Statics - TAM 211

Lecture 39

April 25, 2018

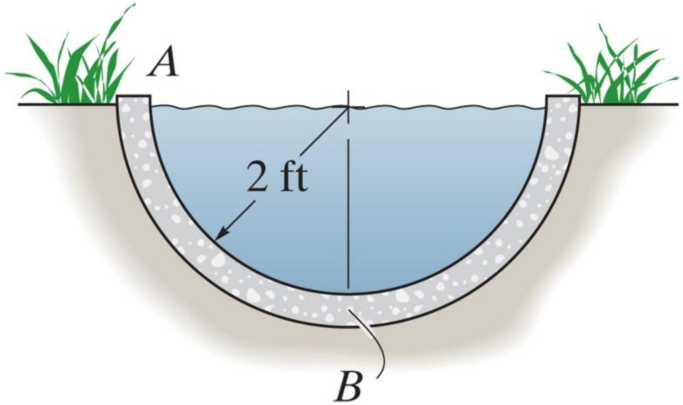
Announcements

- Check ALL of your grades on Compass2g. Report issues
  - Exam grades will be posted later this week
- There will be Discussion Sections next week
- Upcoming deadlines:
  - Quiz 6
    - CBTF (W-F: 4/25-27)
    - CoG thru 3D Rigid Bodies: Lectures 29-36
  - Tuesday (5/1)
    - PL HW 15
  - Wednesday (5/2)
    - Written Assignment 6
  - Quiz 7
    - CBTF (Thurs-Tues: 5/3-8)
    - 50 minutes
    - Fluid Pressure - Virtual Work

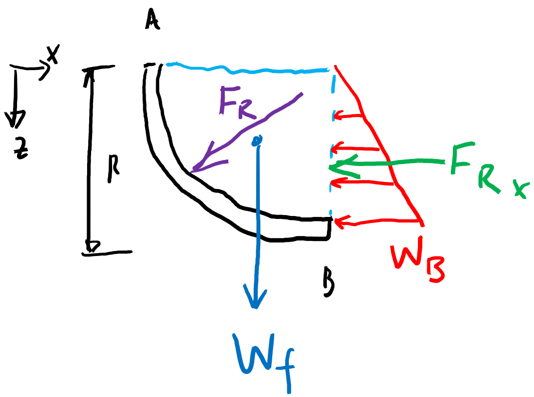
Chart of centroid locations for different geometries - Attachment in CBTF quiz

Chapter 9 Part II – Fluid Pressure

Chap 9.5



The semicircular drainage pipe is filled with water. Determine the resultant force that the water exerts on the side AB of the pipe per foot of pipe length. The specific weight of the water is $\gamma = 62.4\ \text{lb/ft}^3$



$$F_{Rx} = \frac{W_B R}{2} = \frac{p_B b R}{2} = \frac{\gamma R^2 b}{2} \qquad (p_B = \gamma R)$$

Triangle

$$\frac{F_{Rx}}{b} = \frac{\gamma R^2}{2} = \boxed{124.8\ \frac{\text{lb}}{\text{ft}}}$$

$$W_f = \gamma \forall = \gamma A b = \gamma\left(\frac{\pi R^2}{4}\right) b$$

$$\frac{W_f}{b} = \frac{\gamma \pi R^2}{4} = \boxed{196.6\ \frac{\text{lb}}{\text{ft}}}$$

$$F_R = \sqrt{F_{Rx}^2 + W_f^2} = \frac{\gamma R^2 b}{2}\sqrt{1 + \frac{\pi}{2}}$$

$$\frac{F_R}{b} = \frac{\gamma R^2}{2}\sqrt{1 + \frac{\pi}{2}}$$

specific weight, density

$$\gamma = \rho g$$

$$p = \rho g z = \gamma z$$

$$p = \gamma R$$

Chapter 11: Virtual Work

Goals and Objectives

- Introduce the principle of virtual work
- Show how it applies to determining the equilibrium configuration of a series of pin-connected members

Aside: Recall from Physics: Energy, work and power

- Mechanical energy [joule (J)]:
  - Capacity of a body to do work
- Work [joule (J)]:
  - Energy change over a period of time
- Power [watt (W)]:
  - Rate at which work is done or energy is expended
- Joule = Watt * second

Aside: Mechanical energy [joule (J)]:

- Capacity of a body to do work
- Measure of the state of a body as to its ability to do work at an instant in time
- Kinetic energy:
  - Translational:
  - Rotational:
- Potential energy:
  - Gravitational:
  - Elastic:

$$KE_{trans} = \frac{1}{2} m v^2$$

$$KE_{rot} = \frac{1}{2} I_O \omega^2$$

$$PE_{grav} = mgh$$

$$PE_{elas} = \frac{1}{2} k x^2$$

Aside: Work [joule (J)]:

- Energy change over a period of time as a result of a force (or moment) acting through a translational (or rotational) displacement

$$U_{trans} = \int_{r1}^{r2} F\, dr \qquad\qquad U_{rot} = \int_{\theta 1}^{\theta 2} M\, d\theta$$

- Measure of energy flow from one body to another
  - Requires time to elapse
  - e.g., Energy flows from A to B ⟶ A does work on B

- ~~Power~~ Work generated by a force (or moment) is the dot product of the force and translational (rotational - angular) ~~velocity~~ displacement at the point of application of the force

$$U_{trans} = \boldsymbol{F} \bullet \boldsymbol{r} \qquad\qquad U_{rot} = \boldsymbol{M} \bullet \boldsymbol{\theta}$$

Aside: Power [watt (W)]:

- Rate at which work is done or energy is expended

$$P = \frac{dW}{dt}$$

- Alternatively, work is the integral of power (area under the power curve)

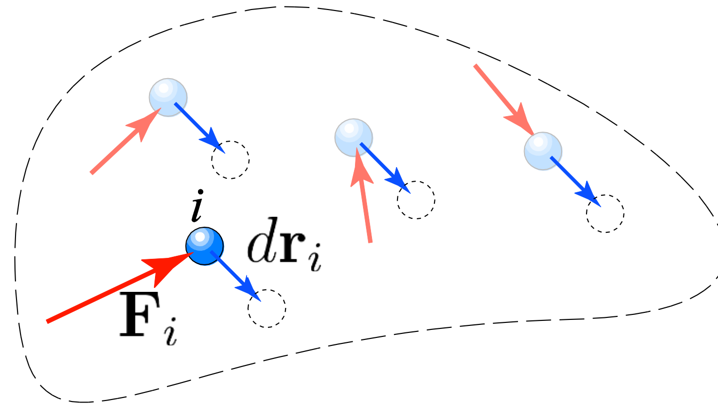
$$W = \int_{t1}^{t2} P dt$$

- Power generated by a force (or moment) is the dot product of the force and translational (rotational - angular) velocity at the point of application of the force

$$P_{trans} = \boldsymbol{F} \cdot \boldsymbol{v} \qquad P_{rot} = \boldsymbol{M} \cdot \boldsymbol{\omega}$$

Definition of Work (U)

Work of a force

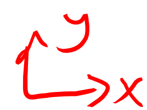


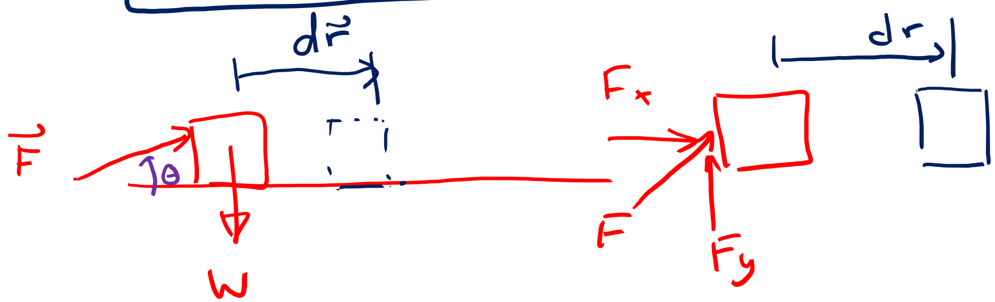
A force does work when it undergoes a displacement in the direction of the line of action.

The work dU produced by the force $\boldsymbol{F}$ when it undergoes a differential displacement $d\boldsymbol{r}$ is given by

$$dU = \mathbf{F} \cdot d\mathbf{r}$$

Scalar





ONLY consider forces in direction of displacement to do work:

$$dU = F_x\, dr$$

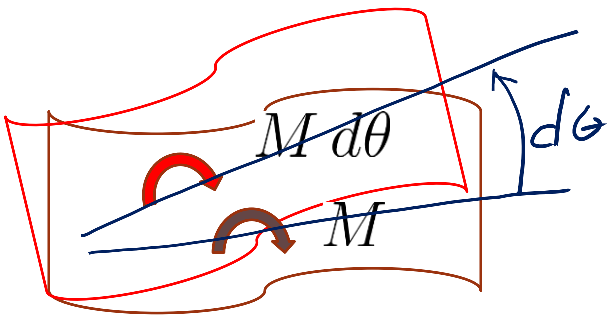
$$F_x = (F \cos\theta)\, dr$$

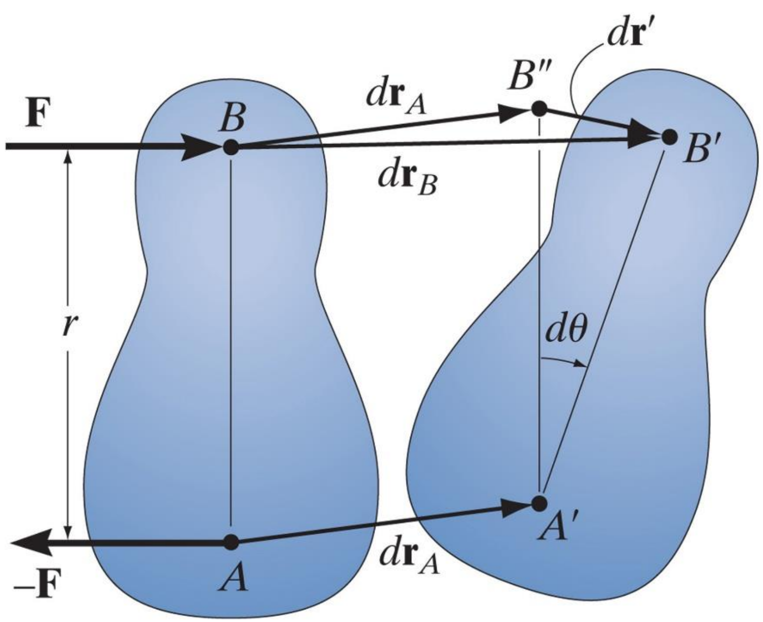
Note: W does no work because ⊥ to dr

Definition of Work (U)

Work of a couple moment

$$dU = M\mathbf{k} \cdot d\theta\,\mathbf{k} = M\,d\theta$$



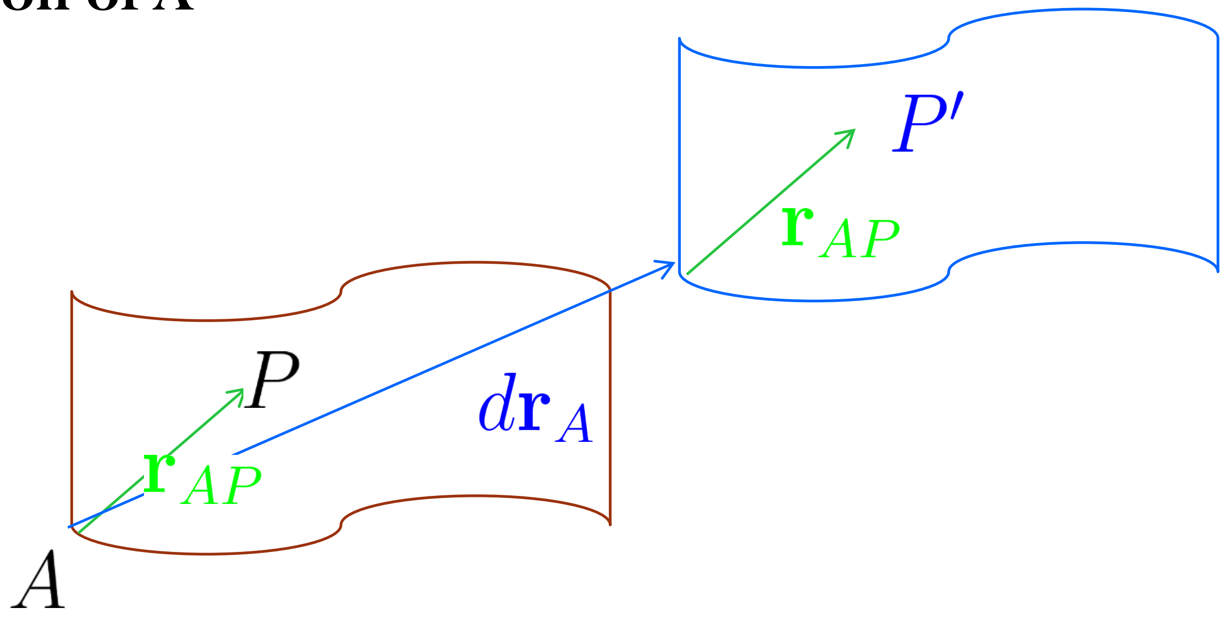


Incremental Displacement

Rigid body displacement of P = translation of A + rotation about A

$$d\mathbf{r}_P = \boxed{d\mathbf{r}_A} + d\theta\, \mathbf{k} \times \mathbf{r}_{AP}$$

Translation of A

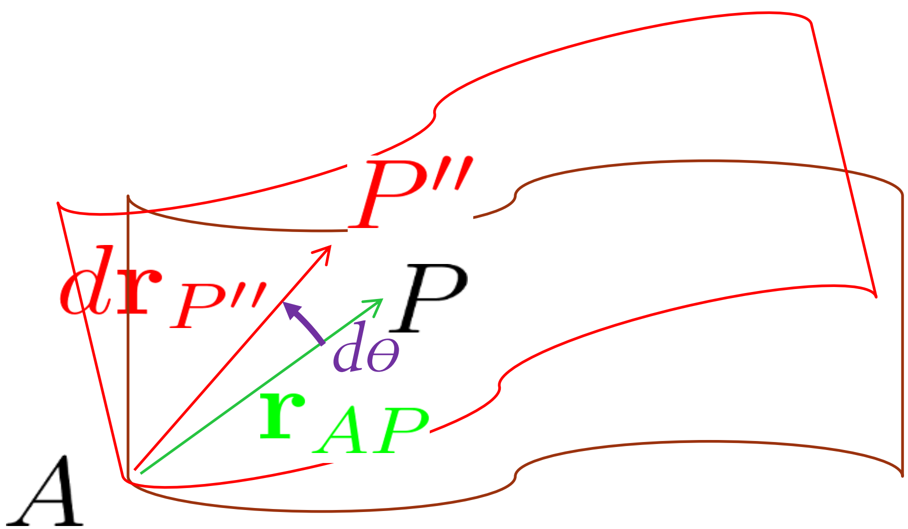


Incremental Displacement

Rigid body displacement of P = translation of A + rotation about A

$$d\mathbf{r}_P = d\mathbf{r}_A + \overbrace{d\theta\,\mathbf{k} \times \mathbf{r}_{AP}}^{d\mathbf{r}_{P''}}$$

Rotation about A

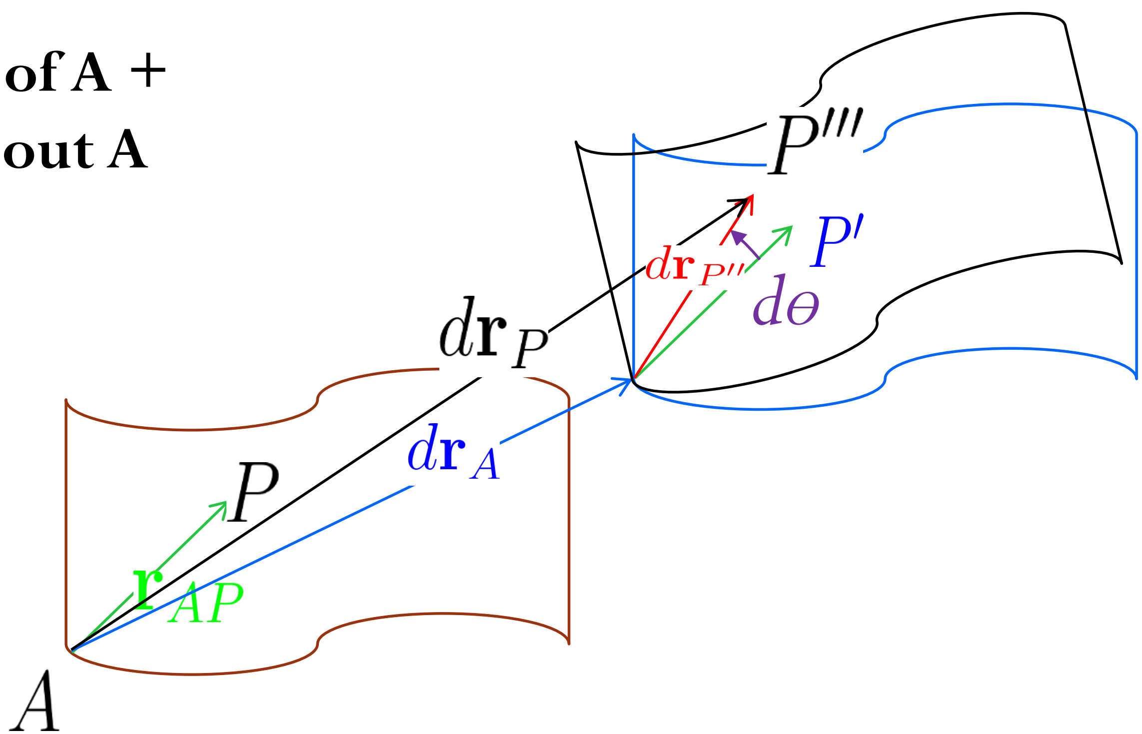


Incremental Displacement

Rigid body displacement of P = translation of A + rotation about A

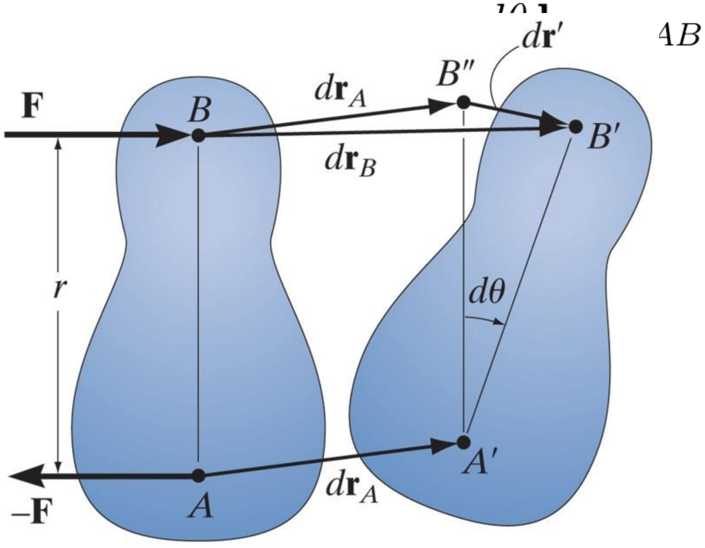
$$d\mathbf{r}_P = d\mathbf{r}_A + d\theta\,\mathbf{k} \times \mathbf{r}_{AP}$$

Translation of A +
Rotation about A



Definition of Work

Work of couple moment



$$d\mathbf{r}_P = d\mathbf{r}_A + d\theta\,\mathbf{k} \times \mathbf{r}_{AP}$$

$$\begin{aligned}
dU &= \sum_i \mathbf{F}_i \cdot d\mathbf{r}_i \\
&= \mathbf{F}_A \cdot d\mathbf{r}_A + \mathbf{F}_B \cdot d\mathbf{r}_B \\
&= -\mathbf{F} \cdot (d\mathbf{r}_A + d\theta\,\mathbf{k} \times \mathbf{r}_{AA}) + \mathbf{F} \cdot (d\mathbf{r}_A + d\theta\,\mathbf{k} \times \mathbf{r}_{AB}) \\
&= \mathbf{F} \cdot (d\theta\,\mathbf{k} \times \mathbf{r}_{AB}) \\
&= d\theta\,\mathbf{k} \cdot (\mathbf{r}_{AB} \times \mathbf{F}) \\
&= d\theta\,\mathbf{k} \cdot \mathbf{M}
\end{aligned}$$

$$\therefore\ dU = M\mathbf{k} \cdot d\theta\,\mathbf{k} = M\,d\theta$$

The couple forces do no work during the translation $d\boldsymbol{r}_A$

Work due to rotation