Chapter 6: Structural Analysis

Goals and Objectives

- Determine the forces in members of a truss using the method of joints
- Determine zero-force members
- Determine the forces in members of a truss using the method of sections

Simple trusses

Trusses are commonly used to support roofs.

A more challenging question is, that for a given load, how can we design the trusses' geometry to minimize cost?

Scaffolding

An understanding of statics is critical for predicting and analyzing possible modes of failure.

Buckling of slender members in compression is always a consideration in structural analysis.

Simple trusses

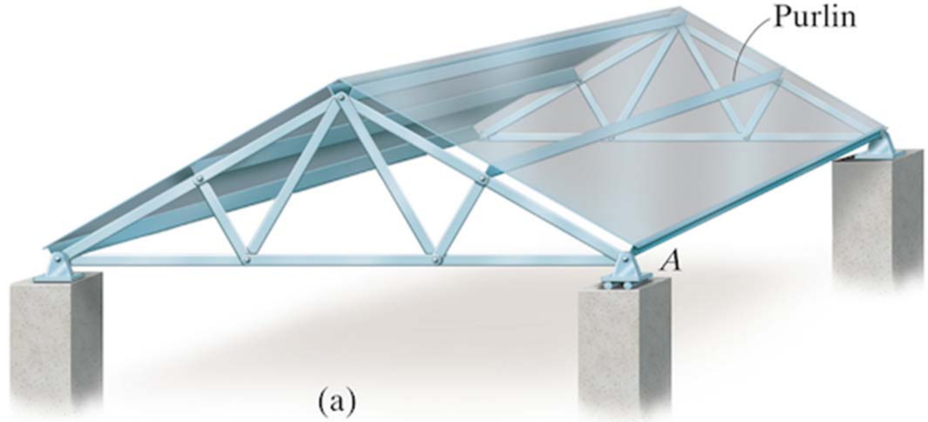
Truss:

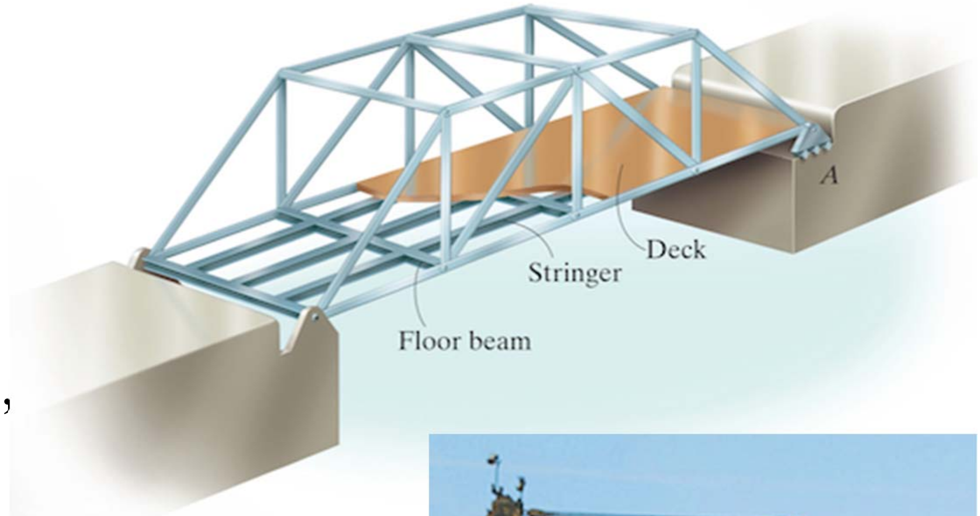
- Structure composed of slender members joined together at end points
- Transmit loads to supports

Assumption of trusses

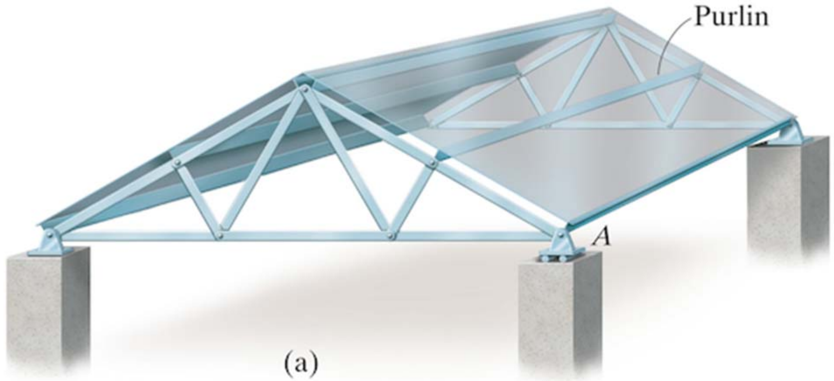
- Loading applied at joints, with negligible weight (If weight included, vertical and split at joints)
- Members joined by smooth pins

Result: all truss members are two-force members, and therefore the force acting at the end of each member will be directed along the axis of the member

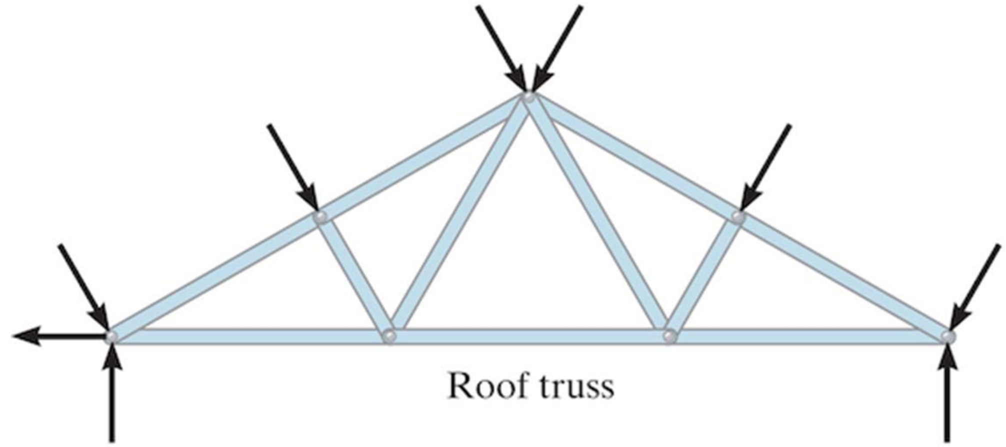




Roof trusses

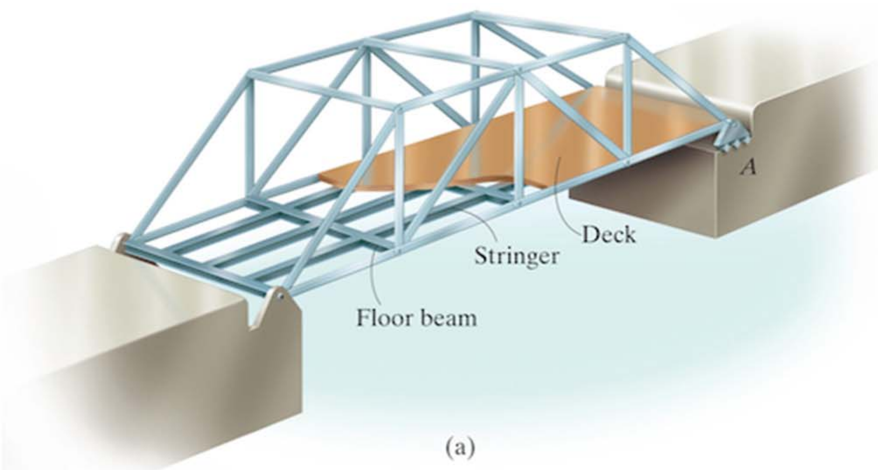


(a)

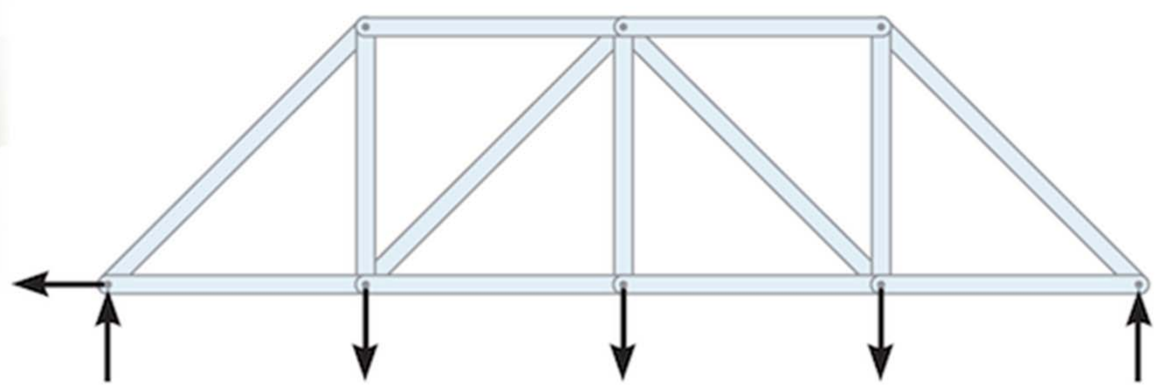


Load on roof transmitted to purlins, and from purlins to roof trusses at joints.

Bridge trusses



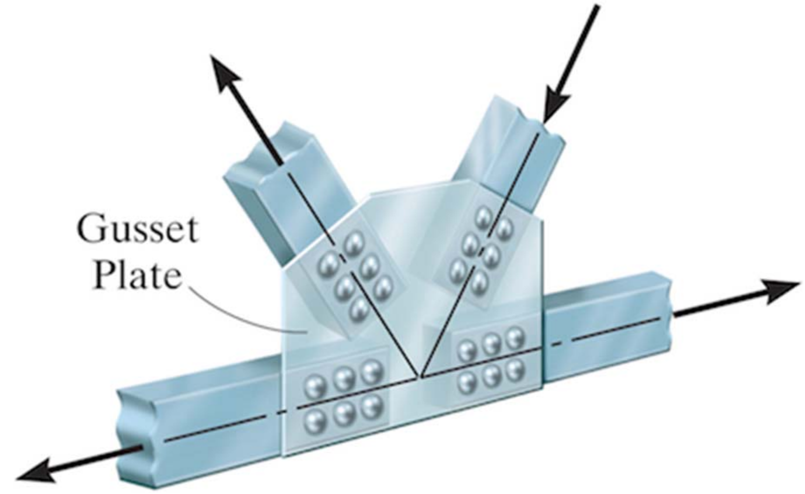
(a)

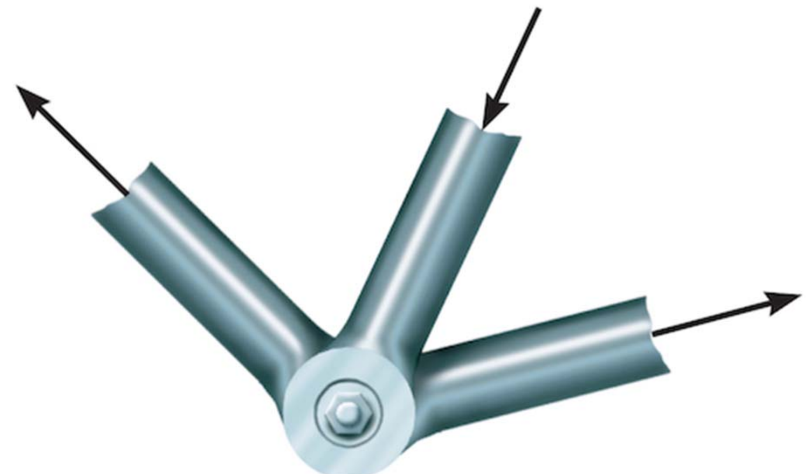


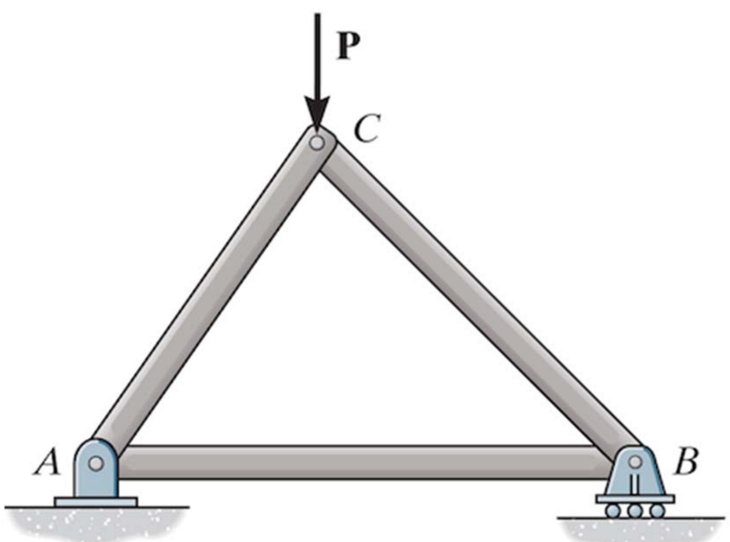
Load on deck transmitted to stringers, and from stringers to floor beams, and from floor beams to bridge trusses at joints.

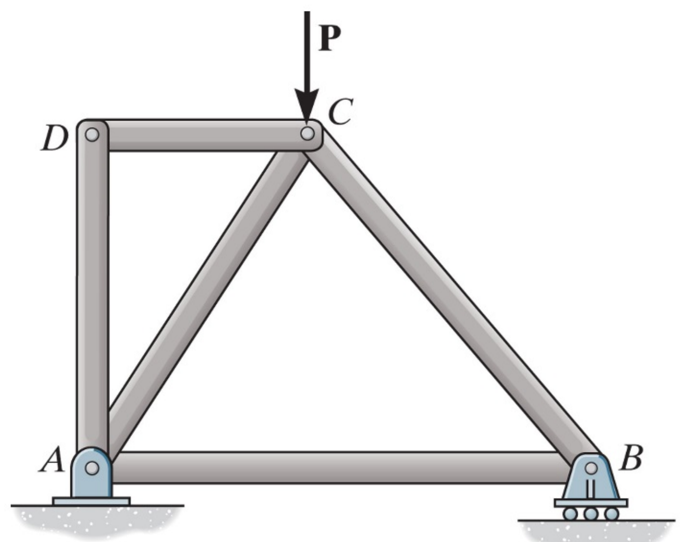
Truss joints

- Bolting or welding of the ends of the members to a gusset plates or passing a large bolt through each of the members
- Properly aligned gusset plates equivalent to pins (i.e., no moments) from coplanar, concurrent forces
- Simple trusses built from triangular members



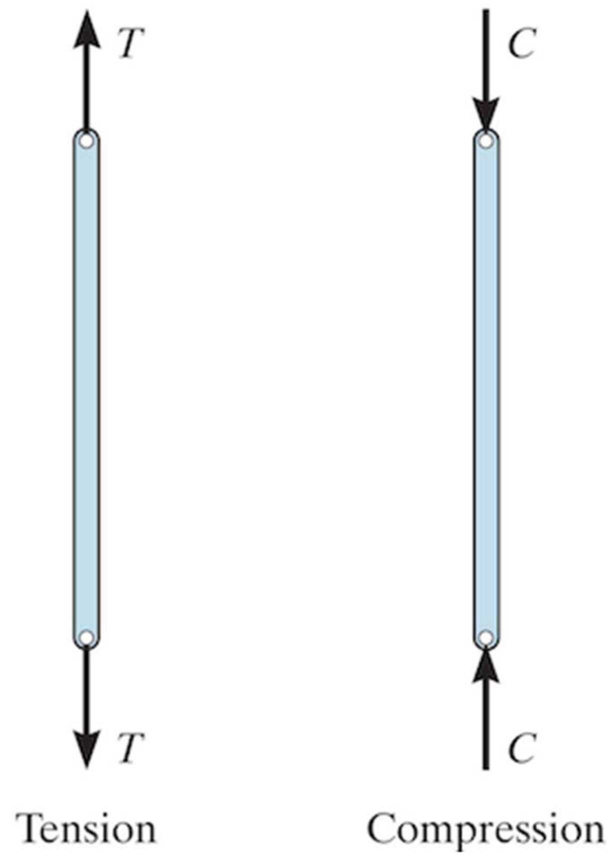






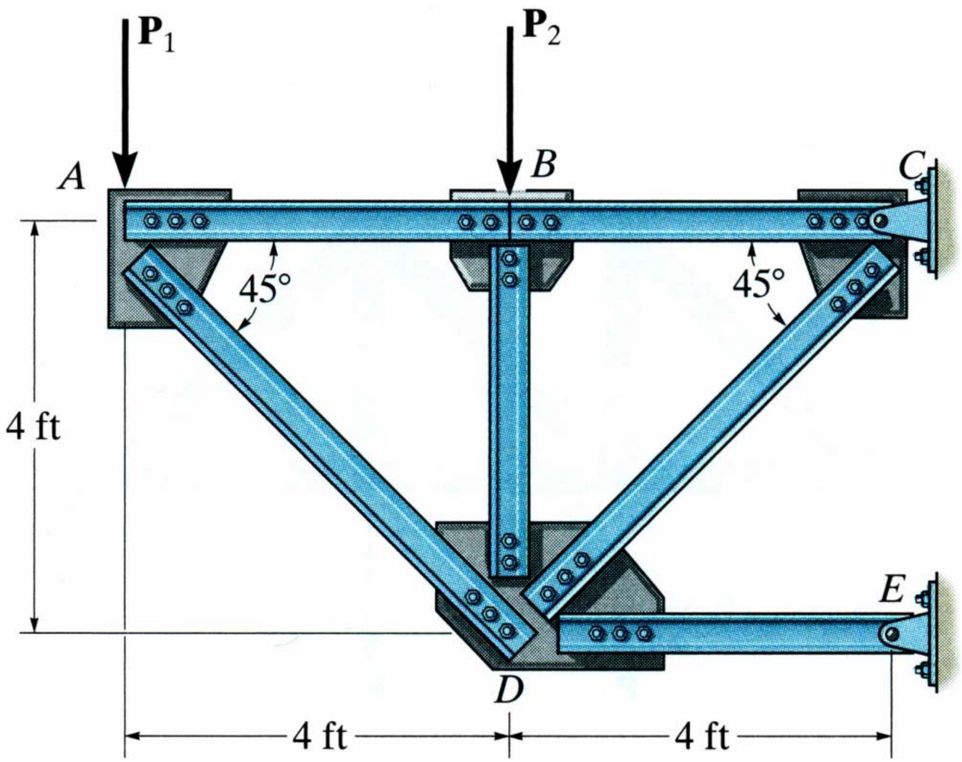
Method of joints

- Entire truss is in equilibrium if and only if all individual pieces (truss members and connecting pins) are in equilibrium.
- Truss members are two-force members: equilibrium satisfied by equal, opposite, collinear forces.
  - Tension: member has forces elongating.
  - Compression: member has forces shortening.
- Pins in equilibrium: $\sum F_x = 0$ and $\sum F_y = 0$



Procedure for analysis:

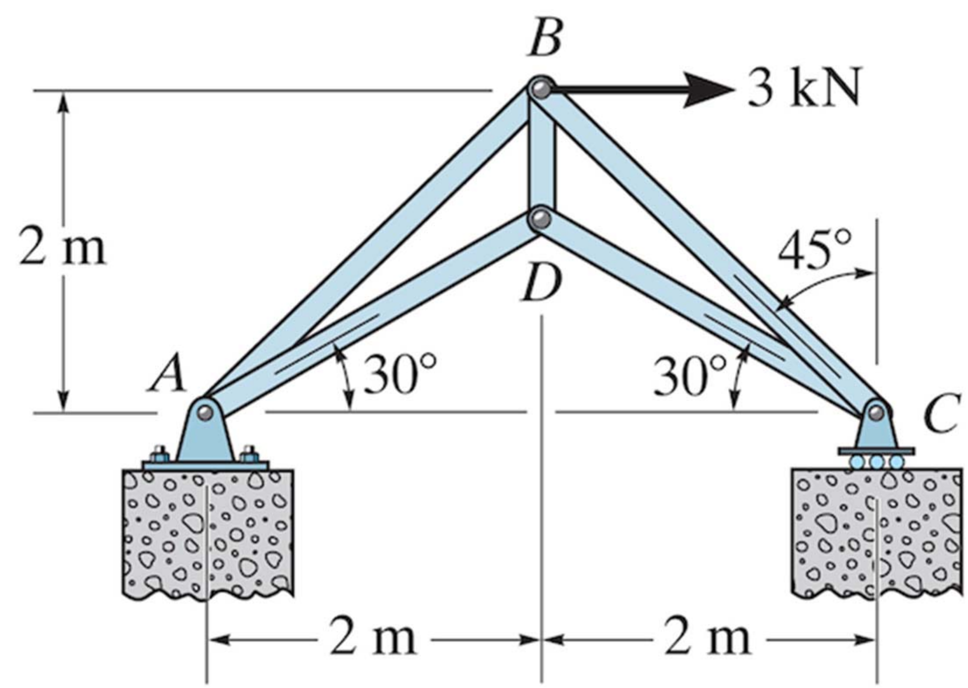
- Free-body diagram for each joint
- Start with joints with at least 1 known force and 1-2 unknown forces.
- Generates two equations, 1-2 unknowns for each joint.
- Assume the unknown force members to be in *tension; i.e. the forces "pull" on the pin*. Numerical solutions will yield positive scalars for members in tension and negative scalar for members in compression.



Example 1)

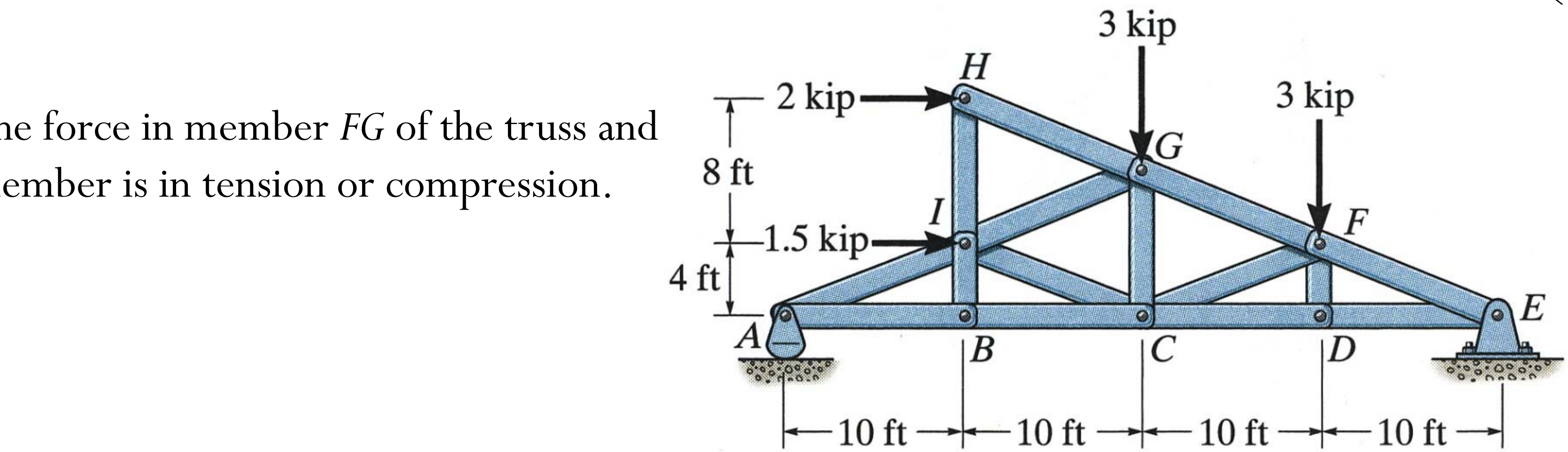
The truss, used to support a balcony, is subjected to the loading shown. Approximate each joint as a pin and determine the force in each member. State whether the members are in tension or compression.

We will determine the force in each member of the truss and indicate whether the members are in tension or compression.



Example 2)

Determine the force in member *FG* of the truss and state if the member is in tension or compression.

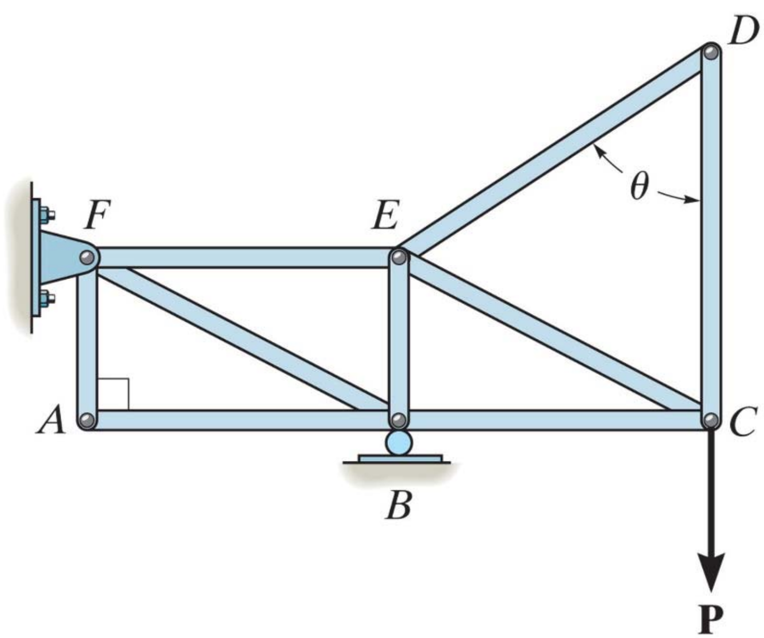


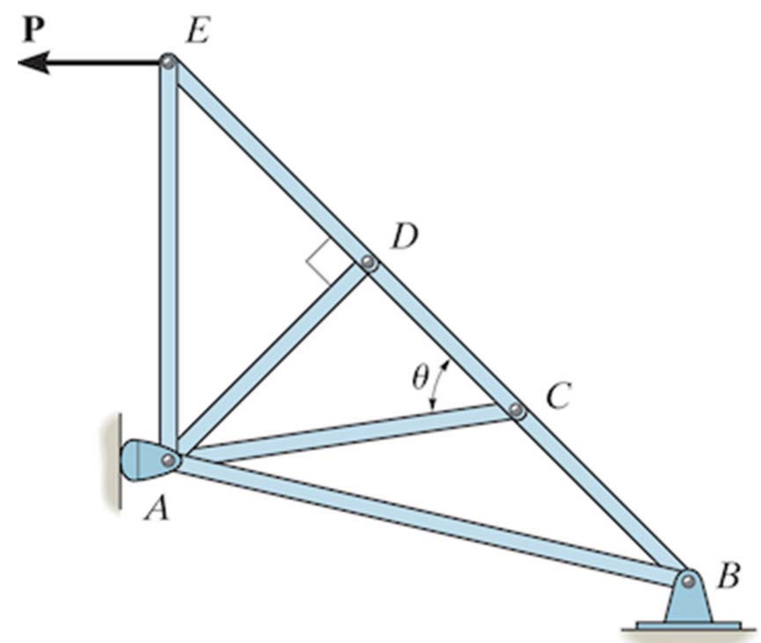
Zero-force members

- Particular members in a structure may experience no force for certain loads.
- Zero-force members are used to increase instability
- Identifying members with zero-force can expedite analysis.

Two situations:

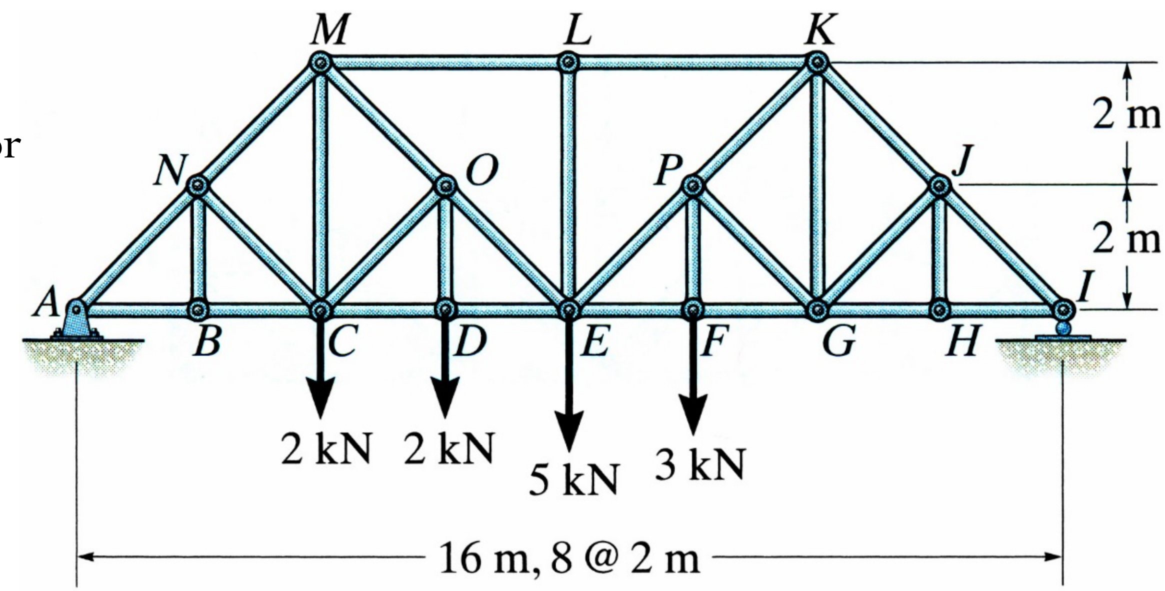
- Joint with two non-collinear members, no external or support reaction applied to the joint → **Both members are zero-force members**.
- Joint with two collinear member, plus third non-collinear, no loads applied to the joint → **Non-collinear member is a zero-force member**.





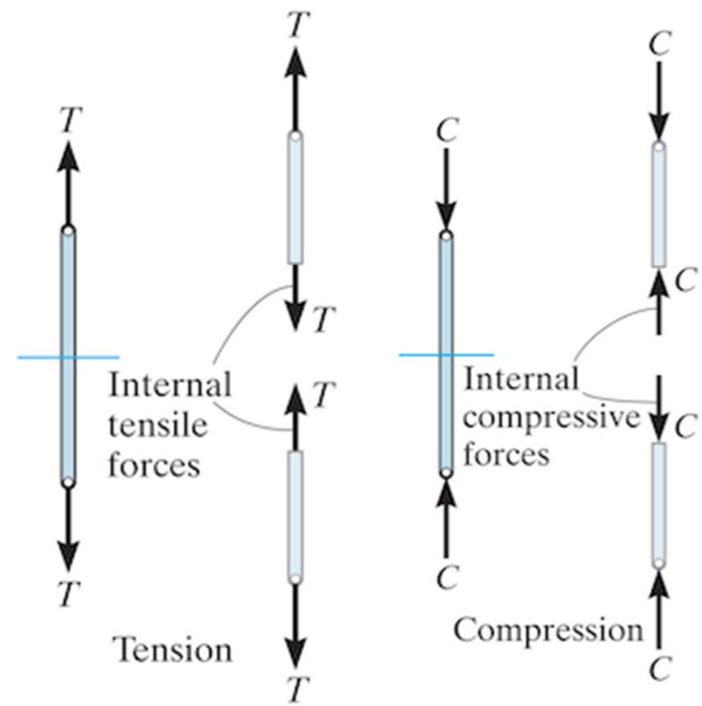
Example 3)

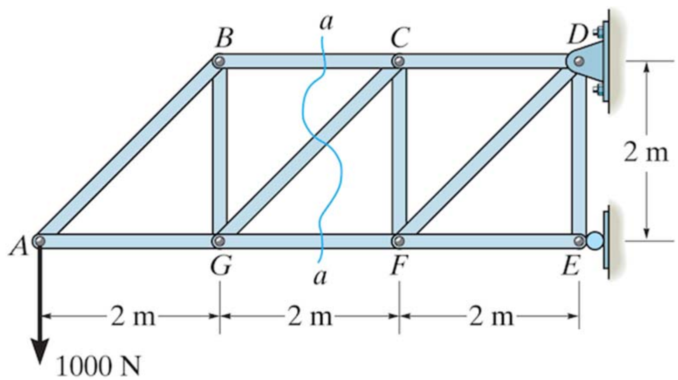
Determine the force in member GC of the truss and state if the member is in tension or compression.

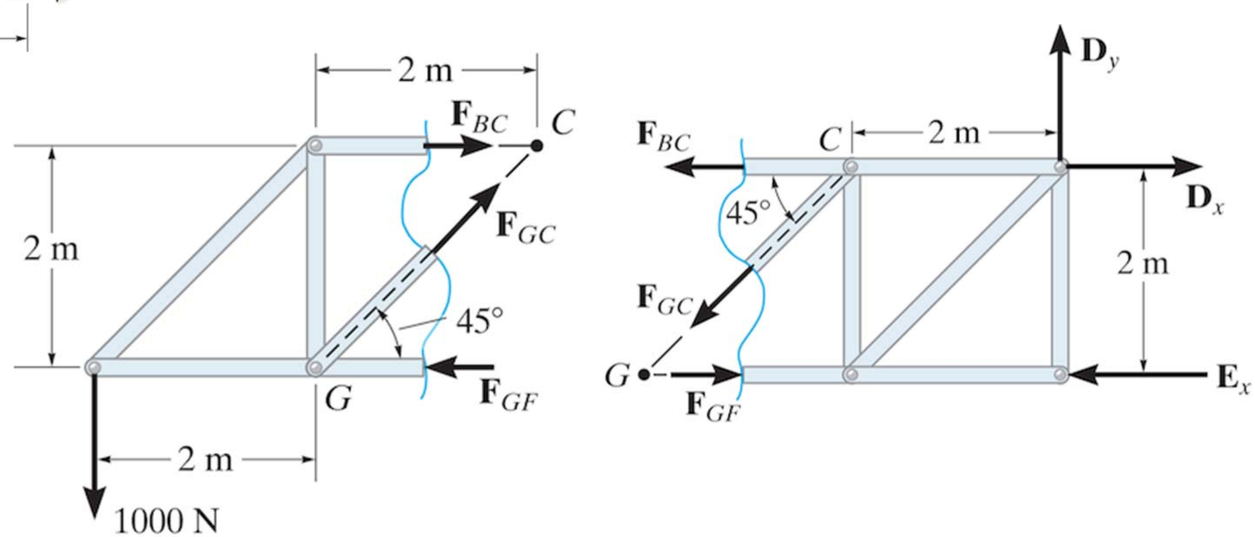


Method of sections

- Determine external support reactions
- "Cut" the structure at a section of interest into two separate pieces and set either part into force and moment equilibrium (your cut should be such that you have up to three unknowns)



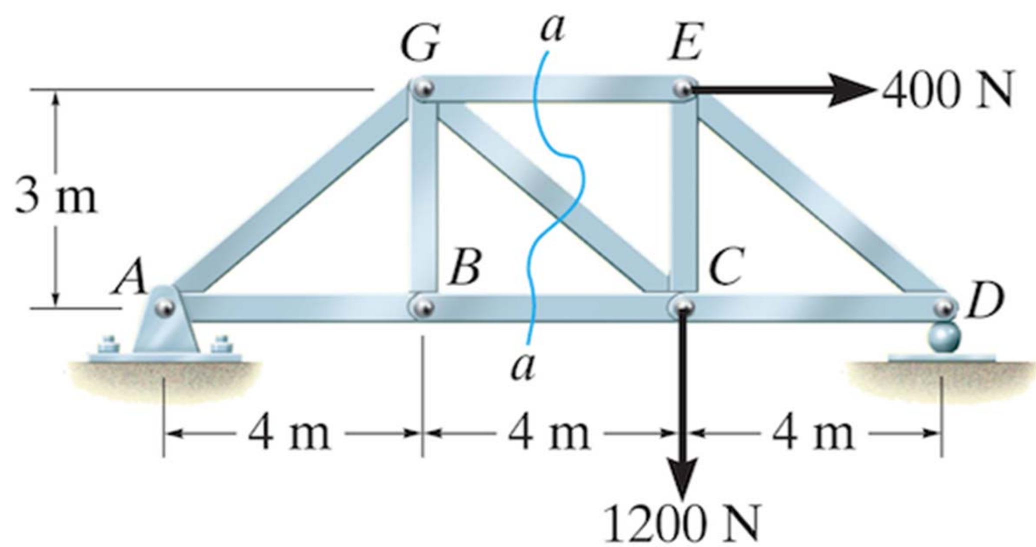


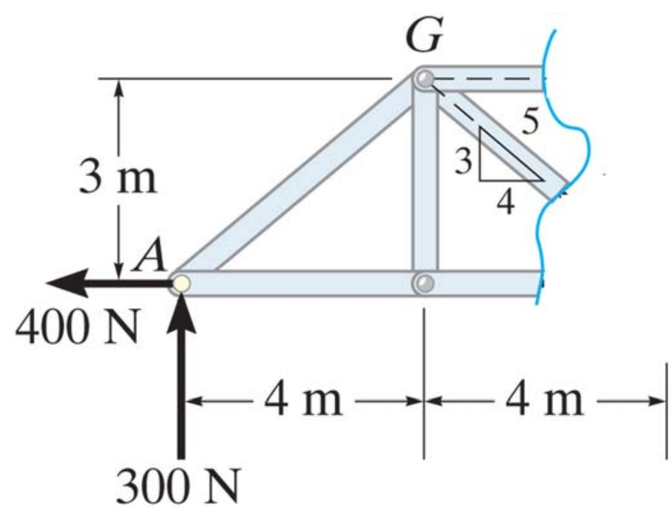


- Determine equilibrium equations (e.g.,moment around point of intersection of two lines)
- Assume all internal loads are tensile.

Example 4)

Determine the force in member GC of the truss and state if the member is in tension or compression.





Frames and machines

Frames and machines are two common types of structures that have at least **one multi-force member** (Recall that trusses have nothing but two-force members).

Frames are generally **stationary** and used to support various external loads.

Machines contain **moving parts** and are designed to alter the effect of forces

Frames and machines

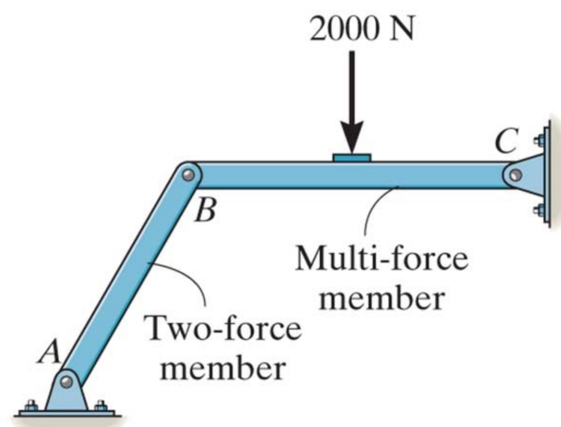
The members can be truss elements, beams, pulleys, cables, and other components. The general solution method is the same:

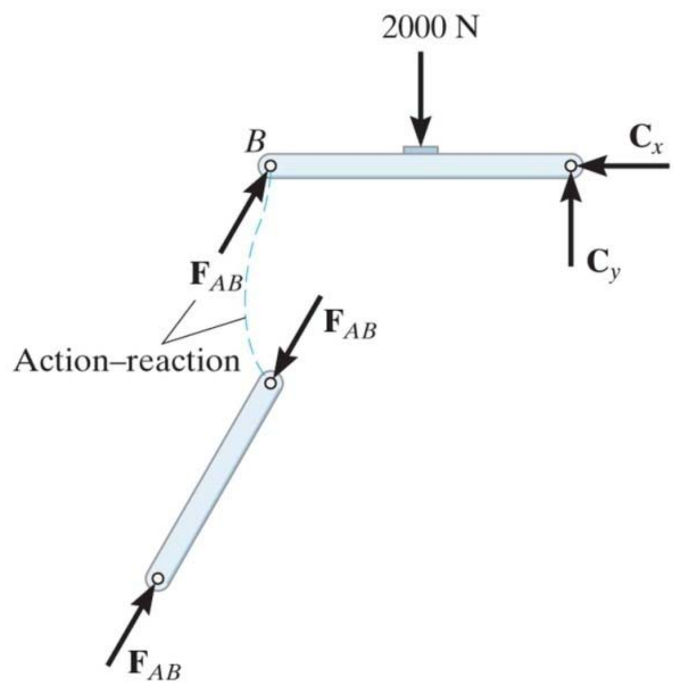
1. Set the entire structure into external equilibrium. This step will generally produce more unknowns than there are relevant equations of equilibrium.

2. Identify two-force members

3. Isolate various part(s) of the structure, setting each part into equilibrium. The sought forces or couples must appear in one or more free-body diagrams.

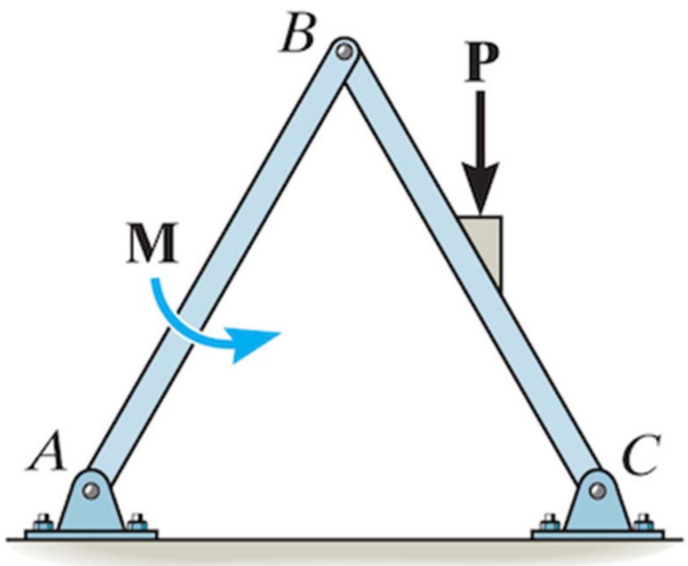
4. Solve for the requested unknowns. Look for ways to form single equations and single unknowns.

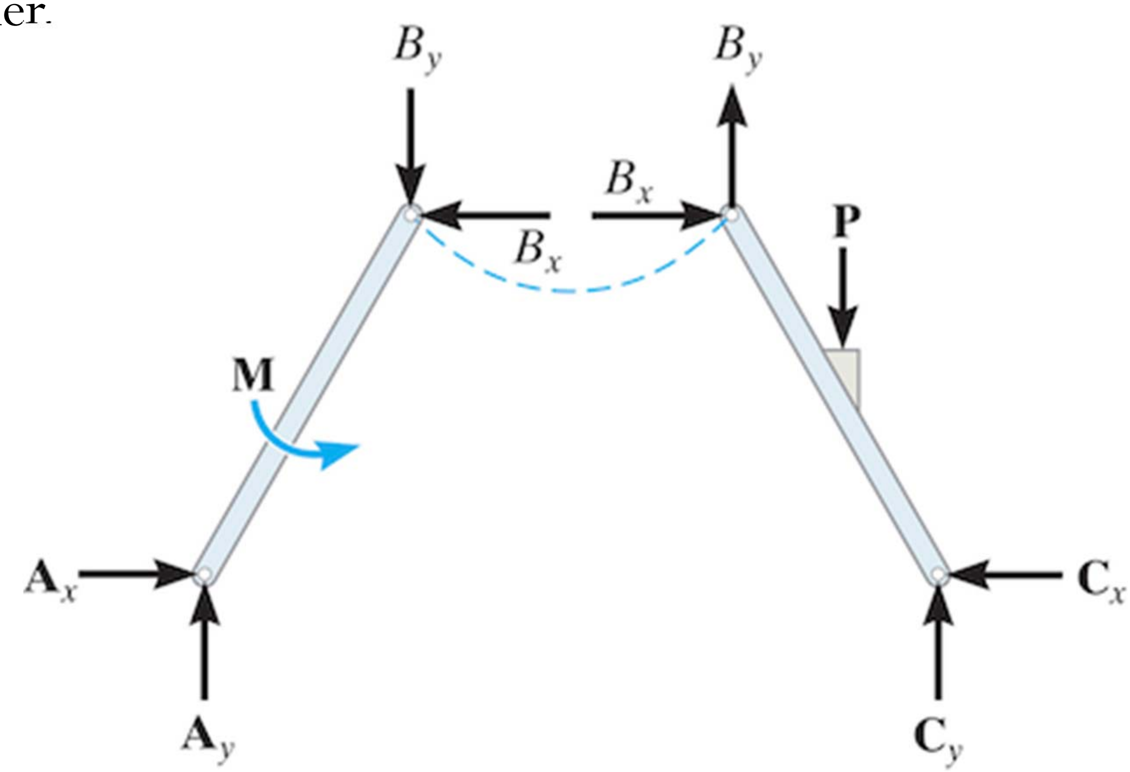


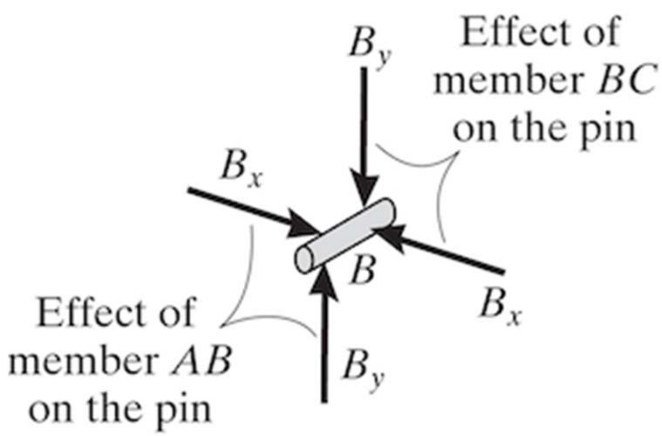


Problems are going to be **challenging** since there are usually several unknowns. A lot of practice is needed to develop good strategies and ease of solving these problems.

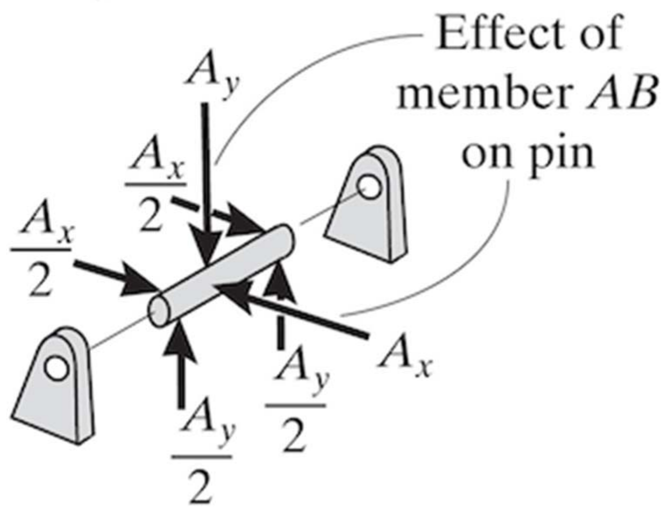
For the frame, draw the free-body diagram of (a) each member, (b) the pins at *B* and *A*, and (c) the two members connected together.





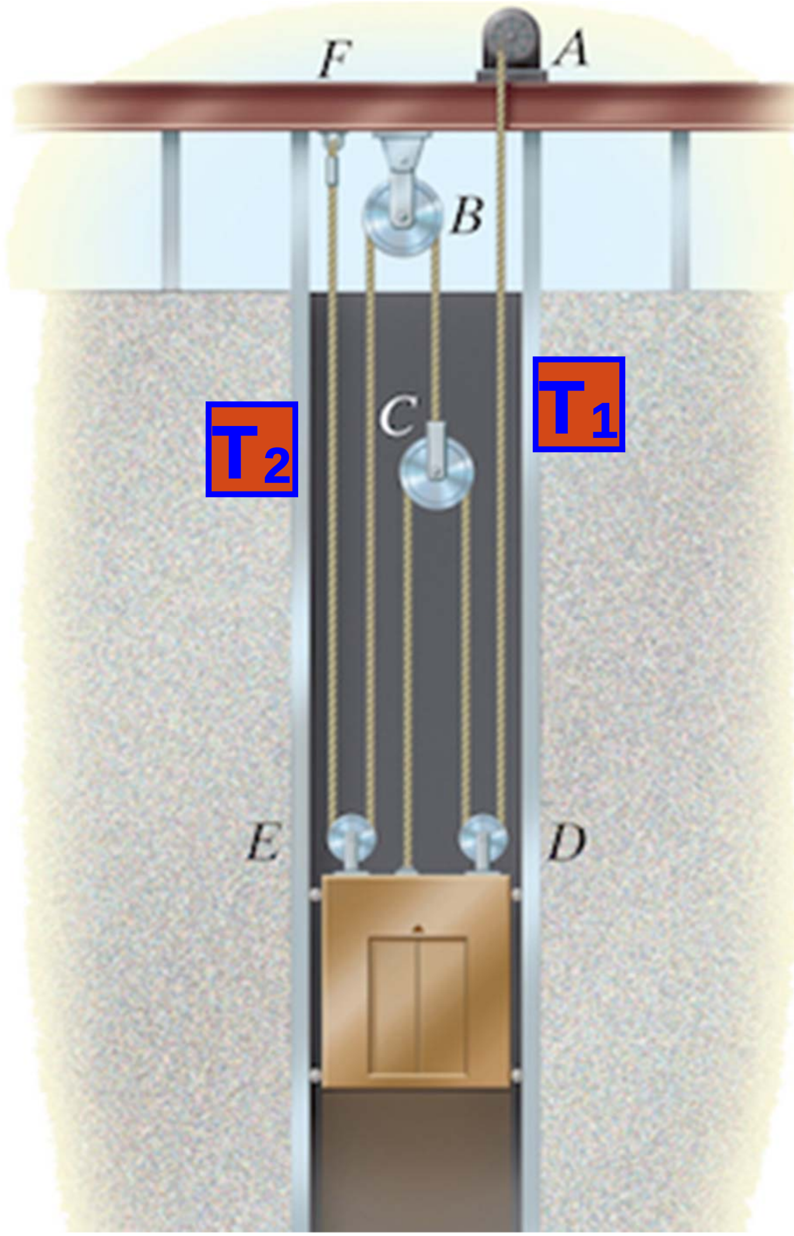


Pin *B*



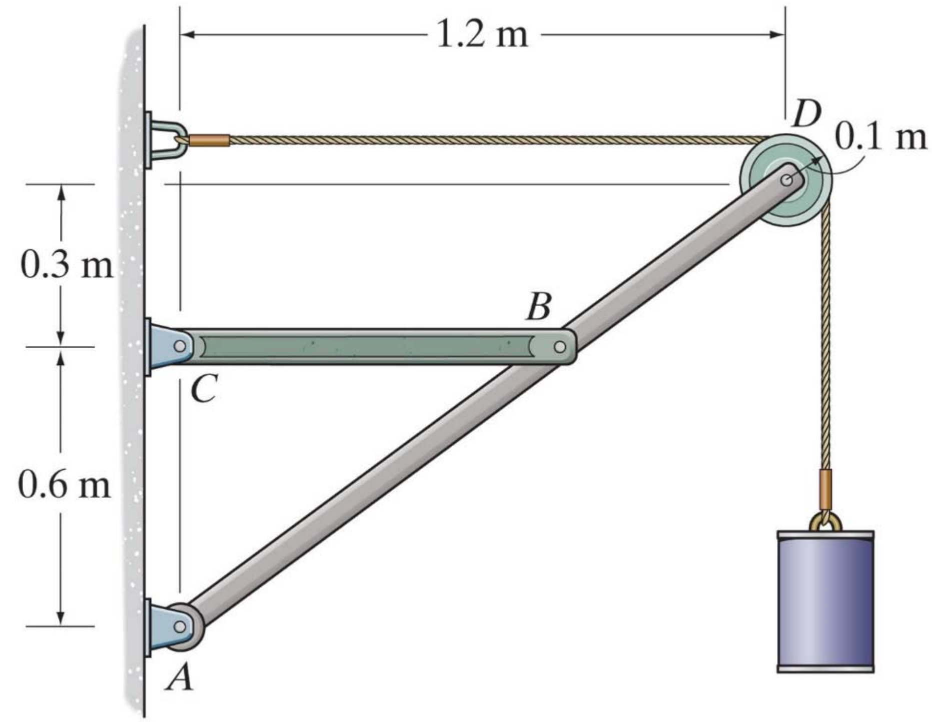
Pin *A*

A 500 kg elevator car is being hoisted by a motor using a pulley system. If the car travels at a constant speed, determine the force developed in the cables. Neglect the cable and pulley masses.



Example 5)

The frame supports a 50kg cylinder. Determine the horizontal and vertical components of reaction at A and the force at C



Example 6)

The compound beam shown is pin-connected at B. Determine the components of reaction at its supports. Neglect its weight and thickness.