Statics - TAM 211

Lecture 37

April 20, 2018

Chap 9.5

Announcements

- Upcoming deadlines:
  - Monday (4/23)
    - Mastering Engineering Tutorial 15
  - Tuesday (4/24)
    - PL HW 14
  - Quiz 6
    - CBTF (4/25-27)
  - Written Assignment 6
    - **Wednesday May 2**

The Submarine Sports Car
Price $2,000,000
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Chapter 9 Part II – Fluid Pressure

Chap 9.5

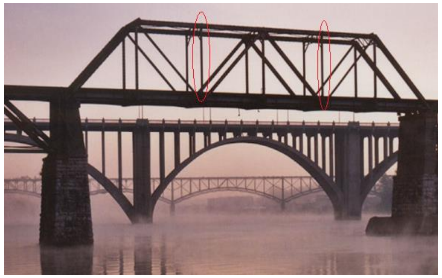
Goal and objective

- Present a method for finding the resultant force of a pressure loading caused by a fluid

Mechanics is a branch of the physical sciences that is concerned with the **state of rest or motion of bodies that are subjected to the action of forces**

SOLIDS

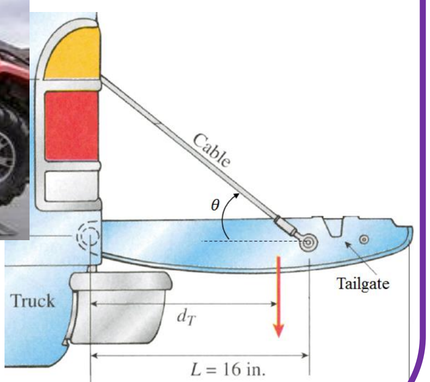
Rigid Bodies



TAM 210/211: Statics

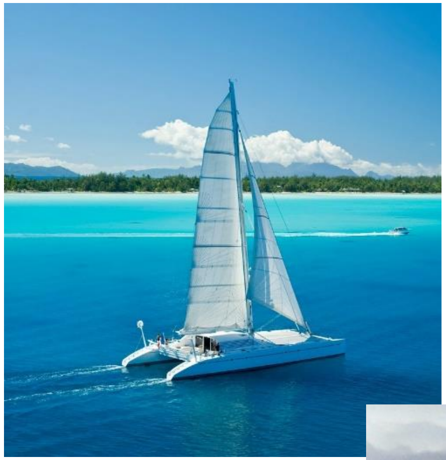
TAM212: Dynamics

Deformable Bodies



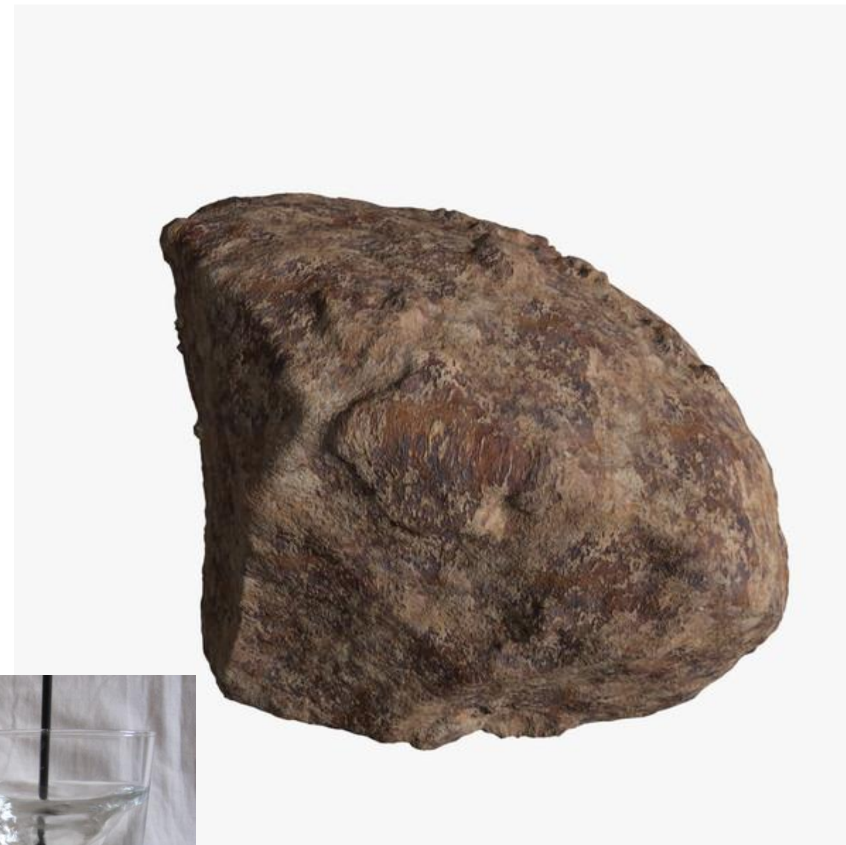
TAM 251: Solid Mechanics

FLUIDS



What Makes a Fluid or Solid?

Honey



Rock

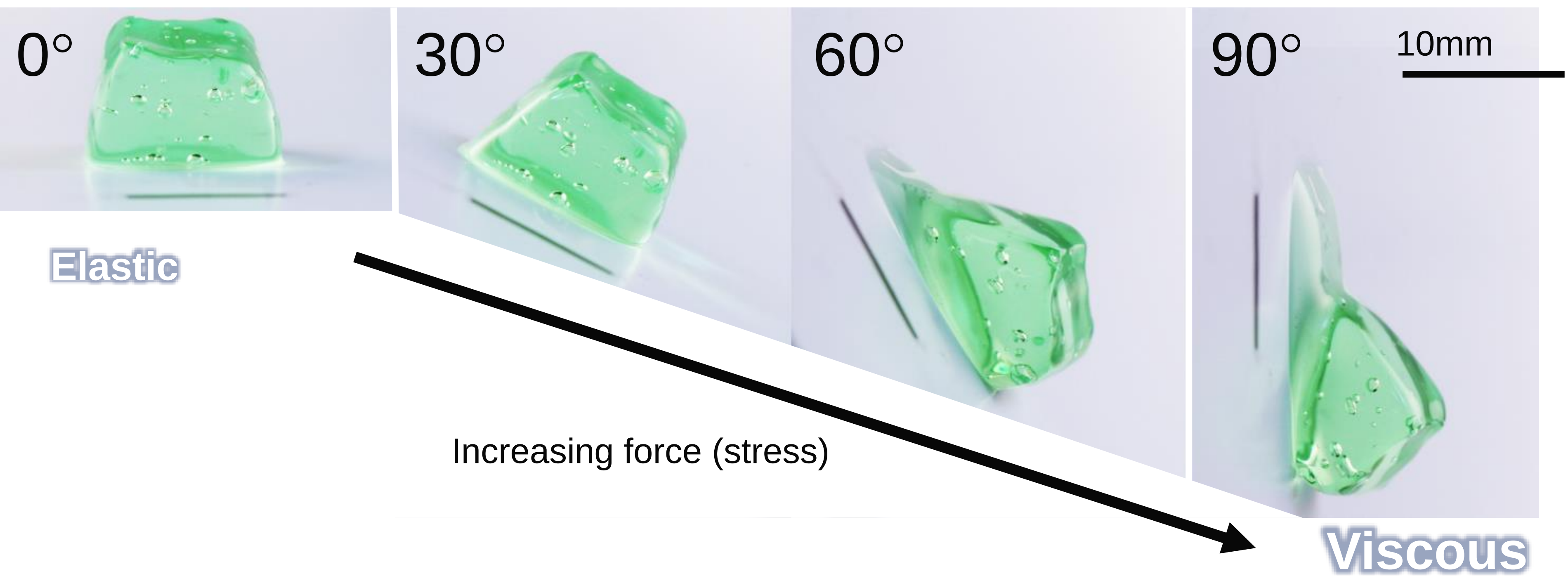
Viscosity

What is Sand?

Particles swollen with water – ‘Squishy Baff’

Aloe Gel

Yield-Stress Fluid

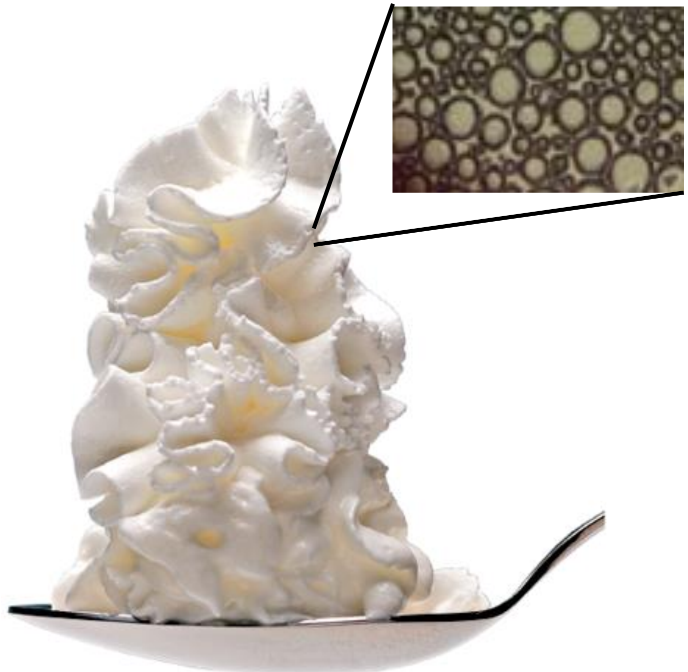


They act like a solid…

But they flow like a fluid once enough stress is applied.

Whipping cream (liquid) + air (gas) = Foam (solid)

with compressed air



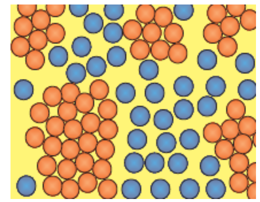
mechanical beating

They look like a fluid…

[Video]

cornstarch + water =

(small, hard particles)



But they may bear static loads like solids

Summary

Water takes shape of its container. Rock does not.

Water and rock fit classical definitions of fluid and solid, respectively

Sand and Squishy Baff take the shape of containers, but are composed of solid particles

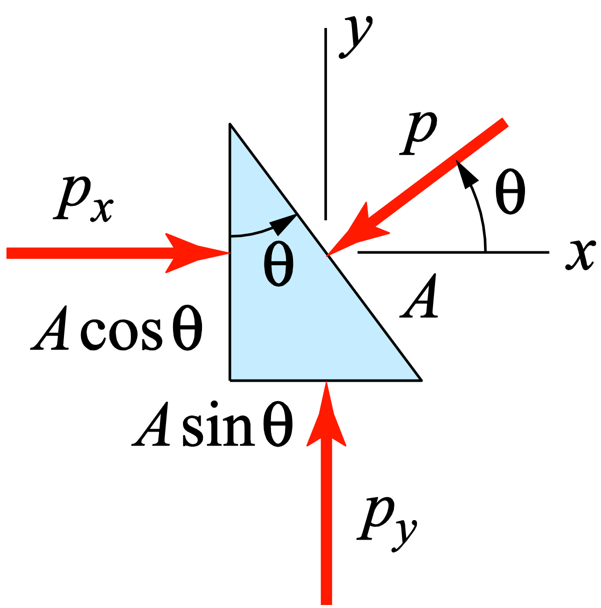
Sand and Squishy Baff are granular materials, which have properties of both fluids and solid

The aloe gel holds its shape and can trap air bubbles, until a certain amount of stress is applied.

Aloe gel is a suspension of particles, which is able to bear static load like a solid but behaves like a fluid when “enough” stress is applied.

Fluids

Pascal's law: A fluid at rest creates a pressure p at a point that is the ***same*** in ***all*** directions. Recall: $p = F/A$, or $F = pA$



For equilibrium of **an infinitesimal element**,

$$\Sigma F_x = 0: \quad p_x(A\cos\theta) - pA\cos\theta = 0 \quad \Rightarrow \quad p_x = p,$$

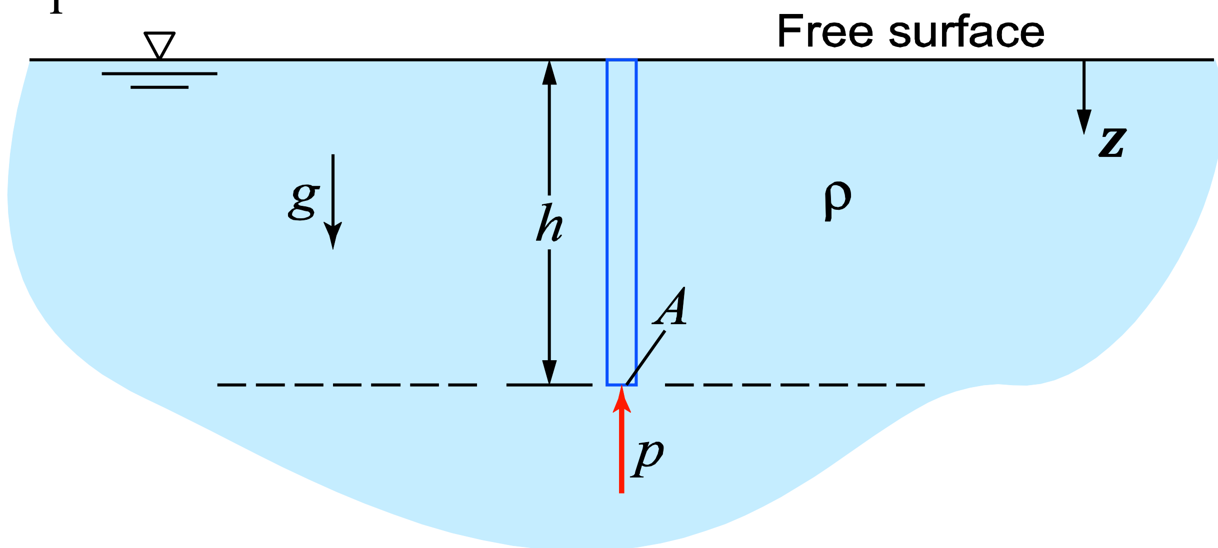
$$\Sigma F_y = 0: \quad p_y(A\sin\theta) - pA\sin\theta = 0 \quad \Rightarrow \quad p_y = p.$$

Thus, $p_x = p_y = p$ for any angle θ. The Pascal's law holds for fluids, but not solids.

Incompressible: An incompressible fluid is one for which the mass density ρ is independent of the pressure p. Liquids are generally considered incompressible. Gases are compressible, but may be approximated as incompressible if the pressure variations are relatively small.

Fluid Pressure

For an incompressible fluid at rest with mass density ρ, the pressure varies linearly with depth z



Summing forces in the vertical direction gives

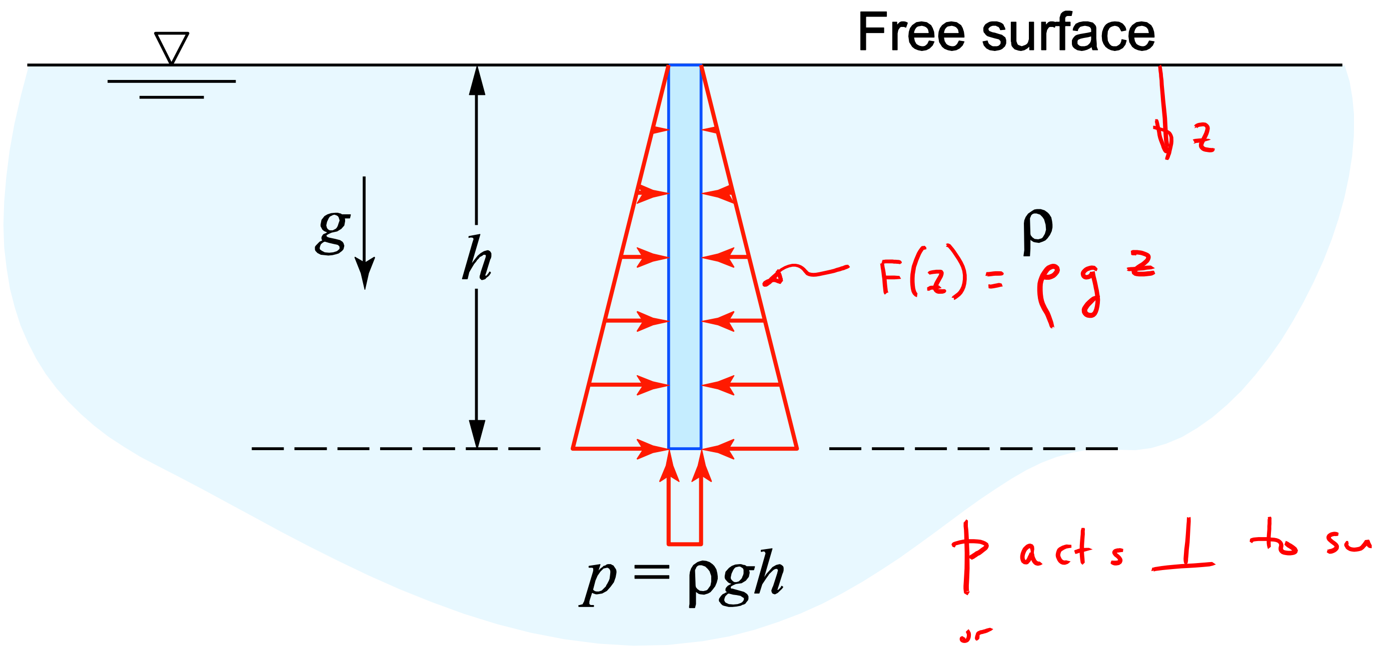
$$\Sigma F_{\substack{\cancel{x}\\ z}} = 0: \quad mg - pA = 0 \quad \Rightarrow \quad \big(\rho(Ah)\big)g - pA = 0 \quad \text{or} \quad p = \rho g h\,.$$

In general, this result is written as $\boxed{p = \rho\, g\, z = \gamma\, z}$

where $\gamma = \rho\, g$ is called the specific weight (weight per unit volume).

For fresh water: $\gamma = 62.4\ \text{lb/ft}^3\ (9810\ \text{N/m}^3)$

Observe that the pressure varies *linearly* from the free surface, and is *constant* along any horizontal plane (since h is constant):

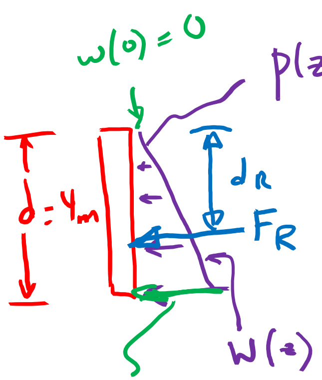


p acts ⊥ to surface

or

$F_{pressure}$

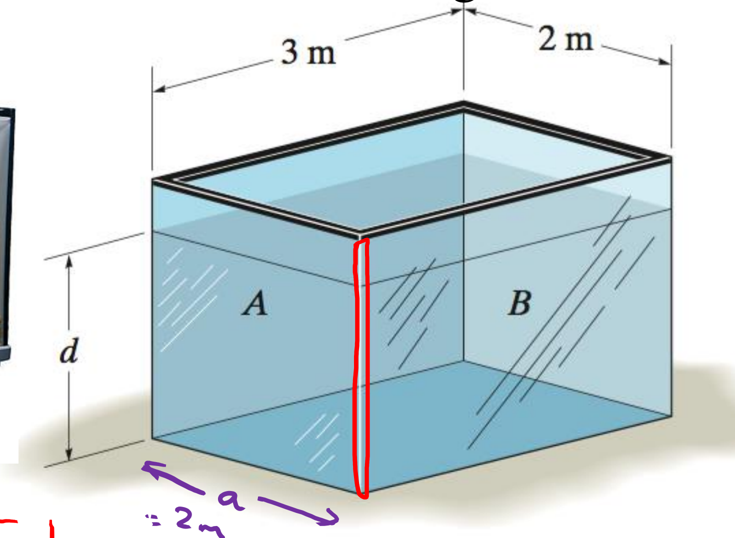
The tank is filled with water to a depth of $d = 4$ m. Determine the resultant force the water exerts on side A of the tank. ($\rho = 1000\ \mathrm{kg/m^3}$)



$p(z) = \rho g z = \dfrac{\text{Force}}{\text{Area}}$

$z = 0$

$+z$



$W(z) = \dfrac{\text{Force}}{\text{length}}$ (assuming uniform "depth of face (surface)")

$w(d) = \rho g d a$

$= p(z) \cdot a = \rho g z \, a$

$\boxed{\Rightarrow \text{Distributed load due to fluid pressure} = (\text{pressure}) * (\text{surface width})}$

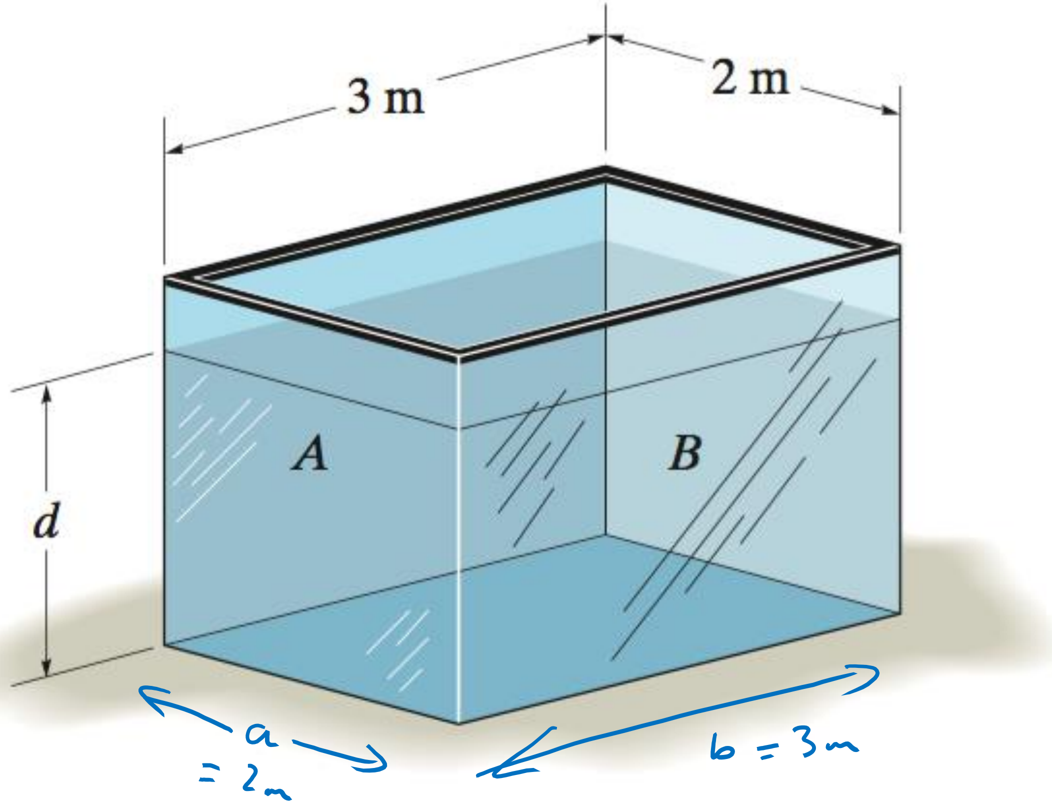
$F_R = \frac{1}{2}(d) \cdot w(d) = \frac{1}{2} d (\rho g d a) = \frac{1}{2} \rho g a d^2$

$= \frac{1}{2}\left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(2m)(4m)^2$

$\boxed{F_R = 157\ kN}$

$\boxed{d_R = ? = \frac{2}{3} d}$

The tank is filled with water to a depth of $d = 4$ m. Determine the resultant force the water exerts on side B of the tank. ($\rho = 1000\ \mathrm{kg/m^3}$)



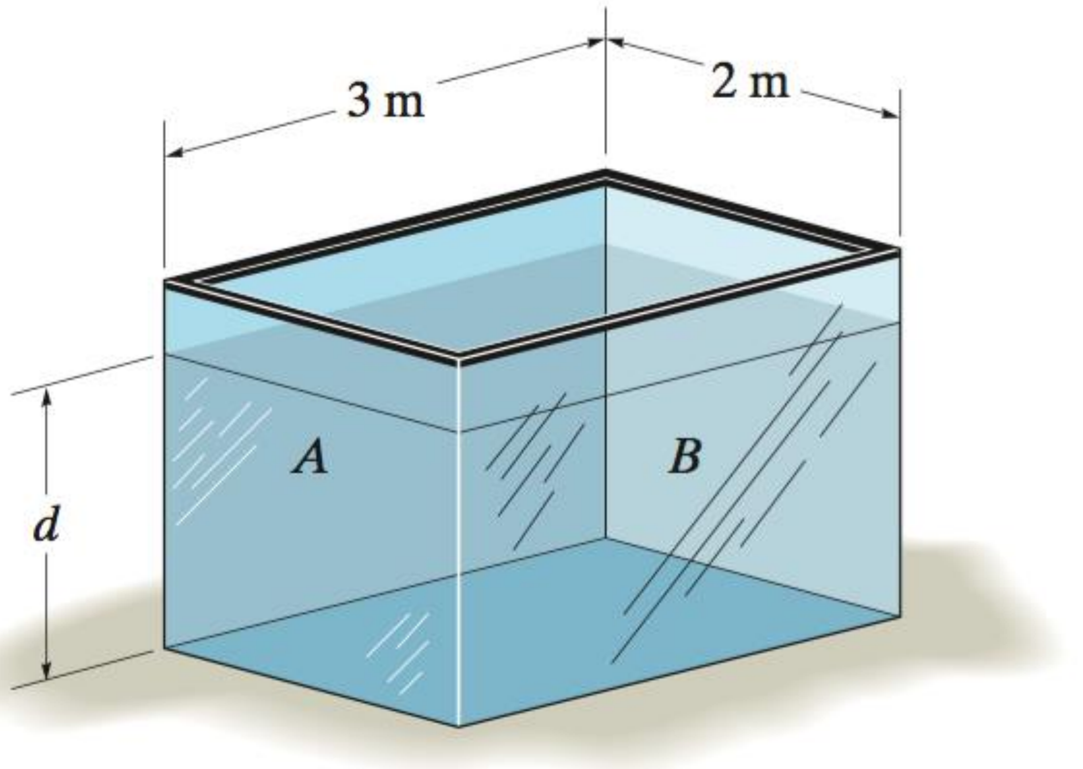
$$F_R = \frac{1}{2} d \cdot w(d)$$

$$= \frac{1}{2} d \left(\rho g d \cdot \text{face width} \right)$$

$b = 3m$

$$F_{R_B} = 235 \text{ kN}$$

If the tank is filled with oil instead, what depth d should it reach so that it creates the same resultant forces on side A. ($\rho = 900\ \mathrm{kg/m^3}$)



F_{water} $\qquad$ F_{oil}

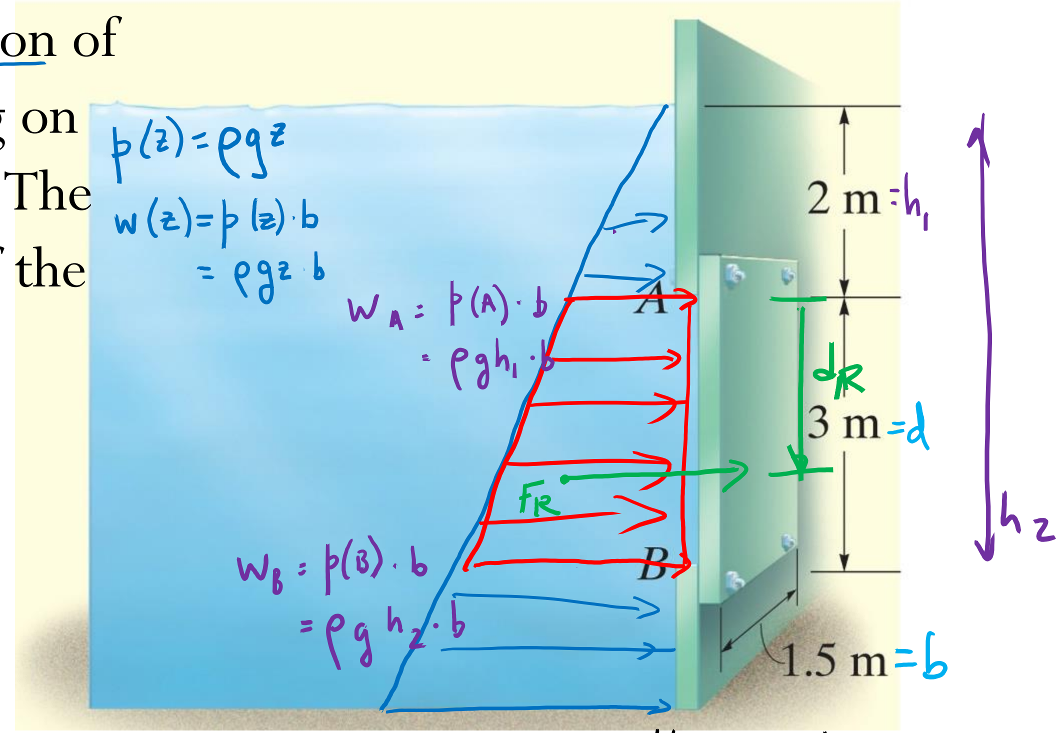
$$\cancel{\tfrac{1}{2}}\rho_w \cancel{g} \cancel{\alpha} d_w^2 = \cancel{\tfrac{1}{2}}\rho_o \cancel{g} \cancel{\alpha} d_o^2$$

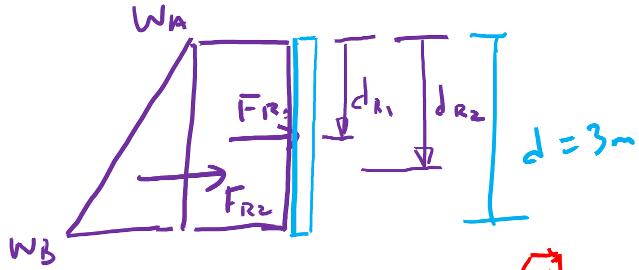
$$d_o = \sqrt{\frac{\rho_w}{\rho_o}}\, d_w$$

$$= 1.05\, d_w$$

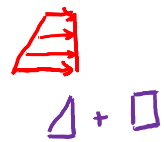
$$d_o = 4.22\ m$$

Determine the magnitude and location of the resultant hydrostatic force acting on the submerged rectangular plate *AB*. The plate has width 1.5m. The density of the water is 1000 kg/m^3





Convert load trapezoid into 2 simpler load shapes



$$F_R = F_{R_1} + F_{R_2} = 154.5\text{ N}$$

$$d_R = \frac{d_{R_1} F_{R_1} + d_{R_2} F_{R_2}}{F_R} \quad \text{from } \left(\sum M_R\right)_A = \sum M_A$$

$d_R = 1.71$ m below point A

Corrections to notes written in class:
For $w(z)$, w_A, w_B, I incorrectly had written $w = p \cdot d$, must be $w = p \cdot b$
width