Statics - TAM 211

Lecture 5

September 19, 2018

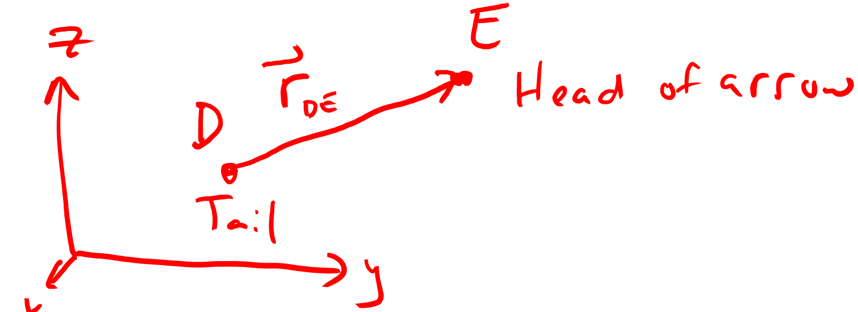
Announcements

- Use the Blackboard Discussion Board if you have questions.
- Videos with more practice of resultant forces have been uploaded to Blackboard
- No class on Monday September 24 (Mid-Autumn Festival)

- Upcoming deadlines:
- Friday (Sept 21)
  - Written Assignment 1
  - Find on Schedule
  - You can SCAN your WA at RC
  - Submit on Blackboard
- Tuesday (9/26)
  - Prairie Learn HW2
- ~~Wednesday (9/27)~~
  - Quiz 1
  - 6-7 pm
  - Computer Lab
  - No personal calculator, must use computer

Quiz 1 must be rescheduled due to ZJUI seminar speaker
Check back for new day/time

Recap of Lecture 4



- Position vectors

$$\vec r_{DE} = (x_E - x_D)\hat i + (y_E - y_D)\hat j + (z_E - z_D)\hat k$$

D: Tail, E: Head

- Force vector directed along a line

$$\vec F = F\,\vec u$$

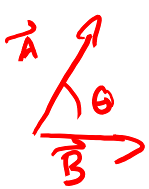
$$\vec u = \frac{\vec r}{|\vec r|}$$

- Dot (scalar) product

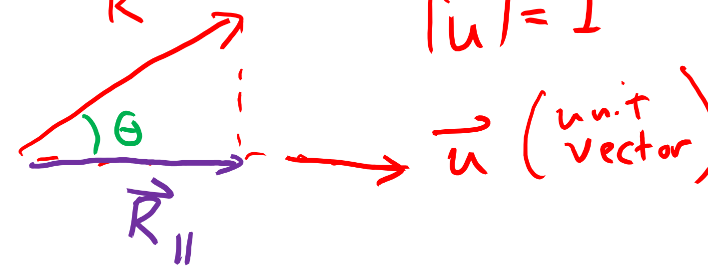
$$\vec A \cdot \vec B = C = |\vec A|\,|\vec B| \cos\theta$$

$$= \sum_{i=x,y,z} (A_i B_i)$$

$$= A_x B_x + A_y B_y + A_z B_z$$



$\boxed{\text{PROJECTIONS}}$



$$|\vec u| = 1$$

$$\vec R_{\parallel} = \underbrace{(\vec R \cdot \vec u)}_{|\vec R_{\parallel}|}\,\vec u$$

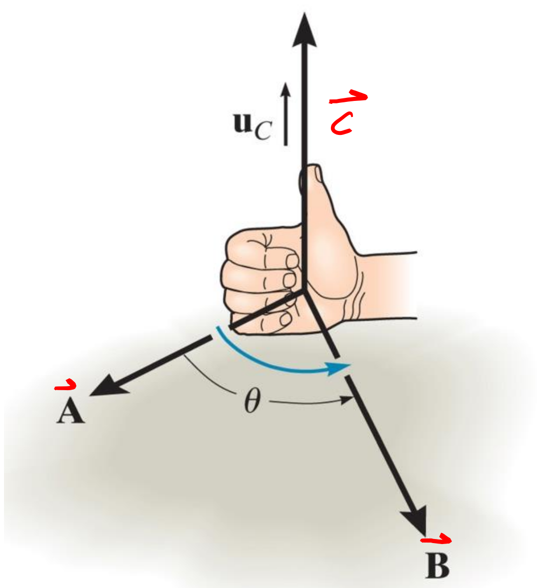
$$|\vec R_{\parallel}| = |\vec R|\,\underbrace{|\vec u|}_{1} \cos\theta$$

$$= R \cos\theta$$

Cross (or vector) product

The cross product of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ yields the vector $\mathbf{C}$, which is written

$$\boxed{\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}}$$



The magnitude of vector $\mathbf{C}$ is given by:

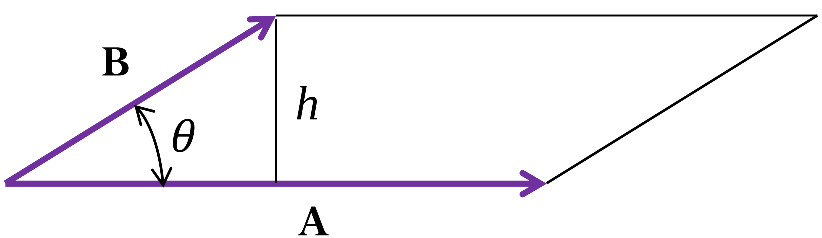
$$C = |\boldsymbol{C}| = |\boldsymbol{A}|\,|\boldsymbol{B}|\sin(\theta)$$

The vector $\mathbf{C}$ is perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$ (specified by the **right-hand rule**). Hence,

$$\vec{\boldsymbol{C}} = \underbrace{A\,B\,\sin(\theta)}_{\text{magnitude}}\,\vec{\boldsymbol{u}}_c$$

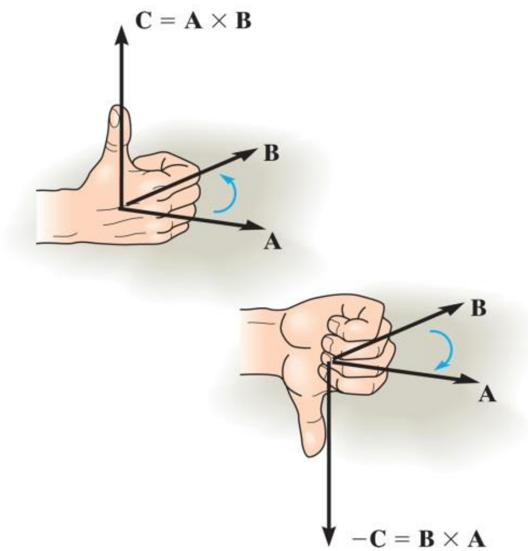
direction of unit vector $\vec{u}_c$

Geometric definition of the cross product: the magnitude of the cross product is given by the area of a parallelogram



$$Area = |\boldsymbol{A}|\,h = |\boldsymbol{A}||\boldsymbol{B}|\sin(\theta)$$
$$= |\boldsymbol{A} \times \boldsymbol{B}|$$

Cross (or vector) product



Laws of operation:

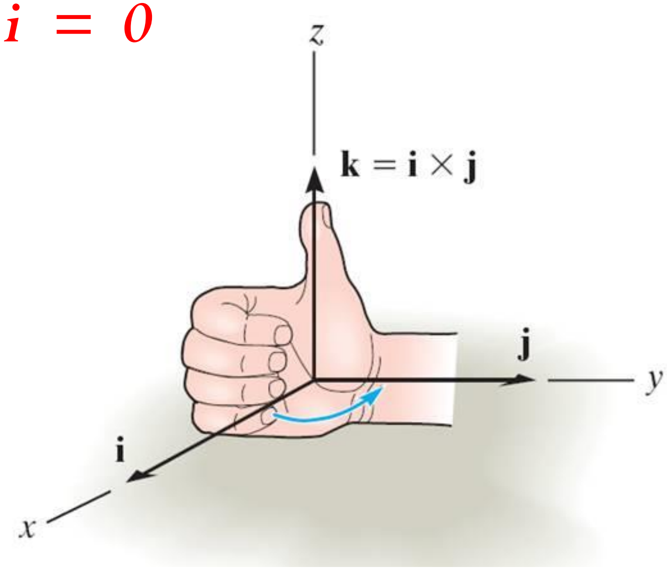
$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$$

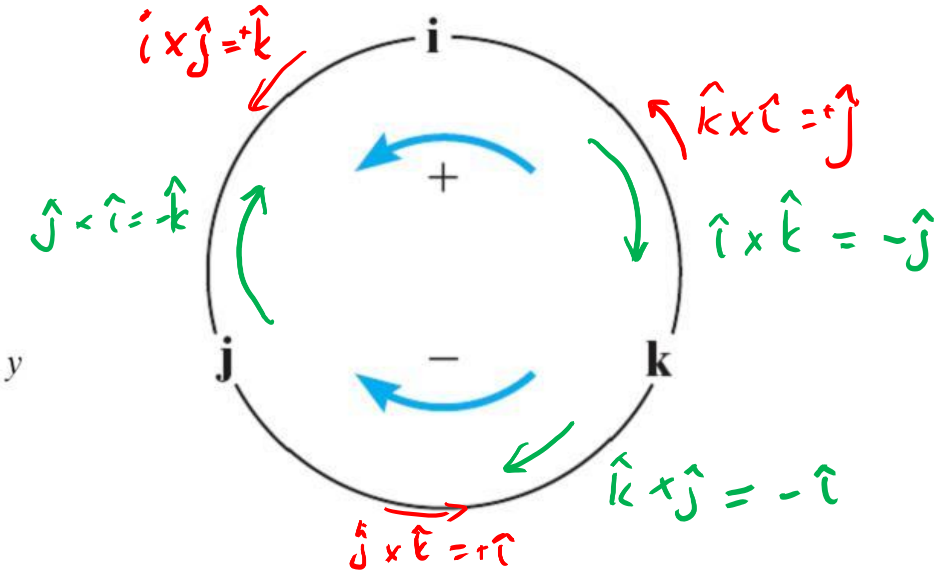
$$\alpha(\boldsymbol{A} \times \boldsymbol{B}) = (\alpha \boldsymbol{A}) \times \boldsymbol{B} = \boldsymbol{A} \times (\alpha \boldsymbol{B}) = (\boldsymbol{A} \times \boldsymbol{B})\alpha$$

$$\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D}) = \boldsymbol{A} \times \boldsymbol{B} + \boldsymbol{A} \times \boldsymbol{D}$$

Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $i \times i = 0$





Considering the cross product in Cartesian coordinates

$$
\begin{aligned}
\boldsymbol{A} \times \boldsymbol{B} &= (A_x\,\boldsymbol{i} + A_y\,\boldsymbol{j} + A_z\,\boldsymbol{k}) \times (B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k}) \\
&= +A_xB_x\overset{0}{\cancel{(\boldsymbol{i} \times \boldsymbol{i})}} + A_xB_y(\overset{+\hat{k}}{\boldsymbol{i} \times \boldsymbol{j}}) + A_xB_z(\overset{-\hat{j}}{\boldsymbol{i} \times \boldsymbol{k}}) \\
&\quad +A_yB_x(\boldsymbol{j} \times \boldsymbol{i}) + A_yB_y\cancel{(\boldsymbol{j} \times \boldsymbol{j})} + A_yB_z(\boldsymbol{j} \times \boldsymbol{k}) \\
&\quad +A_zB_x(\boldsymbol{k} \times \boldsymbol{i}) + A_zB_y(\boldsymbol{k} \times \boldsymbol{j}) + A_zB_z\cancel{(\boldsymbol{k} \times \boldsymbol{k})} \\
&= (A_yB_z - A_zB_y)\boldsymbol{i} - (A_xB_z - A_zB_x)\boldsymbol{j} + (A_xB_y - A_yB_x)\boldsymbol{k}
\end{aligned}
$$

Cross (or vector) product

Also, the cross product can be written as a determinant.

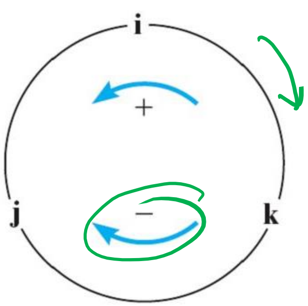
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be determined using 2 × 2 determinants.

Determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

For element **i**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{i}(A_yB_z - A_zB_y)$ ①



$\hat{i} \times \hat{k} = -\hat{j}$

For element **j**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\mathbf{j}(A_xB_z - A_zB_x)$ ②

Remember the negative sign ★

For element **k**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{k}(A_xB_y - A_yB_x)$ ③

$$\boxed{\boldsymbol{A} \times \boldsymbol{B} = (A_yB_z - A_zB_y)\boldsymbol{i} - (A_xB_z - A_zB_x)\boldsymbol{j} + (A_xB_y - A_yB_x)\boldsymbol{k}}$$

① - ② + ③

Chapter 3: Equilibrium of a particle

Goals and Objectives

- Practice following general procedure for analysis.
- Introduce the concept of a free-body diagram for an object modeled as a particle.
- Solve particle equilibrium problems using the equations of equilibrium.

General procedure for analysis

1. Read the problem carefully; write it down carefully.
2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.
3. Apply principles needed.
4. Solve problem symbolically. Make sure equations are dimensionally homogeneous
5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).
6. See if answer is reasonable. check the value

Most effective way to learn engineering mechanics is to *solve problems!*

Equilibrium of a particle

According to Newton's first law of motion, a particle will be in **equilibrium** (that is, it will remain at rest or continue to move with constant velocity) if and only if

v = Const
$\Rightarrow$ a = 0

$$\sum_i \vec{F}_i = 0$$

where $\vec{\boldsymbol{F}}$ is the resultant force vector of all forces acting on a particle.

3-Dimensional forces: equilibrium requires

$$\sum \vec{F} = \sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k} = 0 \Rightarrow \underbrace{\begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{array}}_{\text{3 eqns to use}}$$

only magnitudes

Equilibrium of a particle (cont)

2D

Coplanar forces: if all forces are acting in a single plane, such as the "xy" plane, then the equilibrium condition becomes

$$\sum \vec{F} = \sum F_x \hat{i} + \sum F_y \hat{j}$$

$$\Rightarrow \quad \underbrace{\begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \end{array}}_{\text{ONLY HAVE 2 EQNS}}$$

Free body diagram

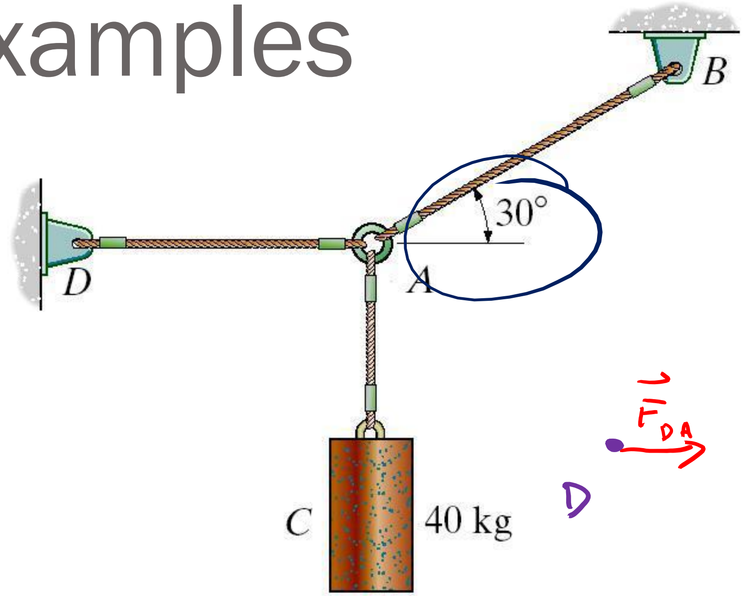
Drawing of a body, or part of a body, on which all forces acting on the body are shown.

- Key to writing the equations of equilibrium.
- Can draw for any object/subsystem of system. Pick the most appropriate object. (Equal & opposite forces on interacting bodies.)

- ❑ Draw Outlined Shape: image object free of its surroundings
  - ❑ Sometimes may collapse large object into point mass
- ❑ Establish x, y, z axes in any suitable orientation
  - ❑ Show positive directions for translation and rotation
- ❑ Show all forces acting on the object at points of application
- ❑ Write out equations of equilibrium (EoE) for the appropriate body (bodies)
- ❑ Label all known and unknown forces
- ❑ Sense ("direction") of unknown force can be assumed. If solution is negative, then the sense is reverse of that shown on FBD

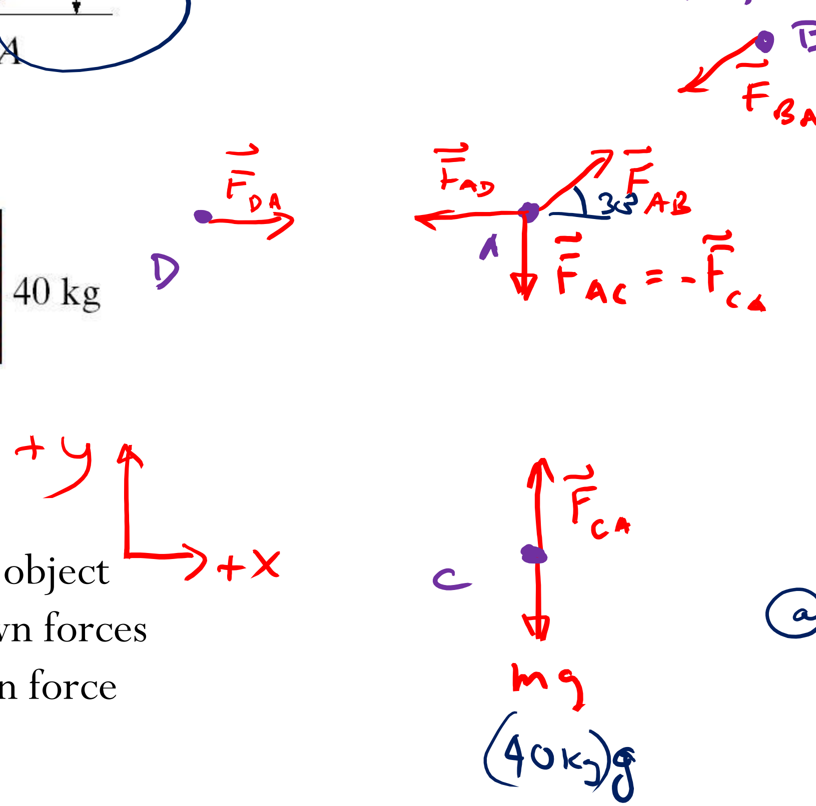
Examples

Find the tension in the cables for a given mass.



- [x] Draw Outlined Shape
- [x] Establish x, y, z axes
- [x] Show all forces acting on object
- [ ] write eqns
- [ ] Label known and unknown forces
- [ ] Assume sense of unknown force

Four Points A, B, C, D → Four FBD



@ Pt. A:

$\Sigma F_x = 0$

① $-F_{AD} + F_{AB}\cos\theta = 0$

$\Sigma F_y = 0$

② $-F_{AC} + F_{AB}\sin\theta = 0$

3 unknowns, 2 eqns
⇒ Indeterminate Problem

@ Pt. C:

$\Sigma F_y = 0$

③ $F_{CA} - mg = 0$

1 unknown, 1 eqn

④ Note that $F_{CA} = -F_{AC}$

1 eqn

∴ Have total of 4 unknowns ($F_{AB}, F_{AC}, F_{AD}, F_{CA}$)
& eqns (1-4)
⇒ Determinate Problem

A determinate problem means that it is possible to determine (or solve) for all unknowns.

Determinate : # unknowns = # eqns

Indeterminate (can not solve for all unkn):

unknowns > # eqns