Statics - TAM 211

Lecture 39

April 25, 2018

Announcements

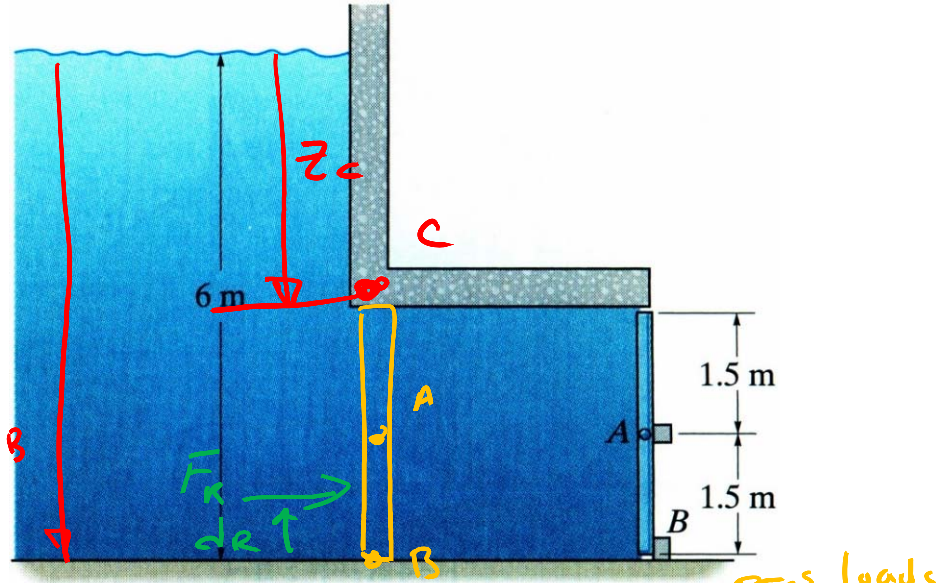
- Check ALL of your grades on Compass2g. Report issues
  - Exam grades will be posted later this week
- There will be Discussion Sections next week
- Upcoming deadlines:
  - Quiz 6
    - CBTF (W-F: 4/25-27)
    - CoG thru 3D Rigid Bodies: Lectures 29-36
  - Tuesday (5/1)
    - PL HW 15
  - Wednesday (5/2)
    - Written Assignment 6
  - Quiz 7
    - CBTF (Thurs-Tues: 5/3-8)
    - 50 minutes
    - Fluid Pressure - Virtual Work

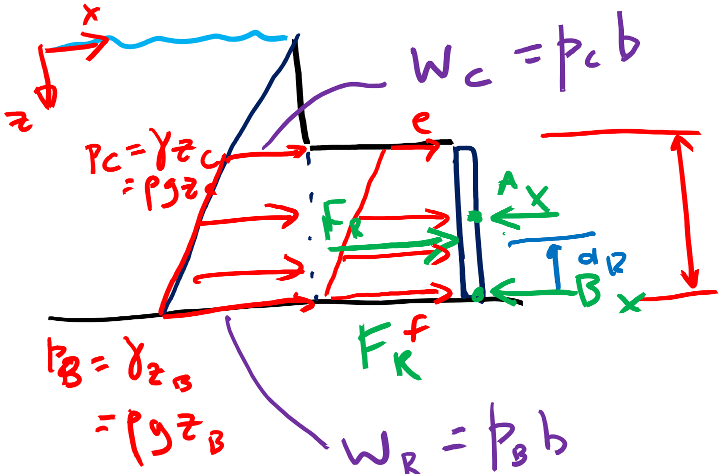
Chapter 9 Part II – Fluid Pressure

Chap 9.5

2m wide rectangular gate is pinned at its center A and prevented from rotating by block at B. Determine reactions at supports due to hydrostatic pressure. Water density is 1000 kg/m[3]

$b = 2\,m$





Now ok, needed to include "b" when write as pressure in the F_R eqn.

orange same as original? Y/N (A / B) pres loads iclicker

Trapezoid: $A = \frac{1}{2} h (e+f)$, $\bar{c} = \frac{1}{3} h \left(\frac{2e+f}{e+f} \right)$

$W = \text{Area shape}$ $\quad W = p \cdot b$

$F_R = \frac{1}{2} h (W_c + W_b) = \frac{1}{2} h (p_c + p_b)\,b \Rightarrow F_R = 284.9\,kN$ (verify)

Now ok, needed to include "b" when write as pressure in the F_R eqn.

$W_c = p_c b = \rho g z_c b$

$W_B = \rho g z_B b$

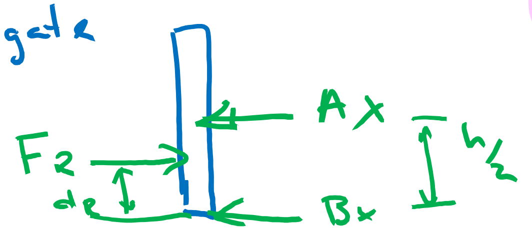
Centroid:

$d_R = \frac{1}{3} h \left(\frac{2W_c + W_B}{W_c + W_B} \right) = \frac{1}{3} h \left(\frac{2p_c + p_B}{p_c + p_B} \right) \Rightarrow d_R = \frac{4}{3}\,m$

ok $\left(\frac{b}{b}\right) = 1$ No "b" in this expression since b's cancel

Note where d_R is relative to

FBD gate



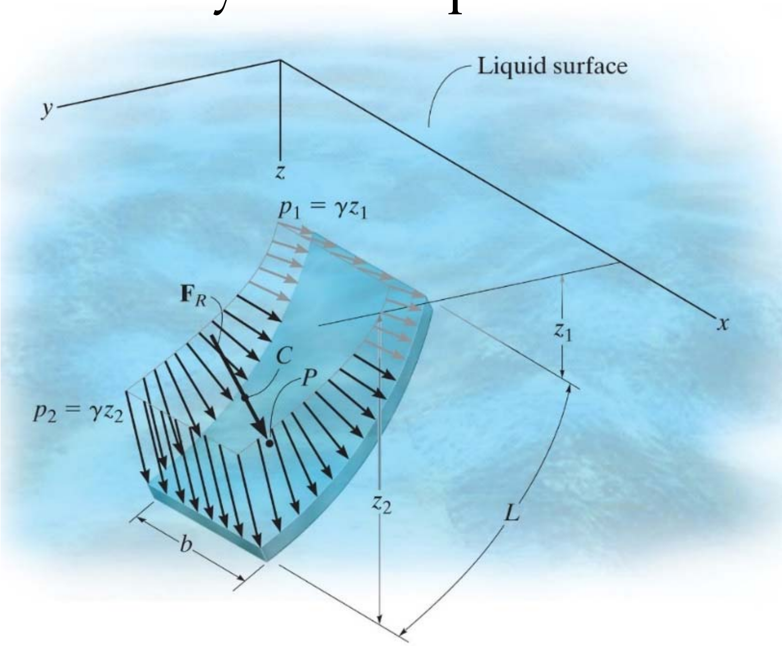
$+\circlearrowleft \sum M_B: \; -F_R d_R + A_x \frac{h}{2} = 0 \Rightarrow A_x = 235.4\,kN$

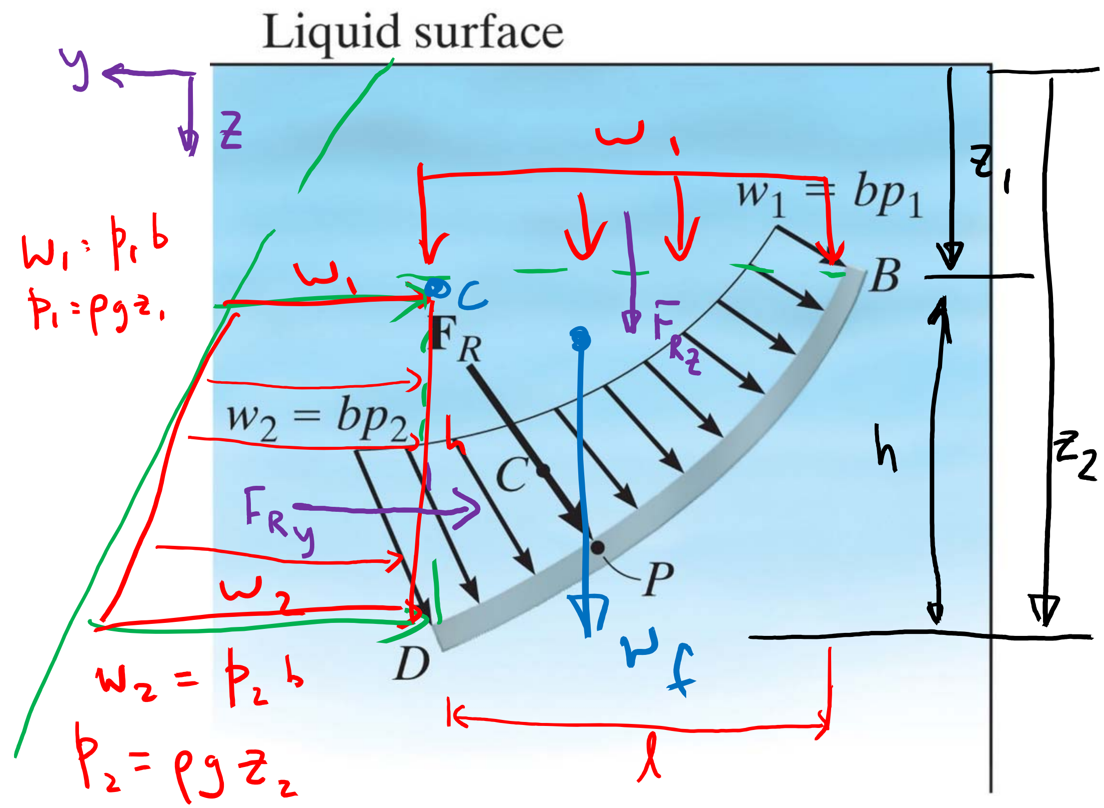
$\sum F_x: \; F_R - A_x - B_x = 0 \Rightarrow B_x = 29.5\,kN$

No "b" in this expression since b's cancel

Fluid Pressure of a curved plate with constant width

For an incompressible fluid at rest with mass density , the pressure varies linearly with depth *z*





$$\vec{F}_R = \Sigma \vec{F}_z + \Sigma \vec{F}_y$$

$$F_{R_y} = \frac{1}{2} h (e+f) = \frac{1}{2} h (w_1 + w_2)$$

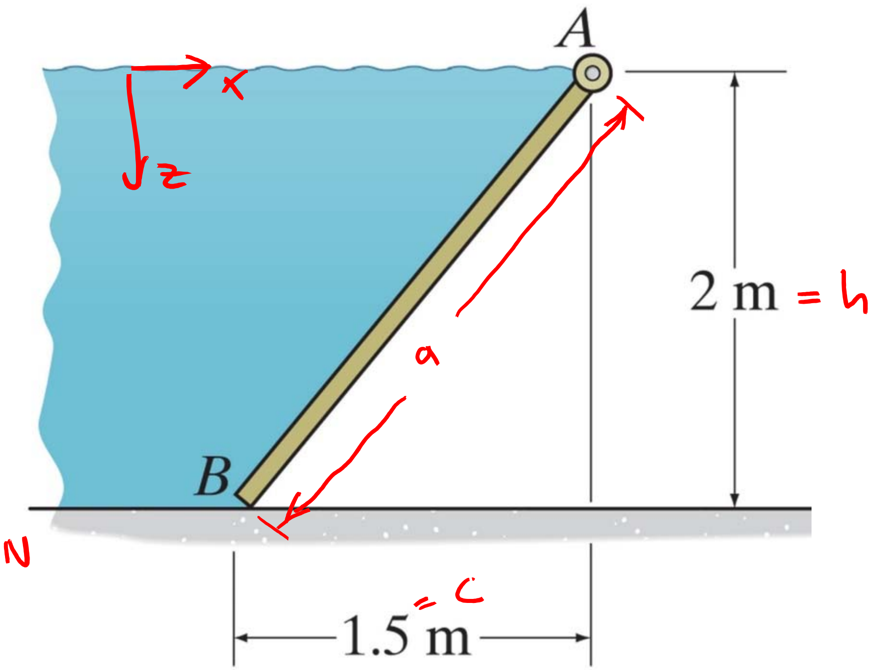
$$\Sigma F_z = F_{R_z} + W_f$$

$$F_{R_z} = w_1 l \quad , \quad W_f = \gamma \, Vol = \gamma A_{BCD} \, b$$

$$F_R = \sqrt{(F_{R_z} + W_f)^2 + F_{R_y}^2}$$

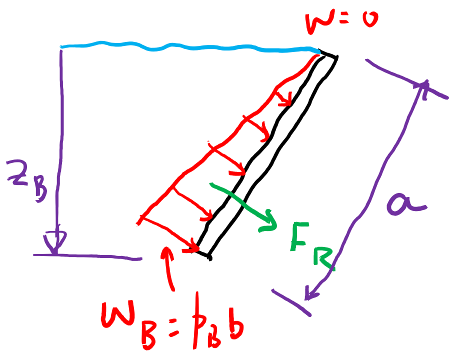
Determine the magnitude of the resultant hydrostatic force acting on the gate AB. The gate has width 1.5m.

b = 1.5m



2 solution approaches:

① Perpendicular load:

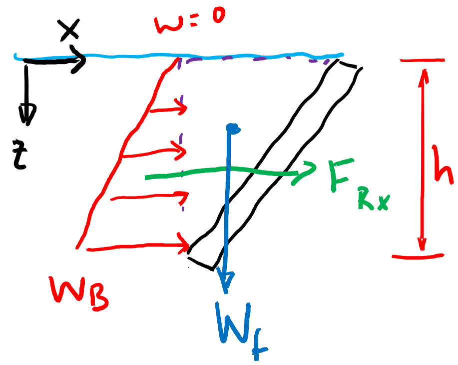


Triangular load

$$F_R = \frac{w_B a}{2} = \frac{p_B b a}{2}$$

$$F_R = \frac{\rho g z_B b a}{2} = 36.8\ \text{kN}$$

② Separate into x, z components:



Triangle load:

$$F_{Rx} = \frac{w_b h}{2} = \frac{\rho g b h^2}{2}, \quad W_f = \gamma \cdot Vol = \rho g A_{tri} b = \rho g \frac{ch}{2} b$$

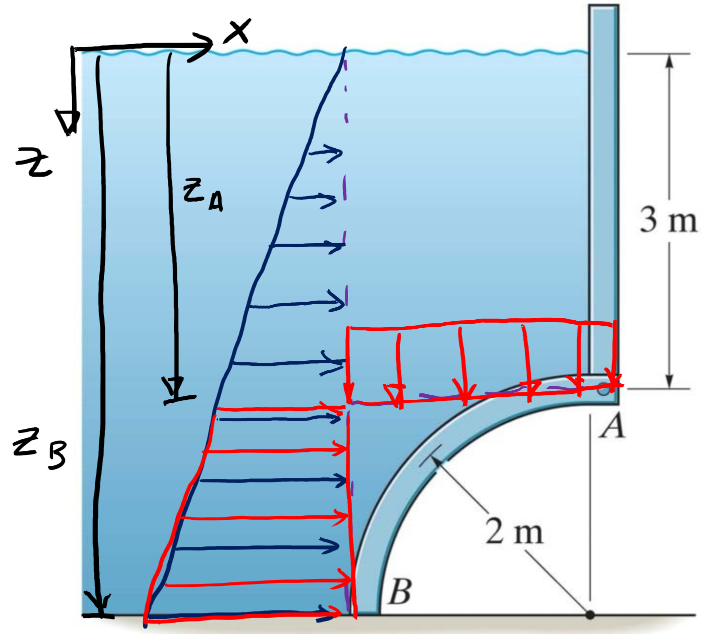
$$F_R = \sqrt{F_{Rx}^2 + W_f^2} = \frac{\rho g b h}{2}\sqrt{h^2+c^2}$$

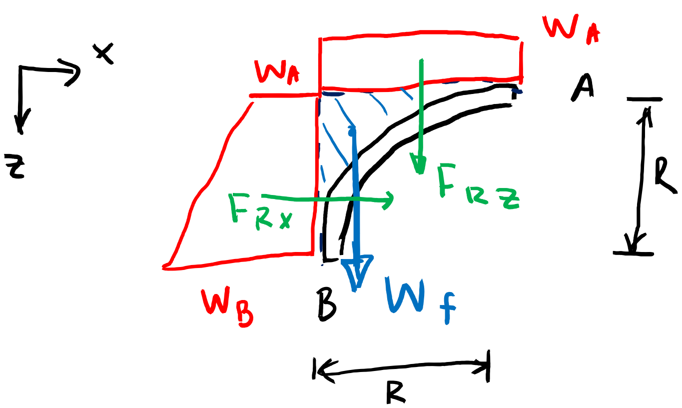
$$F_R = \frac{\rho g h b a}{2}$$

✓ same as before since $h = z_B$

The arched surface AB is shaped in the form of a quarter circle. If it is 8 m long, determine the horizontal and vertical components of the resultant force caused by the water acting on the surface.

$b = 8\text{ m}$





Rectangle: $F_{Rz} = W_A R = p_A b R = \rho g z_A b R = 470.9\text{ kN}$

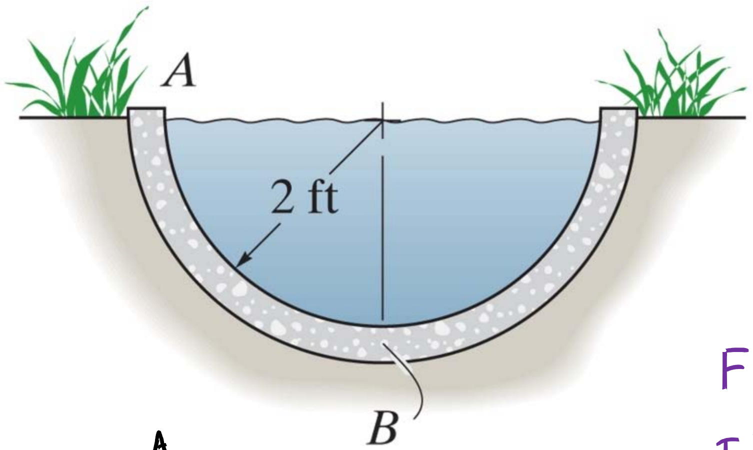
Weight of water: $W_f = \gamma V = \rho g A b$, $A = R^2 - \frac{\pi R^4}{2}$ $\Rightarrow$ $W_f = 67.4\text{ kN}$

Trapezoid: $F_{Rx} = \frac{1}{2} R (w_A + w_B) = \frac{R}{2} b (p_A + p_B) = 627.8\text{ kN}$

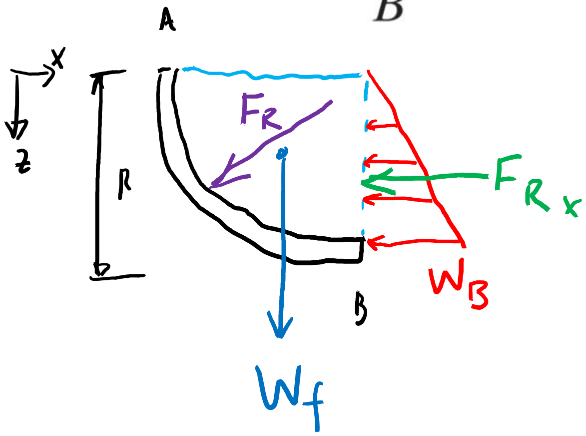
$\therefore$ $\Sigma F_{vert} = F_{Rz} + W_f \Rightarrow$ $F_{vert} = 538.3\text{ kN}$

$\Sigma F_{hor} = F_{Rx} \Rightarrow$ $F_{hor} = 627.8\text{ kN}$

$F_R = \sqrt{F_V^2 + F_H^2} = 827.0\text{ kN}$



The semicircular drainage pipe is filled with water. Determine the resultant force that the water exerts on the side AB of the pipe per foot of pipe length. The specific weight of the water is $\gamma = 62.4\ \text{lb/ft}^3$



$$F_{Rx} = \frac{w_B R}{2} = \frac{p_B b R}{2} = \frac{\gamma R^2 b}{2}$$

Triangle

$$\frac{F_{Rx}}{b} = \frac{\gamma R^2}{2} = 124.8 \frac{\text{lb}}{\text{ft}}$$

$$W_f = \gamma V = \gamma A b = \gamma\left(\frac{\pi R^2}{4}\right) b$$

$$\frac{W_f}{b} = \frac{\gamma \pi R^2}{4} = 196.6 \frac{\text{lb}}{\text{ft}}$$

$$F_R = \sqrt{F_{Rx}^2 + W_f^2} = \frac{\gamma R^2 b}{2} \sqrt{1 + \frac{\pi}{2}}$$

$$\frac{F_R}{b} = \frac{\gamma R^2}{2} \sqrt{1 + \frac{\pi}{2}}$$

Chapter 11: Virtual Work

Goals and Objectives

- Introduce the principle of virtual work

- Show how it applies to determining the equilibrium configuration of a series of pin-connected members

Energy, work and power

- Mechanical energy [joule (J)]:
  - Capacity of a body to do work
- Work [joule (J)]:
  - Energy change over a period of time
- Power [watt (W)]:
  - Rate at which work is done or energy is expended
- Joule = Watt * second

Mechanical energy [joule (J)]:

- Capacity of a body to do work

- Measure of the state of a body as to its ability to do work at an instant in time

- Kinetic energy:
  - Translational:
  - Rotational:

- Potential energy:
  - Gravitational:
  - Elastic:

$$KE_{trans} = \frac{1}{2}mv^2$$

$$KE_{rot} = \frac{1}{2}I_O\omega^2$$

$$PE_{grav} = mgh$$

$$PE_{elas} = \frac{1}{2}kx^2$$

Work [joule (J)]:

- Energy change over a period of time as a result of a force (or moment) acting through a translational (or rotational) displacement

$$U_{trans} = \int_{r1}^{r2} F\, dr \qquad U_{rot} = \int_{\theta 1}^{\theta 2} M\, d\theta$$

- Measure of energy flow from one body to another
  - Requires time to elapse
  - e.g., Energy flows from A to B $\rightarrow$ A does work on B

- Power generated by a force (or moment) is the dot product of the force and translational (rotational - angular) velocity at the point of application of the force

$$U_{trans} = \boldsymbol{F} \bullet \boldsymbol{r} \qquad U_{rot} = \boldsymbol{M} \bullet \boldsymbol{\theta}$$

Power [watt (W)]:

- Rate at which work is done or energy is expended

$$P = \frac{dW}{dt}$$

- Alternatively, work is the integral of power (area under the power curve)

$$W = \int_{t1}^{t2} P dt$$

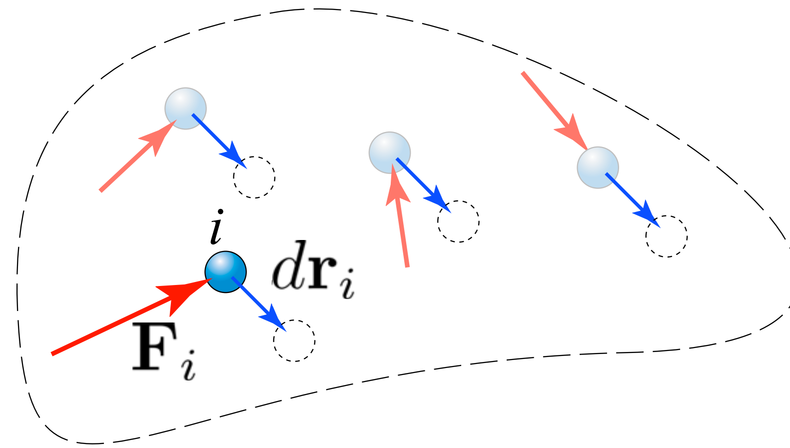
- Power generated by a force (or moment) is the dot product of the force and translational (rotational - angular) velocity at the point of application of the force

$$P_{trans} = \boldsymbol{F} \bullet \boldsymbol{v} \qquad P_{rot} = \boldsymbol{M} \bullet \boldsymbol{\omega}$$

Definition of Work (U)

Work of a force

A force does work when it undergoes a displacement in the direction of the line of action.



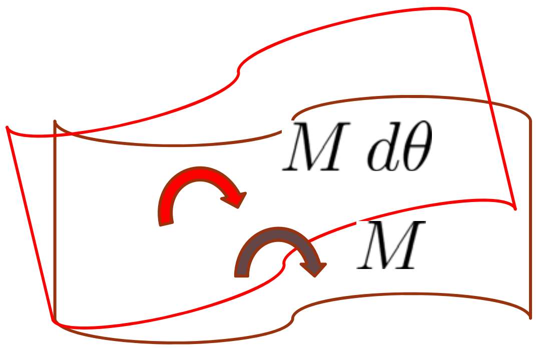
The work dU produced by the force $\boldsymbol{F}$ when it undergoes a differential displacement $d\boldsymbol{r}$ is given by

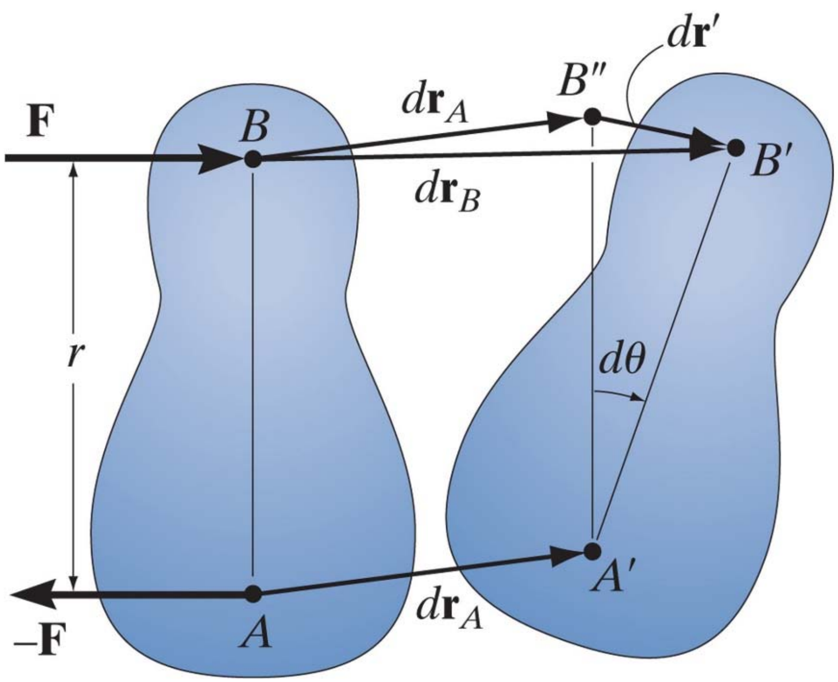
$$dU = \mathbf{F} \cdot d\mathbf{r}$$

Definition of Work (U)

Work of a couple moment

$$dU = M\mathbf{k} \cdot d\theta\,\mathbf{k} = M\,d\theta$$



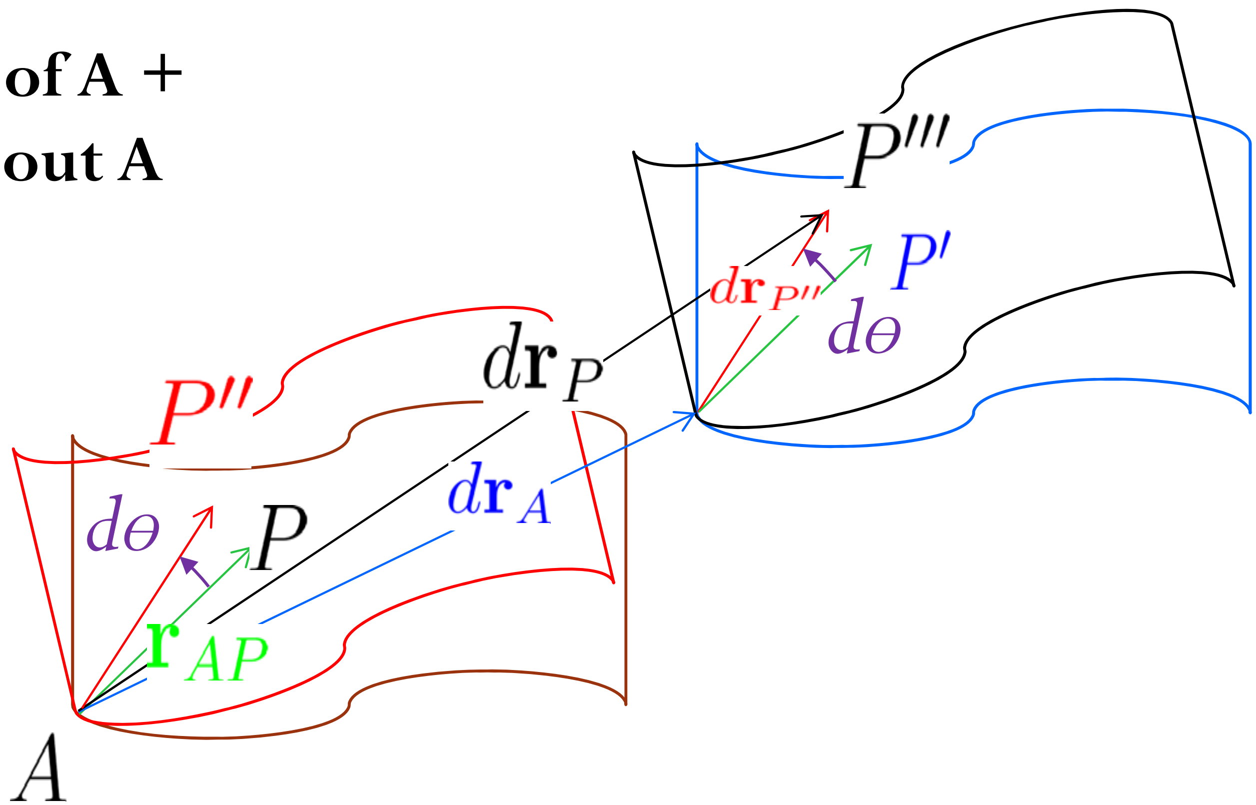


Incremental Displacement

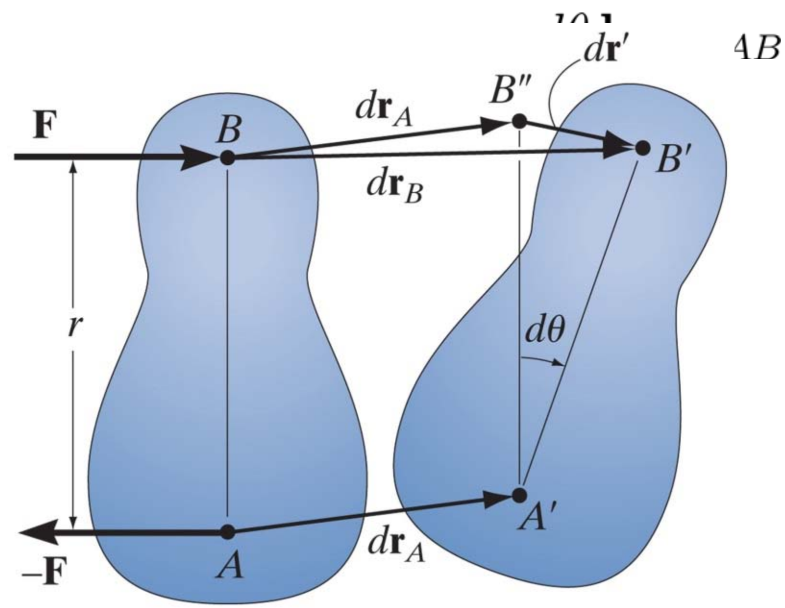
Rigid body displacement of P = translation of A + rotation about A

$$d\mathbf{r}_P = d\mathbf{r}_A + d\theta\,\mathbf{k} \times \mathbf{r}_{AP}$$

Translation of A +
Rotation about A



Definition of Work



Work of couple moment

$$d\mathbf{r}_P = d\mathbf{r}_A + d\theta\,\mathbf{k} \times \mathbf{r}_{AP}$$

$$\begin{aligned} dU &= \sum_i \mathbf{F}_i \cdot d\mathbf{r}_i \\ &= \mathbf{F}_A \cdot d\mathbf{r}_A + \mathbf{F}_B \cdot d\mathbf{r}_B \\ &= -\mathbf{F} \cdot (d\mathbf{r}_A + d\theta\,\mathbf{k} \times \mathbf{r}_{AA}) + \mathbf{F} \cdot (d\mathbf{r}_A + d\theta\,\mathbf{k} \times \mathbf{r}_{AB}) \\ &= \mathbf{F} \cdot (d\theta\,\mathbf{k} \times \mathbf{r}_{AB}) \\ &= d\theta\,\mathbf{k} \cdot (\mathbf{r}_{AB} \times \mathbf{F}) \\ &= d\theta\,\mathbf{k} \cdot \mathbf{M} \end{aligned}$$

$$\therefore \; dU = M\mathbf{k} \cdot d\theta\,\mathbf{k} = M\,d\theta$$

The couple forces do no work during the translation $d\mathbf{r}_A$

Work due to rotation

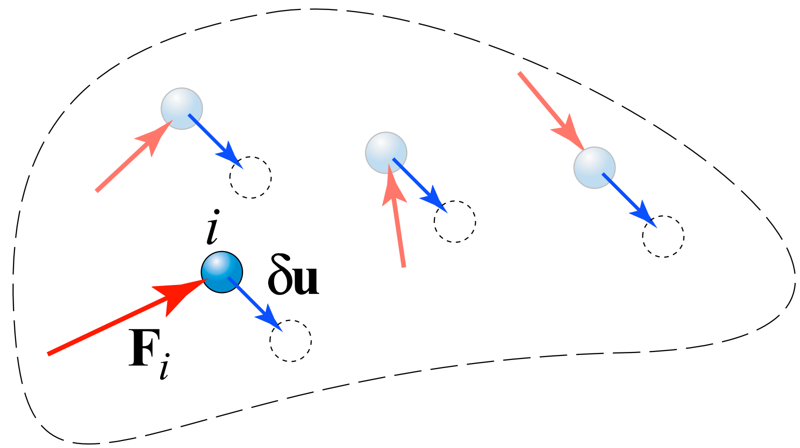
Virtual Displacements

A *virtual displacement* is a conceptually possible displacement *or* rotation of all *or* part of a system of particles. The movement is assumed to be possible, but actually does not exist. These "movements" are first-order differential quantity denoted by the symbol δ (for example, $\delta \mathrm{r}$ and $\delta \theta$.

Principle of Virtual Work

The principle of virtual work states that if a body is in equilibrium, then the algebraic sum of the virtual work done by all the forces and couple moments acting on the body is zero for any virtual displacement of the body. Thus,

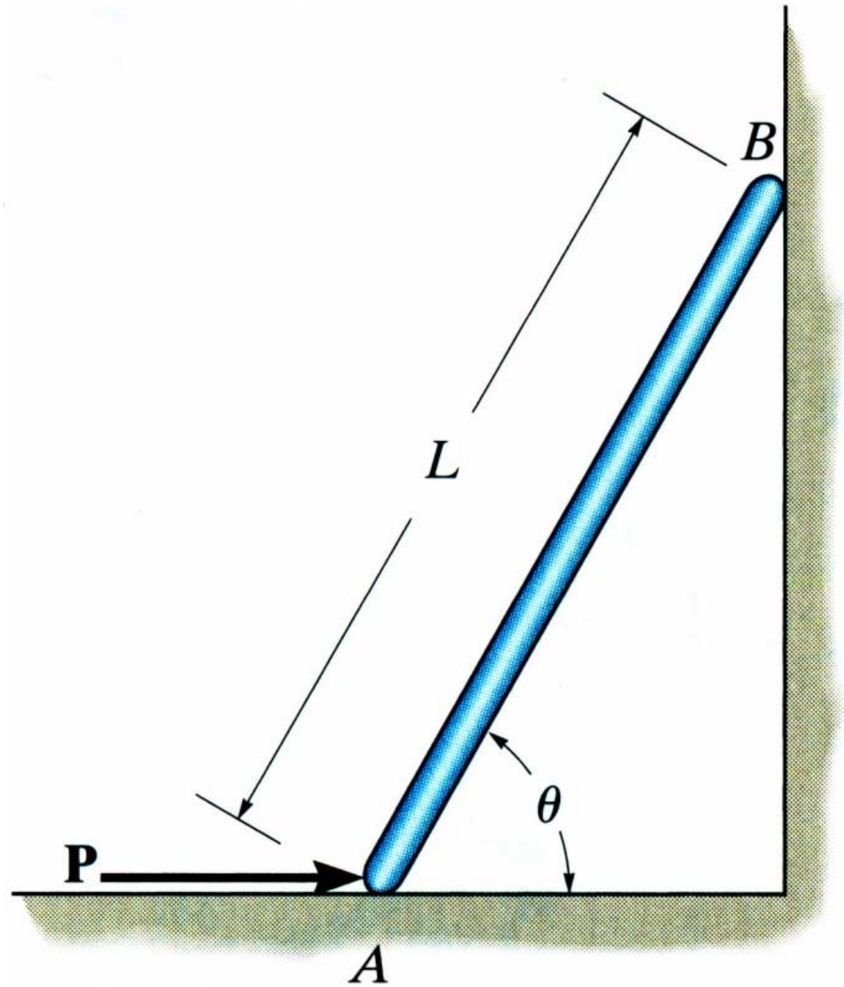
$$\delta U = 0$$



$$\delta U = \Sigma(\boldsymbol{F} \cdot \delta \boldsymbol{u}) + \Sigma(\boldsymbol{M} \cdot \delta \boldsymbol{\theta}) = 0$$

For 2D:

$$\delta U = \Sigma(\boldsymbol{F} \cdot \delta \boldsymbol{u}) + \Sigma(M\, \delta\theta) = 0$$

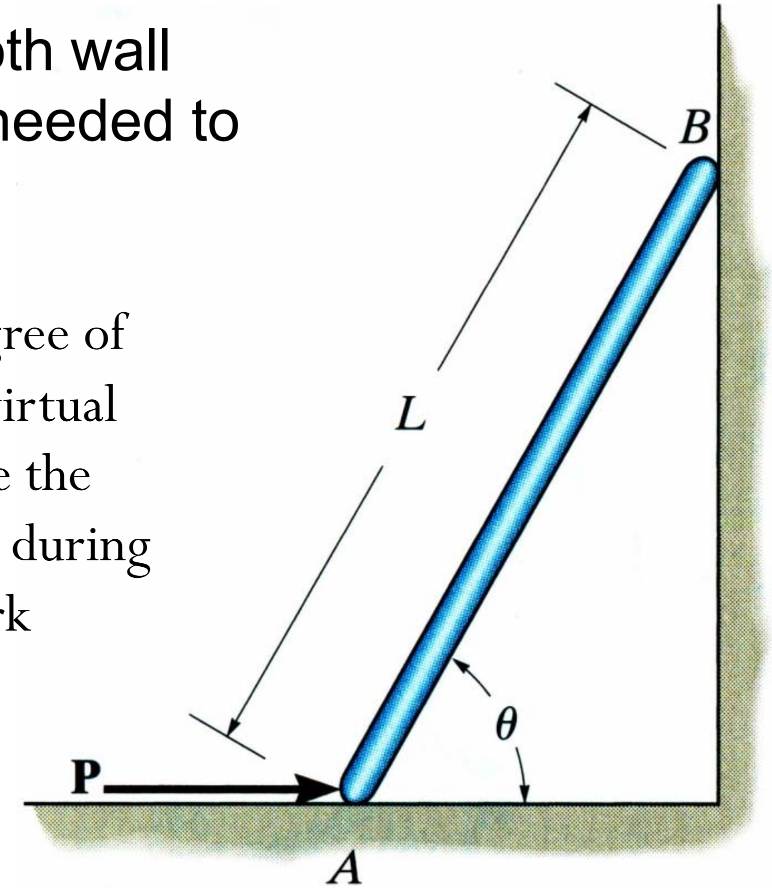


The thin rod of weight W rests against the smooth wall and floor. Determine the magnitude of force P needed to hold it in equilibrium.

Procedure for Analysis

1. Draw FBD of the entire system and provide coordinate system
2. Sketch the "deflected position" of the system
3. Define position coordinates measured from a <u>fixed</u> point and select the parallel line of action component and <u>remove forces that do no work</u>
4. <u>Differentiate</u> position coordinates to obtain virtual displacement
5. Write the virtual work equation and express the virtual work of each force/ couple moment
6. Factor out the comment virtual displacement term and solve

The thin rod of weight *W* rests against the smooth wall and floor. Determine the magnitude of force *P* needed to hold it in equilibrium.



Use the principle of virtual work. This problem has one degree of freedom, which we can take as the angle θ. Let $\delta\theta$ be the virtual rotation of the rod, such that the rod slides at A and B. Since the contact at A and B are smooth, the only forces that do work during the virtual displacements are P and W. Then the virtual work becomes: