Statics - TAM 211

Lecture 7

September 26, 2018

Announcements

- No classes September 29 – October 7 for National Day Holiday
  - Saturday Sept 29 make-up class cancelled to accommodate for Quiz 1
  - Sunday Sept 30 make-up class cancelled to accommodate for Quiz 2
- Upcoming deadlines:
- Thursday (9/28)
  - Quiz 1, 6-7 pm
  - Computer Lab (D211 for ME, D331 for CEE)
  - No personal calculator, must use computer
- Friday (Sept 28)
  - Written Assignment 2
- Tuesday (10/9)
  - Prairie Learn HW3

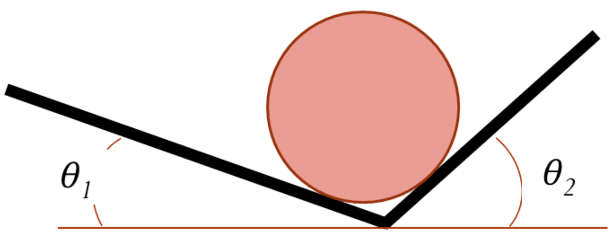
www.reddit.com/r/StarWars

Recap: Idealizations

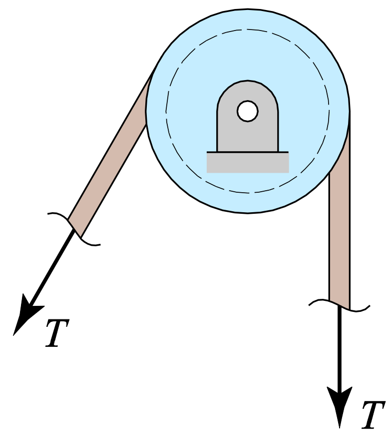
Smooth surfaces: regarded as frictionless; force is perpendicular to surface

Pulleys: (usually) regarded as frictionless; tension around pulley is same on either side.

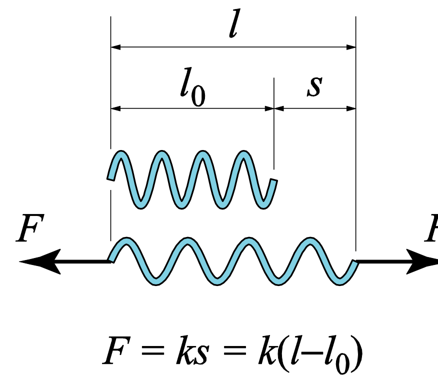
Springs: (usually) regarded as linearly elastic; tension is proportional to *change* in length s.



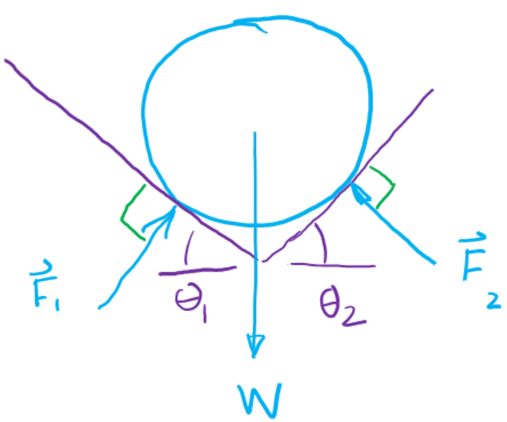
Smooth surface

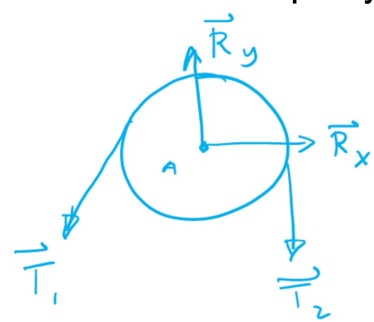


Frictionless pulley



Linearly elastic spring





$|\vec{T}_1| = |\vec{T}_2|$

Assuming cable is massless & rigid

Magnitudes are same

Directions do not need to be the same

$s = l_f - l_0$

if $s > 0 \rightarrow$ elongation

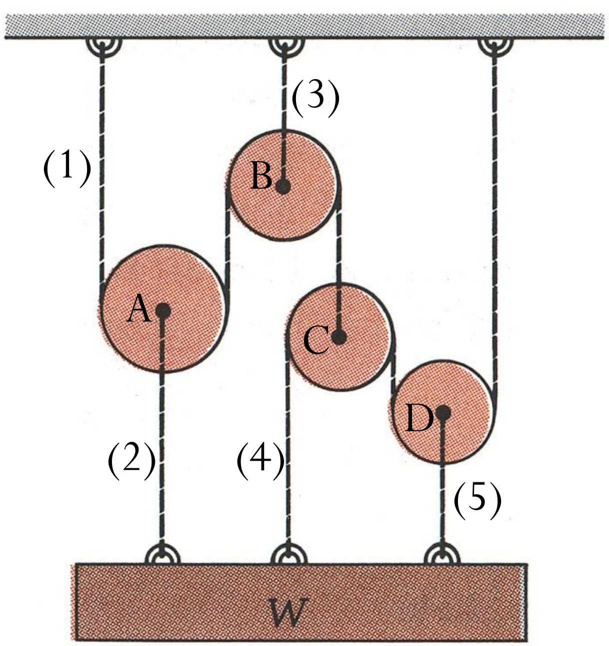
if $s < 0 \rightarrow$ compression

Recap: Equilibrium of a system of particles

Some practical engineering problems involve the statics of interacting or interconnected particles. To solve them, we use Newton's first law

$$\Sigma \mathbf{F} = \mathbf{0}$$

on selected multiple free-body diagrams of particles or groups of particles.



The five ropes can each take 1500 N without breaking. How heavy can W be without breaking any?

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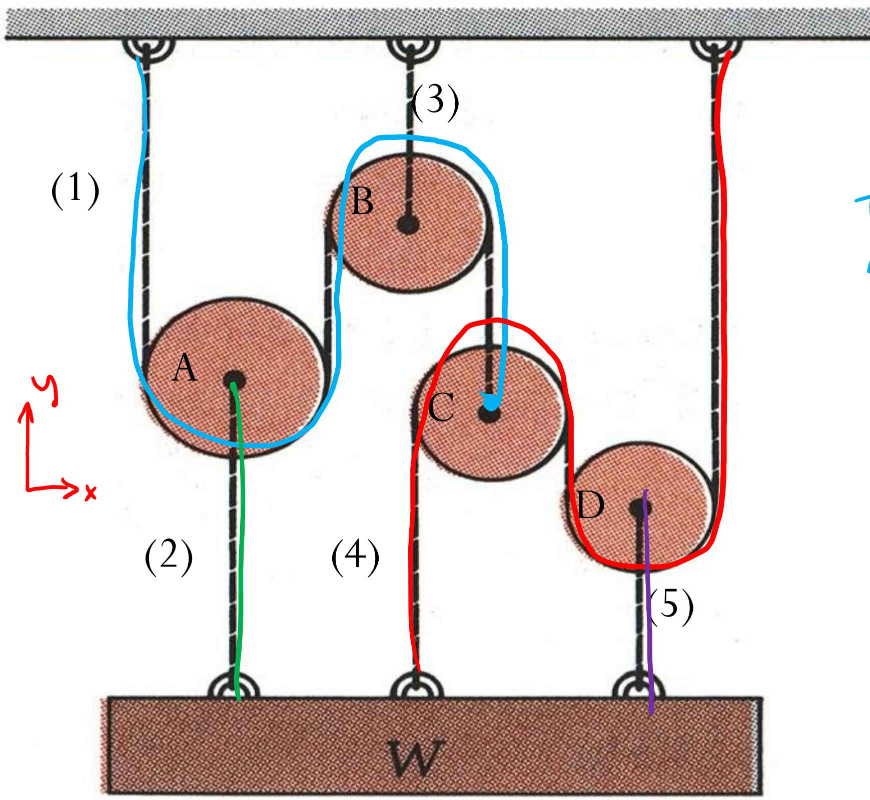
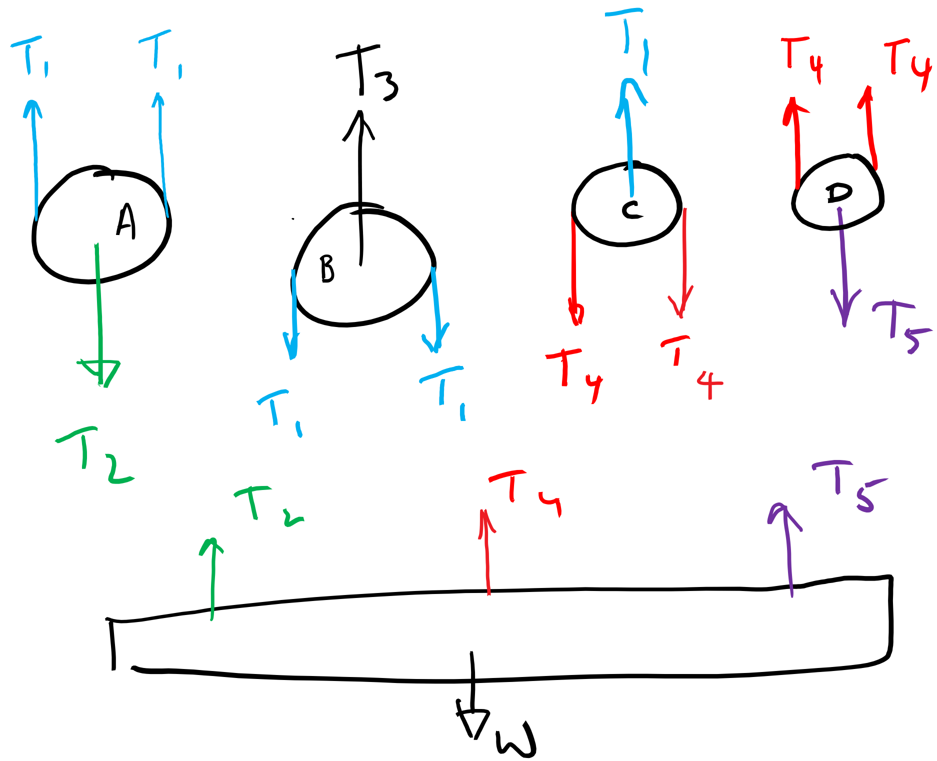


Note: No pin jt reaction forces at center of pulleys because these pulleys are not secured to a fixed (or grounded) pin jt.



$\Sigma F_y = 0$

write eqns of equilibrium from each FBD

A: $2T_1 - T_2 = 0$

B: $-2T_1 + T_3 = 0$

C: $T_1 - 2T_4 = 0$

D: $2T_4 - T_5 = 0$

W: $T_2 + T_4 + T_5 - W = 0$

Chose a rope that will support the largest load. Then set its load to the breakage limit.

$T_2 = T_3 = 2T_1 = 4T_4 = 2T_5$

T_2 or T_3 are possible choices. **Pick T_2.**

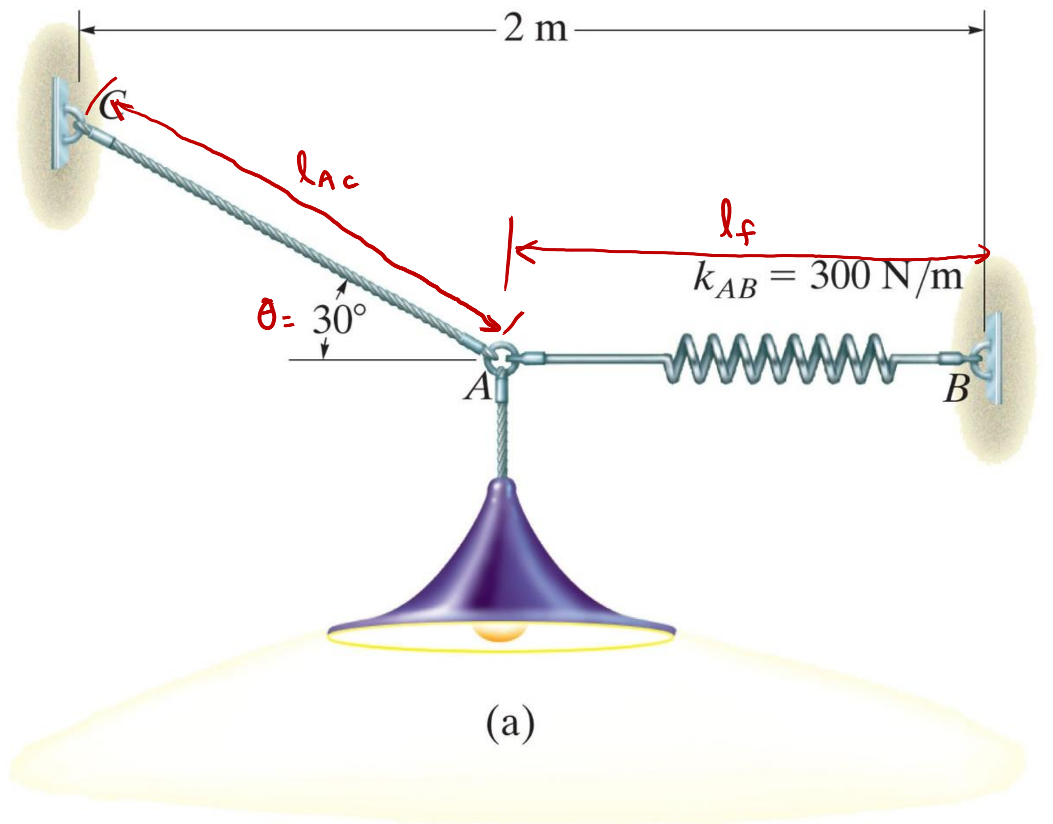
$\therefore$ If $T_2 = 1500$ N,

$\Longrightarrow W = T_2 + 0.25\,T_2 + 0.5\,T_2$

Then $\boxed{W = 2625 \text{ N}}$

Determine the required length of cord AC so that the 8-kg lamp can be suspended in the position shown.

The undeformed spring length is 0.4 m and has a stiffness of 300 N/m.



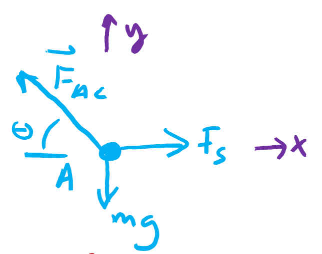
(a)

Given: $m = 8\,kg$, $l_o = 0.4\,m$, $k_{AB} = 300\,N/m$

$\theta = 30°$

Find: l_{AC}

Soln: FBD of A



ΣF_x: $F_s - F_{AC}\cos\theta = 0$ ①

ΣF_y: $F_{AC}\sin\theta - mg = 0$ ②

$F_s = k_{AB}\,s = k_{AB}(l_f - l_o)$ ③

③ into ① : $k_{AB}(l_f - l_o) - F_{AC}\cos\theta = 0$

insert ② : $k_{AB}(l_f - l_o) - \left(\frac{mg}{\sin\theta}\right)\cos\theta = 0 \Rightarrow l_f = \left(\frac{mg}{k_{AB}}\right)\frac{\cos\theta}{\sin\theta} + l_o = 0.853\,m$

Use geometrical constraint: $2m = l_f + l_{AC}\cos\theta$

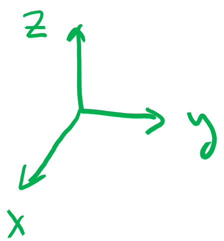
$$l_{AC} = \frac{2m - l_f}{\cos\theta} = \boxed{1.32\,m = l_{AC}}$$

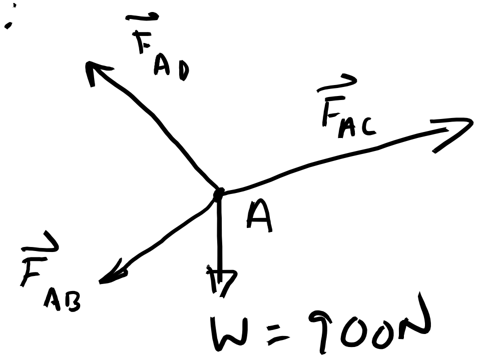
3D force systems

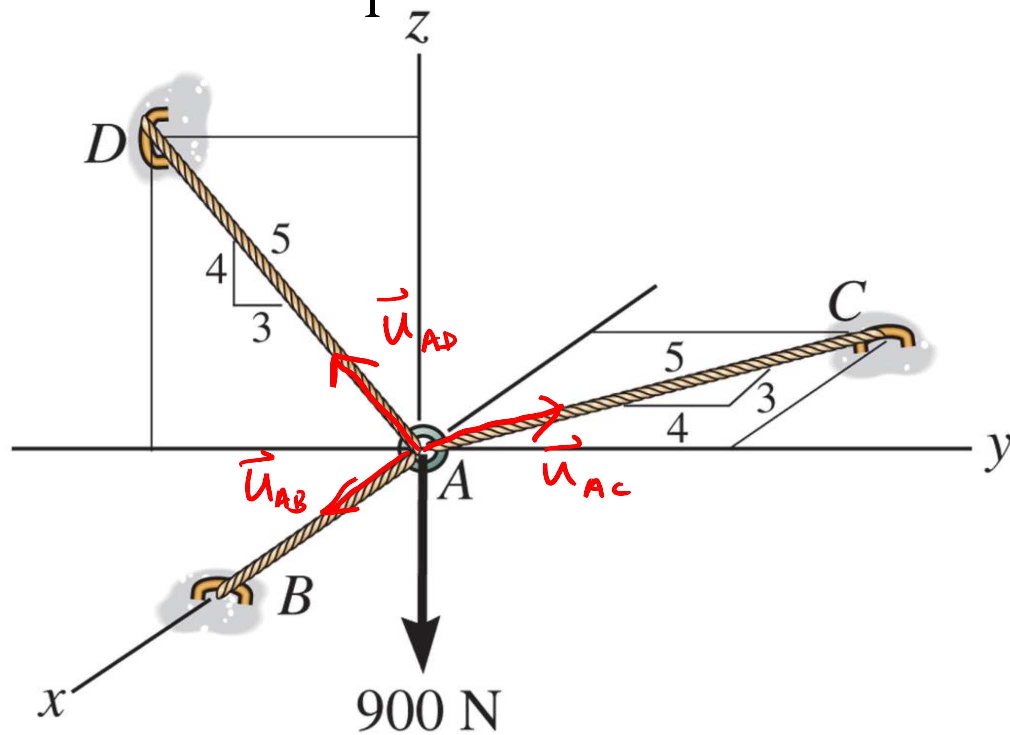
Use $\sum \vec{F_x} = 0, \sum \vec{F_y} = 0, \sum \vec{F_z} = 0$

Find the tension developed in each cable

① Draw FBD @ A:







② Use $\vec{F} = F\vec{u}$, $\vec{u} = \frac{\vec{r}}{|\vec{r}|}$

$\vec{u}_{AB} = 1\hat{i}$

$\vec{u}_{AC} = -\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$

$\vec{u}_{AD} = 0\hat{i} - \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}$

③ Write Eqn of Equil:

$$\left.\begin{aligned} \Sigma F_x &: F_{AB} - F_{AC}\left(\tfrac{3}{5}\right) = 0 \\ \Sigma F_y &: -F_{AD}\left(\tfrac{3}{5}\right) + F_{AC}\left(\tfrac{4}{5}\right) = 0 \\ \Sigma F_z &: F_{AD}\,\tfrac{4}{5}) - 900 = 0 \end{aligned}\right\} \Rightarrow$$

$F_{AB} = ?$

$F_{AC} = ?$

$F_{AD} = ?$

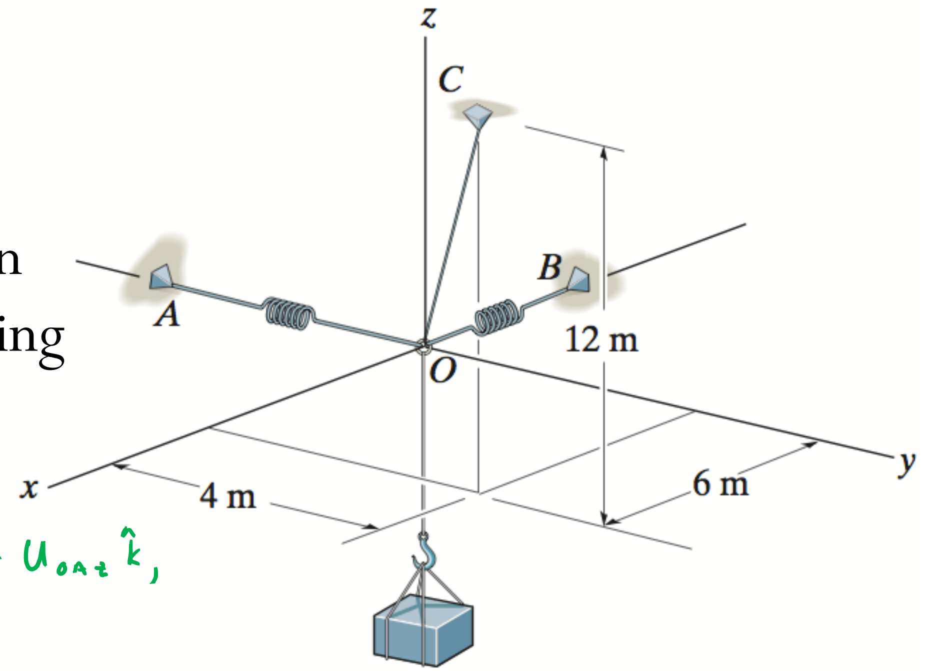
Solve for the magnitudes (tensions) of the 3 cables

If wanted the forces, then compute the vectors. $\vec{F}_{AB} = F_{AB}\vec{u}_{AB}$, etc.

check: $F_{AB} = 506N$, $F_{AC} = 1125N$, $F_{AD} = 844N$

Example – 3D

Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of $k = 360$ N-m.



Check solution: If $\vec{u}_{OA} = u_{OAx}\hat{\imath} + u_{OAy}\hat{\jmath} + u_{OAz}\hat{k}$,

then $s_{OA} = \dfrac{F_{OC}\, u_{OAy}}{k} = 218$ mm

$s_{OB} = \dfrac{F_{OC}\, u_{OAx}}{k} = 327$ mm

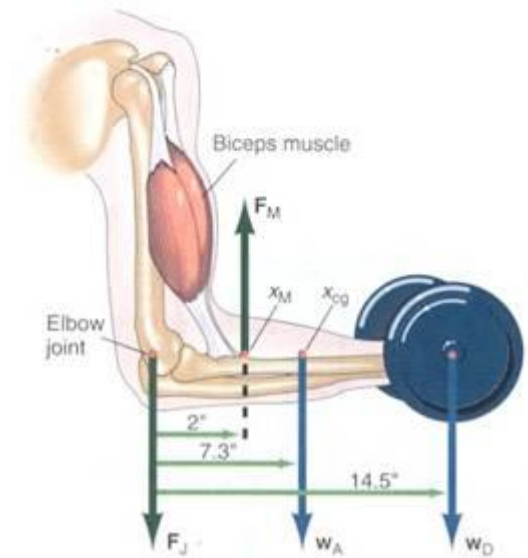
Chapter 4: Force System Resultants

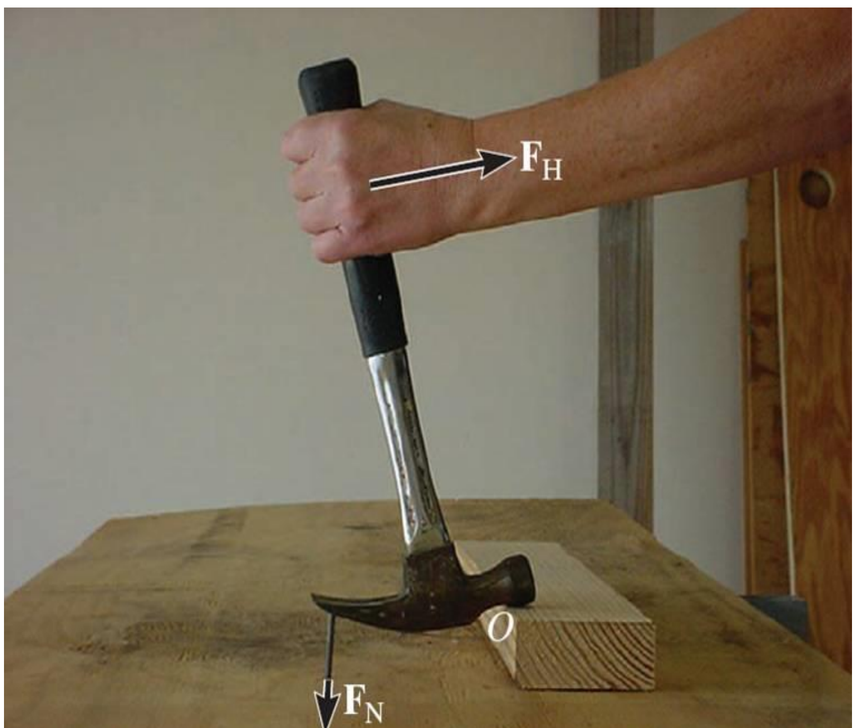
Goals and Objectives

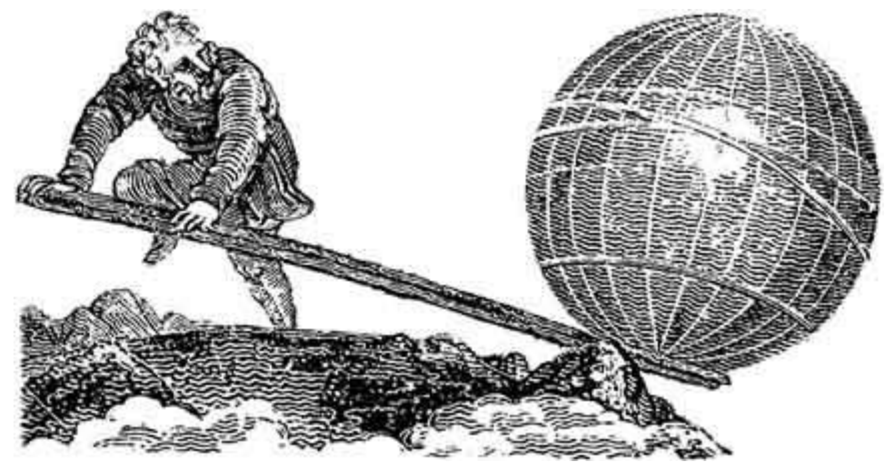
- Discuss the concept of the <u>moment of a force</u> and show how to calculate it in two and three dimensions
- How to find the <u>moment about a specified axis</u>
- Define the <u>moment of a couple</u>
- Finding <u>equivalence force and moment systems</u>
- Reduction of <u>distributed loading</u>

Moment of a force

The **moment of a force about a point** provides a measure of the **tendency for rotation** (sometimes called a torque).



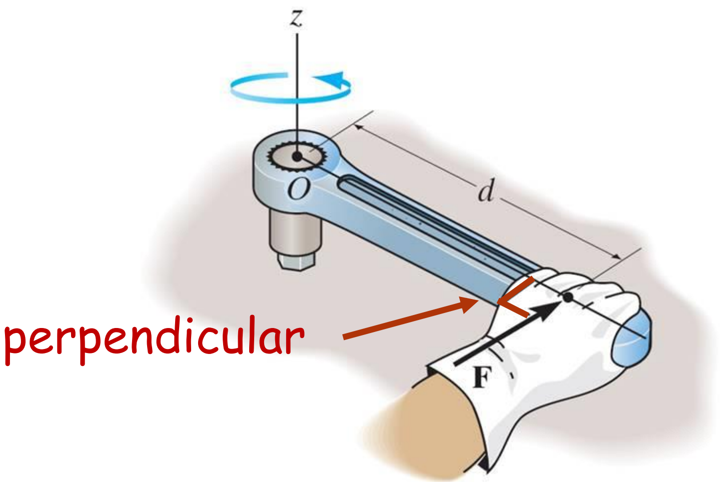


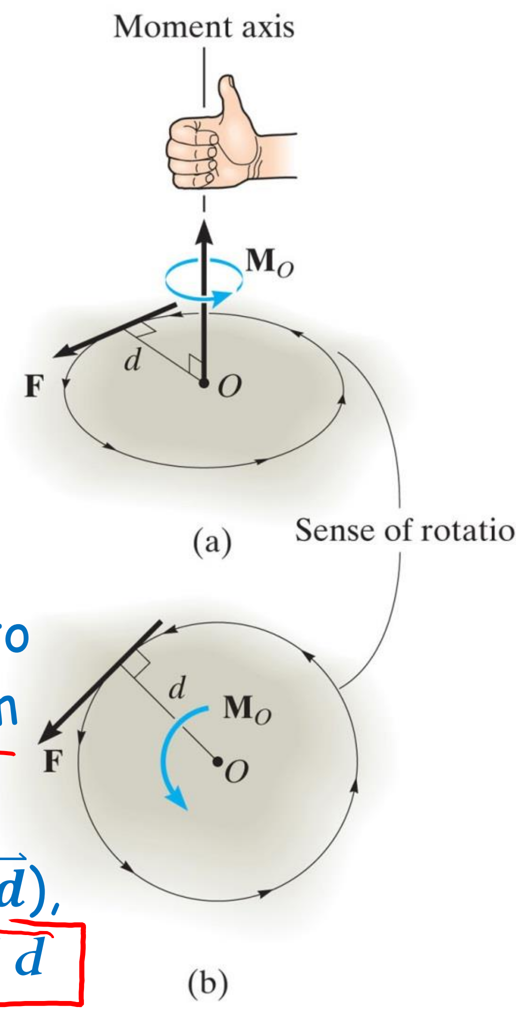


Moment 1.A very brief period of time. An exact point in time. 2. Importance. **3. A turning effect produced by a force acting at a distance on an object.** Oxford Dictionary

Moment of a force – scalar formulation

The **moment of a force about a point** provides a measure of the **tendency for rotation** (sometimes called a torque).





Direction: Moment about point O $\vec{M_O}$ is perpendicular to the plane that contains the force $\vec{F}$ and its moment arm $\vec{d}$. The right-hand rule is used to define the sense.

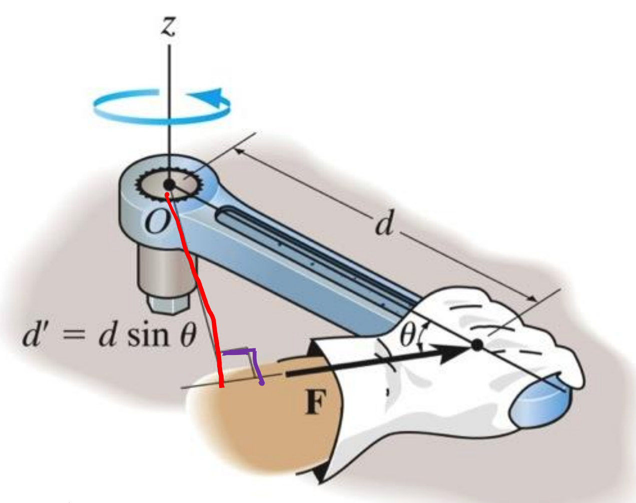
Magnitude: In a 2D case (where $\vec{F}$ is perpendicular to $\vec{d}$), the magnitude of the moment about point O is $\boxed{M_O = F\,d}$

Moment of a force – vector formulation

The moment of a force $\vec{F}$ about point O, or actually about the moment axis passing through O and perpendicular to the plane containing O and $\vec{F}$, can be expressed using the cross (vector) product, namely:

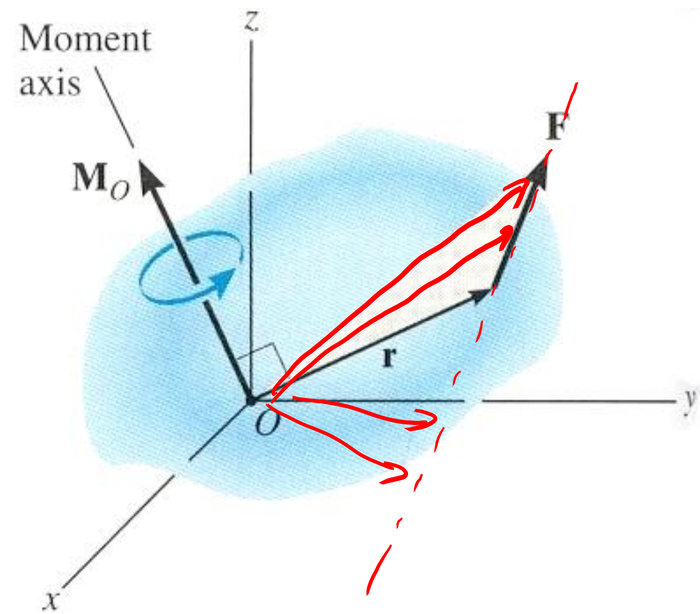
$$\overrightarrow{M_O} = \vec{r} \times \vec{F}$$

where $\vec{r}$ is the position vector directed from O to any point on the line of action of $\vec{F}$.

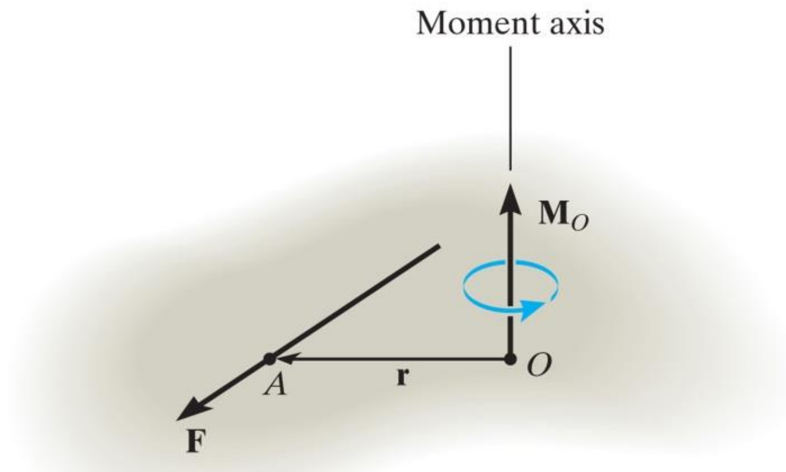


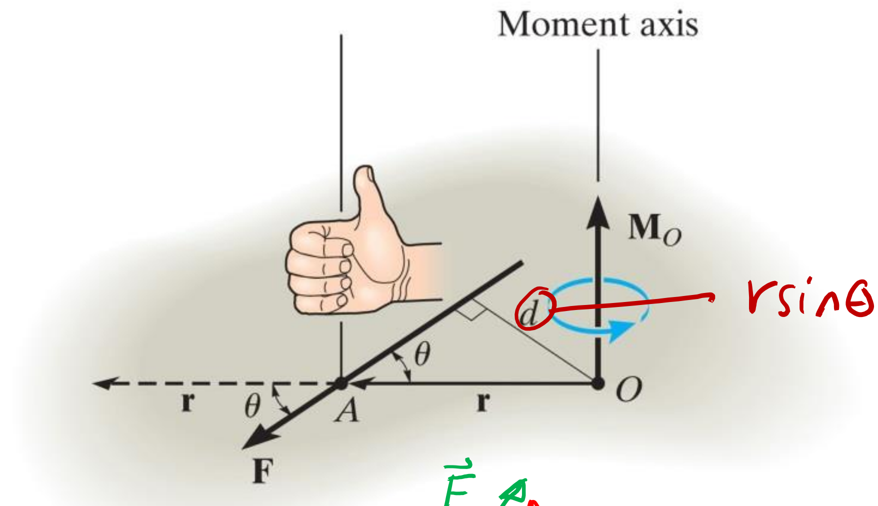
$M_o = F d'$

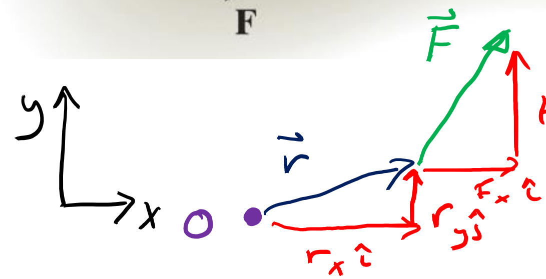
$\theta \neq 90°$



Moment of a force – vector formulation







Use cross product: $\overrightarrow{M_O} = \vec{r} \times \vec{F}$

Direction: Defined by right hand rule.

$$\overrightarrow{M_O} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = (r_yF_z - r_zF_y)\hat{\imath} - (r_xF_z - r_zF_x)\hat{\jmath} + \overbrace{(r_xF_y - r_yF_x)\hat{k}}^{\text{2D } (x,y) \text{ version}}$$

Magnitude: recall $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\theta$

$$M_O = \left|\overrightarrow{M_O}\right| = |\vec{r}|\left|\vec{F}\right| \sin\theta = F(\overbrace{r\sin\theta}^{d}) = Fd$$

⊥ distance from O to $\vec F$

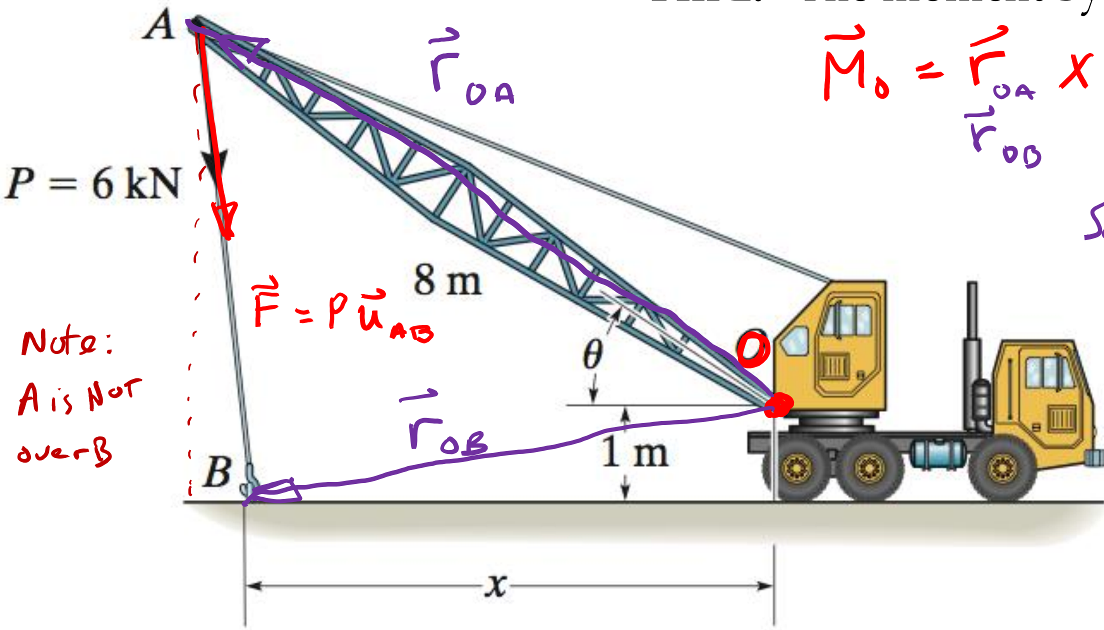
Example

Given: The angle $\theta = 30°$ and $x = 10$ m.

Find: The moment by $\vec{P}$ about point O. $\Rightarrow$ Find $\vec{M}_o$

$\vec{M}_o = \vec{r}_{OA} \times \vec{F}$

$\vec{r}_{OB}$



Soln: $M_o \cong 48$ kNm

$\vec{M}_o$ in $+\hat{k}$ direction

$+\circlearrowleft$