Statics - TAM 211

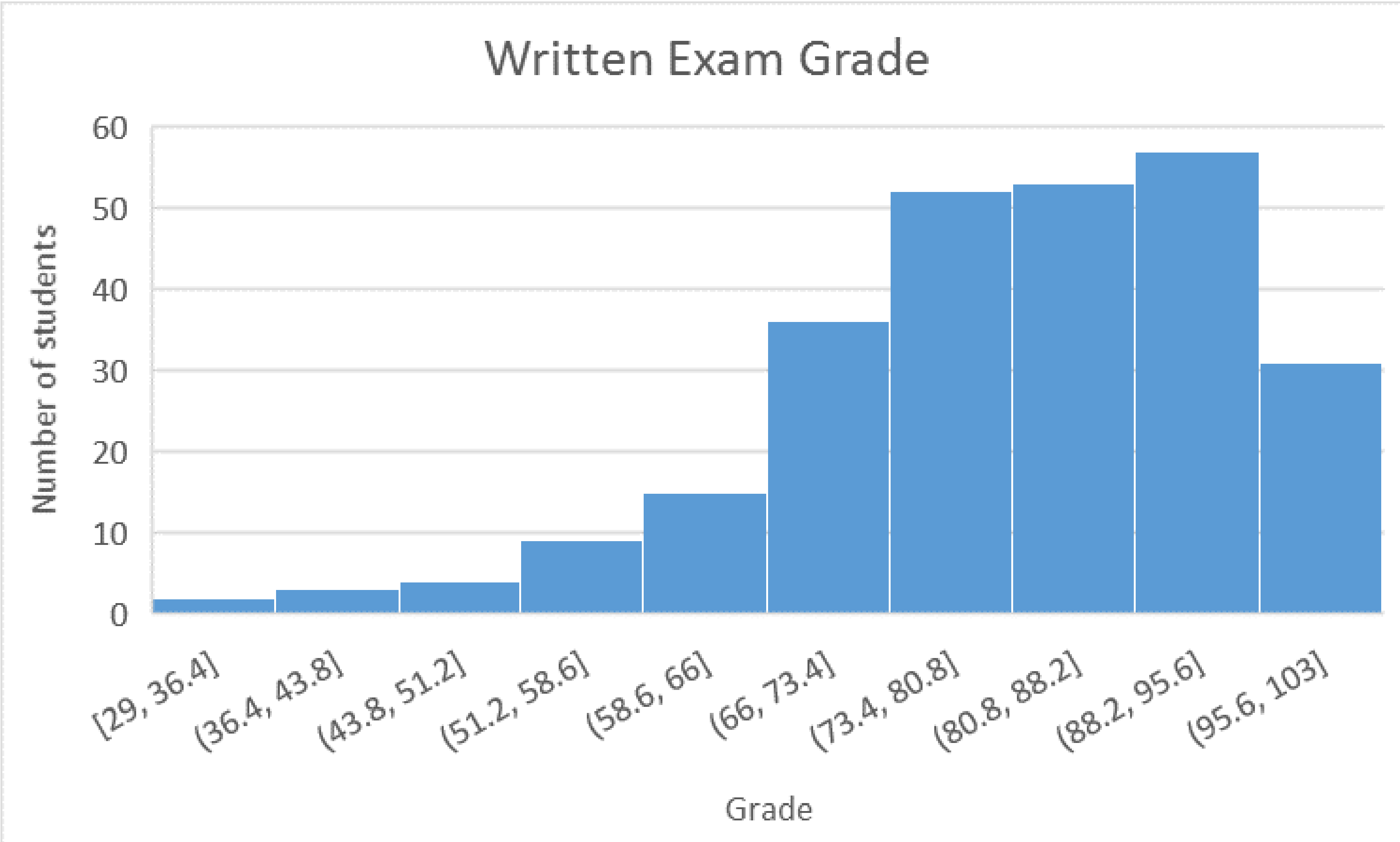
Lecture 40

April 27, 2018

Announcements

- Check ALL of your grades on Compass2g. Report issues
  - Exam grades are now posted
- There will be Discussion Sections next week
- Upcoming deadlines:
  - Quiz 6
    - CBTF (W-F: 4/25-27)
    - CoG thru 3D Rigid Bodies: Lectures 29-36

    Chart of centroid locations for different geometries - Attachment in CBTF quiz
  - Tuesday (5/1)
    - PL HW 15
  - Wednesday (5/2)
    - Written Assignment 6
  - Quiz 7
    - CBTF (Thurs-Tues: 5/3-8)
    - 50 minutes
    - Fluid Pressure - Virtual Work



Mean: 81.3
Median: 82.0
Standard deviation: 17.6
Minimum: 29
Maximum: 102

Chapter 11: Virtual Work

Goals and Objectives

- Introduce the principle of virtual work

- Show how it applies to determining the equilibrium configuration of a series of pin-connected members

"Work-Energy Method" for deriving equations of equilibrium.

$$\delta U = \sum(\vec{F}\cdot\delta\vec{r}) + \sum(\vec{M}\cdot\delta\vec{\theta}) = 0$$

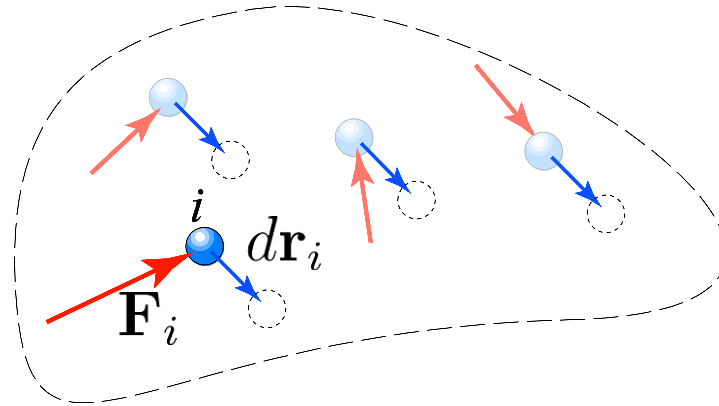
Throughout this course, we have been using the "Force-Balance Method" for deriving E of E.

$$\sum\vec{F} = 0, \quad \sum\vec{M} = 0$$

Recap: Definition of Work (U)

Work of a force

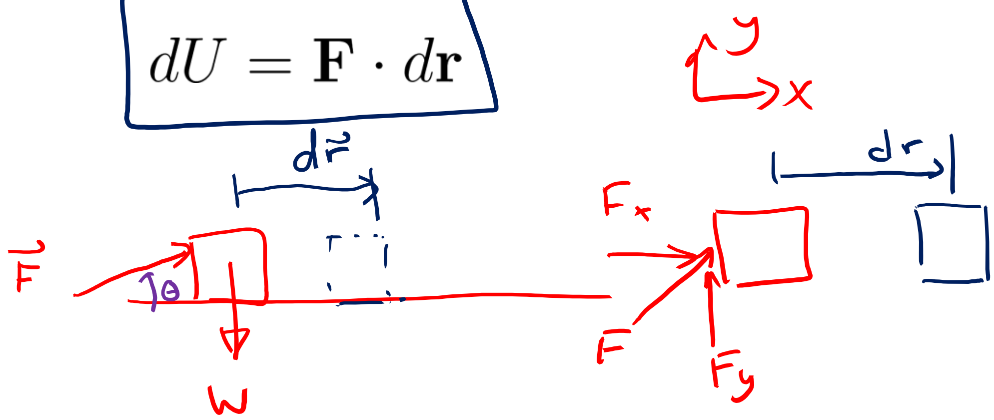
A force does work when it undergoes a displacement in the direction of the line of action.



The work dU produced by the force $\boldsymbol{F}$ when it undergoes a differential displacement $d\boldsymbol{r}$ is given by

Scalar

$$\boxed{dU = \mathbf{F} \cdot d\mathbf{r}}$$



ONLY Look at Force in direction of displacement to do work :

$$dU = F_x \, dr$$

$$dU = F_x \, dr = (F \cos\theta) \, dr$$

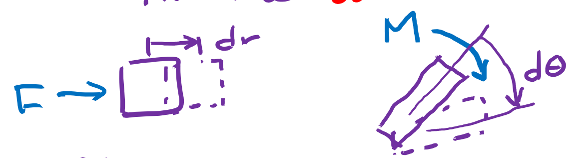
Note: W does no work because ⊥ to dr

Definition of Work (U)

Work of a couple moment

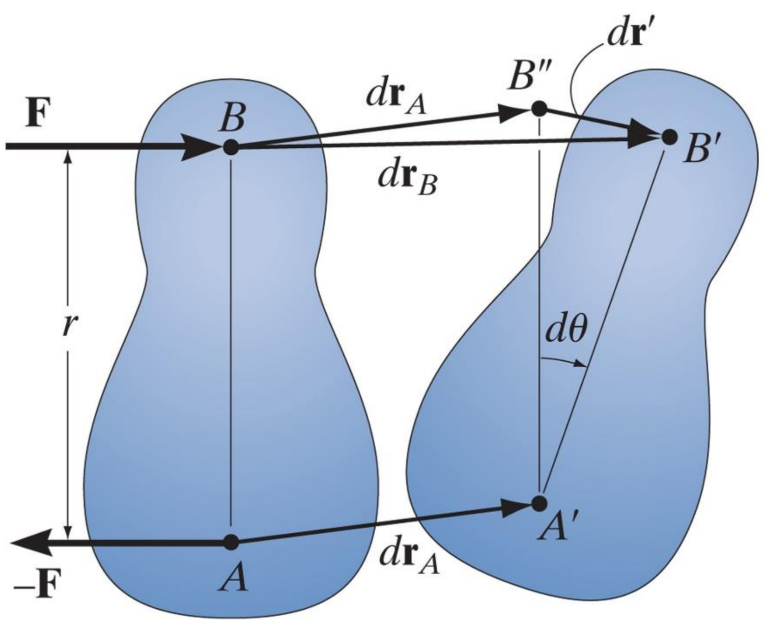
$$dU = M\mathbf{k} \cdot d\theta\,\mathbf{k} = M\,d\theta$$

Positive Work: Force/moment are in the same direction as displacement

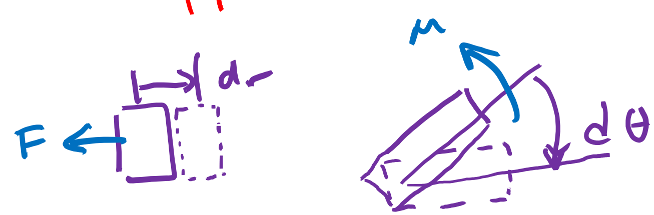


+Fdr

+Mdθ



Negative Work: F/M are opposite direction as disp.



-Fdr

- Mdθ

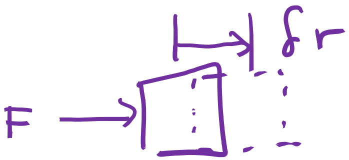
Virtual Displacements

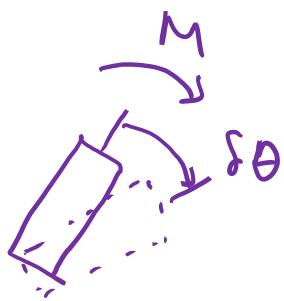
A *virtual displacement* is a conceptually possible displacement *or* rotation of all *or* part of a system of particles. The movement is assumed to be possible, but actually does not exist. These "movements" are first-order differential quantity denoted by the symbol δ (for example, δr and $\delta\theta$.

lower case delta

Virtual Displacement $\Rightarrow$ Statics (Not moving) $\therefore$ theoretically no displacement.

Mental Game





Principle of Virtual Work

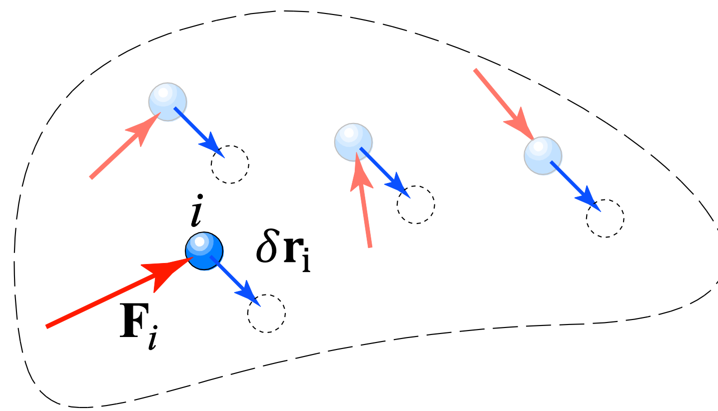
The principle of virtual work states that if a body is in equilibrium, then the algebraic sum of the virtual work done by all the forces and couple moments acting on the body is zero for any virtual displacement of the body. Thus,

virtual work

$$\delta U = 0$$

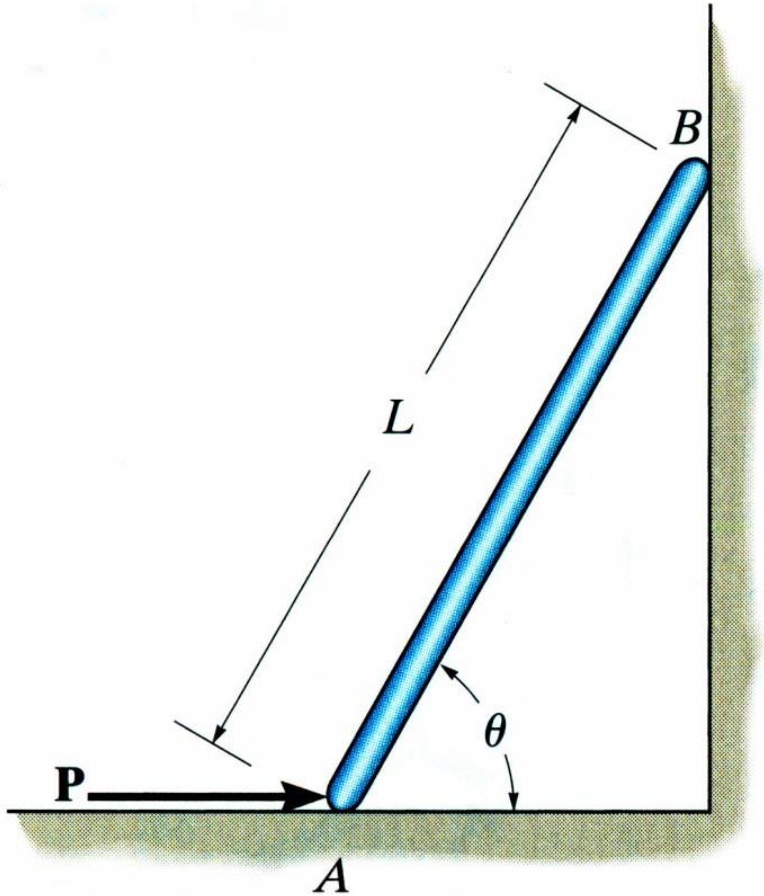
super, super small → 0

$$\delta U = \Sigma(\vec{\boldsymbol{F}} \cdot \delta \vec{\boldsymbol{r}}) + \Sigma(\vec{\boldsymbol{M}} \cdot \delta \vec{\boldsymbol{\theta}}) = 0$$



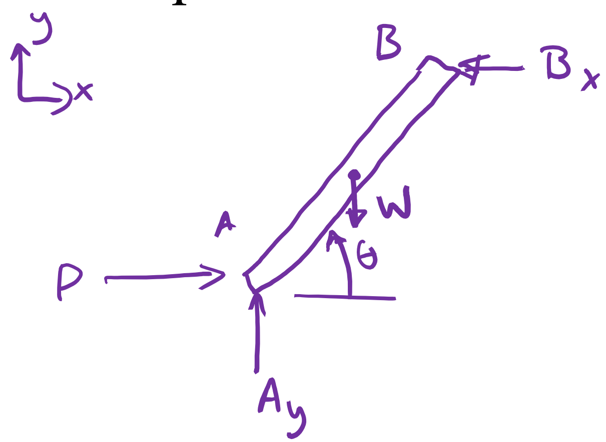
For 2D:

$$\delta U = \Sigma(\underset{x,\,y}{\vec{\boldsymbol{F}}} \cdot \delta \vec{\mathbf{r}}) + \Sigma(\underset{z}{M}\ \delta\theta) = 0$$



The thin rod of weight W rests against the smooth wall and floor. Determine the magnitude of force P needed to hold it in equilibrium.

assume known



UNKNOWNS:

P, A_y, B_x

Previously we have used Force-Balance Method to find unknowns:

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$

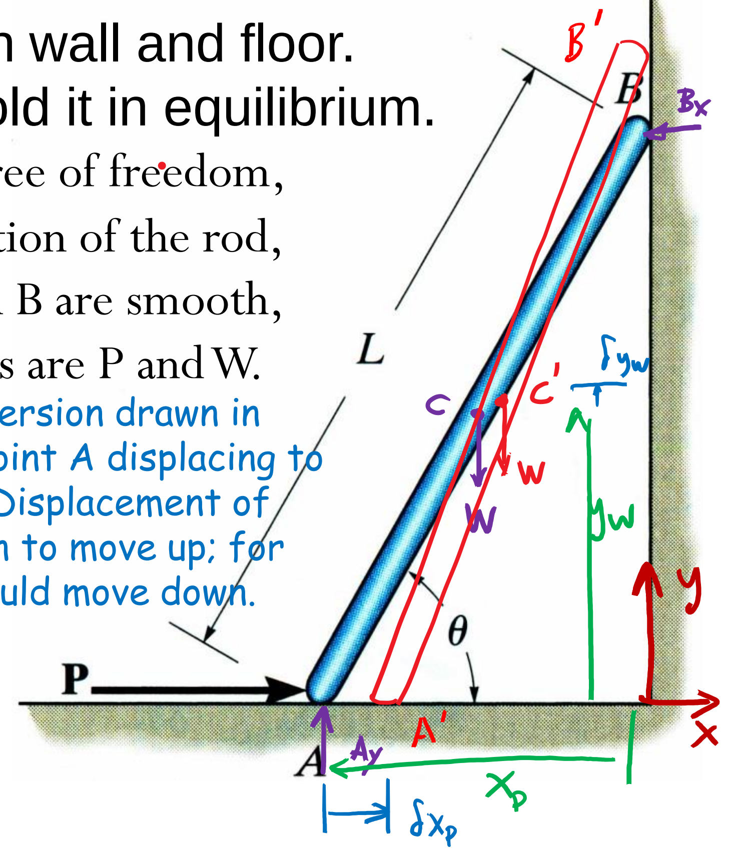
Procedure for Analysis

1. Draw FBD of the entire system and provide coordinate system
2. Sketch the "deflected position" of the system
3. Define position coordinates measured from a <u>fixed</u> point and select the parallel line of action component and <u>remove forces that do no work</u>
4. <u>Differentiate</u> position coordinates to obtain virtual displacement
5. Write the virtual work equation and express the virtual work of each force/ couple moment
6. Factor out the comment virtual displacement term and solve

The thin rod of weight *W* rests against the smooth wall and floor. Determine the magnitude of force *P* needed to hold it in equilibrium.

Use the principle of virtual work. This problem has one degree of freedom, which we can take as the angle θ. Let $\delta\theta$ be the virtual rotation of the rod, such that the rod slides at A and B. Since the contact at A and B are smooth, the only forces that do work during the virtual displacements are P and W. Then the virtual work becomes:

FBD has been revised from version drawn in class, which was drawn for point A displacing to left instead of to the right. Displacement of point C was incorrectly drawn to move up; for movement of A to left, C should move down.



$$\delta U = \sum \vec{F}\cdot\delta\vec{r} = 0$$

$$(\vec{P}\cdot\delta\vec{r}_P) + (\vec{W}\cdot\delta\vec{r}_W) = 0$$

$$(P\hat{i}\cdot\delta x_P\hat{i}) + (-W\hat{j}\cdot(\delta x_W\hat{i} + \delta y_W\hat{j})) = 0$$

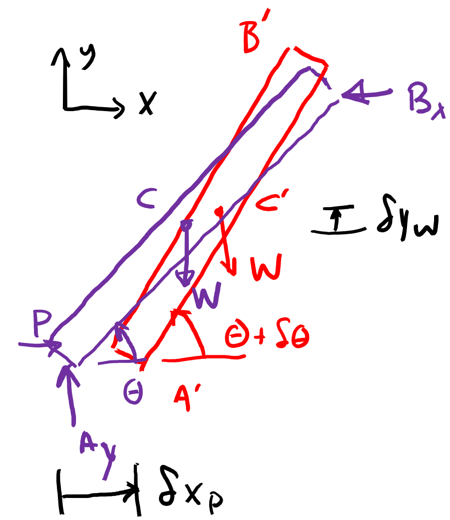
$$P\,\delta x_P - W\,\delta y_W = 0$$

$\vec{x}_P$ points in $-\hat{i}$ direction

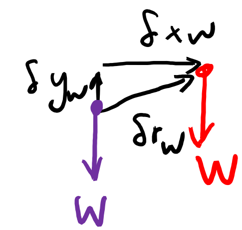
$$x_P = -L\cos\theta \Rightarrow \delta x_P = L\sin\theta\,\delta\theta$$

$$y_W = \frac{L}{2}\sin\theta \Rightarrow \delta y_W = \frac{L}{2}\cos\theta\,\delta\theta$$

FBD of rod:



Displacement of C:



$$\therefore P(L\sin\theta\,\delta\theta) - W\left(\frac{L}{2}\cos\theta\,\delta\theta\right) = 0$$

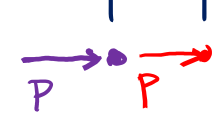
factor out $\delta\theta \neq 0$

$$\Rightarrow PL\sin\theta - \frac{WL}{2}\cos\theta = 0$$

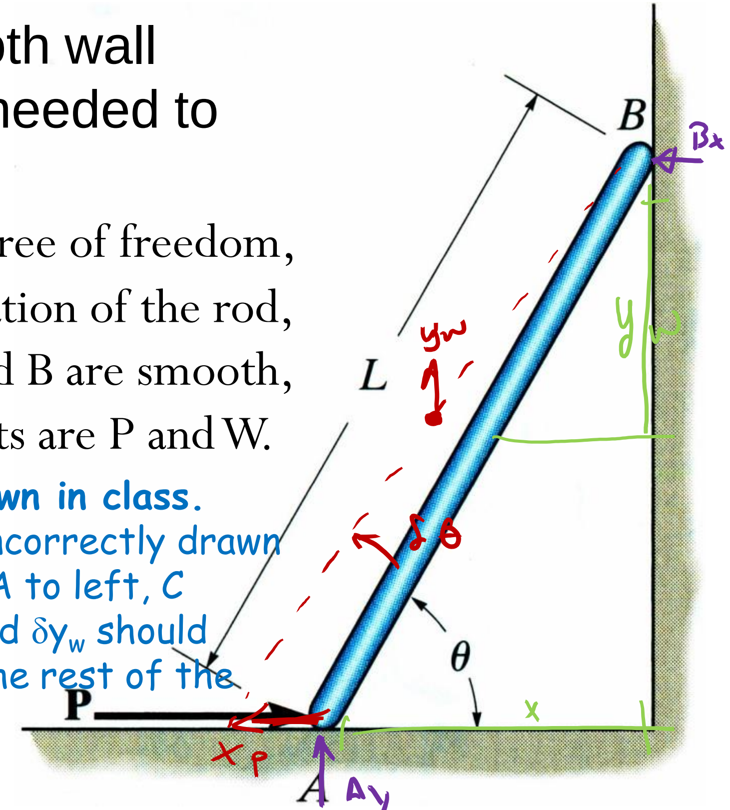
$$\Rightarrow \boxed{P = \frac{W}{2\tan\theta}}$$

Displacement of A:

$|\delta r_P| = \delta x_P$

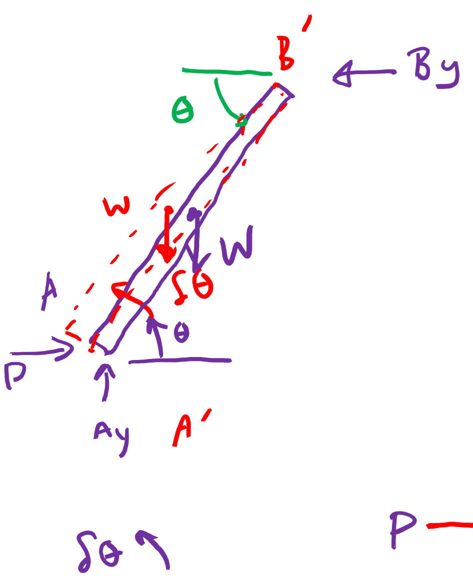


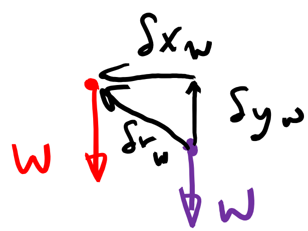
The thin rod of weight W rests against the smooth wall and floor. Determine the magnitude of force P needed to hold it in equilibrium.

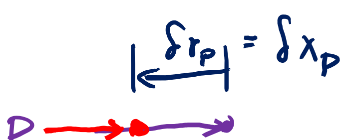


Use the principle of virtual work. This problem has one degree of freedom, which we can take as the angle θ. Let $\delta\theta$ be the virtual rotation of the rod, such that the rod slides at A and B. Since the contact at A and B are smooth, the only forces that do work during the virtual displacements are P and W. Then the virtual work becomes:

These are original notes drawn in class.
Displacement of point C was incorrectly drawn to move up; for movement of A to left, C should move down. Thus, W and δy_w should point in the same direction. The rest of the solution is correct as written.







$$\delta U = \sum \vec{F} \cdot d\vec{r} = 0$$

$$= -P\,\delta x_p + (W\,\delta y_w) = 0$$

$$x_p = -L\cos\theta \Rightarrow \delta x_p = L\sin\theta\,\delta\theta$$

$$y_w = \frac{L}{2}\sin\theta \Rightarrow \delta y_w = \frac{L}{2}\cos\theta\,\delta\theta$$

$$\therefore -P(L\sin\theta\,\delta\theta) + W\left(\frac{L}{2}\cos\theta\,\delta\theta\right) = 0$$

factor out $\delta\theta \neq 0$

$$\Rightarrow -PL\sin\theta + \frac{WL}{2}\cos\theta = 0$$

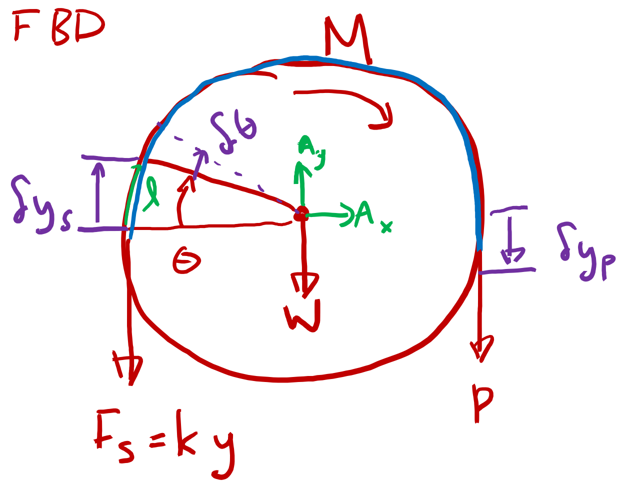
$$\Rightarrow \boxed{P = \frac{W}{2\tan\theta}}$$

W =

Disk of 10 lb is subjected to a vertical force P = 8 lb and a couple moment M = 8 lb ft. Determine disk's rotation θ if the end of the spring wraps around the periphery of the disk as the disk turns. The spring is originally unstretched.

KNOWN: W, M, P

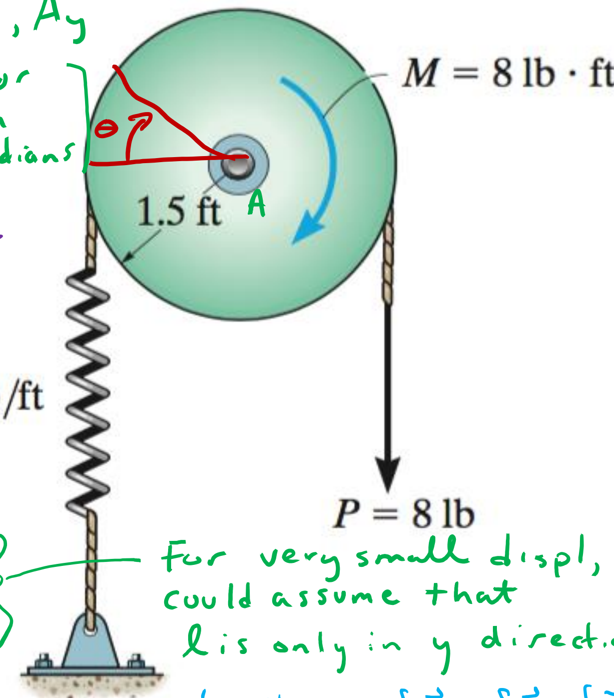
UNKNOWNS: $F_s(y, \theta)$, A_x, A_y



Arc length: $l = r\theta$ [for θ in radians]

$\delta y_s = \delta y_p$ same cable

Weight W does no work (pinned) no displ.

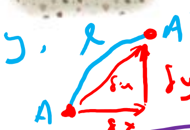


$l = r\theta = y$

$\delta y = r\,\delta\theta$

For very small displ, could assume that l is only in y direction.

Alternatively, if A moves to A'



then $\delta\vec{u} = \delta\vec{x} + \delta\vec{y}$ but only δy does work wrt F_s

$$\delta U = \sum \vec{F}\,\delta\vec{r} + M\delta\theta = 0$$

$$-F_s\,\delta y_s + P\,\delta y_p + M\,\delta\theta = 0$$

$$-F_s(r\,\delta\theta) + P(r\,\delta\theta) + M\,\delta\theta = 0$$

$$\underbrace{(-F_s r + P r + M)}_{=0}\,\delta\theta = 0$$

$$F_s = \frac{Pr + M}{r} = ky = k(r\theta) \Rightarrow \boxed{\theta = \frac{Pr + M}{r^2 k}}$$ radians

$\sum M_A: \; M - F_s r + P r = 0$ Compare to force-balance equilibrium eqn method ✓ same