Statics - TAM 211

Lecture 4

September 17, 2018

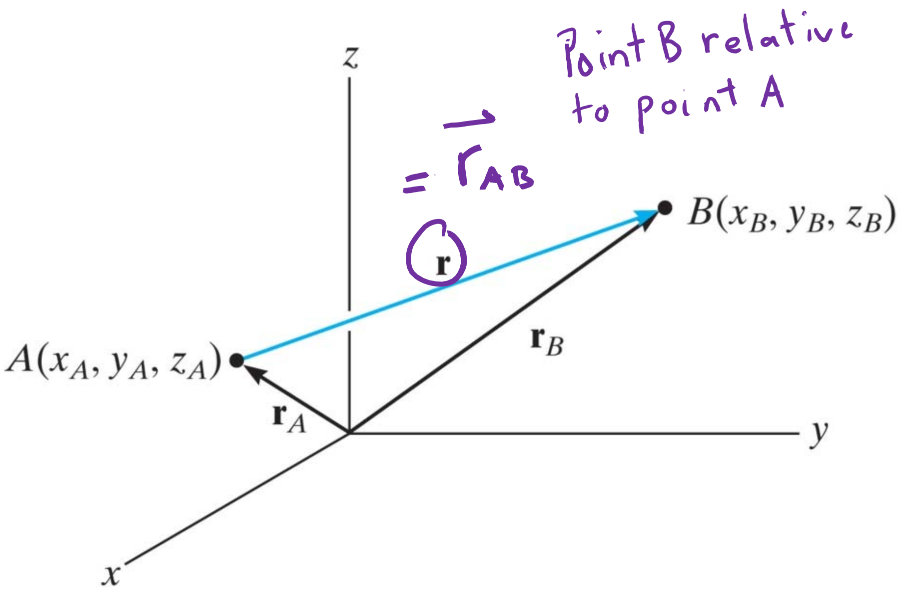
Announcements

- Upcoming deadlines:
- Tuesday (Sept 18)
  - HW1
  - Find on PrairieLearn
- Friday (Sept 21)
  - Written Assignment 1
  - Find on Schedule
  - Submit on Blackboard

giphy.com

Recap from Lecture 3

- Position vector



$$
\begin{aligned}
\boldsymbol{r} &= \boldsymbol{r}_B - \boldsymbol{r}_A \\
&= (x_B\,\boldsymbol{i} + y_B\,\boldsymbol{j} + z_B\,\boldsymbol{k}) - (x_A\,\boldsymbol{i} + y_A\,\boldsymbol{j} + z_A\,\boldsymbol{k})
\end{aligned}
$$

$$
\boldsymbol{r} = (x_B - x_A)\,\boldsymbol{i} + (y_B - y_A)\,\boldsymbol{j} + (z_B - z_A)\,\boldsymbol{k}
$$

Thus, the $(\boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k})$ components of the positon vector $\boldsymbol{r}$ may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).

Force vector directed along a line

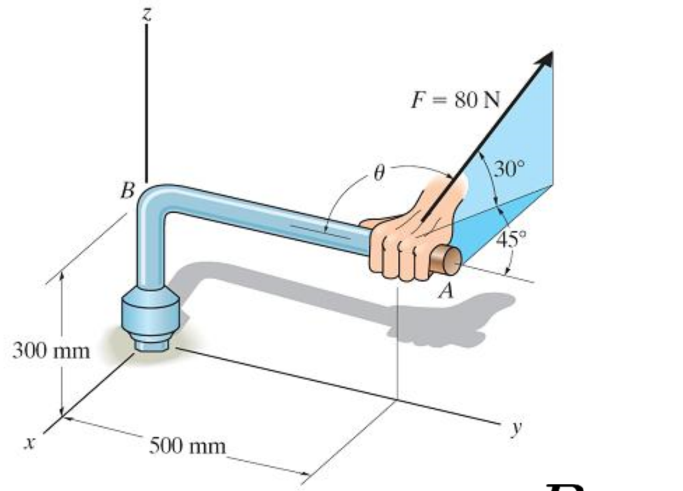
The force vector $\boldsymbol{F}$ acting a long the rope can be defined by the unit vector $\boldsymbol{u}$ (defined the <u>direction</u> of the rope) and the <u>magnitude</u> F of the force.

The unit vector $\boldsymbol{u}$ is specified by the position vector $\boldsymbol{r}$:

Force vector directed along a line

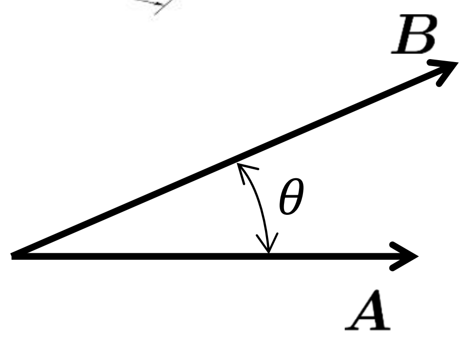
Determine the force vector $\vec{F}$ along the rope.

Dot (or scalar) product



The dot product of vectors **A** and **B** is defined as such

$$\boldsymbol{A} \cdot \boldsymbol{B} =$$

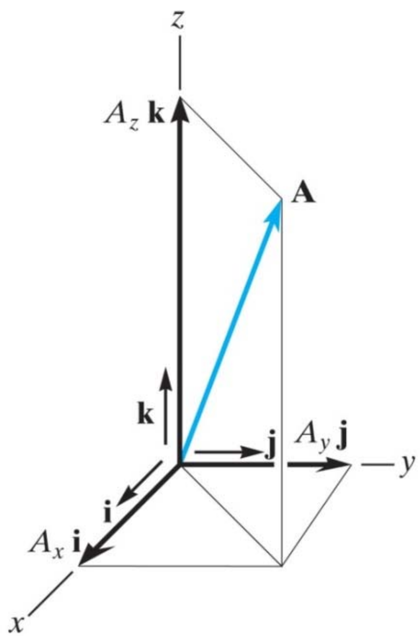


Laws of operation:

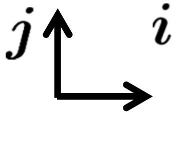
$$\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{B} \cdot \boldsymbol{A}$$
$$\alpha(\boldsymbol{A} \cdot \boldsymbol{B}) = \alpha\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{A} \cdot \alpha\boldsymbol{B}$$
$$\boldsymbol{A} \cdot (\boldsymbol{B} + \boldsymbol{C}) = \boldsymbol{A} \cdot \boldsymbol{B} + \boldsymbol{A} \cdot \boldsymbol{C}$$

Cartesian vector formulation:

$$\boldsymbol{A} \cdot \boldsymbol{B} =$$
$$=$$



Note that:

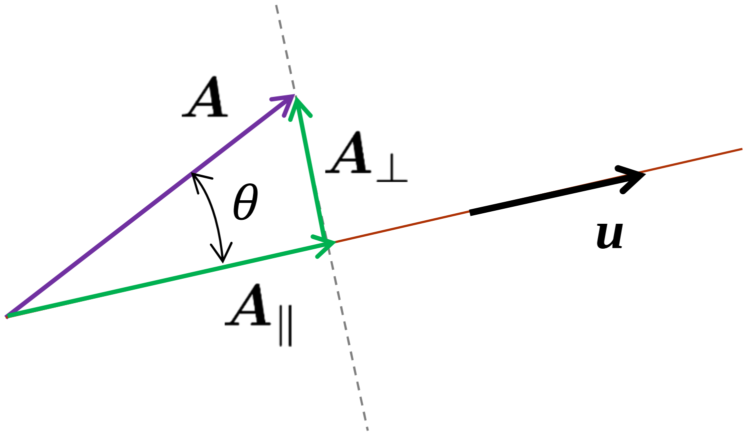


$$\boldsymbol{i} \cdot \boldsymbol{j} = 0$$

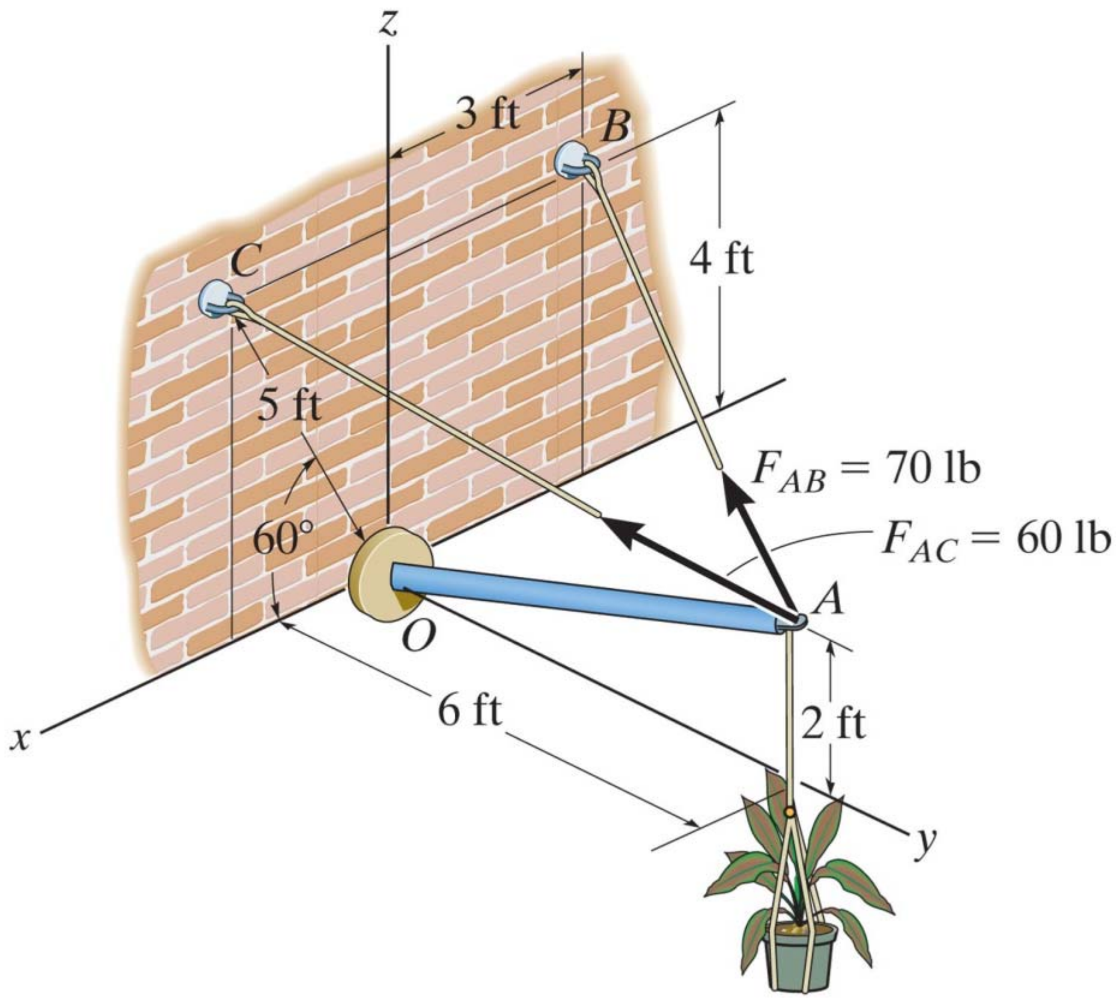
$$\boldsymbol{i} \cdot \boldsymbol{i} = 1$$

Projection of vector onto parallel and perpendicular lines

The scalar component $A_{||}$ of a vector $\boldsymbol{A}$ along (parallel to) a line with unit vector $\boldsymbol{u}$ is given by:



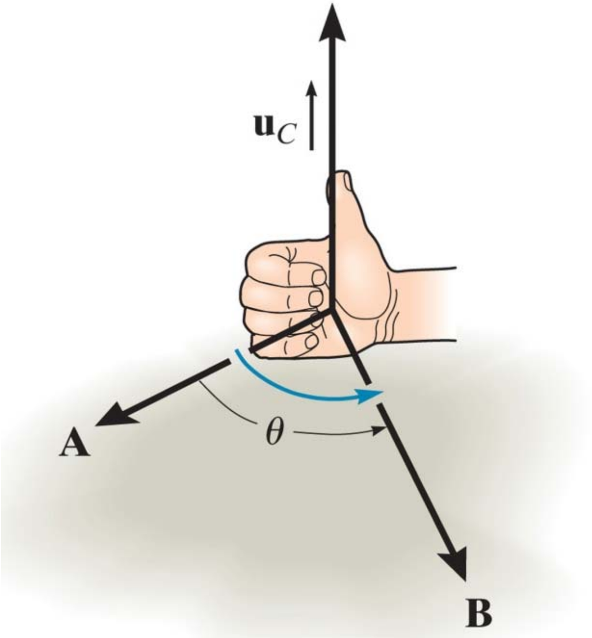
And thus the vector components $\boldsymbol{A}_{||}$ and $\boldsymbol{A}_\perp$ are given by:



Determine the projected component of the force vector F_{AC} along the axis of strut AO. Express your result as a Cartesian vector

Cross (or vector) product

The cross product of vectors **A** and **B** yields the vector **C**, which is written

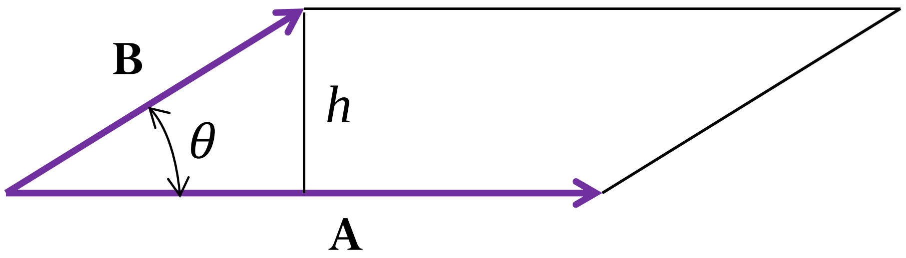


$$\boxed{\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}}$$

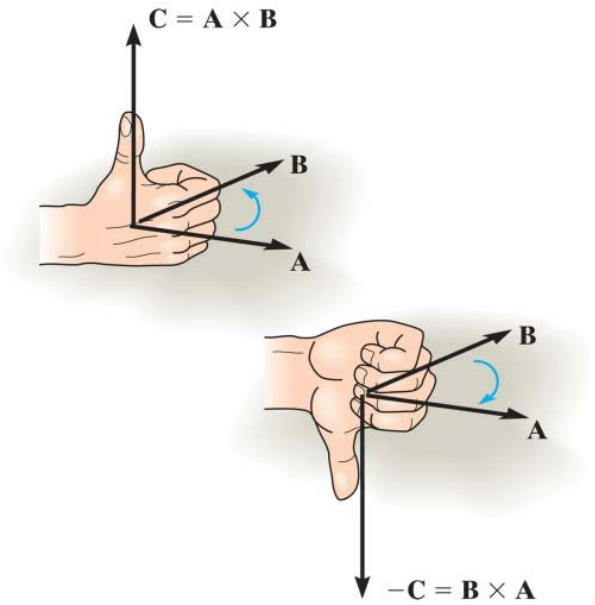
The magnitude of vector **C** is given by:

The vector **C** is perpendicular to the plane containing **A** and **B** (specified by the **right-hand rule**). Hence,

Geometric definition of the cross product: the magnitude of the cross product is given by the area of a parallelogram



Cross (or vector) product



Laws of operation:

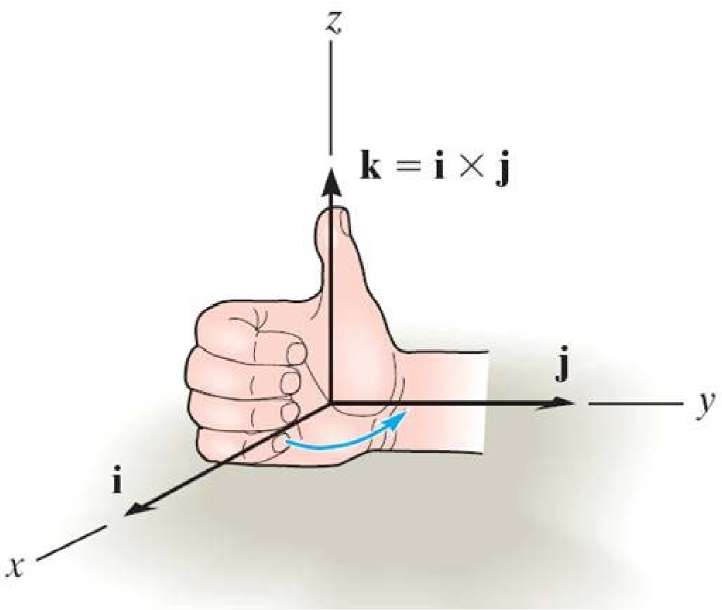
$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$$

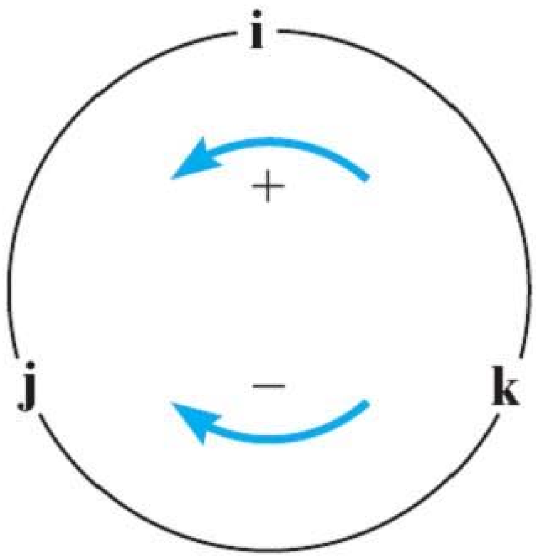
$$\alpha(\boldsymbol{A} \times \boldsymbol{B}) = (\alpha\boldsymbol{A}) \times \boldsymbol{B} = \boldsymbol{A} \times (\alpha\boldsymbol{B}) = (\boldsymbol{A} \times \boldsymbol{B})\alpha$$

$$\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D}) = \boldsymbol{A} \times \boldsymbol{B} + \boldsymbol{A} \times \boldsymbol{D}$$

Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $i \times i = 0$





Considering the cross product in Cartesian coordinates

$\boldsymbol{A} \times \boldsymbol{B}$

Cross (or vector) product

Also, the cross product can be written as a determinant.

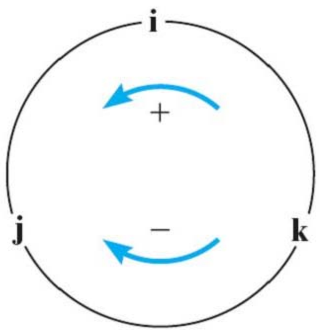
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be determined using 2 × 2 determinants.

For element **i**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

For element **j**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

For element **k**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$



$$\boxed{\boldsymbol{A} \times \boldsymbol{B} = (A_y B_z - A_z B_y)\boldsymbol{i} - (A_x B_z - A_z B_x)\boldsymbol{j} + (A_x B_y - A_y B_x)\boldsymbol{k}}$$