Statics - TAM 211

Lecture 41

April 30, 2018

Announcements

- Check ALL of your grades on Compass2g. Report issues
  - Exam grades are now posted
- There will be Discussion Sections this week
- Upcoming deadlines:
  - Tuesday (5/1)
    - PL HW 15
  - Wednesday (5/2)
    - Written Assignment 6
  - Quiz 7
    - CBTF (Thurs-Tues: 5/3-8)
    - 50 minutes
    - Fluid Pressure - Virtual Work

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Chapter 11: Virtual Work

Goals and Objectives

- Introduce the principle of virtual work
- Show how it applies to determining the equilibrium configuration of a series of pin-connected members

"Work - Energy Method" for deriving equations of equilibrium.

$$\delta U = \sum(\vec{F} \cdot \delta\vec{r}) + \sum(\vec{M} \cdot \delta\vec{\theta}) = 0$$

Throughout this course, we have been using the "Force-Balance Method" for deriving E of E.

$$\sum\vec{F} = 0, \quad \sum\vec{M} = 0$$

Virtual work (Work-Energy Method) is particularly useful for structures with many members, whereas F-B method needs multiple eqns ($\sum\vec{F} = 0$, $\sum\vec{M} = 0$) per member.

Recap: Principle of Virtual Work

The principle of virtual work states that if a body is in equilibrium, then the algebraic sum of the virtual work done by all the forces and couple moments acting on the body is zero for any virtual displacement of the body. Thus,

virtual work

$$\delta U = 0$$

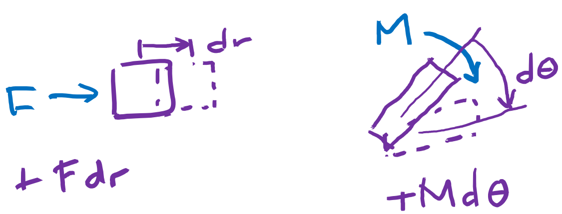
super, super small → 0

$$\delta U = \Sigma(\vec{\boldsymbol{F}} \cdot \delta \vec{\boldsymbol{r}}) + \Sigma(\vec{\boldsymbol{M}} \cdot \delta \vec{\boldsymbol{\theta}}) = 0$$

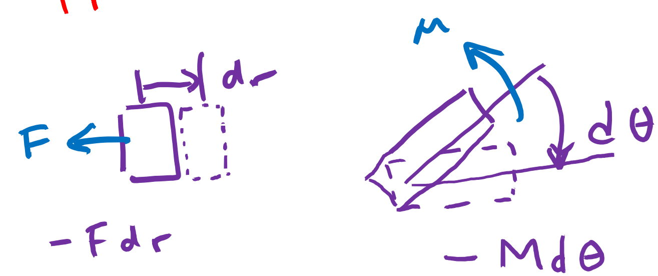
For 2D:

$$\delta U = \Sigma(\underbrace{\vec{\boldsymbol{F}}}_{x,\,y} \cdot \delta \vec{\mathbf{r}}) + \Sigma(\underbrace{M}_{z}\, \delta\theta) = 0$$

Positive Work: Force/moment are in the same direction as displacement



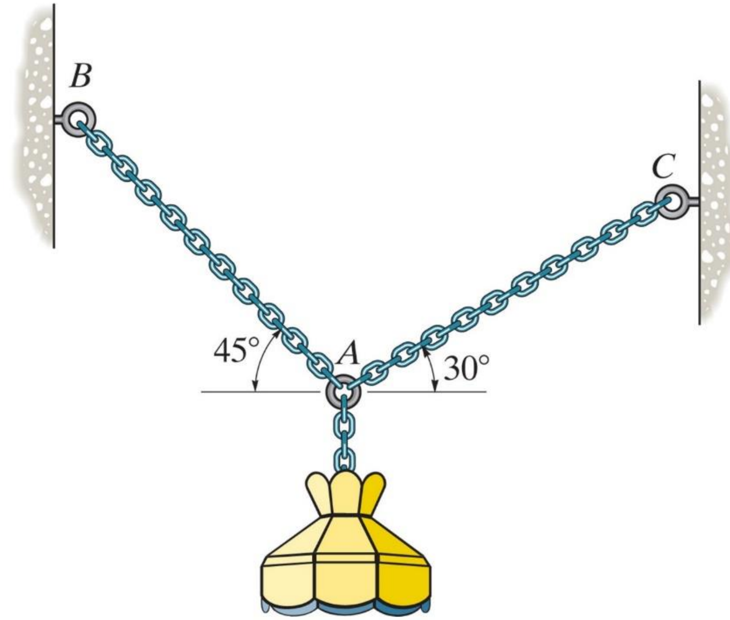
Negative Work: F/M are opposite direction as disp.



Procedure for Analysis

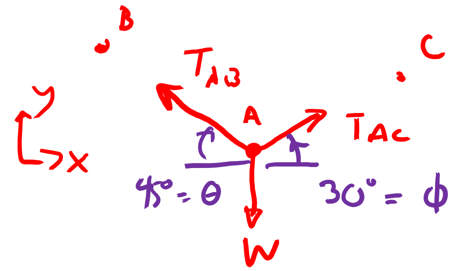
1. Draw FBD of the entire system and provide coordinate system
2. Sketch the "deflected position" of the system
3. Define position coordinates measured from a <u>fixed</u> point and select the parallel line of action component and <u>remove forces that do no work</u>
4. <u>Differentiate</u> position coordinates to obtain virtual displacement
5. Write the virtual work equation and express the virtual work of each force/ couple moment
6. Factor out the comment virtual displacement term and solve

Determine the tension in the cable *AC*. The lamp weighs 10 lb.



Force-Balance Method:

Find: $|\vec{T}_{AC}|$



Unknowns: T_{AC}, T_{AB}

$$\Sigma F_x: \; -T_{ABx} + T_{ACx} = 0$$

$$T_{AB} = \frac{T_{AC}\cos 30°}{\cos 45°} \qquad (1)$$

$$\Sigma F_y: \; T_{ABy} + T_{ACy} - W = 0$$

$$T_{AB} = \frac{W - T_{AC}\sin 30°}{\sin 45°} \qquad (2)$$

$$(1) = (2): \quad \frac{T_{AC}\cos 30°}{\cos 45°} = \frac{W}{\sin 45°} - \frac{T_{AC}\sin 30°}{\sin 45°}$$

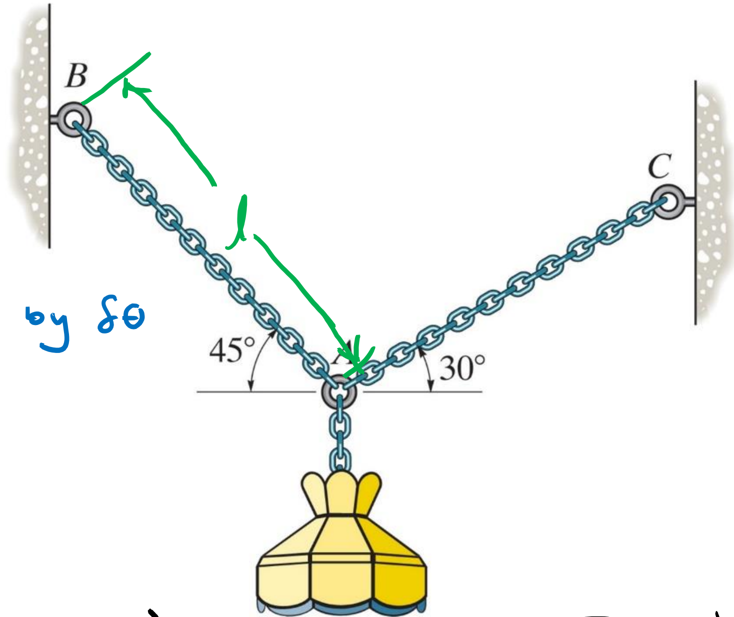
$$T_{AC}\left[\frac{\cos 30°}{\cos 45°} + \frac{\sin 30°}{\sin 45°}\right] = \frac{W}{\sin 45°}$$

$$\boxed{T_{AC} = \frac{W \cos 45}{\cos 30° \sin 45° + \sin 30 \cos 45}}$$

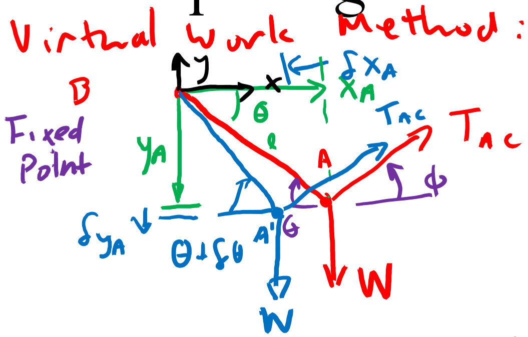
if $W = 10\text{ lb} \Rightarrow \boxed{T_{AC} = 7.3\text{ lb}}$

Determine the tension in the cable AC.

The lamp weighs 10 lb.



Virtual Work Method:



φ stays the same when θ is deflected by $\delta\theta$

$\vec{x}_A = l\cos\theta\,\hat{i} \rightarrow \delta\vec{x}_A = -l\sin\theta\,\delta\theta\,\hat{i}$

$\vec{y}_A = -l\sin\theta\,\hat{j} \rightarrow \delta\vec{y}_A = -l\cos\theta\,\delta\theta\,\hat{j}$

$\vec{W} = -W\hat{j}$

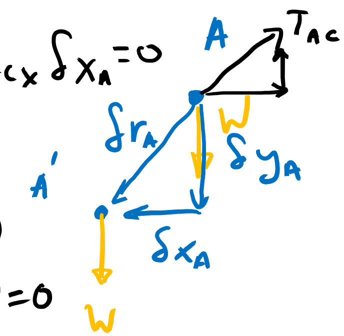
$\vec{T}_{AC} = T_{AC}\cos\phi\,\hat{i} + T_{AC}\sin\phi\,\hat{j}$

$\vec{T}_{AB}$: No force due to cable AB since ~~it does not move.~~

we are assuming that AB is a link within the control volume and $\vec{T}_{AB}$ does not work.

Virtual work:

$$\delta U = \delta \vec{F} \cdot d\vec{r}$$
$$= W\,\delta y_A + T_{AC_y}\,\delta y_A + T_{AC_x}\,\delta x_A = 0$$



$$-W(-l\cos\theta\,\delta\theta) + T_{AC}\sin\phi(-l\cos\theta\,\delta\theta) + T_{AC}\cos\phi(-l\sin\theta\,\delta\theta) = 0$$

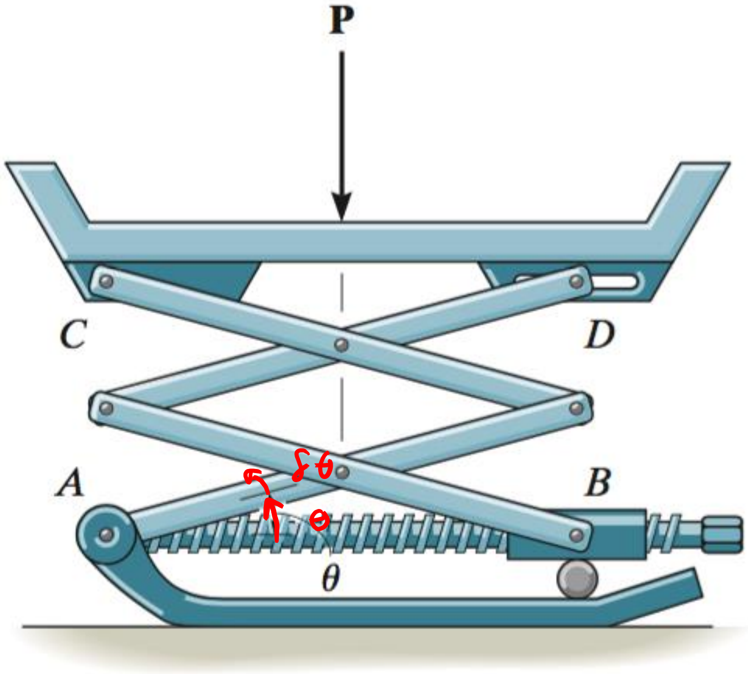
$$\underbrace{(+W\cos\theta - T_{AC}\sin\phi\cos\theta - T_{AC}\cos\phi\sin\theta)}_{\text{set } = 0}\,l\,\delta\theta = 0$$

$$\Rightarrow T_{AC}(\sin\phi\cos\theta + \cos\phi\sin\theta) = W\cos\theta$$

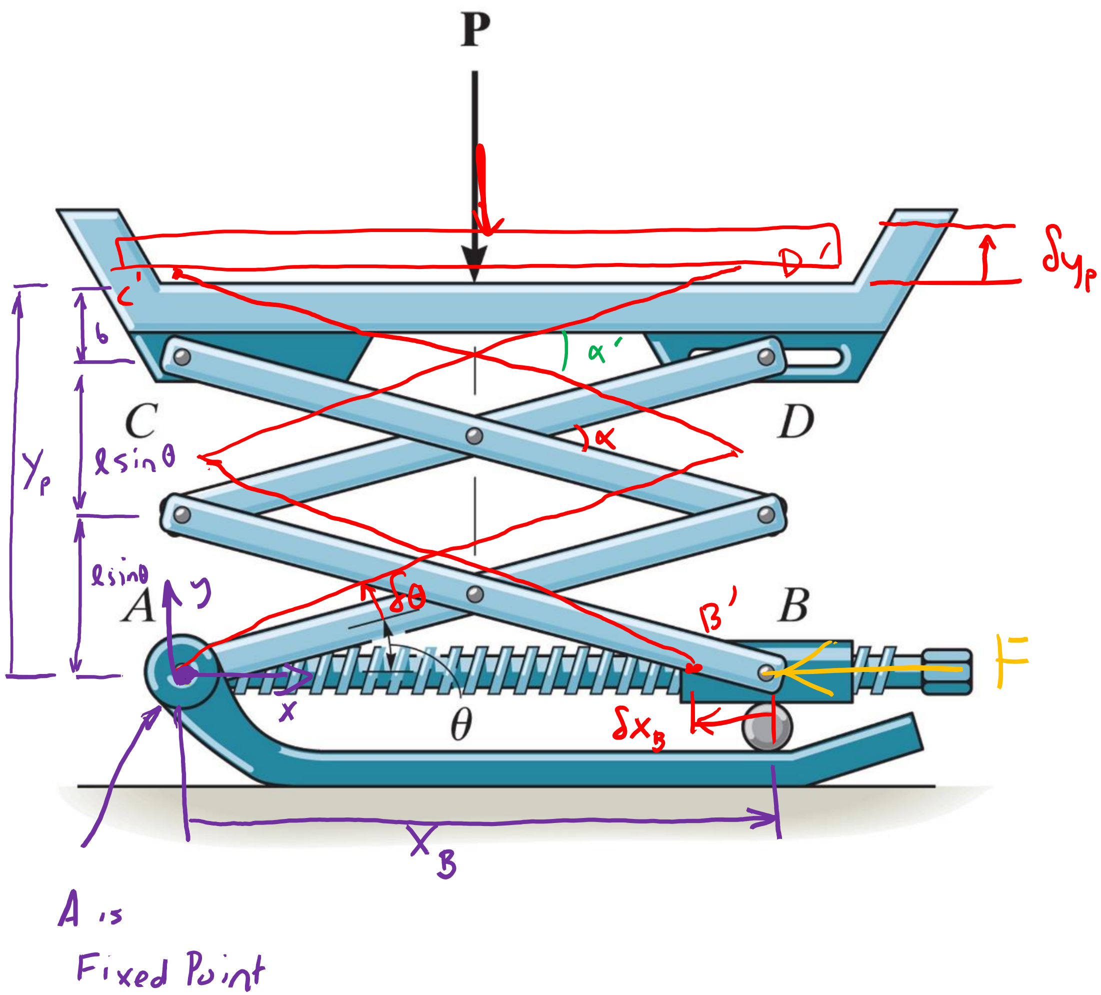
$$\boxed{T_{AC} = \frac{W\cos\theta}{\sin\phi\cos\theta + \cos\phi\sin\theta}}$$

✓ same as before w/ $W = 10$ lb, $\phi = 30°$, $\theta = 45°$

The scissors jack supports a load **P**. Determine axial force in the screw necessary for equilibrium when the jack is in the position shown. Each of the four links has a length *l* and is pin-connected at its center. Points *B* and *D* can move horizontally.



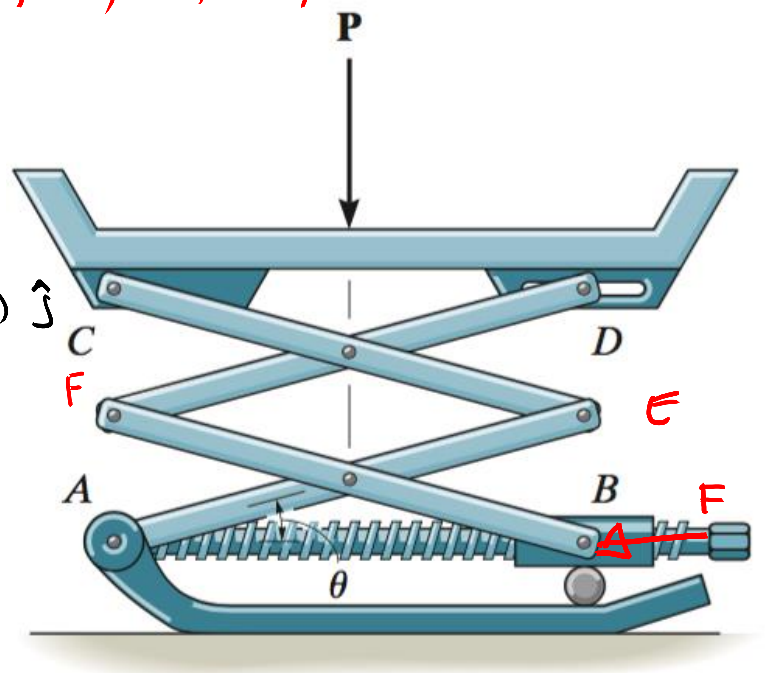
CORRECTION from lecture notes:
α should change with $\delta\theta$ since this structure is a 1 DOF problem



The scissors jack supports a load **P**. Determine axial force in the screw necessary for equilibrium when the jack is in the position shown. Each of the four links has a length *l* and is pin-connected at its center. Points *B* and *D* can move horizontally.

F-B method   Need to look at each member: AE, BF, EC, FD, Platform

$\Rightarrow$ Need many equations to solve for F.



V-W method:

$\vec{X}_B = l\cos\theta\,\hat{i} \Rightarrow \delta\vec{X}_B = -l\sin\theta\,\delta\theta\,\hat{i}$

$\vec{Y}_P = (2l\sin\theta + b)\,\hat{j} \Rightarrow \delta\vec{y}_P = (2l\cos\theta\,\delta\theta + 0)\,\hat{j}$

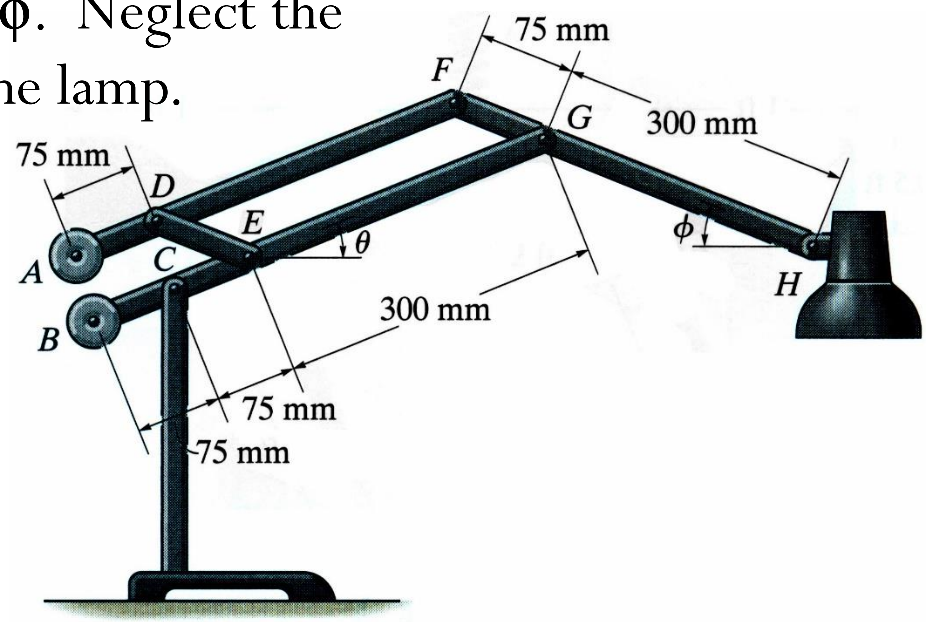
$\delta U = \vec{F}\cdot d\vec{X}_B + \vec{P}\cdot\delta\vec{y}_P = 0$

$\vec{F} = -F\hat{i}, \quad \vec{P} = -P\hat{j}$

$\delta U = -F(-l\sin\theta\,\delta\theta) + (-P)(2l\cos\theta\,\delta\theta) = 0$

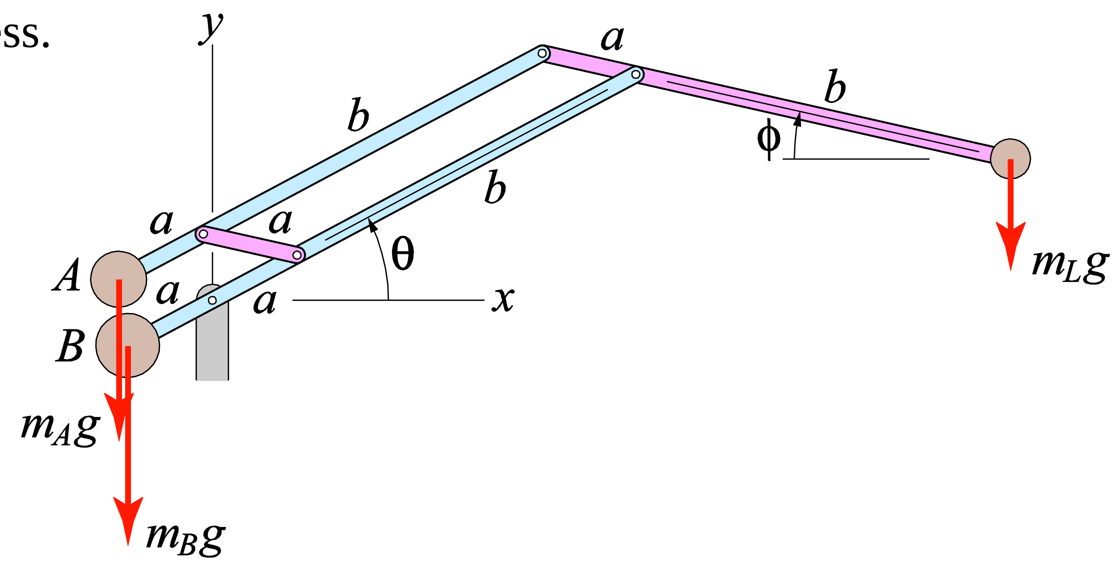
$$\boxed{F = \frac{2P\cos\theta}{\sin\theta}}$$

Determine the mass of A and B required to hold the 400 g desk lamp in balance for any angles θ and ϕ. Neglect the weight of the mechanism and the size of the lamp. Assume that pins are frictionless.



Solution

Use the principle of virtual work. This problem has two degrees of freedom: θ and ϕ. Let $\delta\theta$ and $\delta\phi$ be the virtual displacements from the equilibrium positions. We assume that all the pins are frictionless.



For equilibrium, we require

$$\delta W = -m_L g\, \delta y_L - m_A g\, \delta y_A - m_B g\, \delta y_B = 0$$

Here,

$$y_L = (a+b)\sin\theta - b\sin\phi \quad \Rightarrow \quad \delta y_L = (a+b)\cos\theta\,\delta\theta - b\cos\phi\,\delta\phi$$

$$y_A = a\sin\phi \quad \Rightarrow \quad \delta y_A = a\cos\phi\,\delta\phi$$

$$y_B = -a\sin\theta \quad \Rightarrow \quad \delta y_B = -a\cos\theta\,\delta\theta$$

Substitution gives

$$\delta W = \left(-m_L g(a+b)\cos\theta + m_B g\, a\cos\theta\right)\delta\theta$$
$$+\left(+m_L g\, b\cos\phi - m_A g\, a\cos\phi\right)\delta\phi = 0$$

or

$$\delta W = \left(-m_L g(a+b) + m_B g\, a\right)\cos\theta\,\delta\theta$$
$$+\left(+m_L g\, b - m_A g\, a\right)\cos\phi\,\delta\phi = 0\,.$$

If this relation is to hold for arbitrary $\delta\theta$ and $\delta\phi$, for the general case where $\cos\theta \neq 0$ and $\cos\phi \neq 0$, we require

$$\boxed{m_A = \frac{b}{a} m_L \quad \text{and} \quad m_B = \frac{a+b}{a} m_L\,.}$$

For $a = 75\text{ mm}$, $b = 300\text{ mm}$, and $m_L = 400\text{ g}$,

$$\boxed{\begin{aligned} m_A &= \frac{300}{75}(400\text{ g}) = 1.6\text{ kg}\,, \\ m_B &= \frac{75+300}{75}(400\text{ g}) = 2.0\text{ kg}\,. \end{aligned}}$$