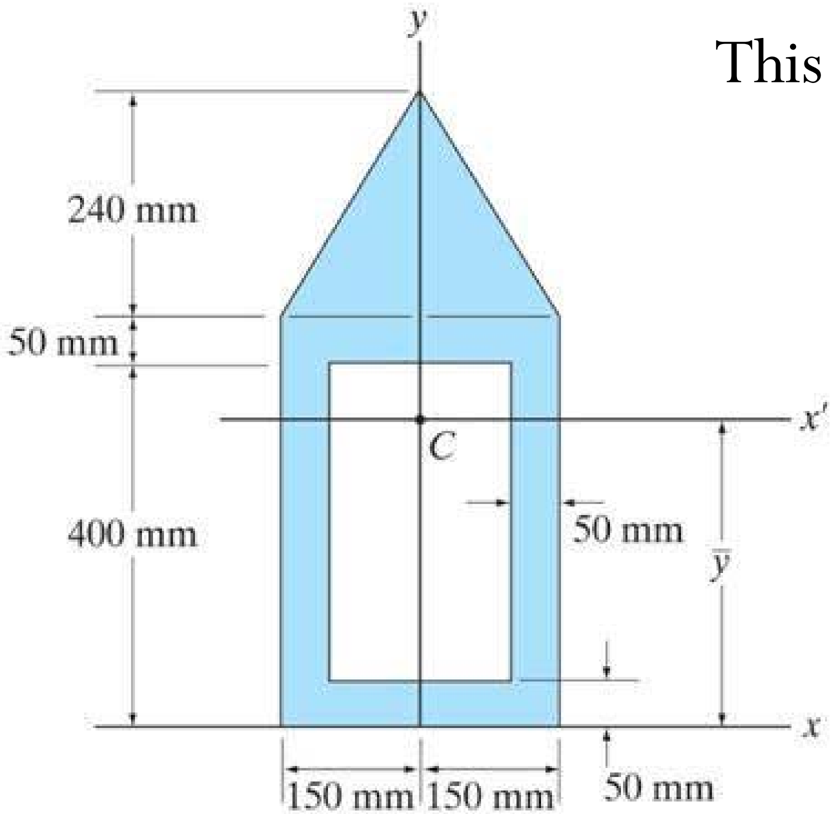
$I_x = \frac{1}{36}bh^3$

Area Moments of Inertia
for common shapes

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Moment of inertia of composite

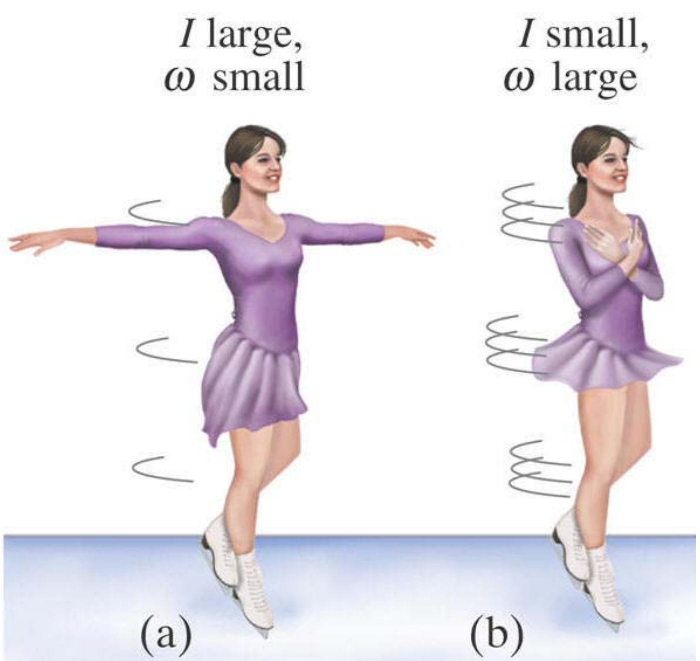
- If individual bodies making up a **composite** body have individual areas *A* and moments of inertia *I* computed through their centroids, then the **composite area** and **moment of inertia** is a sum of the individual component contributions.



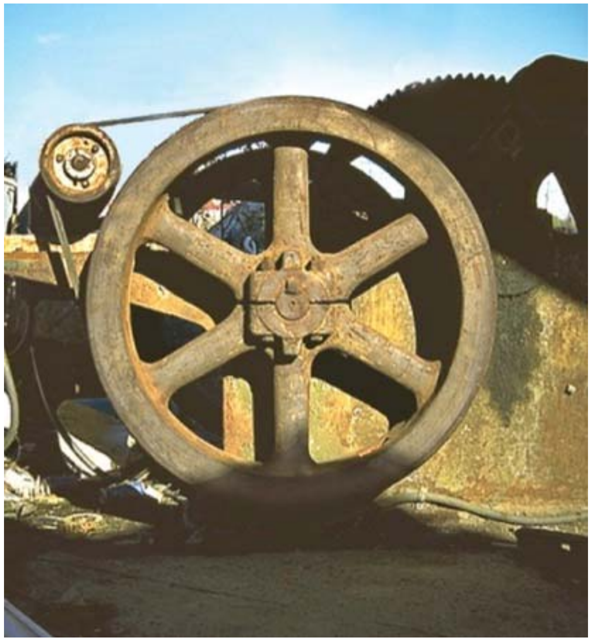
This requires the **parallel axis theorem**:

Mass Moment of Inertia

- **Mass moment of inertia** is the mass property of a rigid body that determines the torque T needed for a desired angular acceleration (α) about an axis of rotation.
- A larger mass moment of inertia around a given axis requires more torque to increase the rotation, or to stop the rotation, of a body about that axis
- Mass moment of inertia depends on the shape and density of the body and is different around different axes of rotation.



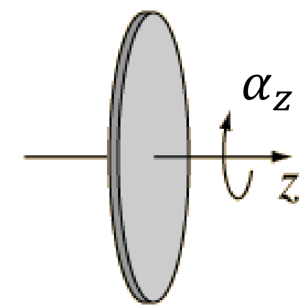
http://ffden-2.phys.uaf.edu/webproj/211_fall_2014/Ariel_Ellison/Ariel_Ellison/Angular.html



Mass Moment of Inertia

Mass Moment of Inertia

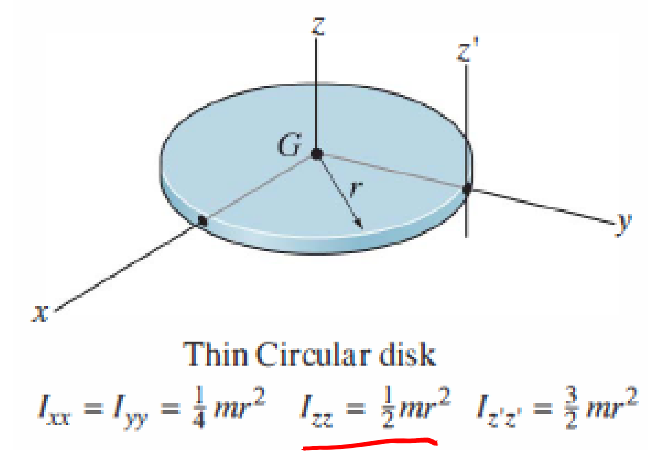
Torque-acceleration relation: $T = I\,\alpha$



where the mass moment of inertia is defined as

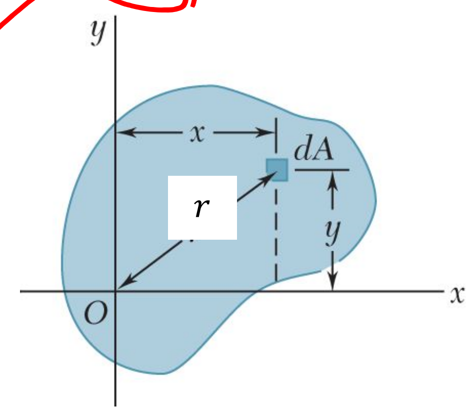
$$I_{zz} = \int \rho\, r^2 dV$$

$I_{zz} = \int r^2\, dm$, if constant ρ



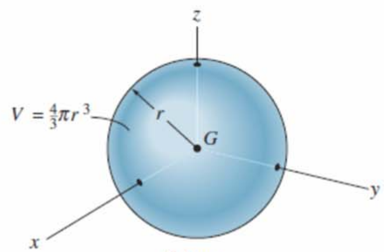
Mass moment of inertia for a disk:

$$\begin{aligned} I_{zz} &= \int \rho\, r^2\, dv = \int_0^t \int_0^{2\pi} \int_0^R \rho\, r^2\, (r\, dr\, d\theta\, dz) \\ &= \rho \int_0^t \int_0^{2\pi} \frac{r^4}{4} d\theta\, dz \\ &= \rho \int_0^t \frac{r^4}{2} \pi\, dz = \rho \frac{r^4}{2} \pi\, t = \frac{r^2}{2} \rho\, \pi\, r^2\, t = \frac{r^2}{2} \rho\, V = \frac{r^2}{2} M \end{aligned}$$



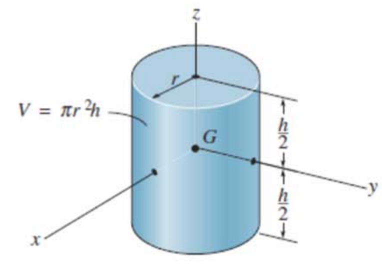
From inside back cover of Hibbler textbook

Center of Gravity and Mass Moment of Inertia of Homogeneous Solids



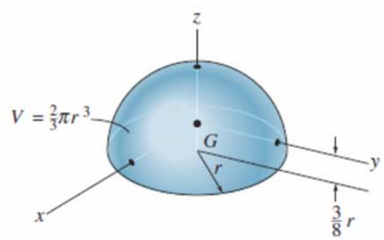
Sphere

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$



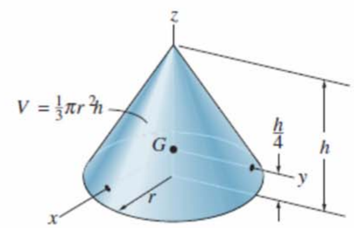
Cylinder

$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2} mr^2$



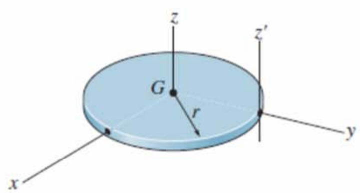
Hemisphere

$I_{xx} = I_{yy} = 0.259mr^2 \quad I_{zz} = \frac{2}{5} mr^2$



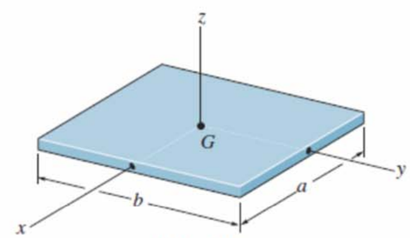
Cone

$I_{xx} = I_{yy} = \frac{3}{80} m (4r^2 + h^2) \quad I_{zz} = \frac{3}{10} mr^2$



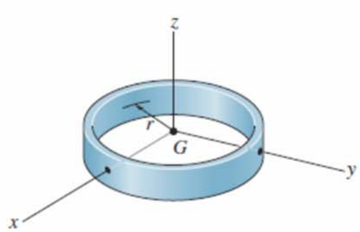
Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \quad I_{zz} = \frac{1}{2} mr^2 \quad I_{z'z'} = \frac{3}{2} mr^2$



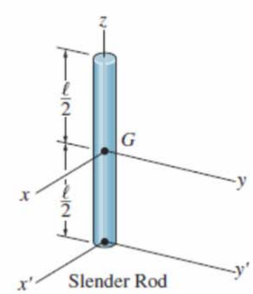
Thin plate

$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$



Thin ring

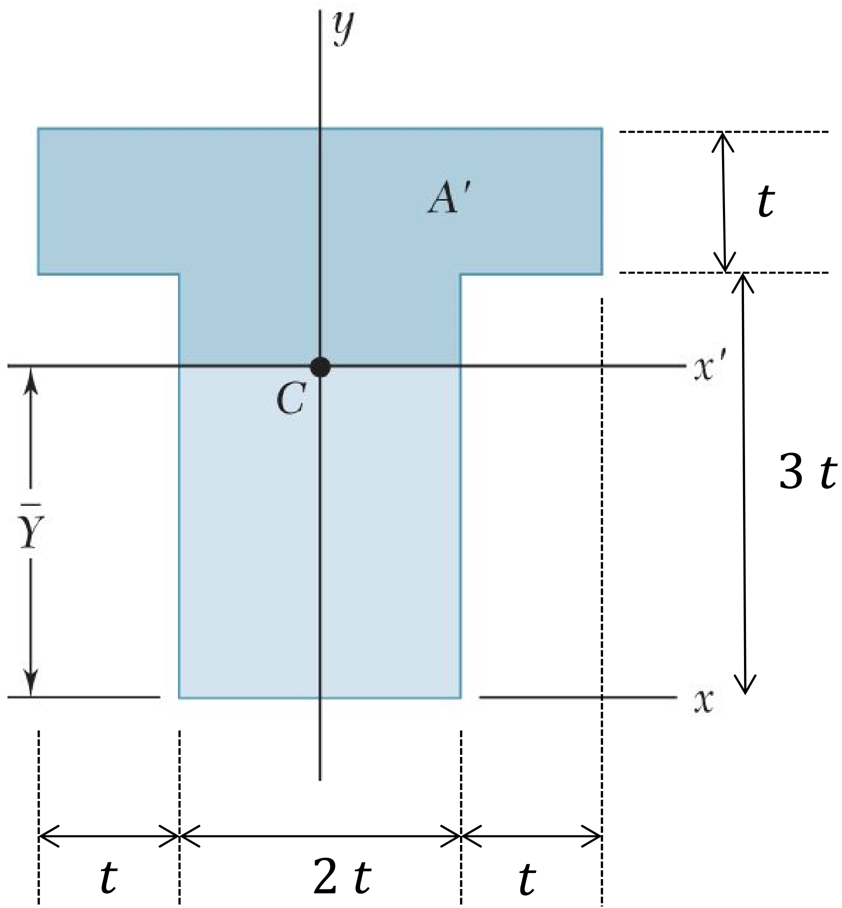
$I_{xx} = I_{yy} = \frac{1}{2} mr^2 \quad I_{zz} = mr^2$



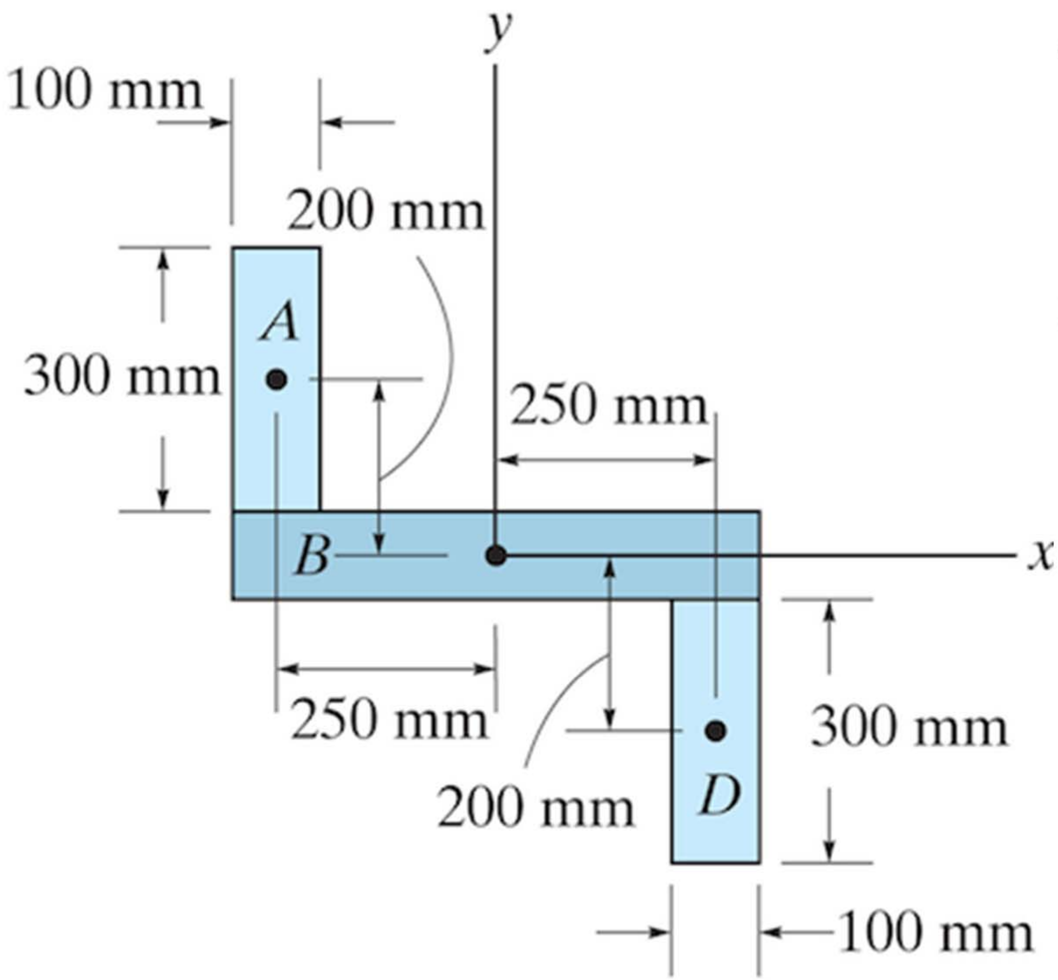
Slender Rod

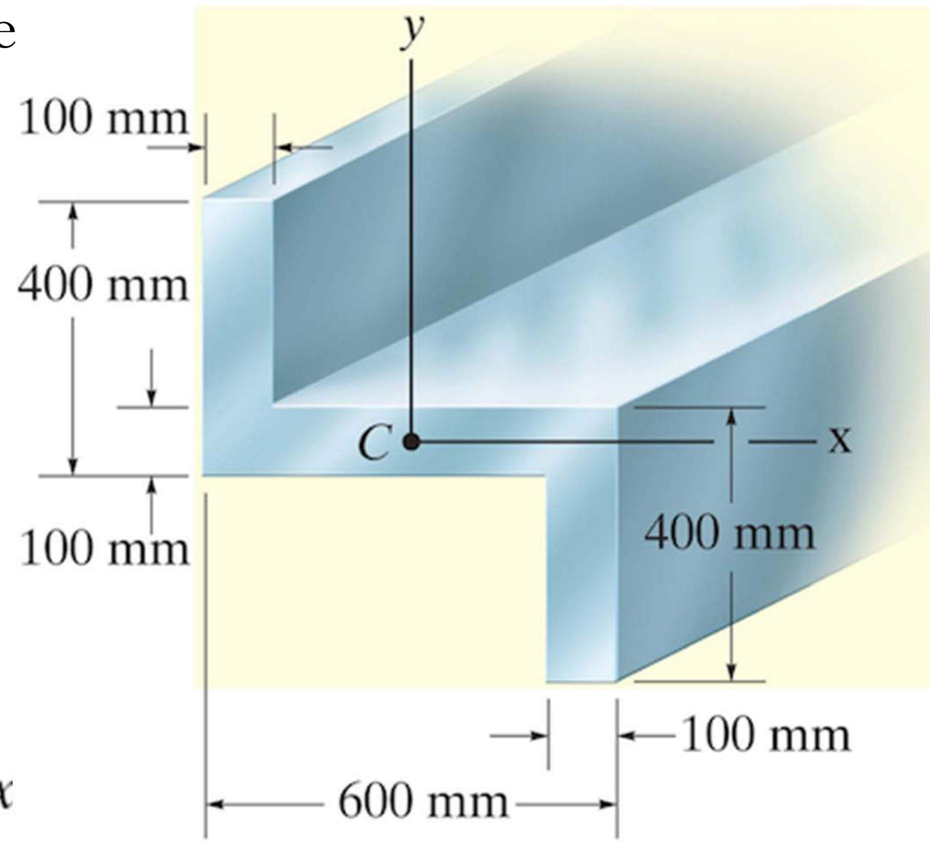
$I_{xx} = I_{yy} = \frac{1}{12} m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m\ell^2 \quad I_{z'z'} = 0$

Find the moment of inertia of the shape about its centroid:

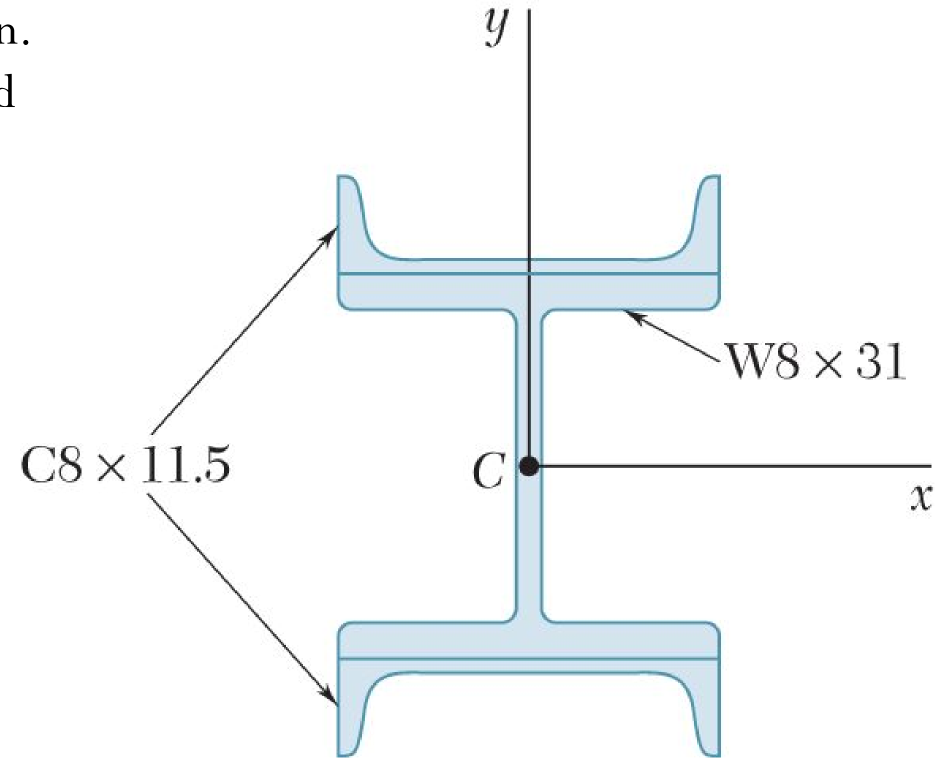


Determine the moment of inertia for the cross-sectional area about the x and y centroidal axes.

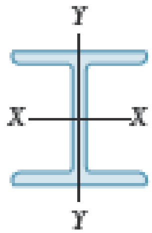
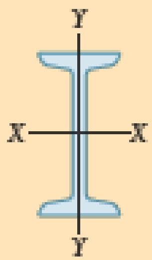
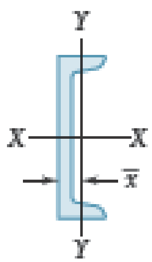
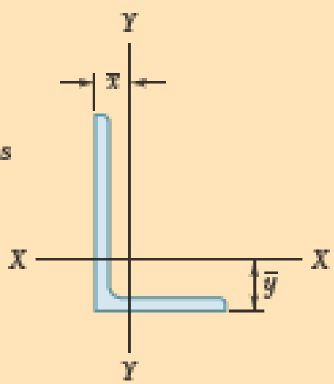




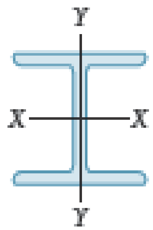
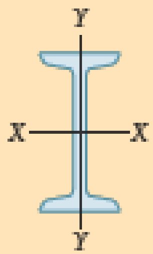
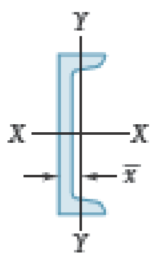
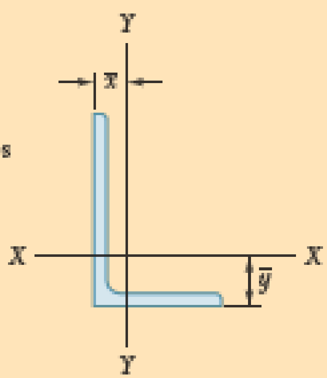
Two channels are welded to a rolled W section as shown. Determine the area moments of inertia of the combined section with respect to the centroidal x and y axes.



English units (inches)

	Designation	Area, in²	Depth, in.	Width, in.	Axis X-X: $\bar{I}_x$, in⁴	Axis X-X: $\bar{k}_x$, in.	Axis X-X: $\bar{y}$, in.	Axis Y-Y: $\bar{I}_y$, in⁴	Axis Y-Y: $\bar{k}_y$, in.	Axis Y-Y: $\bar{x}$, in.
W Shapes (Wide-Flange Shapes) 	W18 × 76†	22.3	18.2	11.0	1330	7.73		152	2.61	
	W16 × 57	16.8	16.4	7.12	758	6.72		43.1	1.60	
	W14 × 38	11.2	14.1	6.77	385	5.87		26.7	1.55	
	W8 × 31	9.12	8.00	8.00	110	3.47		37.1	2.02	
S Shapes (American Standard Shapes) 	S18 × 54.7†	16.0	18.0	6.00	801	7.07		20.7	1.14	
	S12 × 31.8	9.31	12.0	5.00	217	4.83		9.33	1.00	
	S10 × 25.4	7.45	10.0	4.66	123	4.07		6.73	0.950	
	S6 × 12.5	3.66	6.00	3.33	22.0	2.45		1.80	0.702	
C Shapes (American Standard Channels) 	C12 × 20.7†	6.08	12.0	2.94	129	4.61		3.86	0.797	0.698
	C10 × 15.3	4.48	10.0	2.60	67.3	3.87		2.27	0.711	0.634
	C8 × 11.5	3.37	8.00	2.26	32.5	3.11		1.31	0.623	0.572
	C6 × 8.2	2.39	6.00	1.92	13.1	2.34		0.687	0.536	0.512
Angles 	L6 × 6 × 1‡	11.0			35.4	1.79	1.86	35.4	1.79	1.86
	L4 × 4 × $\frac{1}{2}$	3.75			5.52	1.21	1.18	5.52	1.21	1.18
	L3 × 3 × $\frac{1}{4}$	1.44			1.23	0.926	0.836	1.23	0.926	0.836
	L6 × 4 × $\frac{1}{2}$	4.75			17.3	1.91	1.98	6.22	1.14	0.981
	L5 × 3 × $\frac{1}{2}$	3.75			9.43	1.58	1.74	2.55	0.824	0.746
	L3 × 2 × $\frac{1}{4}$	1.19			1.09	0.953	0.980	0.390	0.569	0.487

Metric units (mm)

	Designation	Area mm²	Depth mm	Width mm	Axis X-X $\bar{I}_x$ 10^6 mm⁴	Axis X-X $\bar{k}_x$ mm	Axis X-X $\bar{y}$ mm	Axis Y-Y $\bar{I}_y$ 10^6 mm⁴	Axis Y-Y $\bar{k}_y$ mm	Axis Y-Y $\bar{x}$ mm
W Shapes (Wide-Flange Shapes) 	W460 × 113†	14400	462	279	554	196		63.3	66.3	
	W410 × 85	10800	417	181	316	171		17.9	40.6	
	W360 × 57.8	7230	358	172	160	149		11.1	39.4	
	W200 × 46.1	5880	203	203	45.8	88.1		15.4	51.3	
S Shapes (American Standard Shapes) 	S460 × 81.4†	10300	457	152	333	180		8.62	29.0	
	S310 × 47.3	6010	305	127	90.3	123		3.88	25.4	
	S250 × 37.8	4810	254	118	51.2	103		2.80	24.1	
	S150 × 18.6	2360	152	84.6	9.16	62.2		0.749	17.8	
C Shapes (American Standard Channels) 	C310 × 30.8†	3920	305	74.7	53.7	117		1.61	20.2	17.7
	C250 × 22.8	2890	254	66.0	28.0	98.3		0.945	18.1	16.1
	C200 × 17.1	2170	203	57.4	13.5	79.0		0.545	15.8	14.5
	C150 × 12.2	1540	152	48.8	5.45	59.4		0.286	13.6	13.0
Angles 	L152 × 152 × 25.4‡	7100			14.7	45.5	47.2	14.7	45.5	47.2
	L102 × 102 × 12.7	2420			2.30	30.7	30.0	2.30	30.7	30.0
	L76 × 76 × 6.4	929			0.512	23.5	21.2	0.512	23.5	21.2
	L152 × 102 × 12.7	3060			7.20	48.5	50.3	2.59	29.0	24.9
	L127 × 76 × 12.7	2420			3.93	40.1	44.2	1.06	20.9	18.9
	L76 × 51 × 6.4	768			0.454	24.2	24.9	0.162	14.5	12.4

Chapter 5 Part II – 3-D Rigid Body

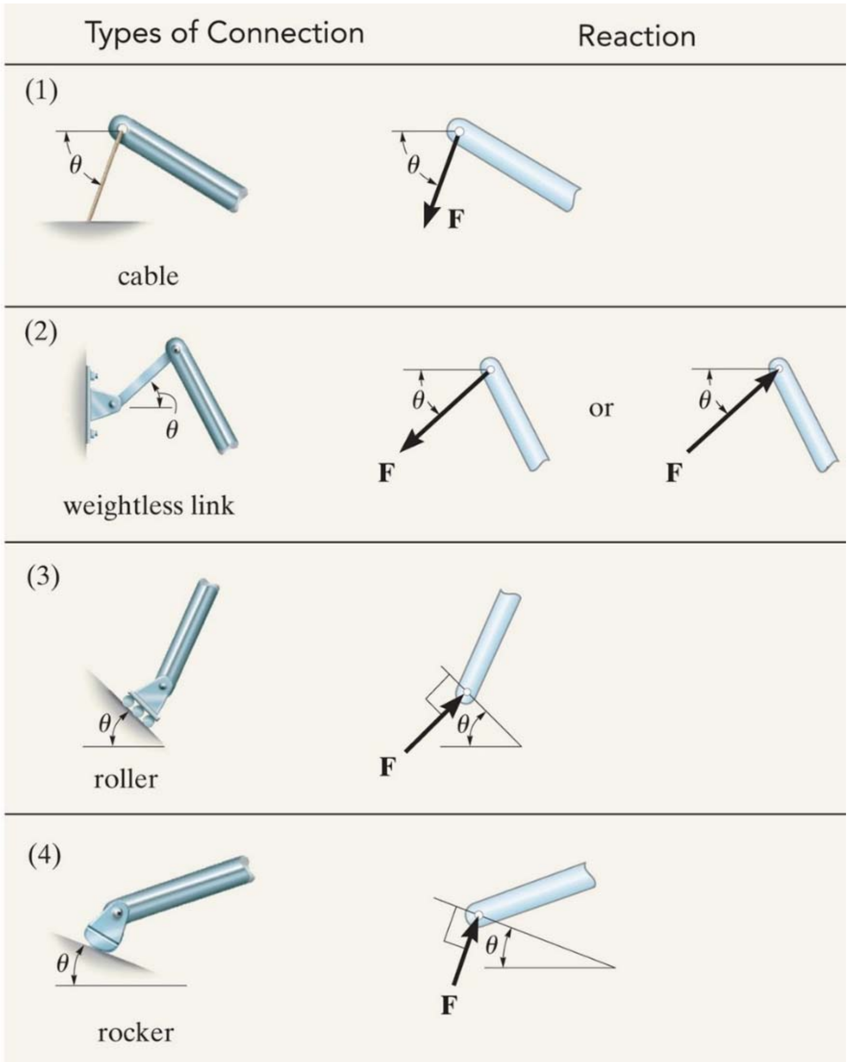
Chap 5.5-5.6

Equilibrium of a rigid body

Now we add the z-axis to the coordinate system!

How many Equations of Equilibriums?

Types of 2D connectors



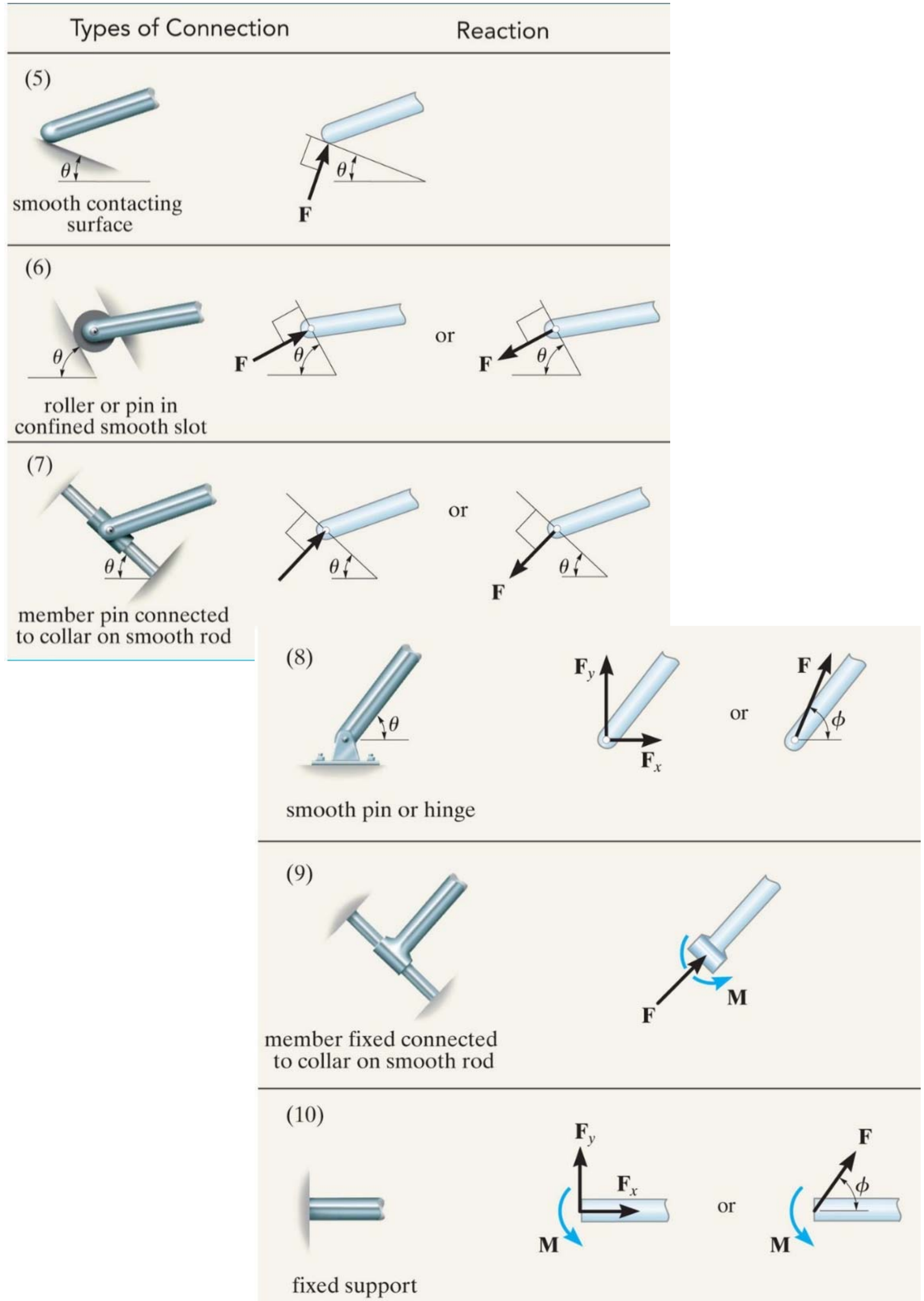


TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		
(2)  smooth surface support		
(3)  roller		

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

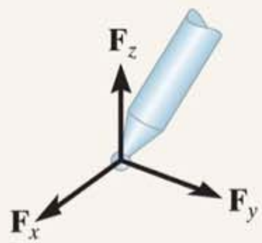
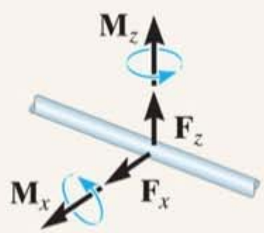
Types of Connection	Reaction	Number of Unknowns
(4)  ball and socket		
(5)  single journal bearing		

TABLE 5–2 Continued

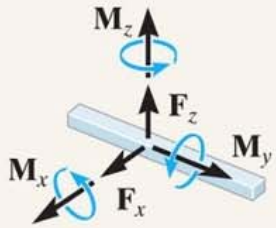
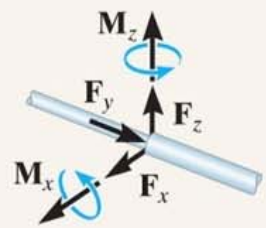
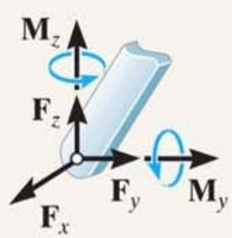
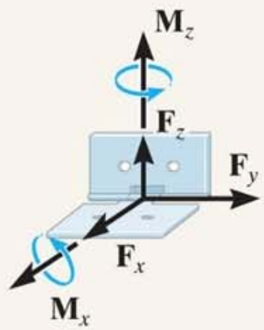
Types of Connection	Reaction	Number of Unknowns
(6)  single journal bearing with square shaft		
(7)  single thrust bearing		
(8)  single smooth pin		

TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(9)  single hinge		
(10)  fixed support