Statics - TAM TAM 211

Lecture 6

September 21, 2018

Announcements

- Zhaoyu Xu (TA) has office hours on Fridays 1-3 pm in Library Cafe.
- Use the Blackboard Discussion Board if you have questions.
- Videos with more practice of resultant forces have been uploaded to Blackboard
- No class on Monday September 24 (Mid-Autumn Festival)

- Upcoming deadlines:
- Friday (Sept 21)
  - Written Assignment 1
- Tuesday (9/26)
  - Prairie Learn HW2
- Thursday (9/28)
  - Note different day!
  - Quiz 1
  - 6-7 pm
  - Computer Lab
  - No personal calculator, must use computer

Chapter 3: Equilibrium of a particle

Goals and Objectives

- Practice following general procedure for analysis.
- Introduce the concept of a free-body diagram for an object modeled as a particle.
- Solve equilibrium problems using the equations of equilibrium.
  - 3D, 2D planar, idealizations (smooth surfaces, pulleys, springs)

Recap: General procedure for analysis

1. Read the problem carefully; write it down carefully.
2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.
3. Apply principles needed.
4. Solve problem symbolically. Make sure equations are dimensionally homogeneous
5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).
6. See if answer is reasonable.

Most effective way to learn engineering mechanics is to ***solve problems!***

Recap: Equilibrium of a particle

3-Dimensional forces: equilibrium requires

$$\sum \boldsymbol{F} = \sum F_x \, \boldsymbol{i} + \sum F_y \, \boldsymbol{j} + \sum F_z \, \boldsymbol{k} = \boldsymbol{0} \quad \Longrightarrow \quad \begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \end{aligned}$$

Planar forces: if all forces are acting in a single plane, such as the "xy" plane, then the equilibrium condition becomes

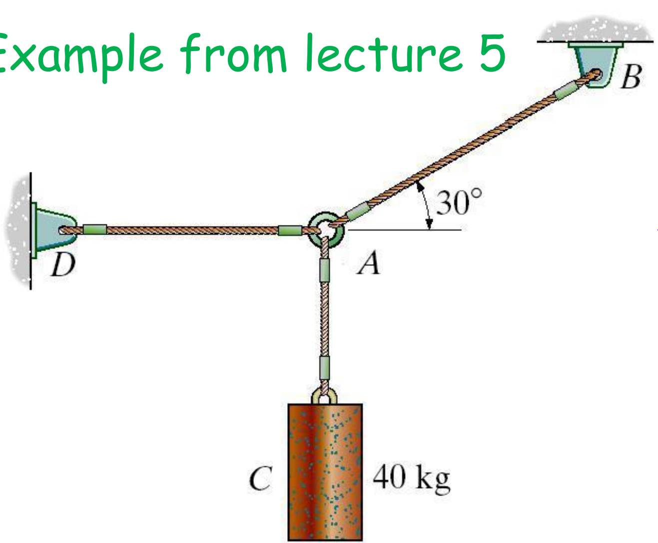
$$\sum \boldsymbol{F} = \sum F_x \, \boldsymbol{i} + \sum F_y \, \boldsymbol{j} = \boldsymbol{0} \quad \Longrightarrow \quad \begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \end{aligned}$$

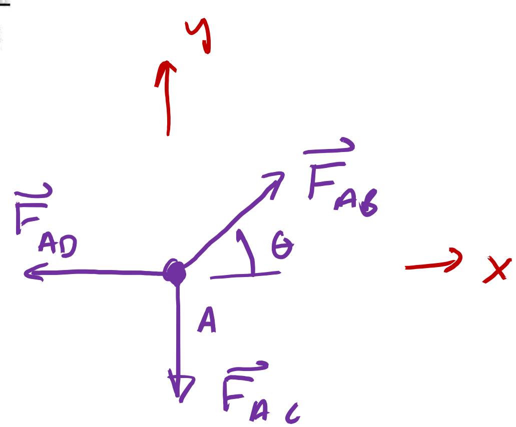
Recap: Free body diagram

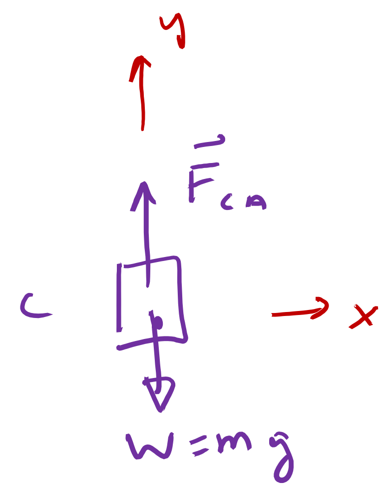
Drawing of a body, or part of a body, on which all forces acting on the body are shown.

- ❑ Draw Outlined Shape: image object free of its surroundings
- ❑ Establish x, y, z axes in any suitable orientation
  - ❑ Show positive directions for translation and rotation
- ❑ Show all forces acting on the object at points of application
- ❑ Label all known and unknown forces
- ❑ Sense ("direction") of unknown force can be assumed. If solution is negative, then the sense is reverse of that shown on FBD

Example from lecture 5



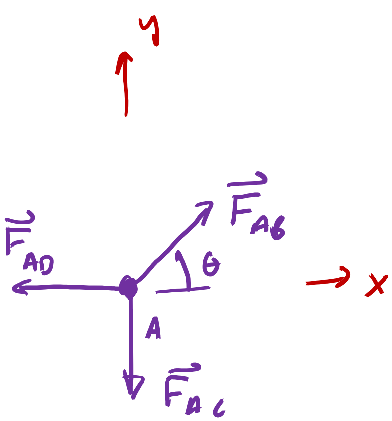




Recap: Equations of equilibrium

- Use FBD to write equilibrium equations in x, y, z directions
  - $\sum \overrightarrow{F_x} = 0$, $\sum \overrightarrow{F_y} = 0$, and if 3D $\sum \overrightarrow{F_z} = 0$,
  - If # equations ≥ # unknown forces, **statically determinate** (can solve for unknowns)
  - If # equations < # unknown forces, **indeterminate** (can **NOT** solve for unknowns), need more equations
- Get more equations from FBD of other bodies in the problem

@ Pt. A:



$\sum F_x = 0$

① $-F_{AD} + F_{AB}\cos\theta = 0$

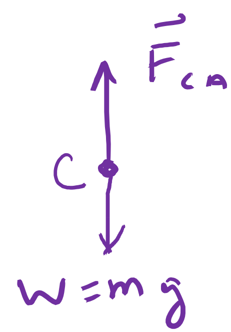
$\sum F_y = 0$

② $-F_{AC} + F_{AB}\sin\theta = 0$

3 unknowns, 2 eqns

⇒ Indeterminate

@ Pt. C:



$\sum F_y = 0$

③ $F_{CA} - mg = 0$

1 unknown, 1 eqn

④ Note that $F_{CA} = -F_{AC}$

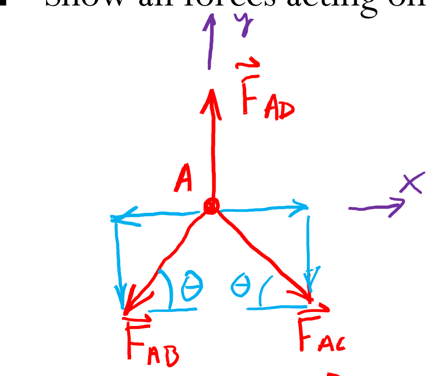
1 eqn

⇒ 4 eqns, 4 unk

⇒ Statically Determinate

Find the forces in cables AB and AC?

- ❑ Draw Outlined Shape
- ❑ Establish x, y, z axes
- ❑ Show all forces acting on object
- ❑ Label known and unknown forces
- ❑ Assume sense of unknown force



Eqns of Equilibrium

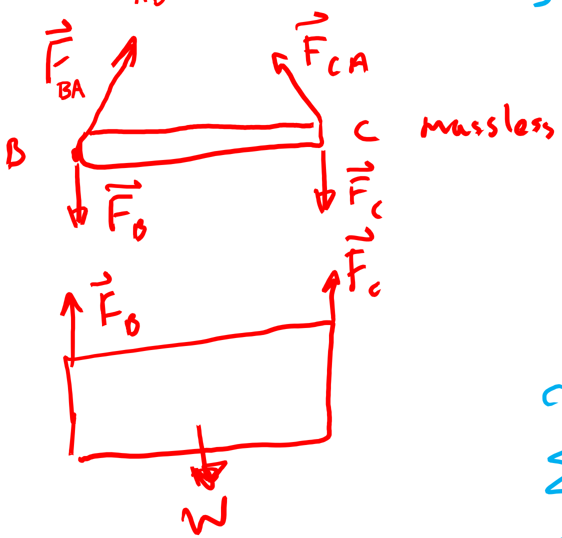
On A:

$$\Sigma F_x : |\vec{F}_{AC}| \cos\theta - |\vec{F}_{AB}| \cos\theta = 0 \quad (1)$$

$$\Sigma F_y : \vec{F}_{AD} - |\vec{F}_{AB}| \sin\theta - F_{AC} \sin\theta = 0 \quad (2)$$

3 unk: $\vec{F}_{AB}$, $\vec{F}_{AC}$, $\vec{F}_{AD}$

2 eqns → Need more eqns



combined object:

$$\Sigma F_x : F_{BA} \cos\theta - F_{CA} \cos\theta = 0 \quad (3)$$

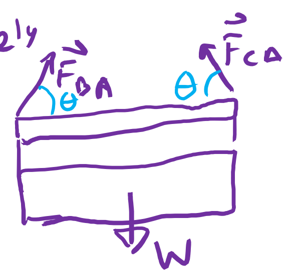
$$\Sigma F_y : F_{BA} \sin\theta + F_{CA} \sin\theta - W = 0 \quad (4)$$

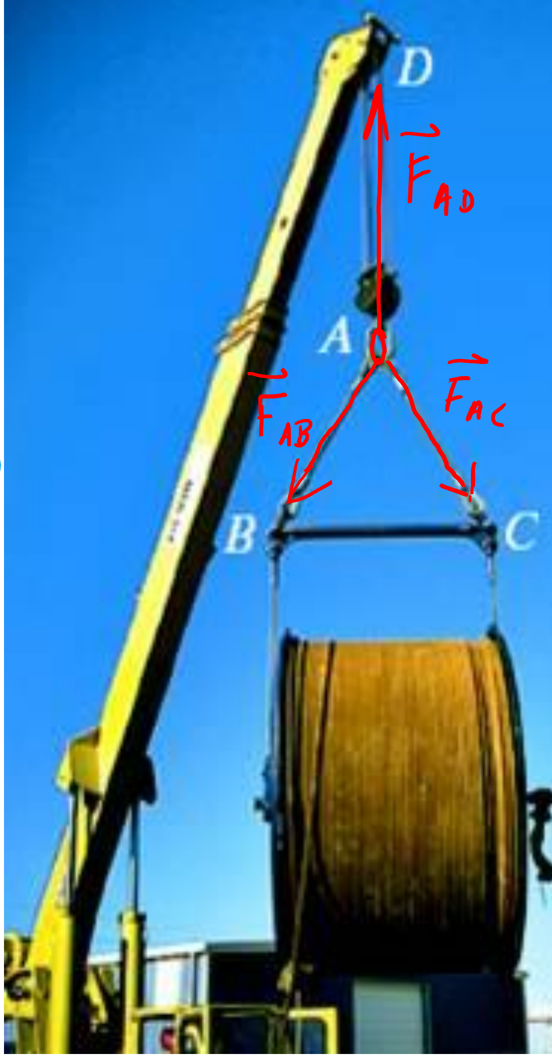
+ 2 unk: F_{BA}, F_{CA}

$F_{BA} = -F_{AB}$ (5) $\quad F_{CA} = -F_{AC}$ (6)

5 unknowns, 6 eqns → Statically Determinate ✓

Alternatively

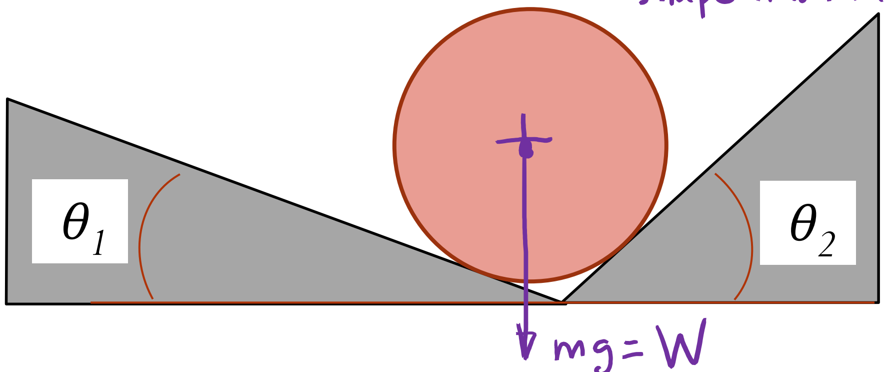




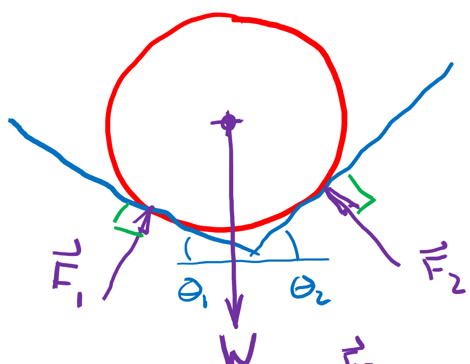
Idealizations

Contact force on a **smooth surface** (with no friction) will be a normal force (i.e., perpendicular to the surface at the point of contact.

Assume shape has mass

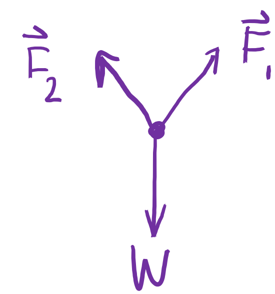


FBD of circular shape:

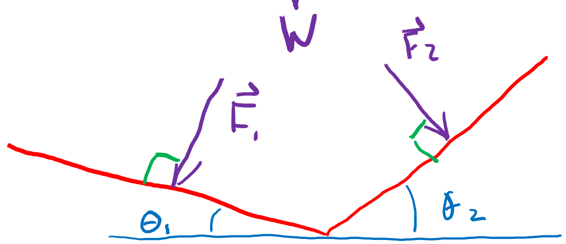


$\Rightarrow$

Can collapse forces to act on a point mass:



FBD of contact surfaces.

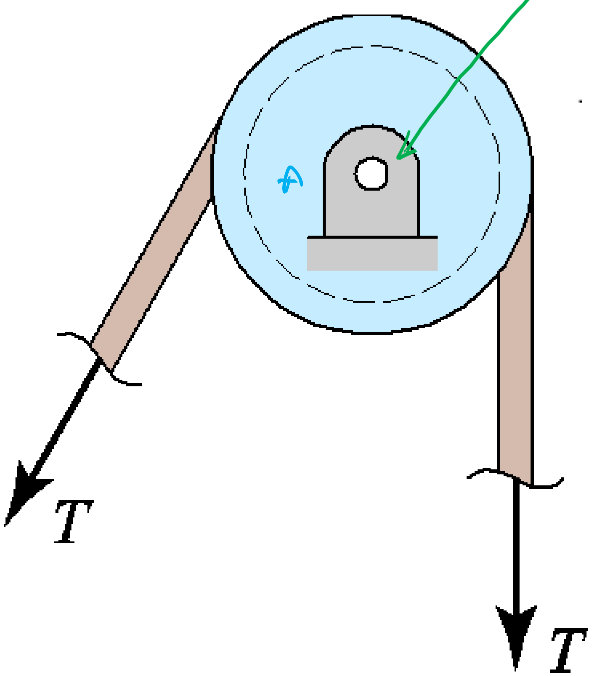


Idealizations

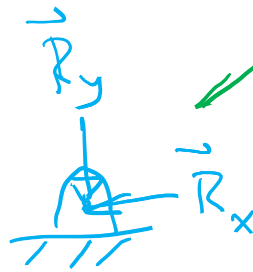
massless &

Pulleys are (usually) regarded as frictionless; then the tension in a rope or cord around the pulley is the same on either side.

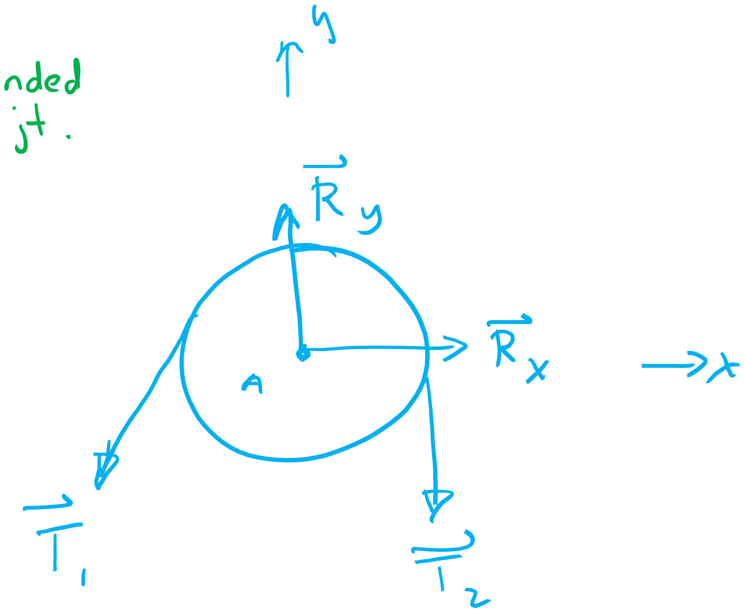
This pulley is secured in position by grounded pin jt.



Frictionless pulley



Reaction Forces on pin jt



$$|\vec{T}_1| = |\vec{T}_2|$$

Assuming cable is massless & rigid

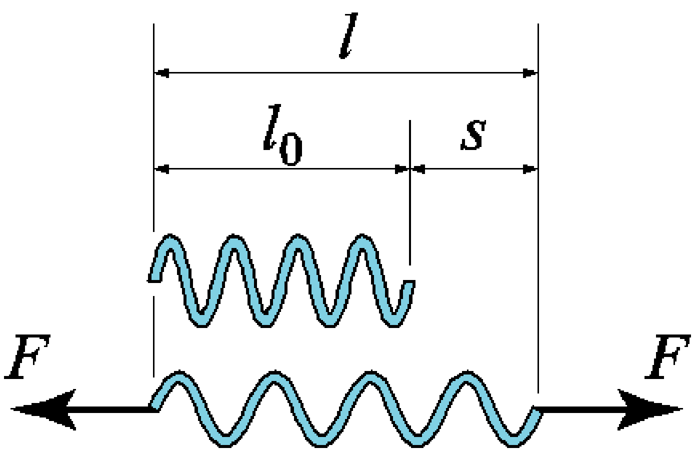
Magnitudes are same

Directions do not need to be the same

Idealizations

Springs are (usually) regarded as linearly elastic; then the tension is proportional to the *change* in length s, where the spring stiffness is k.

massless



$$\boxed{F = ks = k(l - l_0)}$$

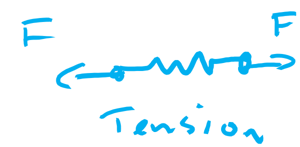
Linearly elastic spring

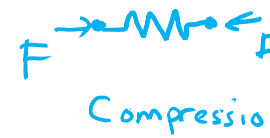
$$\boxed{F_s = ks}$$

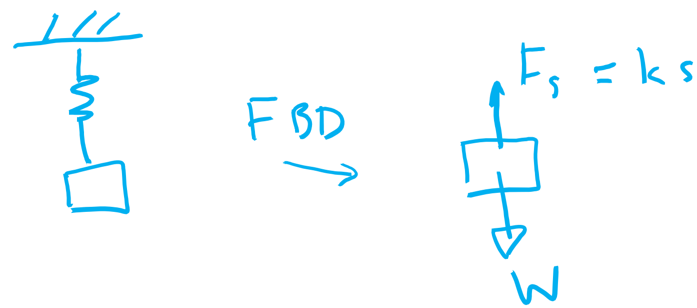
$s = l_f - l_0$

if $s > 0 \rightarrow$ elongation

if $s < 0 \longrightarrow$ compression

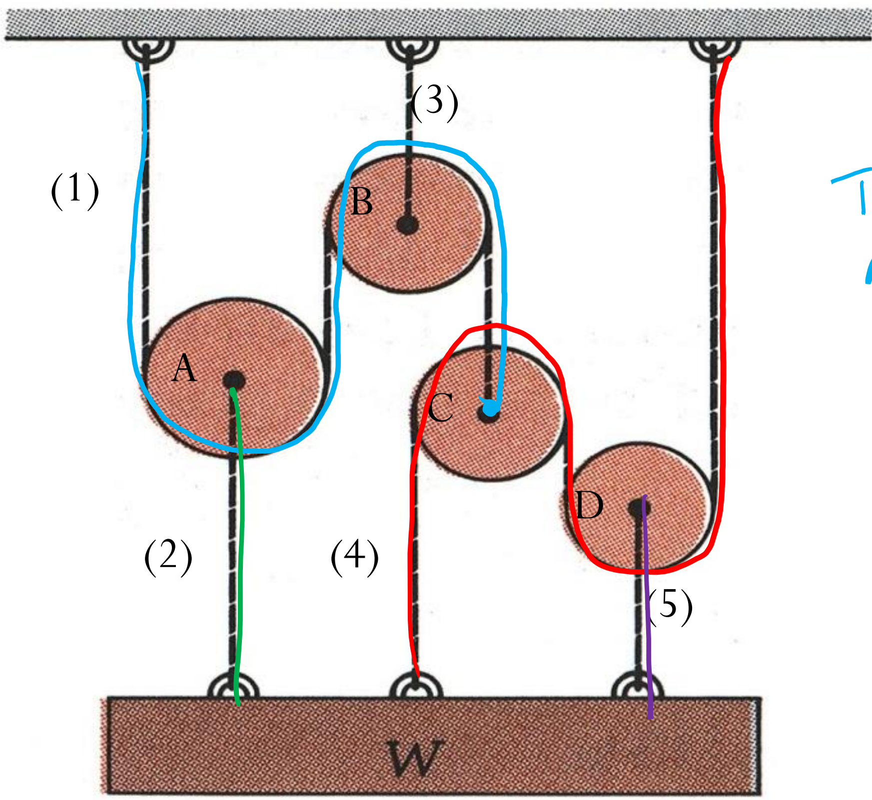




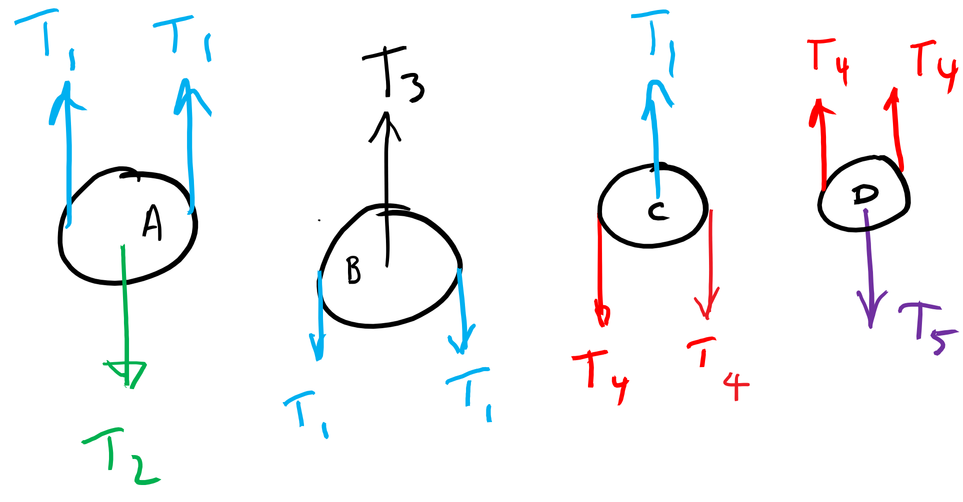


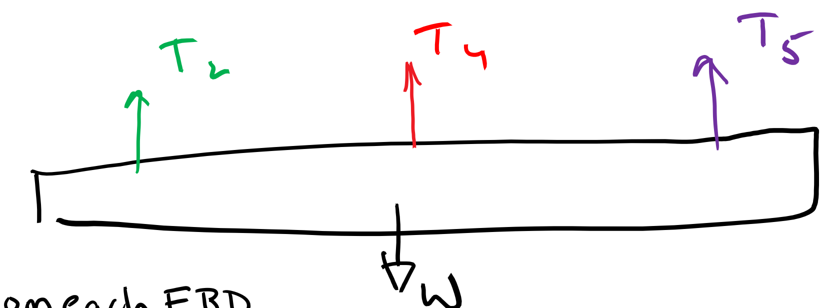
k: spring stiffness

The five ropes can each take 1500 N without breaking. How heavy can *W* be without breaking any?



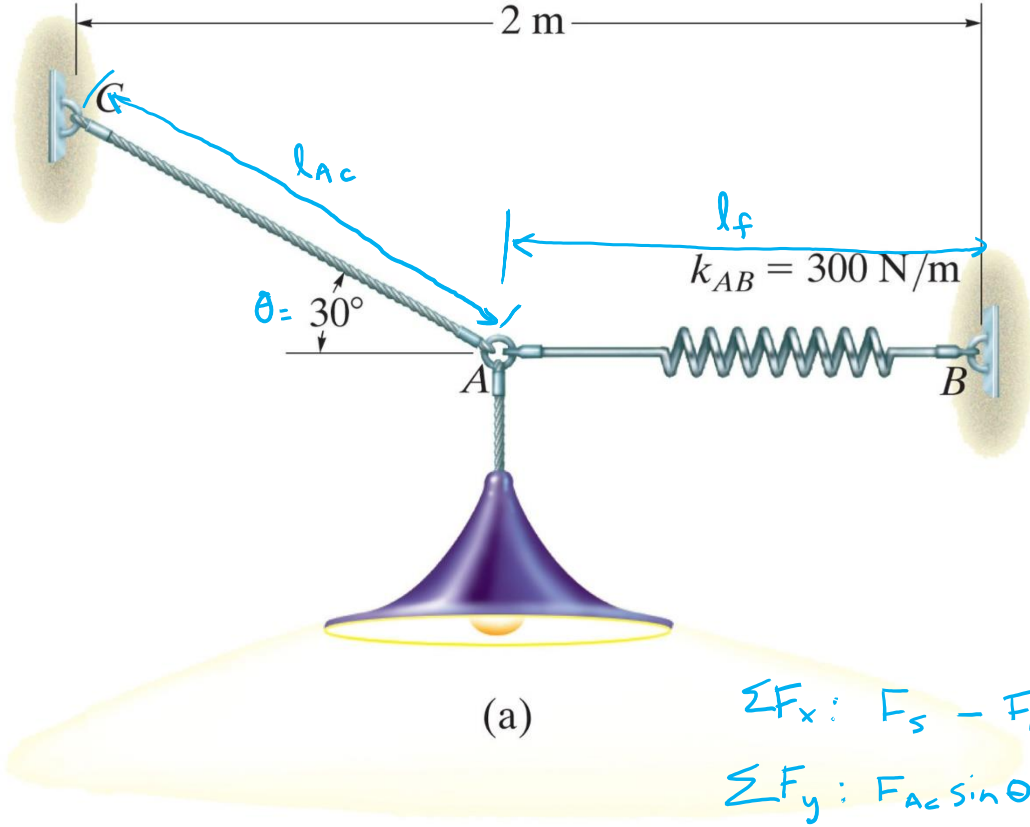
Note: No pin jt reaction forces at center of pulleys because these pulleys are not secured to a fixed (or grounded) pin jt.





$\Sigma F_y = 0$

write eqns of equilibrium from each FBD



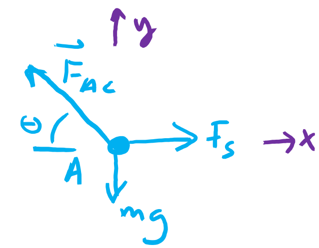
(a)

Determine the required length of cord AC so that the 8-kg lamp can be suspended in the position shown. The undeformed spring length is 0.4 m and has a stiffness of 300 N/m.

Given: $m = 8\,kg$, $l_0 = 0.4\,m$, $k_{AB} = 300\,N/m$
$\theta = 30°$

Find: l_{AC}

Soln: FBD of A



$\Sigma F_x: \; F_s - F_{AC}\cos\theta = 0$ ①

$\Sigma F_y: \; F_{AC}\sin\theta - mg = 0$ ②

$F_s = k_{AB}\,s = k_{AB}(l_f - l_0)$ ③

③ into ①: $k_{AB}(l_f - l_0) - F_{AC}\cos\theta = 0$

insert ②: $k_{AB}(l_f - l_0) - \left(\frac{mg}{\sin\theta}\right)\cos\theta = 0 \Rightarrow l_f = \left(\frac{mg}{k_{AB}}\right)\frac{\cos\theta}{\sin\theta} + l_0 = 0.853\,m$

Use geometrical constraint:

$2m = l_f + l_{AC}\cos\theta$

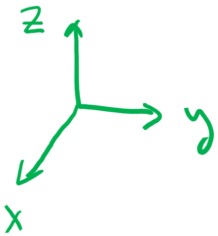
$l_{AC} = \frac{2m - l_f}{\cos\theta} = \boxed{1.32\,m = l_{AC}}$

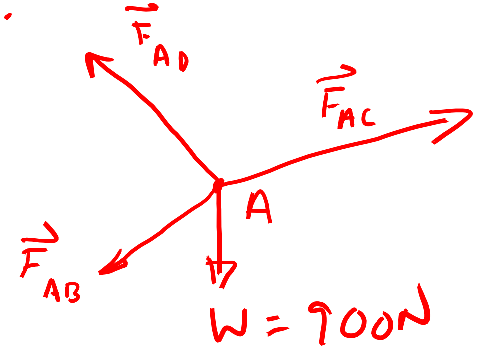
3D force systems

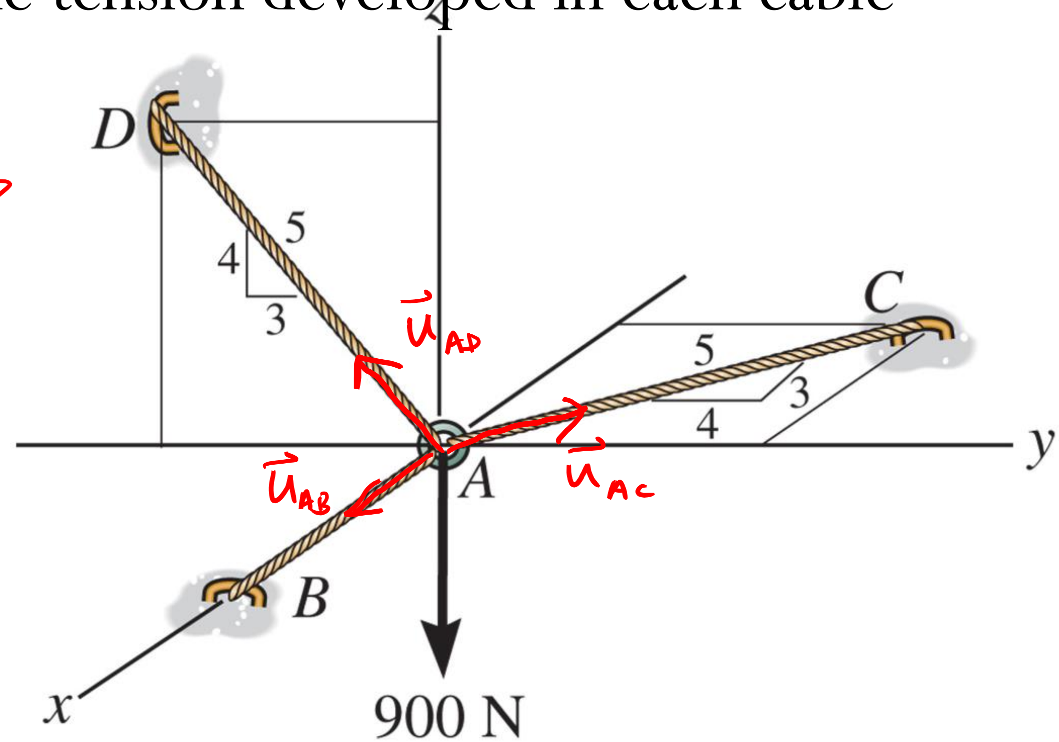
Use $\sum \overrightarrow{F_x} = 0, \sum \overrightarrow{F_y} = 0, \sum \overrightarrow{F_z} = 0$

Find the tension developed in each cable

① Draw FBD @ A:







② Use $\vec{F} = F\vec{u}$, $\vec{u} = \frac{\vec{r}}{|\vec{r}|}$

$\vec{u}_{AB} = 1\hat{i}$

$\vec{u}_{AC} = -\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$

$\vec{u}_{AD} = 0\hat{i} - \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}$

③ Write Eqn of Equil:

$\Sigma F_x: \; F_{AB} - F_{AC}\left(\frac{3}{5}\right) = 0$

$\Sigma F_y: \; -F_{AD}\left(\frac{3}{5}\right) + F_{AC}\left(\frac{4}{5}\right) = 0$

$\Sigma F_z: \; F_{AD}\left(\frac{4}{5}\right) - 900 = 0$

$\Rightarrow$ $F_{AB} = ?$ $F_{AC} = ?$ $F_{AD} = ?$

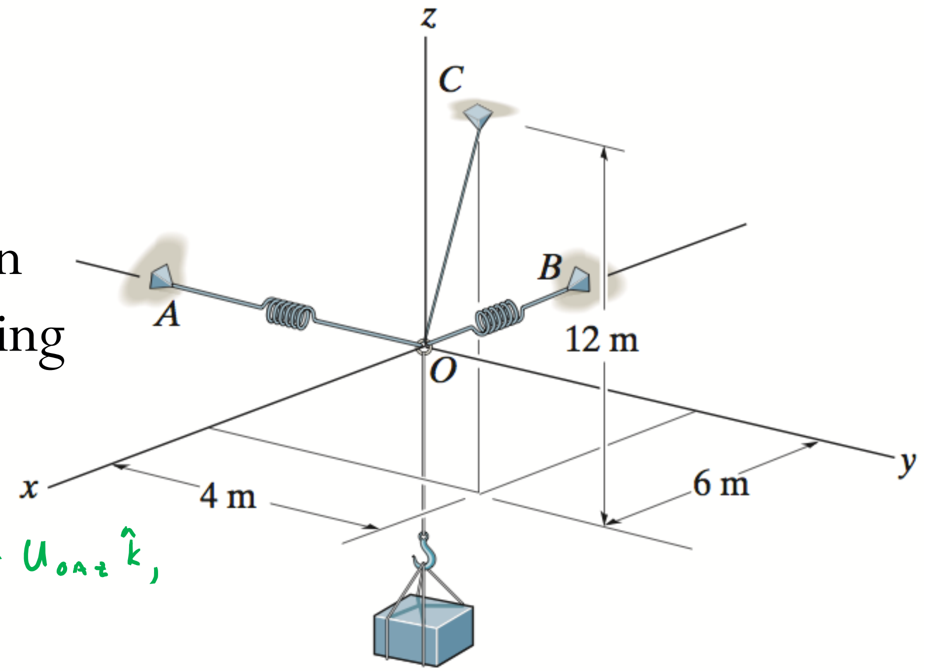
Solve for the magnitudes (tensions) of the 3 cables

If wanted the forces, then compute the vectors. $\vec{F}_{AB} = F_{AB}\vec{u}_{AB}$, etc.

check: $F_{AB} = 506N$, $F_{AC} = 1125N$, $F_{AD} = 844N$

Example – 3D

Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of $k = 360$ N-m.



Check solution: If $\vec{u}_{OA} = u_{OAx}\hat{i} + u_{OAy}\hat{j} + u_{OAz}\hat{k}$,

then $s_{OA} = \dfrac{F_{OC}\, u_{OAy}}{k} = 218 \text{ mm}$

$s_{OB} = \dfrac{F_{OC}\, u_{OAx}}{k} = 327 \text{ mm}$