Statics - TAM 211

Lecture 5

September 19, 2018

Announcements

- Upcoming deadlines:
- Friday (Sept 21)
  - Written Assignment 1
  - Find on Schedule
  - You can SCAN your WA at RC
  - Submit on Blackboard
- Tuesday (9/26)
  - Prairie Learn HW2

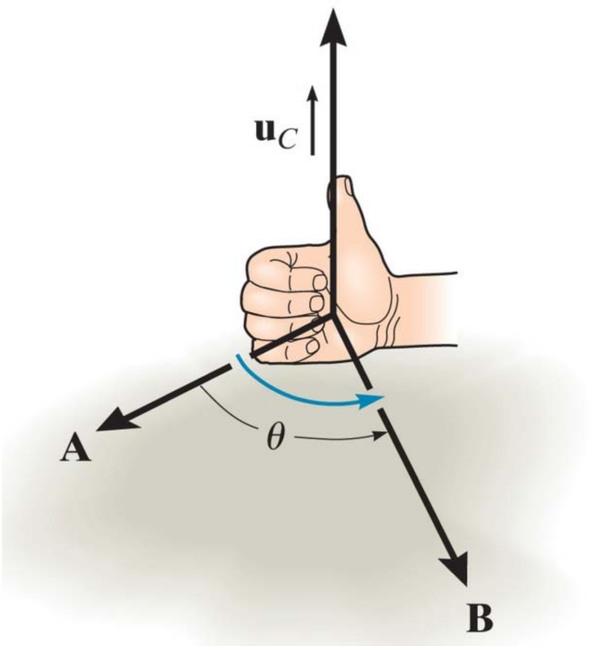
Recap of Lecture 4

- Position vectors
- Force vector directed along a line
- Dot (scalar) product

Cross (or vector) product

The cross product of vectors **A** and **B** yields the vector **C**, which is written

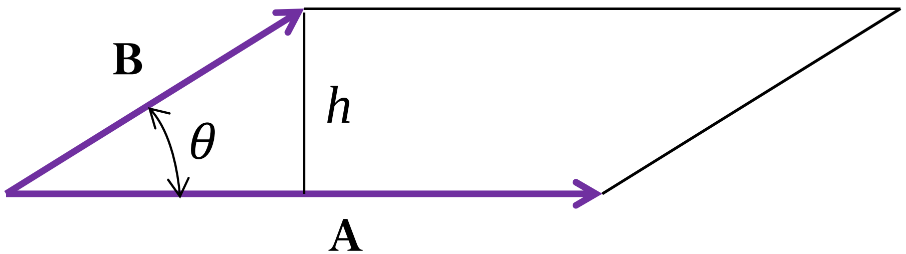
$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$$



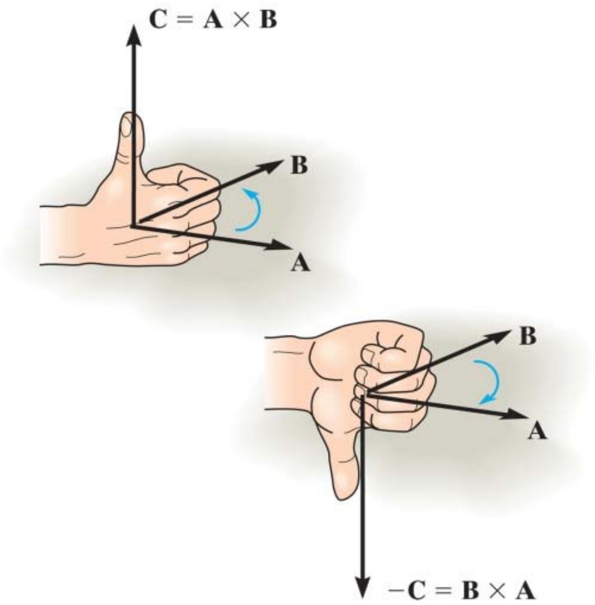
The magnitude of vector **C** is given by:

The vector **C** is perpendicular to the plane containing **A** and **B** (specified by the **right-hand rule**). Hence,

Geometric definition of the cross product: the magnitude of the cross product is given by the area of a parallelogram



Cross (or vector) product



Laws of operation:

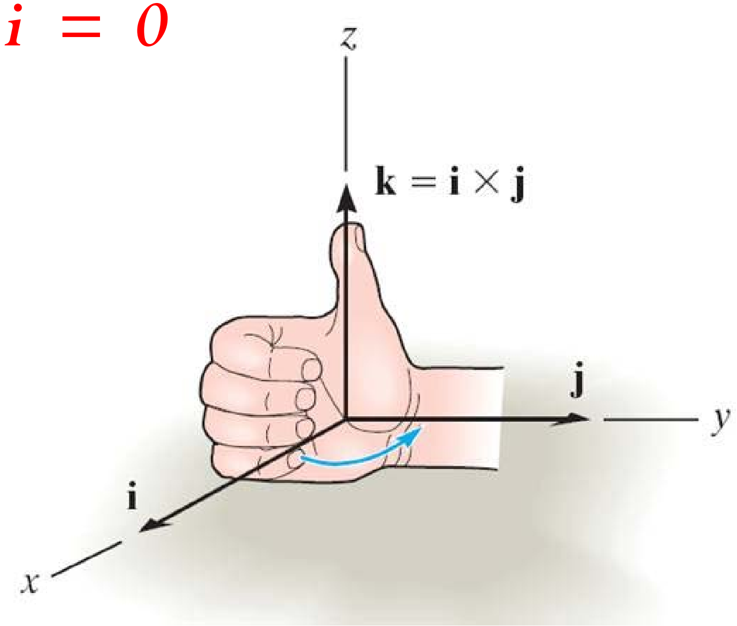
$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$$

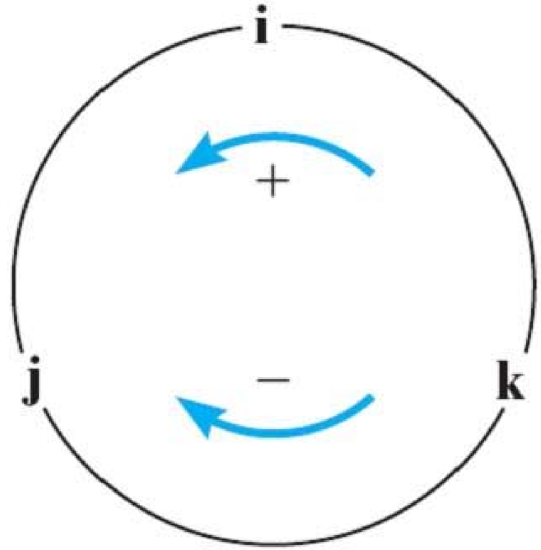
$$\alpha(\boldsymbol{A} \times \boldsymbol{B}) = (\alpha \boldsymbol{A}) \times \boldsymbol{B} = \boldsymbol{A} \times (\alpha \boldsymbol{B}) = (\boldsymbol{A} \times \boldsymbol{B})\alpha$$

$$\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D}) = \boldsymbol{A} \times \boldsymbol{B} + \boldsymbol{A} \times \boldsymbol{D}$$

Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{0}$





Considering the cross product in Cartesian coordinates

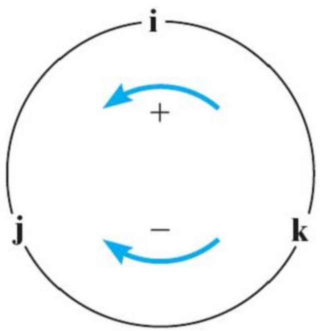
$\boldsymbol{A} \times \boldsymbol{B}$

Cross (or vector) product

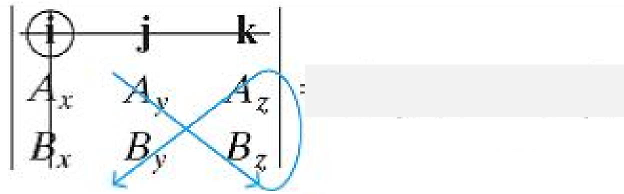
Also, the cross product can be written as a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

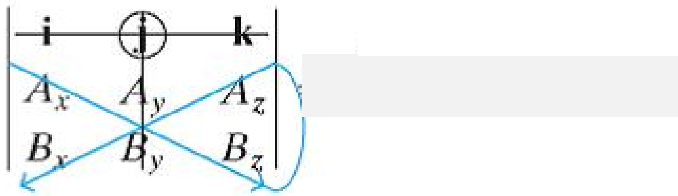
Each component can be determined using 2 × 2 determinants.



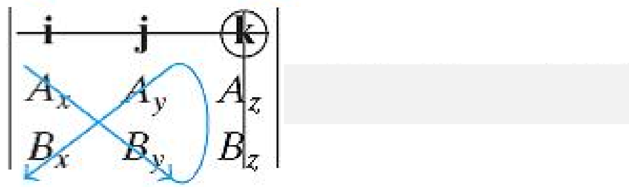
For element **i**:



For element **j**:



For element **k**:



$$\boldsymbol{A} \times \boldsymbol{B} = (A_y B_z - A_z B_y)\boldsymbol{i} - (A_x B_z - A_z B_x)\boldsymbol{j} + (A_x B_y - A_y B_x)\boldsymbol{k}$$

Chapter 3: Equilibrium of a particle

Goals and Objectives

- Practice following general procedure for analysis.

- Introduce the concept of a free-body diagram for an object modeled as a particle.

- Solve particle equilibrium problems using the equations of equilibrium.

General procedure for analysis

1. Read the problem carefully; write it down carefully.
2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.
3. Apply principles needed.
4. Solve problem symbolically. Make sure equations are dimensionally homogeneous
5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).
6. See if answer is reasonable.

Most effective way to learn engineering mechanics is to ***solve problems!***

Equilibrium of a particle

According to Newton's first law of motion, a particle will be in **equilibrium** (that is, it will remain at rest or continue to move with constant velocity) if and only if

where $\vec{\boldsymbol{F}}$ is the resultant force vector of all forces acting on a particle.

3-Dimensional forces: equilibrium requires

Equilibrium of a particle (cont)

Coplanar forces: if all forces are acting in a single plane, such as the "xy" plane, then the equilibrium condition becomes

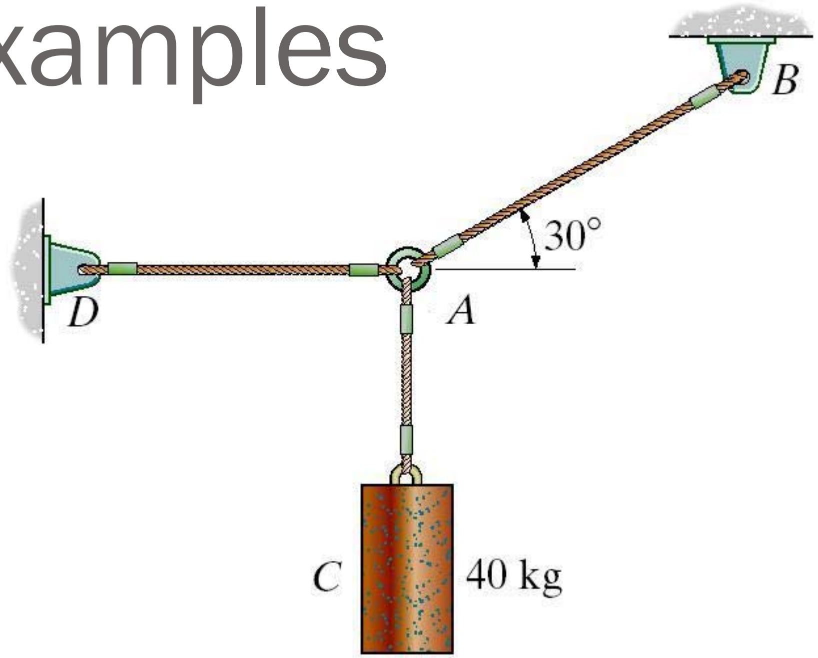
Free body diagram

Drawing of a body, or part of a body, on which all forces acting on the body are shown.

- Key to writing the equations of equilibrium.
- Can draw for any object/subsystem of system. Pick the most appropriate object. (Equal & opposite forces on interacting bodies.)

- ❑ Draw Outlined Shape: image object free of its surroundings
  - ❑ Sometimes may collapse large object into point mass
- ❑ Establish x, y, z axes in any suitable orientation
  - ❑ Show positive directions for translation and rotation
- ❑ Show all forces acting on the object at points of application
- ❑ Label all known and unknown forces
- ❑ Sense ("direction") of unknown force can be assumed. If solution is negative, then the sense is reverse of that shown on FBD

Examples



Find the tension in the cables for a given mass.

- ❑ Draw Outlined Shape
- ❑ Establish x, y, z axes
- ❑ Show all forces acting on object
- ❑ Label known and unknown forces
- ❑ Assume sense of unknown force

Find the forces in cables AB and AC?