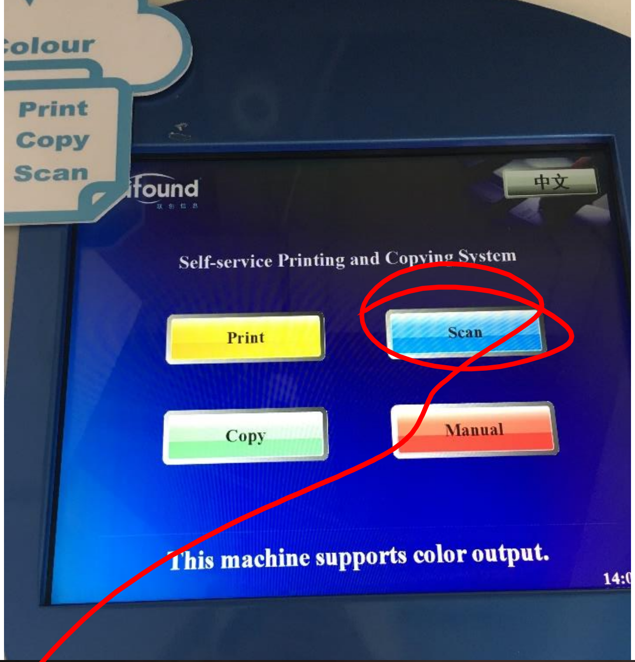
Statics - TAM 211

Lecture 4

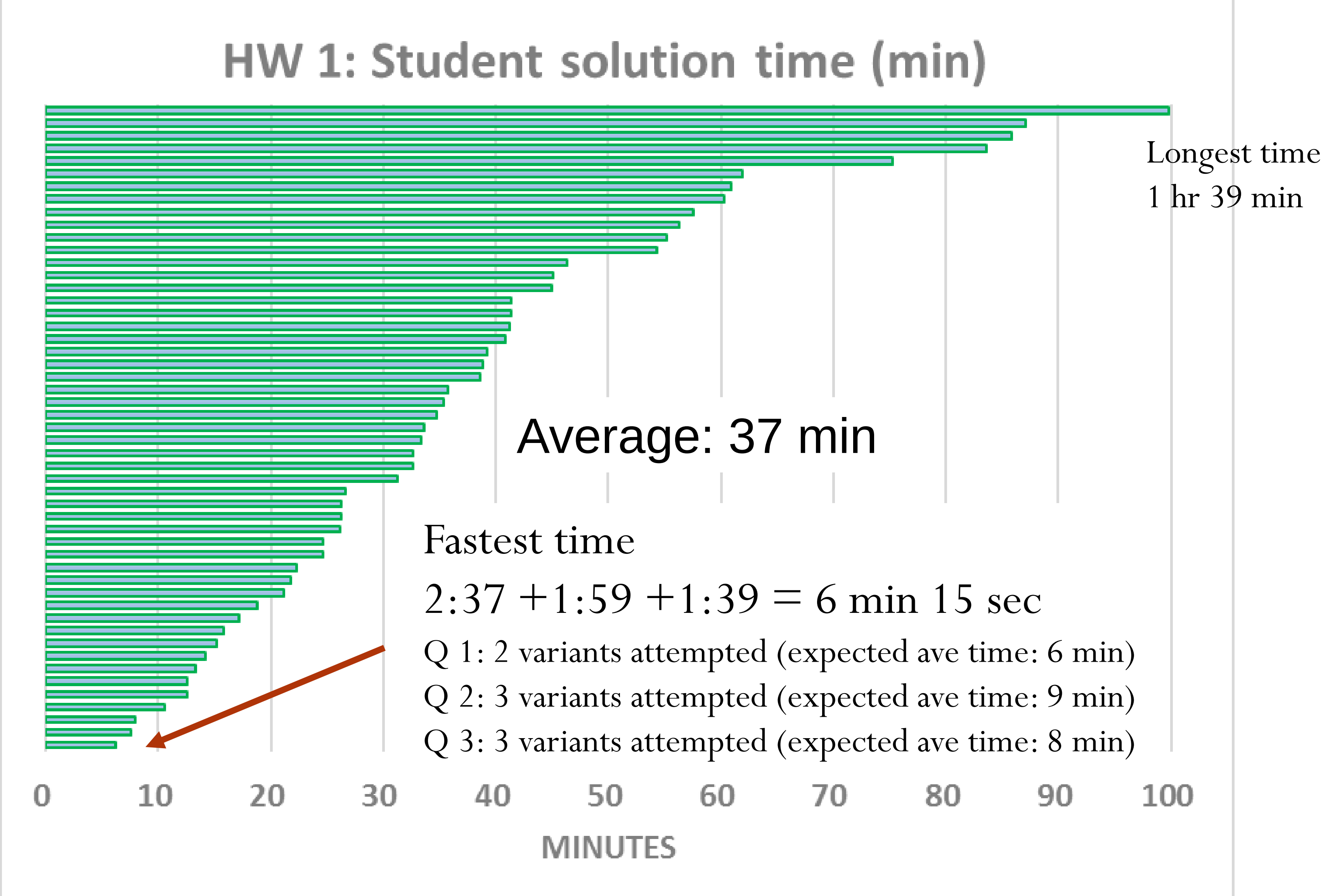
September 17, 2018

Announcements



- Upcoming deadlines:
- Tuesday (Sept 18)
  - HW1
  - Find on [PrairieLearn]
- Friday (Sept 21)
  - Written Assignment 1
  - Find on [Schedule]
  - You can SCAN your WA at RC
  - Submit on Blackboard

giphy.com



- Not trying to solve problems **<u>on your own and copying other's answers</u>** will make taking quizzes ∞ more difficult!
- Similar to students at UIUC. Many continue bad habits and do very poorly in this course.

Student Code & Academic Integrity

"All students are responsible to refrain from infractions of academic integrity, conduct that may lead to suspicion of such infractions, and conduct that aids others in such infractions."

"I did not know" is not an excuse.

The following are academic integrity infractions:

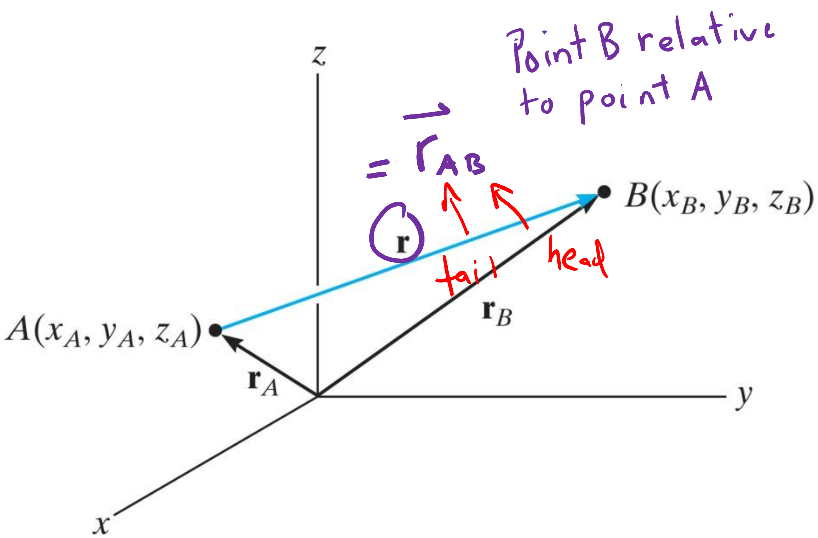
- Cheating - using or attempting to use unauthorized materials
- Plagiarism - representing the words, work, or ideas of another as your own (using non-copyrighted material is also plagiarism)
- Fabrication - falsification or invention of any information, including citations
- Facilitating infractions of academic integrity - helping or attempting to help another commit infraction
- Bribes, Favors, and Threats - actions intended to affect a grade or evaluation
- Academic Interference - tampering, altering or destroying educational material or depriving someone else of access to that material

(source https://provost.illinois.edu/policies/policies/academic-integrity/students-quick-reference-guide-to-academic-integrity/)

– ***Violators will be caught*** – we check!!

Recap from Lecture 3

- Position vector

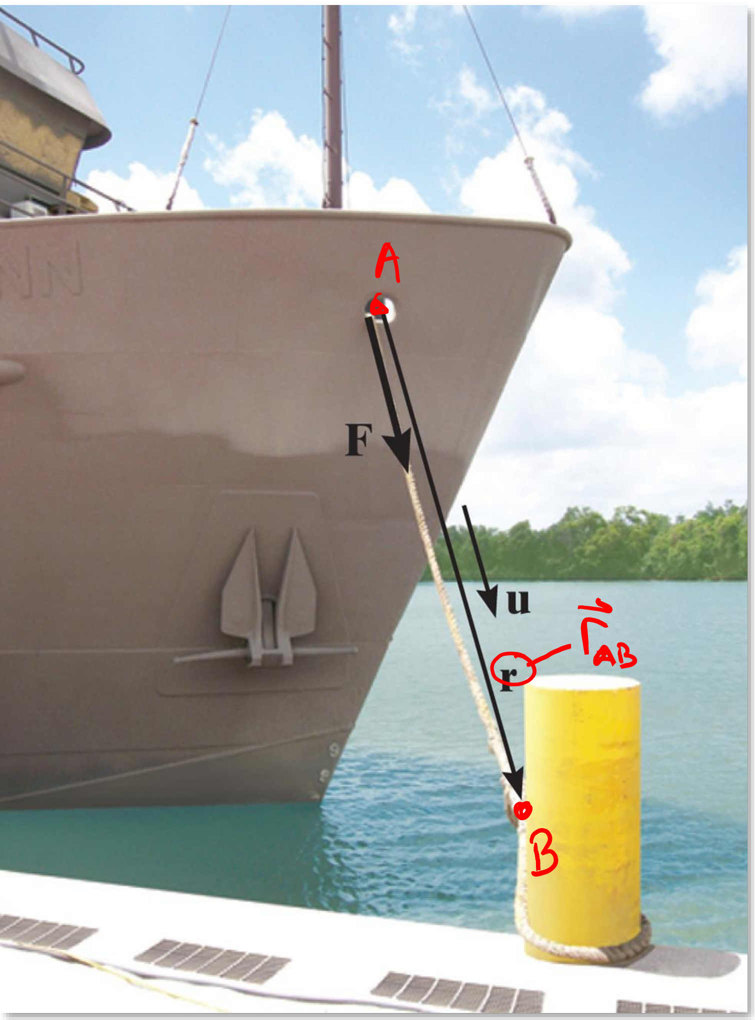


$$\begin{aligned} \boldsymbol{r} &= \boldsymbol{r}_B - \boldsymbol{r}_A \\ &= (x_B\,\boldsymbol{i} + y_B\,\boldsymbol{j} + z_B\,\boldsymbol{k}) - (x_A\,\boldsymbol{i} + y_A\,\boldsymbol{j} + z_A\,\boldsymbol{k}) \end{aligned}$$

$$\boldsymbol{r} = (x_B - x_A)\,\boldsymbol{i} + (y_B - y_A)\,\boldsymbol{j} + (z_B - z_A)\,\boldsymbol{k}$$

Thus, the $(\boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k})$ components of the positon vector $\boldsymbol{r}$ may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).

Force vector directed along a line



The force vector $\boldsymbol{F}$ acting a long the rope can be defined by the unit vector $\boldsymbol{u}$ (defined the <u>direction</u> of the rope) and the <u>magnitude</u> F of the force.

$$\vec{\boldsymbol{F}} = \underbrace{F}_{\text{mag}}\underbrace{\vec{\boldsymbol{u}}}_{\text{direction}}$$

The unit vector $\boldsymbol{u}$ is specified by the position vector $\boldsymbol{r}$:

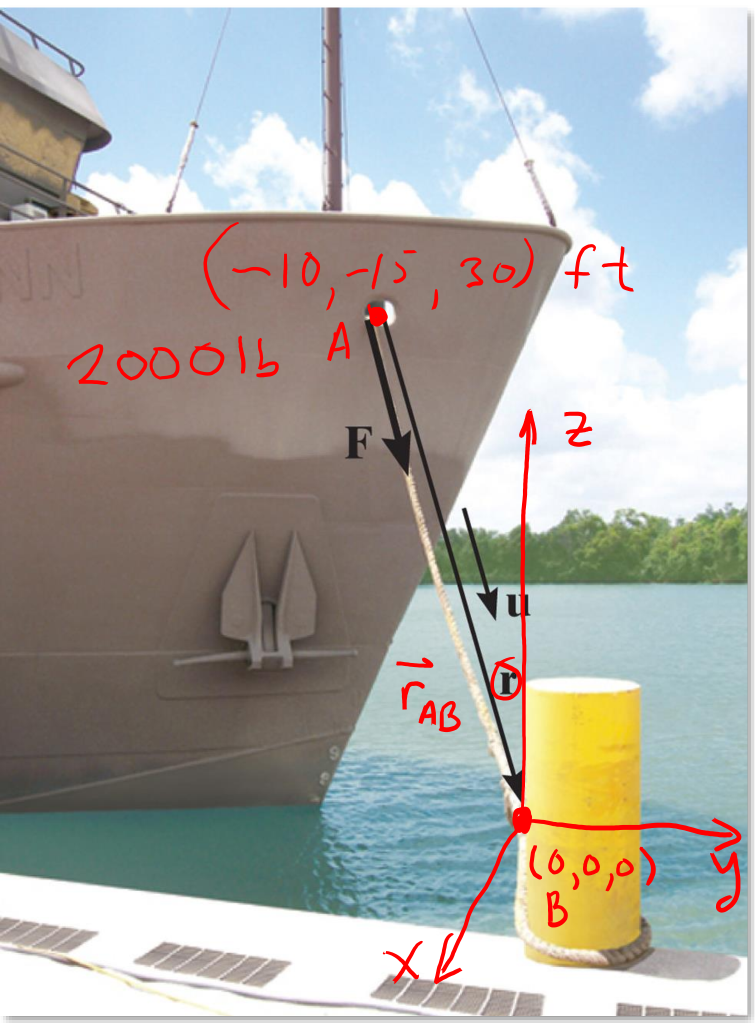
Such that $$\vec{\boldsymbol{u}} = \frac{\vec{\boldsymbol{r}}\ [m]}{|\vec{\boldsymbol{r}}|\ [m]}$$

Note that $\vec{\boldsymbol{u}}$ is unitless and points in the direction of $\vec{\boldsymbol{r}}$.

where

$$\vec{\boldsymbol{r}} = (x_B - x_A)\hat{\boldsymbol{\imath}} + (y_B - y_A)\hat{\boldsymbol{\jmath}} + (z_B - z_A)\hat{\boldsymbol{k}}$$

Force vector directed along a line



Determine the force vector $\vec{F}$ along the rope.

Given: coordinates for pt. A & pt. B & magnitude of F

To solve for $\vec{F}$, use:

$$\vec{F} = |\vec{F}|\,\vec{u} \qquad \vec{u} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = (r_{Bx} - r_{Ax})\hat{i} + (r_{By} - r_{Ay})\hat{j} + (r_{Bz} - r_{Az})\hat{k}$$

$$\vec{r}_{AB} = (0 - (-10))\hat{i} + (0 - (-15))\hat{j} + (0 - (30))\hat{k}$$

$$\vec{r}_{AB} = [10\hat{i} + 15\hat{j} - 30\hat{k}]\ ft$$

$$\therefore\ \vec{F} = (2000\,lb)\left[\frac{10\hat{i} + 15\hat{j} - 30\hat{k}}{\sqrt{10^2 + 15^2 + (-30)^2}}\right]\frac{[ft]}{[ft]}$$

$$= 2000\left[\frac{10\hat{i} + 15\hat{j} - 30\hat{k}}{\sqrt{1325}}\right][lb]$$

$$\boxed{\vec{F} = 549.4\,\hat{i} + 824.2\,\hat{j} - 1648.3\,\hat{k}\ [lb]}$$

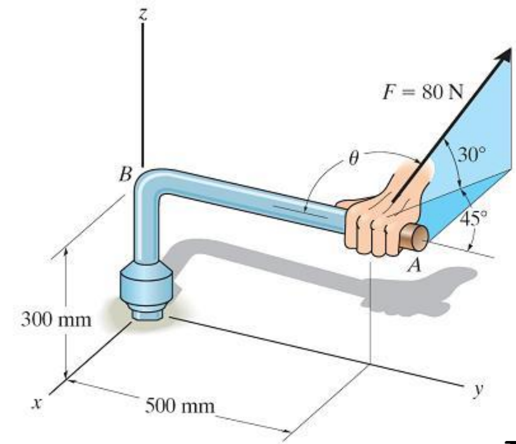
Dot (or scalar) product

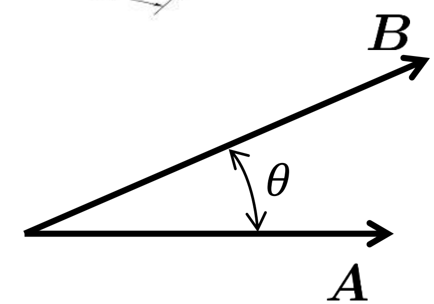
The dot product of vectors **A** and **B** is defined as such

$$\vec{A} \cdot \vec{B} = \quad |\vec{A}||\vec{B}| \cos\theta$$

Uses of dot product:

- Find angle btw 2 vectors $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$
- Find projections $\parallel$ or $\perp$ to a line



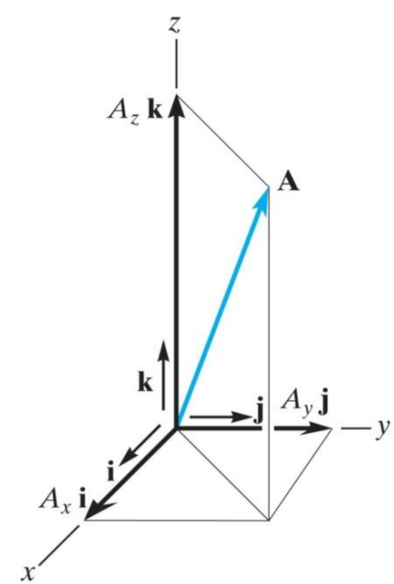


Laws of operation:

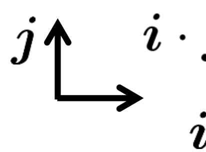
$$\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{B}\cdot\boldsymbol{A}$$
$$\alpha(\boldsymbol{A}\cdot\boldsymbol{B}) = \alpha\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{A}\cdot\alpha\boldsymbol{B}$$
$$\boldsymbol{A}\cdot(\boldsymbol{B}+\boldsymbol{C}) = \boldsymbol{A}\cdot\boldsymbol{B} + \boldsymbol{A}\cdot\boldsymbol{C}$$

Cartesian vector formulation:

$$\begin{aligned} \boldsymbol{A}\cdot\boldsymbol{B} &= (A_x\,\boldsymbol{i} + A_y\,\boldsymbol{j} + A_z\,\boldsymbol{k})\cdot(B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k}) \\ &= A_x\,B_x + A_y\,B_y + A_z\,B_z \end{aligned}$$



Note that:



$$\boldsymbol{i}\cdot\boldsymbol{j} = 0$$

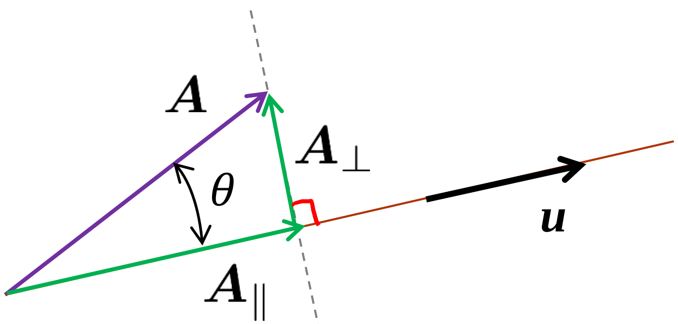
$$\boldsymbol{i}\cdot\boldsymbol{i} = 1$$

Projection of vector onto parallel and perpendicular lines

The scalar component $A_{||}$ of a vector $\boldsymbol{A}$ along (parallel to) a line with unit vector $\boldsymbol{u}$ is given by:

$|\vec{u}| = 1$

$$A_{||} = \boldsymbol{A} \cdot \boldsymbol{u} = |\boldsymbol{A}| \cos(\theta)$$

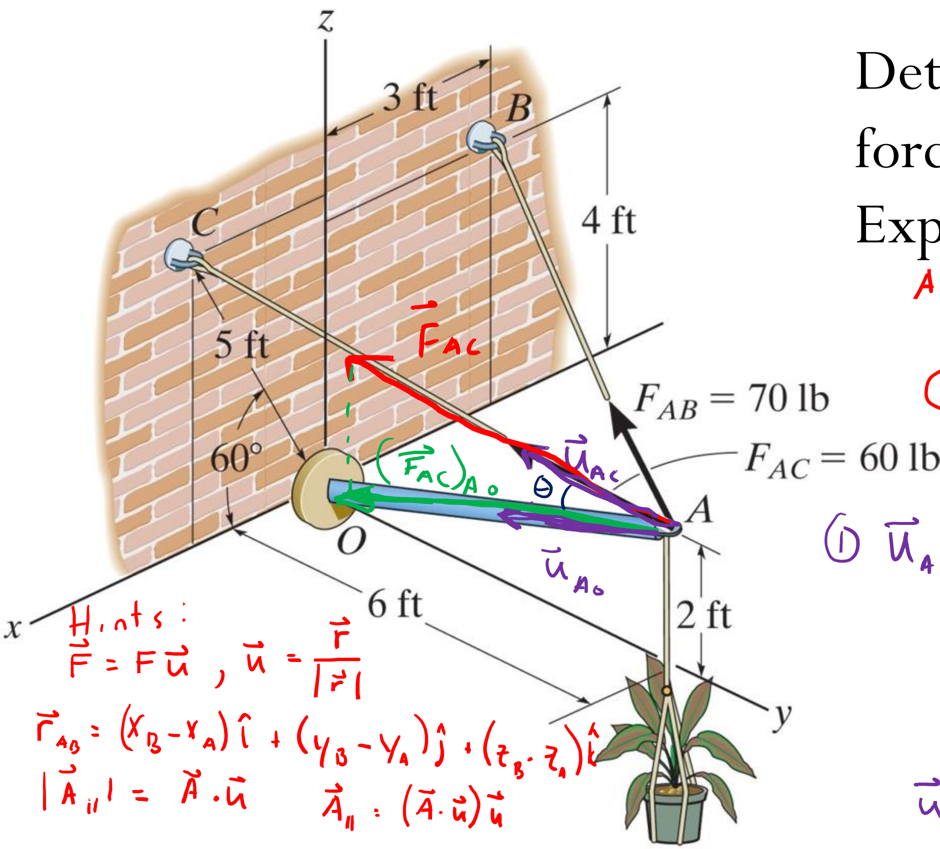


$\vec{A}_{||} + \vec{A}_{\perp} = \vec{A}$

And thus the <u>vector</u> components $\boldsymbol{A}_{||}$ and $\boldsymbol{A}_\perp$ are given by:

$$\vec{\boldsymbol{A}}_{||} = A_{||}\, \vec{\boldsymbol{u}} = \underbrace{(\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{u}})}_{\text{scalar}}\vec{\boldsymbol{u}}$$

$$\vec{\boldsymbol{A}}_\perp = \vec{\boldsymbol{A}} - \vec{\boldsymbol{A}}_{||}$$



Determine the projected component of the force vector F_{AC} along the axis of strut AO. Express your result as a Cartesian vector

Approach:

1. Find unit vectors $\vec{u}_{AC}$ & $\vec{u}_{AO}$
2. Write force vector $\vec{F}_{AC}$ in Cartesian Coord
3. Solve for $\vec{F}_{AC} \cdot \vec{u}_{AO} = (F_{AC})_{AO}$ "$\vec{F}_{AC}$ projected in direction of $\vec{u}_{AO}$"

Hints:

$\vec{F} = F\vec{u}$, $\vec{u} = \frac{\vec{r}}{|\vec{r}|}$

$\vec{r}_{AB} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$

$|\vec{A}_{\parallel}| = \vec{A}\cdot\vec{u}$ $\quad \vec{A}_{\parallel} = (\vec{A}\cdot\vec{u})\vec{u}$

① $$\vec{u}_{AC} = \frac{(x_C - x_A)\hat{i} + (y_C - y_A)\hat{j} + (z_C - z_A)\hat{k}}{|\vec{u}_{AC}|}$$

$$= \frac{5\cos(60)\hat{i} + (-6)\hat{j} + [5\sin(60) - 2]\hat{k}}{|\vec{u}_{AC}|}$$

$$\vec{u}_{AC} = 0.362\hat{i} - 0.869\hat{j} + 0.338\hat{k}$$ (unit vector)

$$= \frac{1}{|\vec{u}_{AC}|}\left(\frac{5}{2}, -6, \frac{5\sqrt{3} - 4}{2}\right)$$ where $|\vec{u}_{AC}| = 6.905$

$$\vec{u}_{AO} = \frac{(x_O - x_A)\hat{i} + (y_O - y_A)\hat{j} + (z_O - z_A)\hat{k}}{|\vec{u}_{AO}|}$$

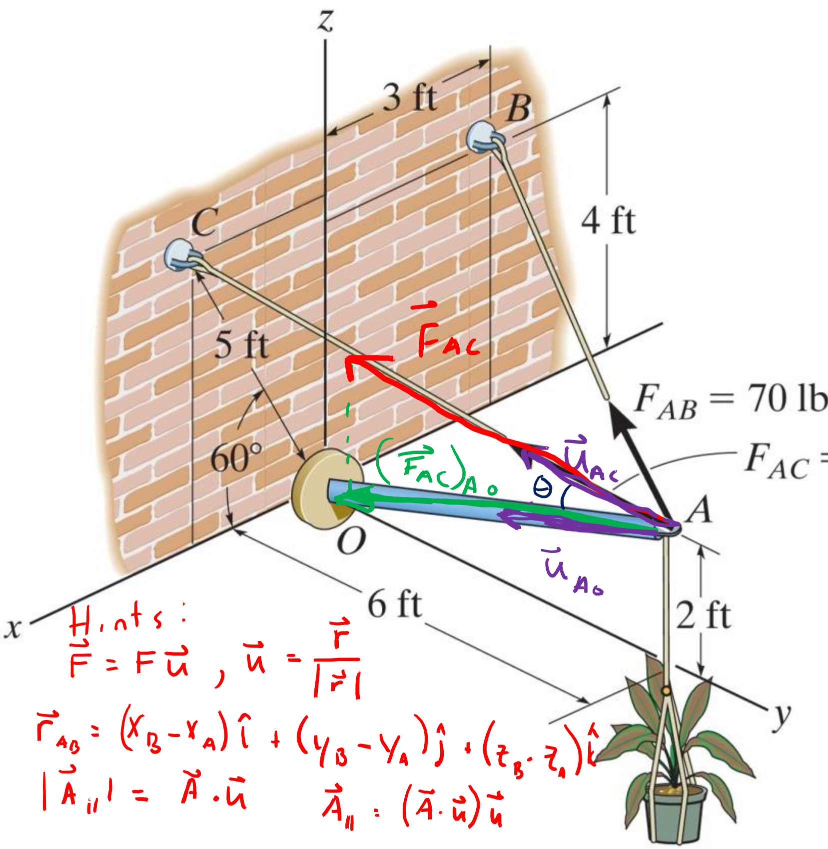
$$= \frac{(-0)\hat{i} + (-6)\hat{j} + (-2)\hat{k}}{|\vec{u}_{AO}|}$$

$$\vec{u}_{AO} = -0.949\hat{j} - 0.316\hat{k}$$ (unit vector)

$$= \left(0, -\frac{3\sqrt{10}}{10}, -\frac{\sqrt{10}}{10}\right)$$

Compare to Liu Yingkai's Solution from Class

Determine the projected component of the force vector F_{AC} along the axis of strut AO. Express your result as a Cartesian vector



Hints:

$\vec{F} = F\vec{u}$, $\vec{u} = \dfrac{\vec{r}}{|\vec{r}|}$

$\vec{r}_{AB} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$

$|\vec{A}_{\parallel}| = \vec{A} \cdot \vec{u}$ $\quad \vec{A}_{\parallel} = (\vec{A} \cdot \vec{u})\vec{u}$

Approach:

1. Find unit vectors $\vec{u}_{AC}$ & $\vec{u}_{AO}$
2. Write force vector $\vec{F}_{AC}$ in Cartesian Coord
3. Solve for $\vec{F}_{AC} \cdot \vec{u}_{AO} = (F_{AC})_{AO}$ "$\vec{F}_{AC}$ projected in direction of $\vec{u}_{AO}$"

(2) $\vec{F}_{AC} = \underbrace{F_{AC}}_{\text{mag}} \underbrace{\vec{u}_{AC}}_{\text{direction}}$

$$\vec{F}_{AC} = (60\text{ lb})[0.362\hat{i} - 0.869\hat{j} + 0.338\hat{k}]$$

$$\cong (21.72, -52.14, 20.28)$$

(3) $(F_{AC})_{AO} = \vec{F}_{AC} \cdot \vec{u}_{AO} = (F_{AC}\vec{u}_{AC}) \cdot \vec{u}_{AO}$

$$= (21.72 * 0) + (-52.14 * -0.949) + (20.28 * -0.316)$$

-6.408

$$\boxed{(F_{AC})_{AO} = 43.07 \text{ lb}}$$

Examples

Given the vectors

$$\begin{aligned} \boldsymbol{A} &= 2\,\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k} \\ \boldsymbol{B} &= 15\,\boldsymbol{i} - 20\,\boldsymbol{j} + 18\,\boldsymbol{k} \\ \boldsymbol{C} &= \boldsymbol{i} + 7\,\boldsymbol{k} \end{aligned}$$

Solve these on your own

Determine:

1. $\boldsymbol{A} + \boldsymbol{B}$
2. $\boldsymbol{B} - \boldsymbol{C}$
3. $\boldsymbol{A} \cdot \boldsymbol{B}$
4. $\boldsymbol{B} \times \boldsymbol{C}$
5. a unit vector in the direction of $\boldsymbol{C}$
6. the direction cosines of $\boldsymbol{B}$