# Statics - TAM 211

**Lecture 38**

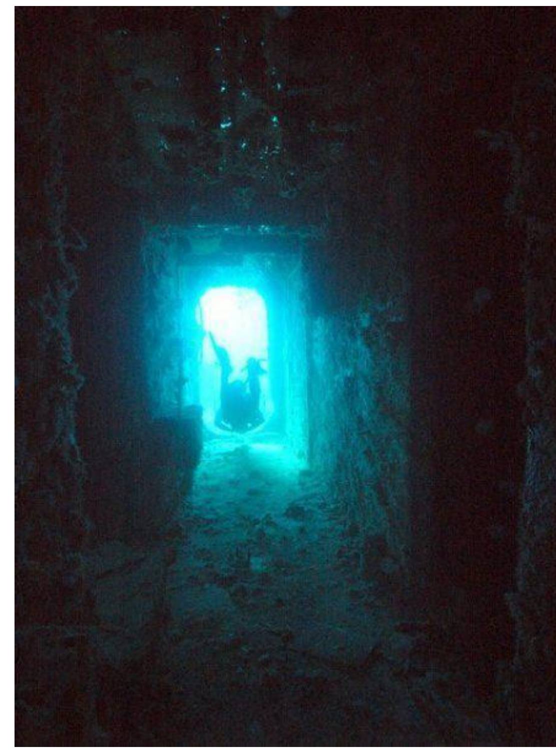
**April 23, 2018**

**Chap 9.5**

# Announcements

- Check ALL of your grades on Compass2g! Report issues
  - Exam grades will be posted later this week
- Upcoming deadlines:
  - Tuesday (4/24)
    - PL HW 14
  - Quiz 6
    - CBTF (W-F: 4/25-27)
    - CoG thru 3D Rigid Bodies: Lectures 29-36
  - Tuesday (5/1)
    - PL HW 15
  - Wednesday (5/2)
    - Written Assignment 6
  - Quiz 7
    - CBTF (Thurs-Tues: 5/3-8)
    - 50 minutes
    - Fluid Pressure - Virtual Work

There will be Discussion sections Next Week



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# Chapter 9 Part II – Fluid Pressure

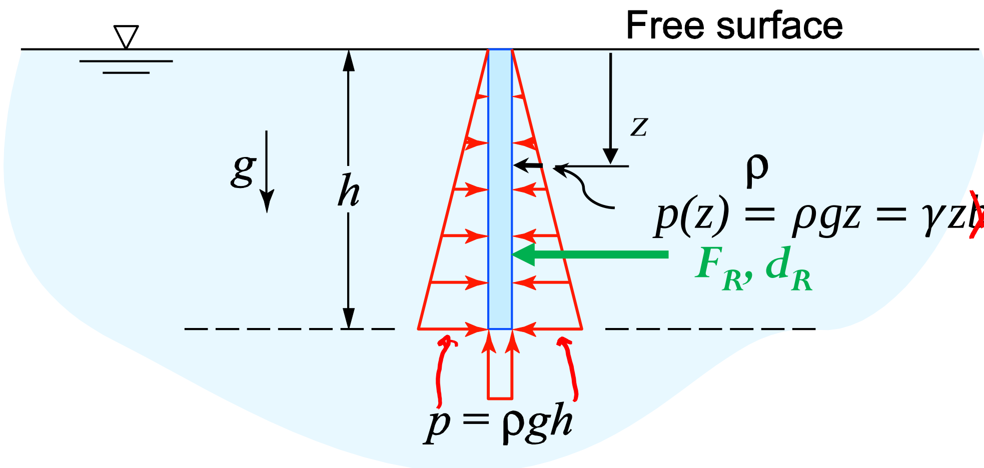
**Chap 9.5**

# Goal and objective

- Present a method for finding the resultant force of a pressure loading caused by a fluid

# Recap: Fluid Pressure

For an incompressible fluid at rest with mass density $\rho$, the pressure varies <u>linearly</u> with depth $z$




Object width is $b$.

$p(z) = \rho g z = \gamma z b$, $w(z) = \rho g z b = \gamma z b$

$p = \left[\frac{Force}{Area}\right]$

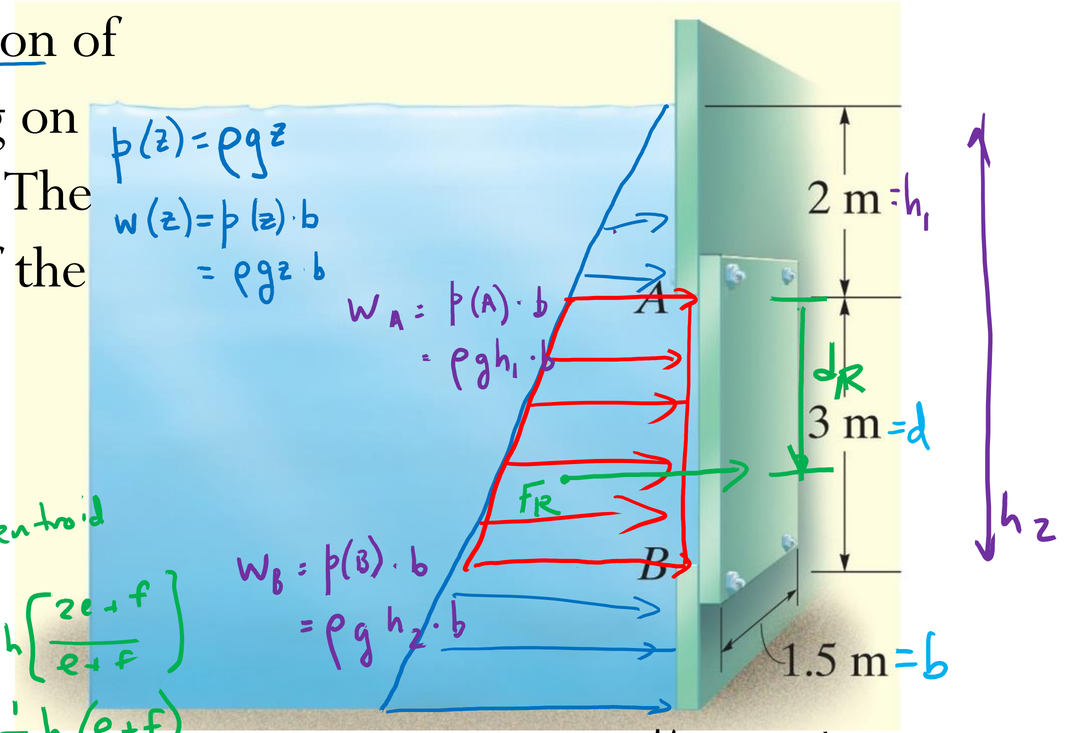
- $p(z) = \rho\, g\, z = \gamma\, z$

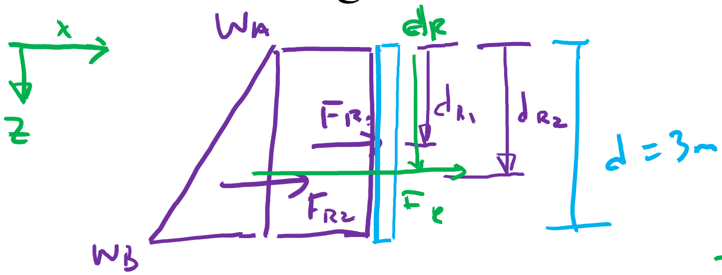
  where $\gamma = \rho\, g$ is called the specific weight (weight per unit volume).

  For fresh water: $\gamma$ =62.4 lb/ft$^3$ (9810 N/m$^3$), $\rho$ = 1000kg/m$^3$

- Pressure $p(z)$ or force due to pressure $F_R$ are always perpendicular to the object's surface.
- Distributed load due to fluid pressure at depth z is due to pressure and uniform width of surface: $w(z) = p(z) \cdot b = \rho g z b = \gamma z b$ $\left[\frac{Force}{length}\right]$
- Determine resultant force (magnitude and ~~direction~~ location): $F_R, d_R$ – location of $\vec{F}_R$
  - If water, this force is called hydrostatic force

Determine the magnitude and location of the resultant hydrostatic force acting on the submerged rectangular plate $AB$. The plate has width 1.5m. The density of the water is 1000 kg/m$^3$



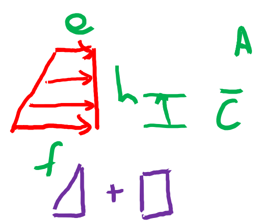

centroid

$$\bar{c} = \frac{1}{3}h\left[\frac{2e+f}{e+f}\right]$$

$$A = \frac{1}{2}h(e+f)$$

Corrections to notes written in class:
For $w(z)$, $w_A$, $w_B$, I incorrectly had written $w = p\cdot d$, must be $w = p \cdot b$ (↑ width)

Convert load trapezoid into 2 simpler load shapes



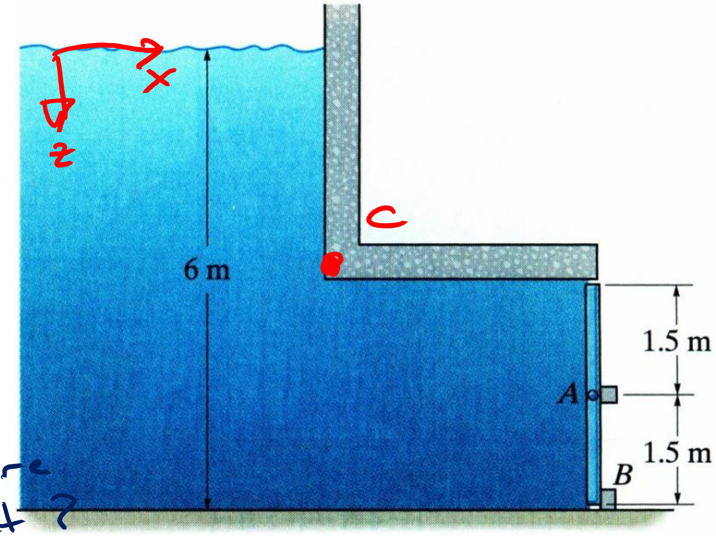

$$F_R = F_{R_1} + F_{R_2} = 154.5\ \text{N}$$

$$d_R = \frac{d_{R_1}F_{R_1} + d_{R_2}F_{R_2}}{F_R} \quad \Big\}\ \text{from } \left(\sum M_R\right)_A = \sum M_A$$

$d_R = 1.71$ m below point A

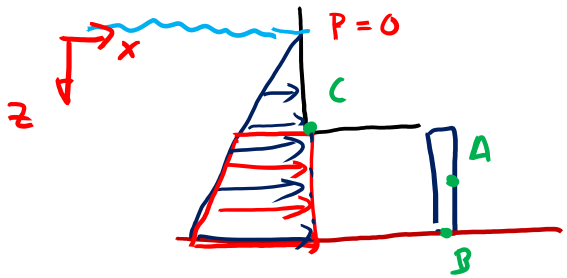
2m wide rectangular gate is pinned at its center A and prevented from rotating by block at B. Determine reactions at supports due to hydrostatic pressure. Water density is 1000 kg/m$^3$

$b = 2\,m$

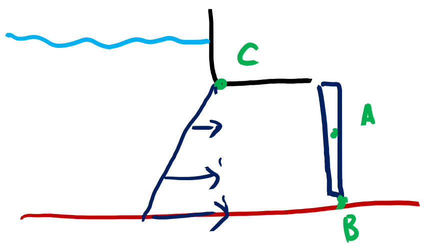



iclicker which FBD for pressure is correct?

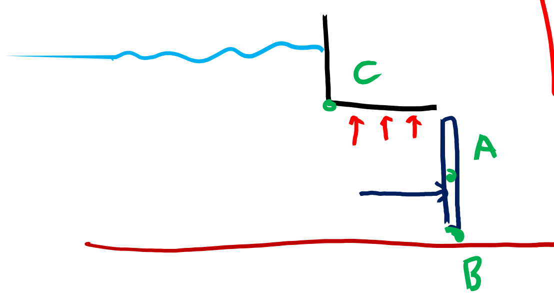
(A)

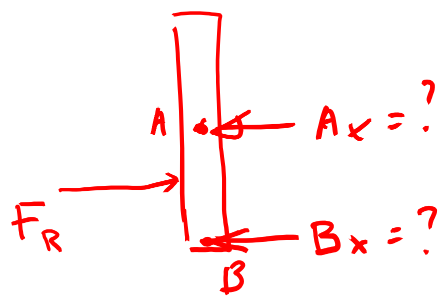



(B)



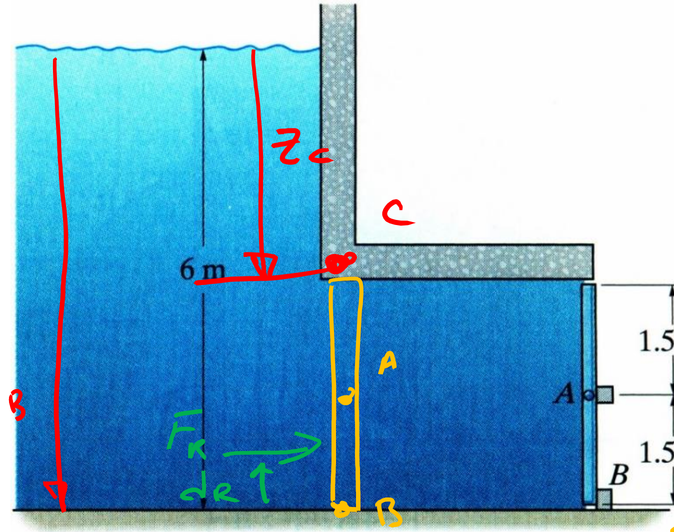

(C)



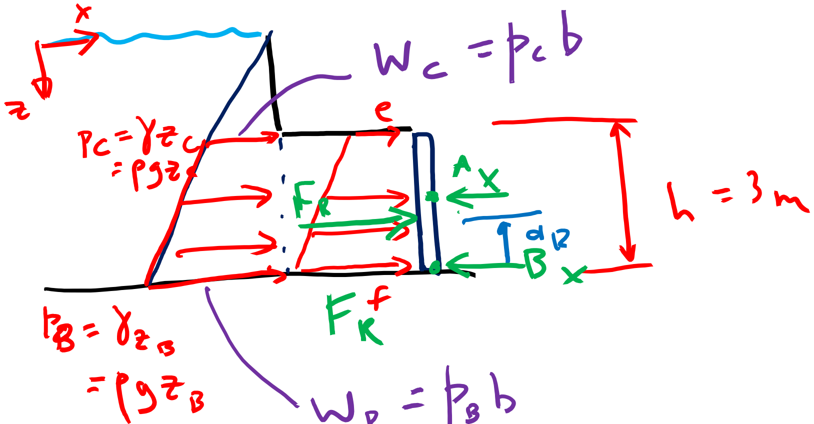



2m wide rectangular gate is pinned at its center A and prevented from rotating by block at B. Determine reactions at supports due to hydrostatic pressure. Water density is 1000 kg/m$^3$

$b = 2\,m$




orange same as pres loads original? Y/N
A B
iclicker




Trapezoid: $A = \frac{1}{2} h (e+f)$, $\bar{c} = \frac{1}{3} h \left( \frac{2e+f}{e+f} \right)$

W = Area shape

$W = p \cdot b$

$W_c = p_c b = \rho g z_c b$

$W_B = \rho g z_B b$

$F_R = \frac{1}{2} h (W_c + W_b) = \frac{1}{2} h (p_c + p_b)\,\mathbf{b} \Rightarrow \boxed{F_R = 284.9\,kN}$

verify

Now ok, needed to include "b" when write as pressure in the $F_R$ eqn.

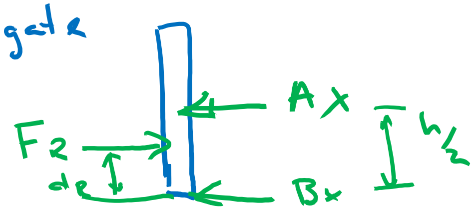
Centroid:

$d_R = \frac{1}{3} h \left( \frac{2W_c + W_B}{W_c + W_B} \right) = \frac{1}{3} h \left( \frac{2p_c + p_B}{p_c + p_B} \right) \Rightarrow \boxed{d_R = \frac{4}{3}\,m}$

ok $\left(\frac{b}{b}\right) = 1$ No "b" in this expression since b's cancel

Note where $d_R$ is relative to

FBD gate

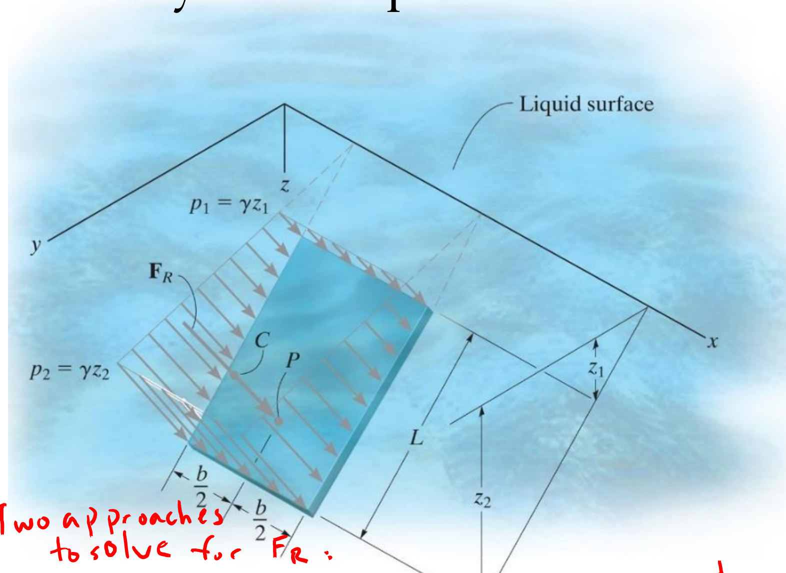


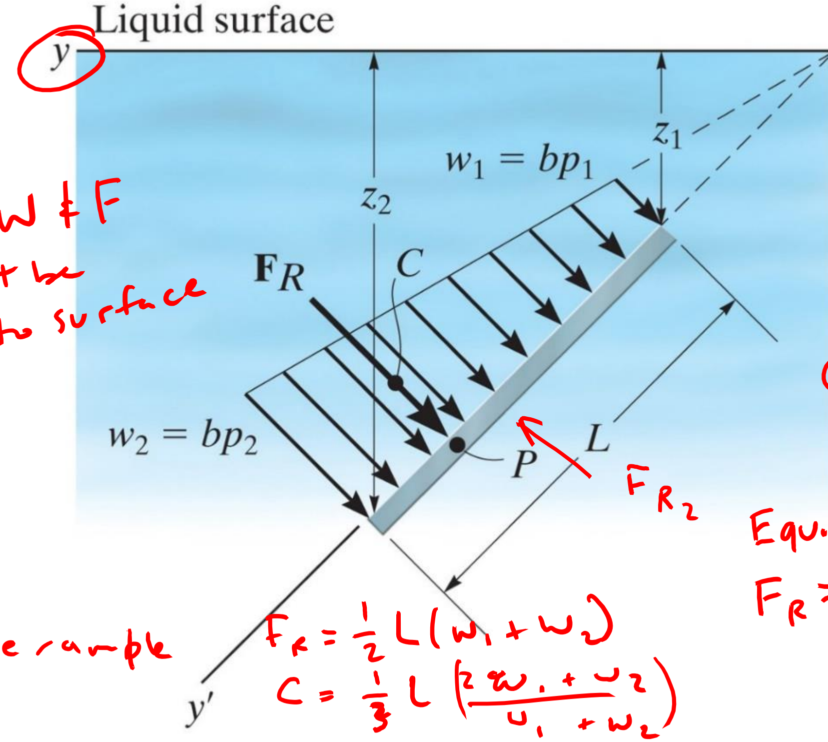

$+\circlearrowleft \sum M_B: -F_R d_R + A_x \frac{h}{2} = 0 \Rightarrow \boxed{A_x = 235.4\,kN}$

$\sum F_x: F_R - A_x - B_x = 0 \Rightarrow \boxed{B_x = 29.5\,kN}$

# Fluid Pressure of a flat plate with constant width

For an incompressible fluid at rest with mass density , the pressure varies linearly with depth **z**




p & w & F must be ⊥ to surface

Equilibrium $F_R = F_{R2}$

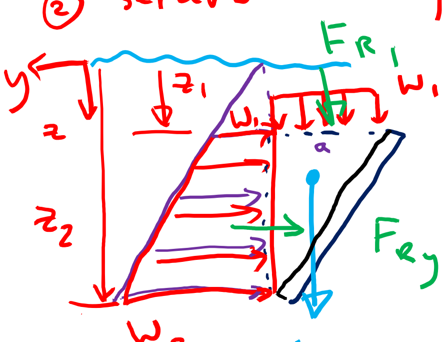
Two approaches to solve for $F_R$:

① Trapezoid $F_R$ trap, $C$ trap see previous example

$F_R = \frac{1}{2} L (w_1 + w_2)$

$C = \frac{1}{3} L \left( \frac{2w_1 + w_2}{w_1 + w_2} \right)$

② separate into y-z components




$F_{R_1} = w_1 a$

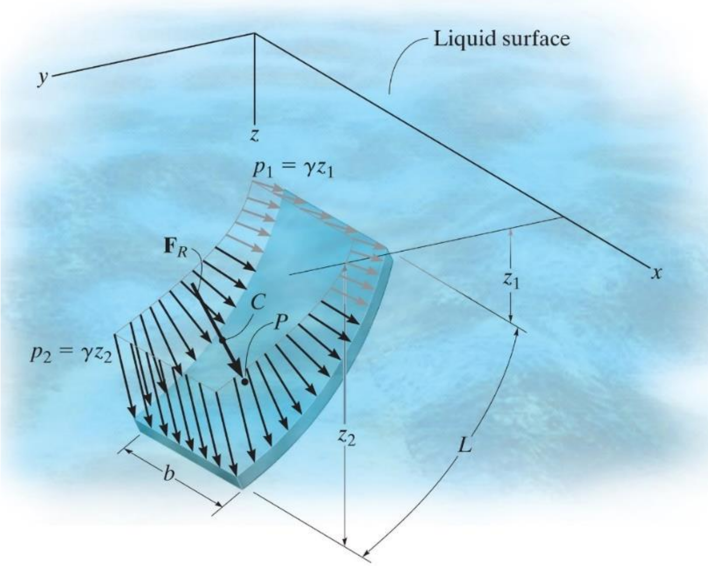
$F_{R_y} = F_{Trapezoid}$

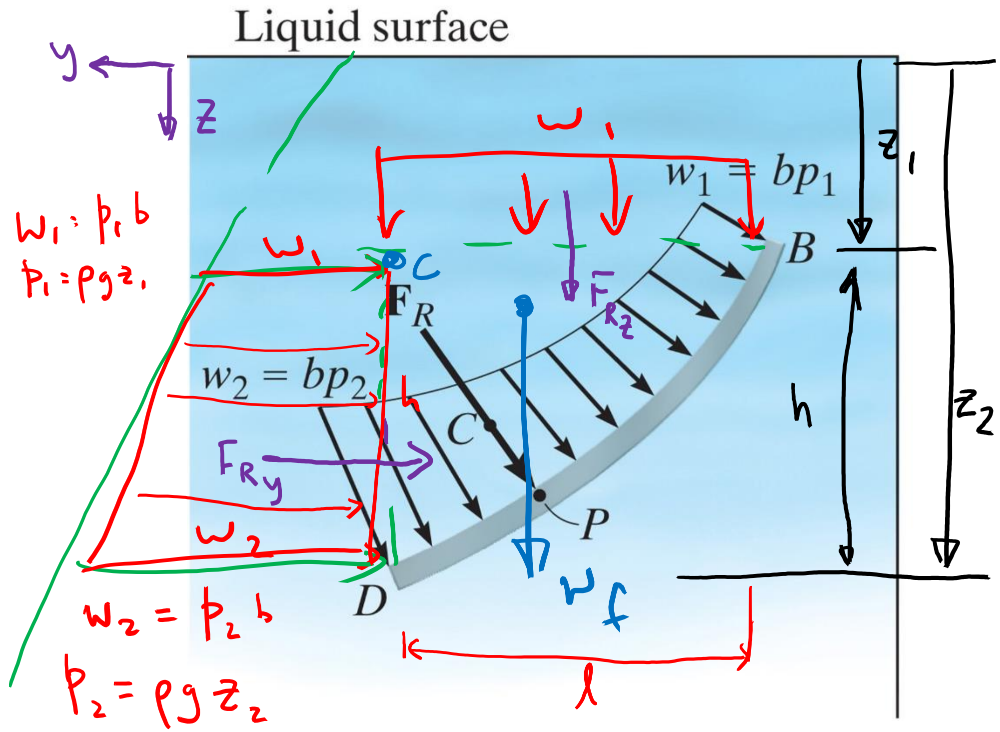
$F_R^2 = (F_{R_1} + W_f)^2 + (F_{R_y})^2$

$W_f = \gamma \cdot \forall ol = \gamma \cdot A \cdot b = \gamma \left(\frac{ac}{2}\right) b$

# Fluid Pressure of a curved plate with constant width

For an incompressible fluid at rest with mass density , the pressure varies linearly with depth $z$




$$\vec{F}_R = \sum \vec{F}_z + \sum \vec{F}_y$$

$$F_{R_y} = \frac{1}{2} h (e+f) = \frac{1}{2} h (w_1 + w_2)$$

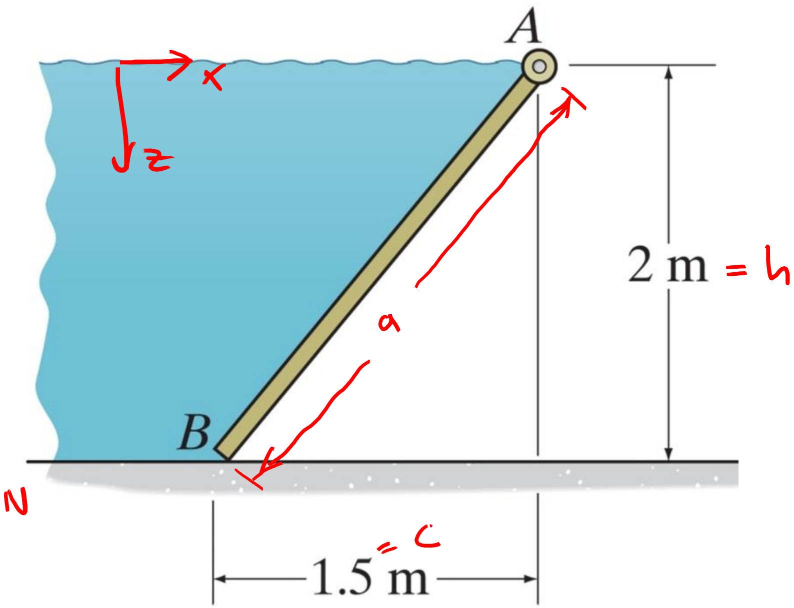
$$\sum F_z = F_{R_z} + W_f$$

$$F_{Rz} = w_1 \ell \quad , \quad W_f = \gamma \forall ol = \gamma A_{BCD}\, b$$

$$F_R = \sqrt{(F_{Rz} + W_f)^2 + F_{R_y}^2}$$

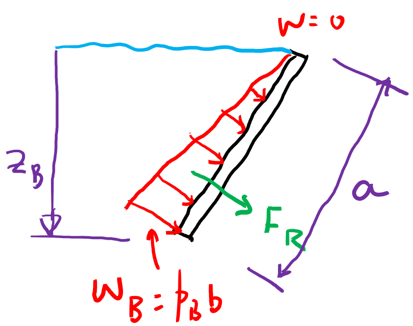
Determine the magnitude of the resultant hydrostatic force acting on the gate AB. The gate has width 1.5m.

$b = 1.5\,m$




2 solution approaches:

① Perpendicular load:

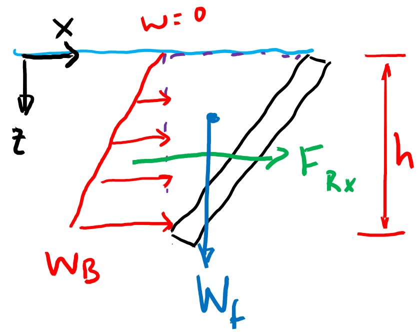



Triangular load

$$F_R = \frac{w_B a}{2} = \frac{p_B b a}{2}$$

$$F_R = \frac{\rho g z_B b a}{2} = 36.8\,kN$$

② Separate into x, z components:




Triangle load:

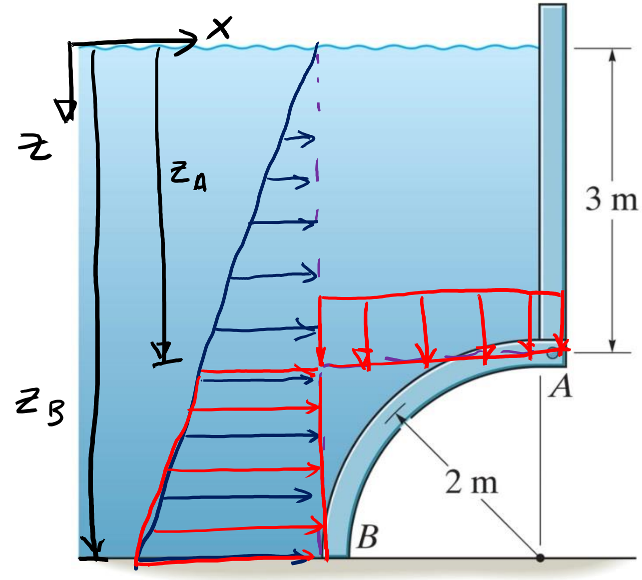
$$F_{Rx} = \frac{w_b h}{2} = \frac{\rho g b h^2}{2}, \quad W_f = \gamma \cdot Vol = \rho g A_{tri} b = \rho g \frac{ch}{2} b$$

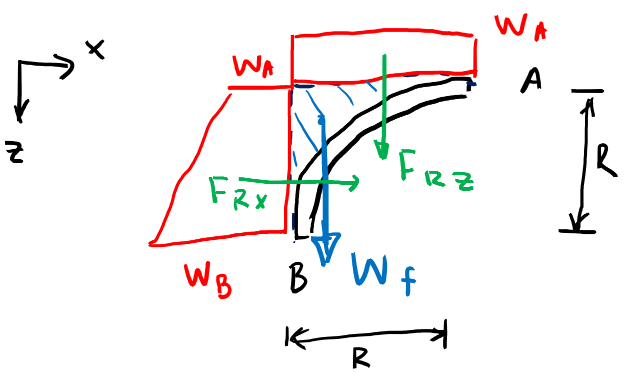
$$F_R = \sqrt{F_{Rx}^2 + W_f^2} = \frac{\rho g b h}{2}\sqrt{h^2+c^2}$$

$$F_R = \frac{\rho g h b a}{2}$$ ✓ same as before since $h = z_B$

The arched surface AB is shaped in the form of a quarter circle. If it is 8 m long, determine the horizontal and vertical components of the resultant force caused by the water acting on the surface.

$b = 8$ m




Rectangle: $F_{Rz} = W_A R = p_A b R = \rho g z_A b R = 470.9 \text{ kN}$

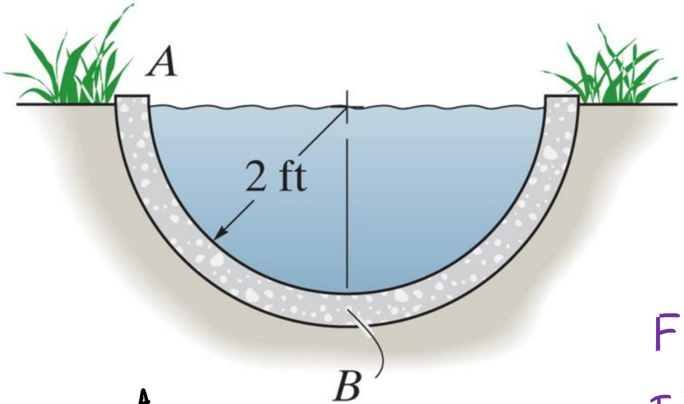
Weight of water: $W_f = \gamma \forall = \rho g A b \; , \; A = R^2 - \frac{\pi R^4}{2} \Rightarrow W_f = 67.4 \text{ kN}$

Trapezoid: $F_{Rx} = \frac{1}{2} R (w_A + w_B) = \frac{R}{2} b (p_A + p_B) = 627.8 \text{ kN}$

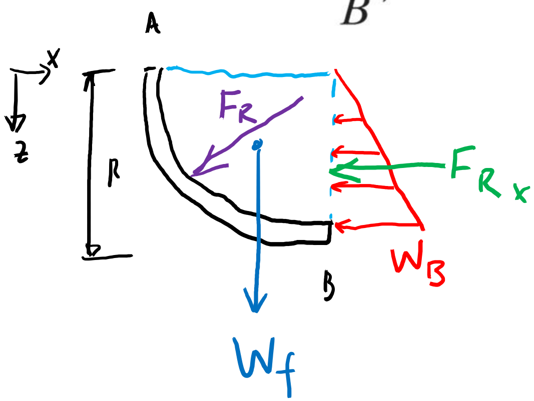
$\therefore \Sigma F_{vert} = F_{Rz} + W_f \Rightarrow \boxed{F_{vert} = 538.3 \text{ kN}}$

$\Sigma F_{hor} = F_{Rx} \Rightarrow \boxed{F_{hor} = 627.8 \text{ kN}}$

$F_R = \sqrt{F_V^2 + F_H^2} = 827.0 \text{ kN}$




The semicircular drainage pipe is filled with water. Determine the resultant force that the water exerts on the side AB of the pipe per foot of pipe length. The specific weight of the water is $\gamma = 62.4\ \text{lb/ft}^3$




$$F_{Rx} = \frac{W_B R}{2} = \frac{p_B b R}{2} = \frac{\gamma R^2 b}{2}$$

Triangle

$$\frac{F_{Rx}}{b} = \frac{\gamma R^2}{2} = \boxed{124.8\ \frac{\text{lb}}{\text{ft}}}$$

$$W_f = \gamma \forall = \gamma A b = \gamma\left(\frac{\pi R^2}{4}\right) b$$

$$\frac{W_f}{b} = \frac{\gamma \pi R^2}{4} = \boxed{196.6\ \frac{\text{lb}}{\text{ft}}}$$

$$F_R = \sqrt{F_{Rx}^2 + W_f^2} = \frac{\gamma R^2 b}{2}\sqrt{1 + \frac{\pi}{2}}$$

$$\frac{F_R}{b} = \frac{\gamma R^2}{2}\sqrt{1 + \frac{\pi}{2}}$$