Statics - TAM 210 & TAM 211

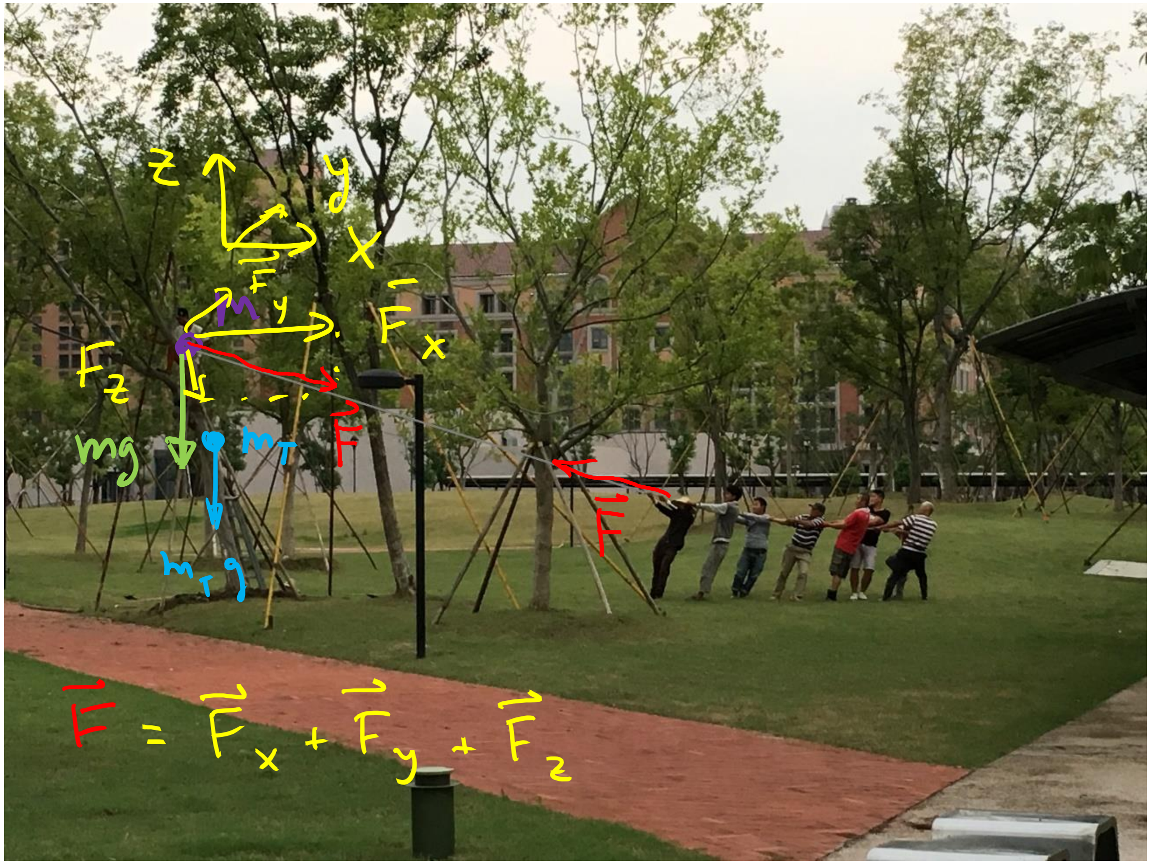
Lecture 3

September 14, 2018

Announcements

- Take practice Quiz 0 on PrairieLearn (not graded)
- MATLAB training sessions TBA (Friday afternoon next 2 weeks)

- Upcoming deadlines:
- Tuesday (Sept 18)
  - HW1
  - Find on PrairieLearn
- Friday (Sept 21)
  - Written Assignment 1
  - Find on Schedule
  - Submit on Blackboard



Chapter 2: Force vectors
Main goals and learning objectives

Define scalars, vectors and vector operations and use them to analyze forces acting on objects

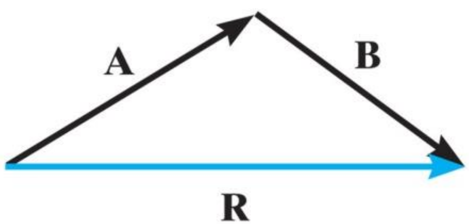
- Add forces and resolve them into components
- Express force and position in Cartesian vector form
- Determine a vector’s magnitude and direction
- Introduce the dot product and use it to find the angle between two vectors or the projection of one vector onto another

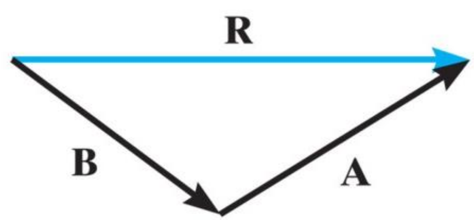
Recap from Lecture 2

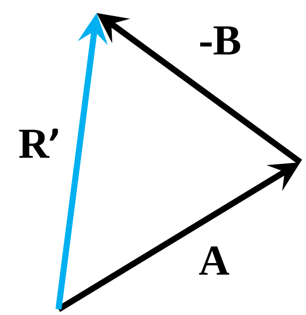
- A force can be treated as a vector, since forces obey all the rules that vectors do.

Half Arrow

$$\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} \qquad \vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}} \qquad \vec{\boldsymbol{R}'} = \vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} = \vec{\boldsymbol{A}} + \left(-\vec{\boldsymbol{B}}\right)$$







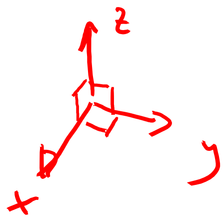
Bold

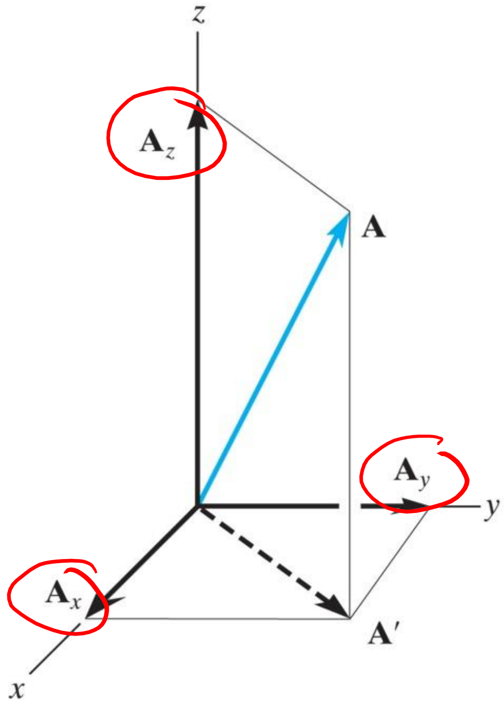
$\mathbf{R} = \mathbf{A} + \mathbf{B}$

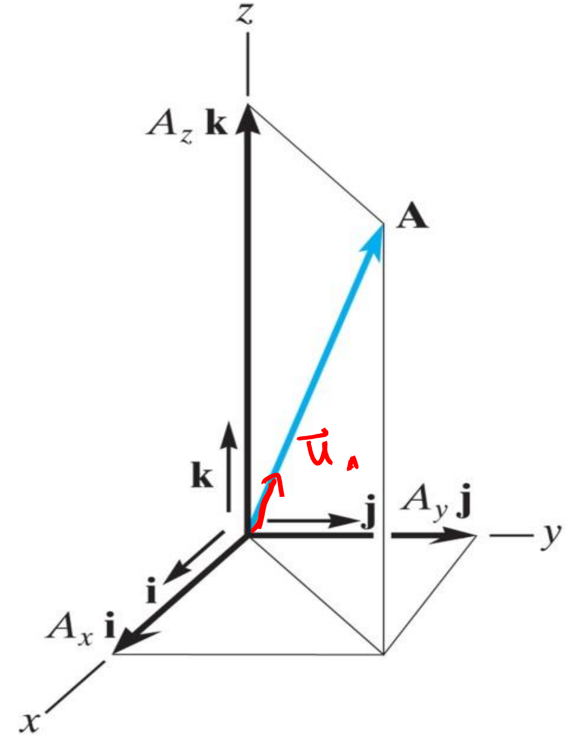
$\mathbf{R} = \mathbf{B} + \mathbf{A}$

Recap

Right-Hand Rule







- Vector representations
  - Rectangular components
  - Cartesian vectors
  - Unit vector

$$\vec{\boldsymbol{A}} = \vec{\boldsymbol{A}_x} + \vec{\boldsymbol{A}_y} + \vec{\boldsymbol{A}_z}$$

$$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} + A_z\widehat{\boldsymbol{k}}$$

$$\vec{\boldsymbol{u}_A} = \frac{\vec{\boldsymbol{A}}}{|\vec{\boldsymbol{A}}|} = \frac{A_x}{|\vec{\boldsymbol{A}}|}\hat{\boldsymbol{\imath}} + \frac{A_y}{|\vec{\boldsymbol{A}}|}\hat{\boldsymbol{\jmath}} + \frac{A_z}{|\vec{\boldsymbol{A}}|}\widehat{\boldsymbol{k}}$$

Basis Vectors

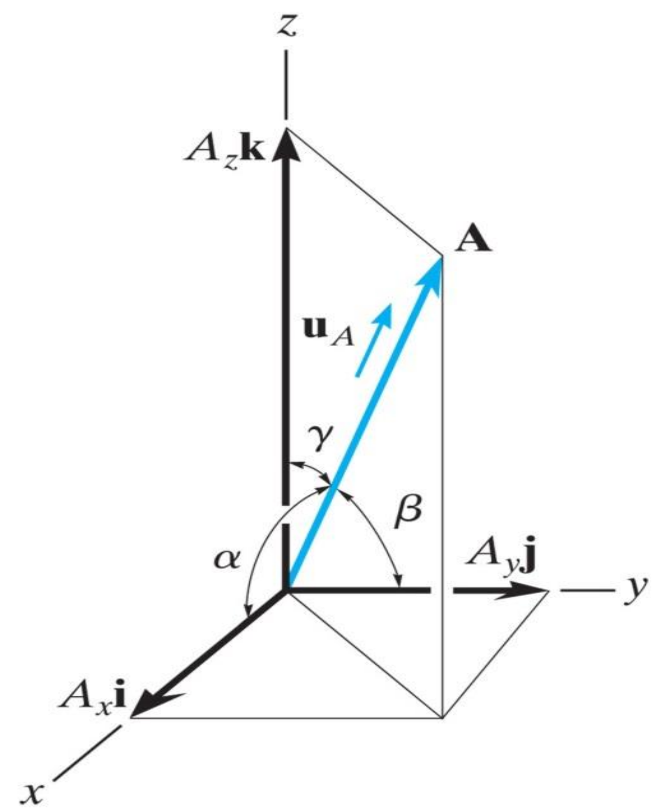
$\hat{i}, \hat{j}, \hat{k}$

unit magnitude

point in x, y, z directions

Recall: Magnitude of a vector (which is a scalar quantity) can be shown as a term with no font modification (A) or vector with norm bars ($|\vec{\boldsymbol{A}}|$), such that $A = |\vec{\boldsymbol{A}}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$\vec{A} = \overrightarrow{A_x} + \overrightarrow{A_y} + \overrightarrow{A_z} \qquad \vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$



- How to define A_x , A_y, A_z?
  - Direction cosines

$$cos(\alpha) = \frac{A_x}{A},\ cos(\beta) = \frac{A_y}{A}, cos(\gamma) = \frac{A_z}{A}$$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$
$$= A\,cos(\alpha)\hat{\imath} + A\,cos(\beta)\hat{\jmath} + A cos(\gamma)\hat{k}$$

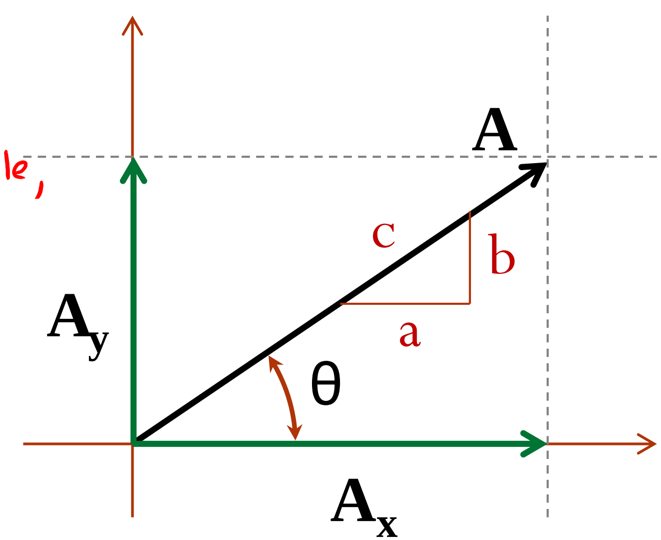
α, β, γ are angles in 3D

  - Rectangular components

In this example, θ is angle in 2D

$$A_x = A\,cos(\theta)\,,\ A_y = A\,sin(\theta)$$

$$A_x = A\left(\frac{a}{c}\right),\ A_y = A\left(\frac{b}{c}\right)$$



The cables attached to the screw eye are subjected to three forces shown.

(a) Express each force vector using the Cartesian vector form (components form).

$$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} + A_z\hat{\boldsymbol{k}} = A\left[\cos(\alpha)\hat{\boldsymbol{\imath}} + \cos(\beta)\hat{\boldsymbol{\jmath}} + \cos(\gamma)\hat{\boldsymbol{k}}\right]$$

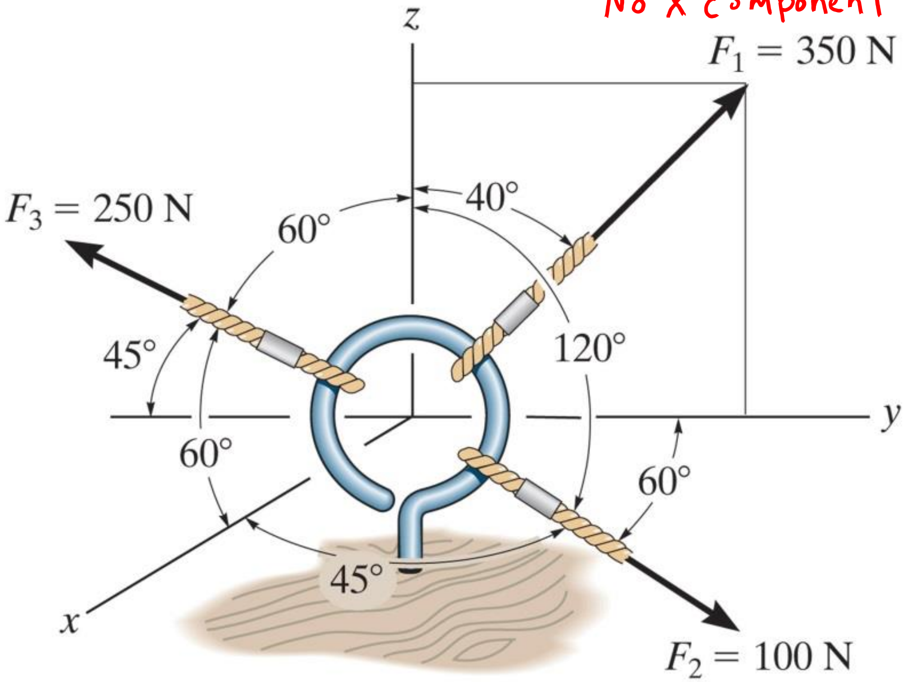
(b) Determine the magnitude of the resultant force vector

$$\left|\vec{\boldsymbol{F}}_R\right| = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$$

(c) Determine the direction cosines of the resultant force vector

$$\cos(\alpha) = \frac{A_x}{A},\ \cos(\beta) = \frac{A_y}{A},\ \cos(\gamma) = \frac{A_z}{A}$$

No x component



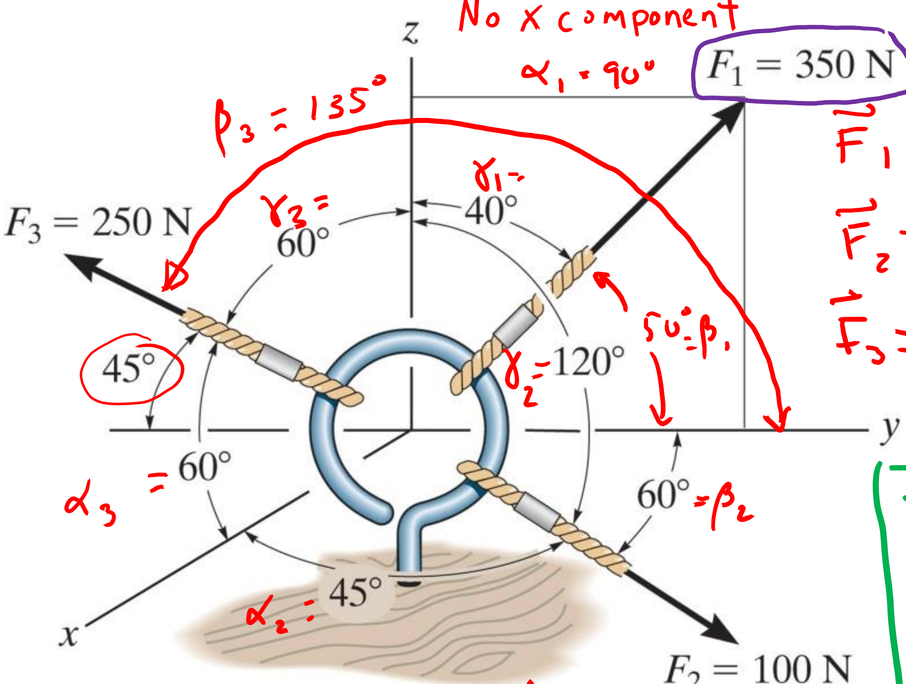
The cables attached to the screw eye are subjected to three forces shown.

(a) Express each force vector using the Cartesian vector form (components form).

$\vec{A} = A_x\hat{\imath}+A_y\hat{\jmath}+A_z\hat{k} = A\left[\cos(\alpha)\hat{\imath} + \cos(\beta)\hat{\jmath} + \cos(\gamma)\hat{k}\right]$

(b) Determine the magnitude of the resultant force vector

$|\vec{F_R}| = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$



$\vec{F}_1 = (350\,N)\left[\cos(90°)\hat{\imath} + \cos(50°)\hat{\jmath} + \cos(40°)\hat{k}\right]$

$\vec{F}_2 = (100\,N)\left[\cos(45°)\hat{\imath} + \cos(60°)\hat{\jmath} + \cos(120°)\hat{k}\right]$

$\vec{F}_3 = (250\,N)\left[\cos(60°)\hat{\imath} + \cos(135°)\hat{\jmath} + \cos(60°)\hat{k}\right]$

or $-\cos(45°)\hat{\jmath}$

$\vec{F}_1 = \left[225\hat{\jmath} + 268\hat{k}\right]N$

$\vec{F}_2 = \left[70.7\hat{\imath} + 50.0\hat{\jmath} - 50.0\hat{k}\right]N$ (a)

$\vec{F}_3 = \left[125\hat{\imath} - 177\hat{\jmath} + 125\hat{k}\right]N$

(b) $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{F}_{Rx} + \vec{F}_{Ry} + \vec{F}_{Rz}$

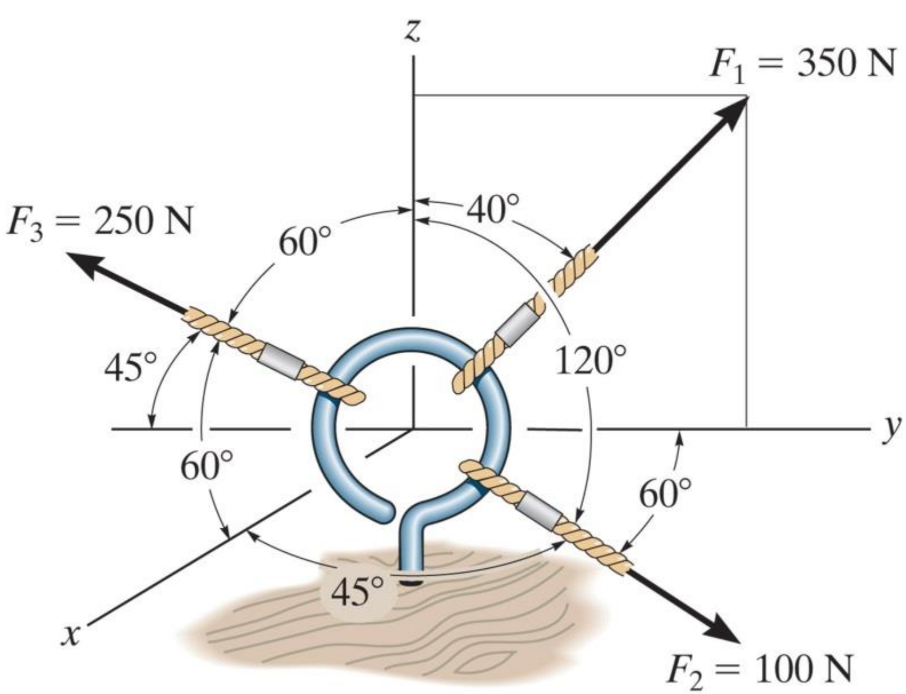
$= (0 + 70.7 + 125)\hat{\imath} + (225 + 50 - 177)\hat{\jmath} + (268 - 50 + 125)\hat{k}$ N

$= 195.71\hat{\imath} + 98.20\hat{\jmath} + 343.12\hat{k}$ N $= |\vec{F}_{Rx}|\hat{\imath} + |\vec{F}_{Ry}|\hat{\jmath} + |\vec{F}_{Rz}|\hat{k}$

$|\vec{F}_R| = \sqrt{(195.71)^2 + (98.20)^2 + (343.12)^2} = 407.03\,N = 407\,N$

$\boxed{|\vec{F}_R| = 407\,N}$ (b) ✓

The cables attached to the screw eye are subjected to three forces shown.
(c) Determine the direction cosines of the resultant force vector



$$cos(\alpha) = \frac{A_x}{A},\ cos(\beta) = \frac{A_y}{A}, cos(\gamma) = \frac{A_z}{A}$$

$$\vec{F}_R = \vec{F}_{Rx} + \vec{F}_{Ry} + \vec{F}_{Rz}$$

$$\cos(\alpha_R) = \frac{|\vec{F}_{Rx}|}{|\vec{F}_R|} = \frac{195.71}{407.03}$$

$$\cos(\beta_R) = \frac{F_{Ry}}{F_R} = \frac{98.20}{407.03}$$

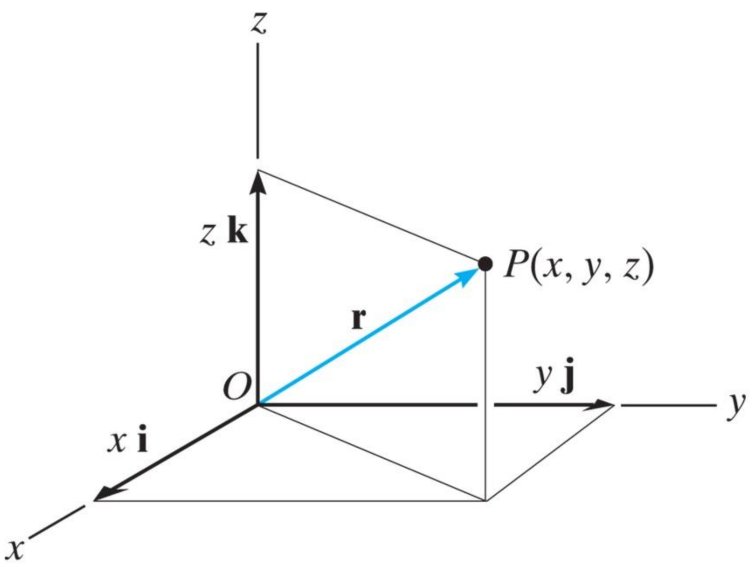
$$\cos(\gamma_R) = \frac{F_{Rz}}{F_R} = \frac{343.12}{407.03}$$

Position vectors

A position vector $\boldsymbol{r}$ is defined as a fixed vector which locates a point in space relative to another point. For example,

$$\vec{\boldsymbol{r}} = x\,\hat{\boldsymbol{\imath}} + y\,\hat{\boldsymbol{\jmath}} + z\,\hat{\boldsymbol{k}}$$

expresses the position of point $P(x,y,z)$ with respect to the origin O.



The position vector $\boldsymbol{r}$ of point $\boldsymbol{B}$ with respect to point $\boldsymbol{A}$ is obtained from:

$$\vec{r}_A + \vec{r} = \vec{r}_B$$

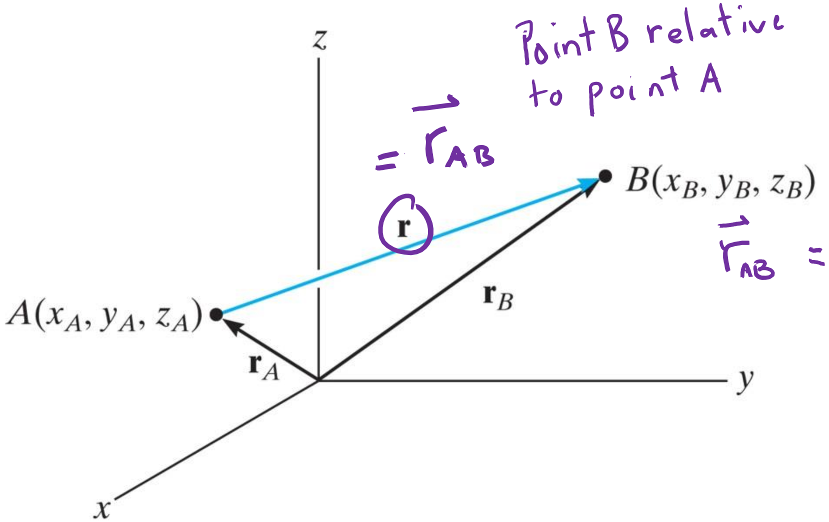
$$\Rightarrow \boxed{\vec{r} = \vec{r}_B - \vec{r}_A}$$

$$\vec{r} = (x_B\hat{\imath} + y_B\hat{\jmath} + z_B\hat{k}) - (x_A\hat{\imath} + y_A\hat{\jmath} + z_A\hat{k})$$

$$\vec{r}_{AB} = \boxed{\vec{r} = (x_B - x_A)\hat{\imath} + (y_B - y_A)\hat{\jmath} + (z_B - z_A)\hat{k}}$$

Point B relative to point A

$= \vec{r}_{AB}$

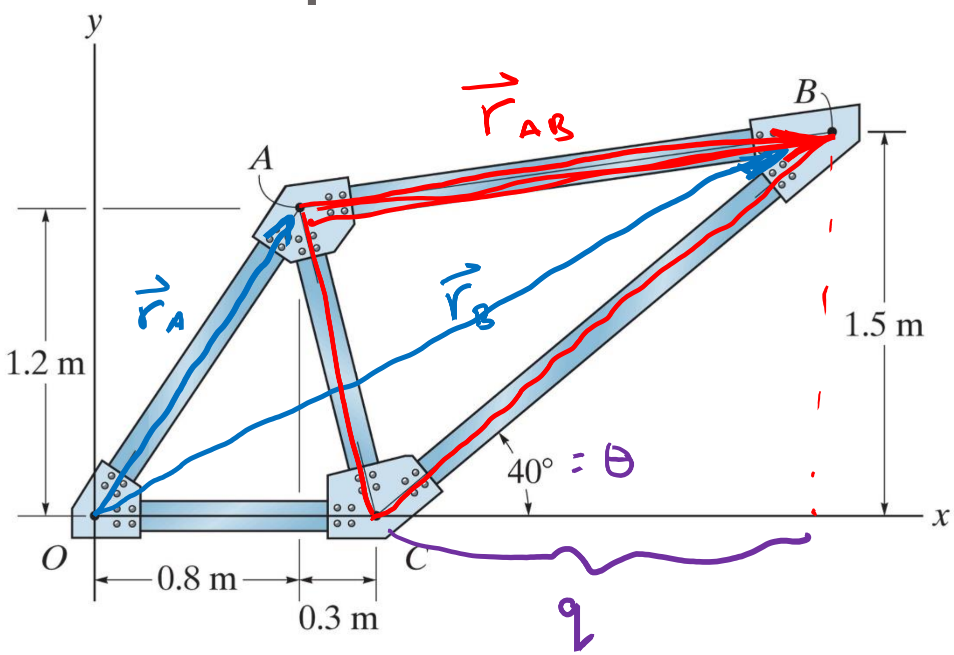


Thus, the $(\boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k})$ components of the positon vector $\boldsymbol{r}$ may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).

Example

Determine the lengths of bars AB, BC and AC.

Length = magnitude



$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

$\vec{r}_A = 0.8\hat{i} + 1.2\hat{j}$ [m]

$\vec{r}_B = ?$

$= (0.8 + 0.3 + q)\hat{i} + 1.5\hat{j}$ [m]

$q = ?$

Use Right Triangle: $\tan 40° = \frac{opp}{adj} = \frac{1.5}{q}$

$q = 1.5/\tan 40° = 1.79 = 1.8$ m

$\therefore \vec{r}_B = 2.9\hat{i} + 1.5\hat{j}$

$\vec{r}_{AB} = 2.1\hat{i} + 0.3\hat{j}$

$\overline{AB} = |\vec{r}_{AB}| = \sqrt{(2.1)^2 + (0.3)^2} = 2.1$ m

$\boxed{\overline{AB} = 2.1}$

Solve for $\overline{BC}$ & $\overline{AC}$ on your own