Statics - TAM 211

Lecture 8

September 28, 2018

Announcements

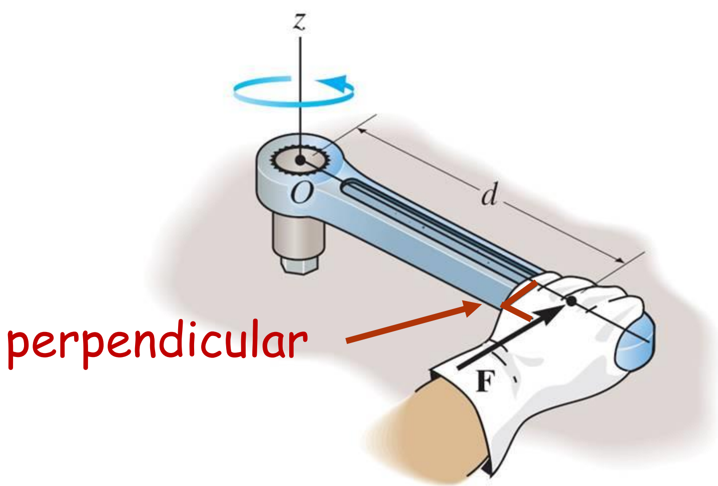
- No classes September 29 – October 7 for National Day Holiday
  - Saturday Sept 29 make-up class cancelled to accommodate for Quiz 1
  - Sunday Sept 30 make-up class cancelled to accommodate for Quiz 2
  - Enjoy your vacation!
- Upcoming deadlines:
- Friday (today)
  - Written Assignment 2
- Tuesday (10/9)
  - Prairie Learn HW3
- Friday (10/12)
  - Written Assignment 3

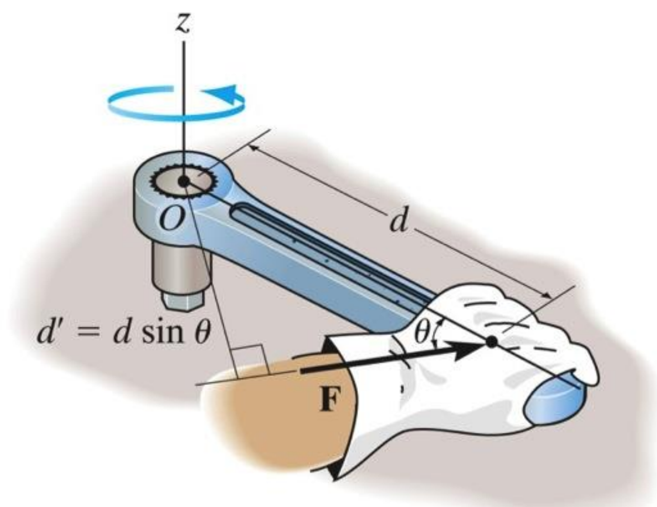
Chapter 4: Force System Resultants

Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- How to find the moment about a specified axis (using Scalar Triple Product)
- Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading

Recap: Moment of a force





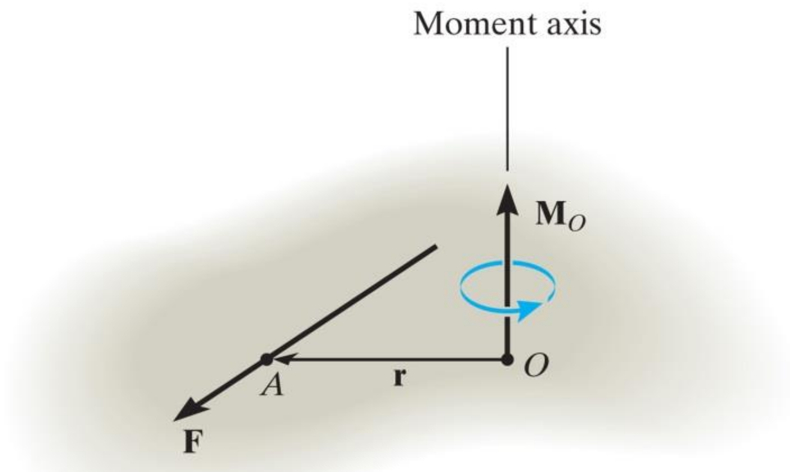
Scalar Formulation: $M_O = F\, d$

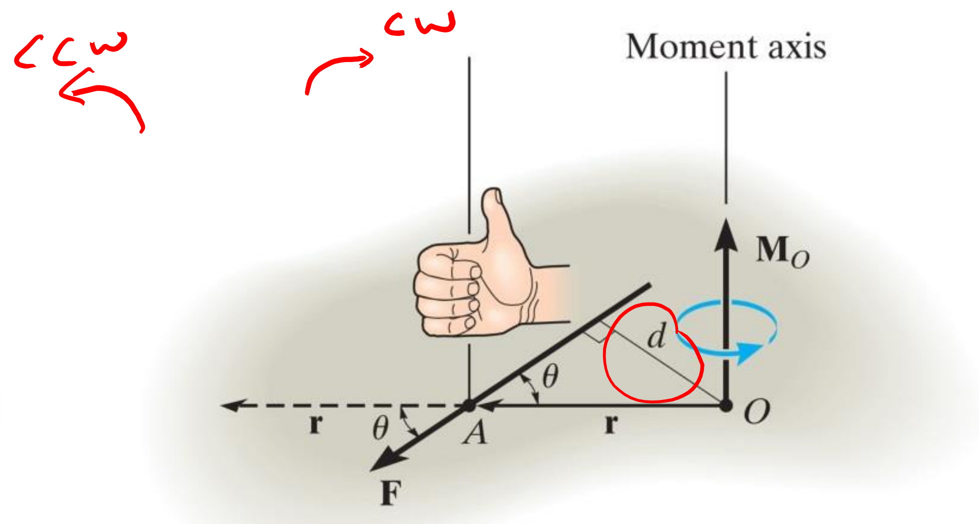
Scalar Formulation: $M_O = F\, d'$

Direction: Moment about point O $\overrightarrow{M_O}$ is perpendicular to the plane that contains the force $\vec{F}$ and its moment arm $\vec{d}$. The right-hand rule is used to define the sense.

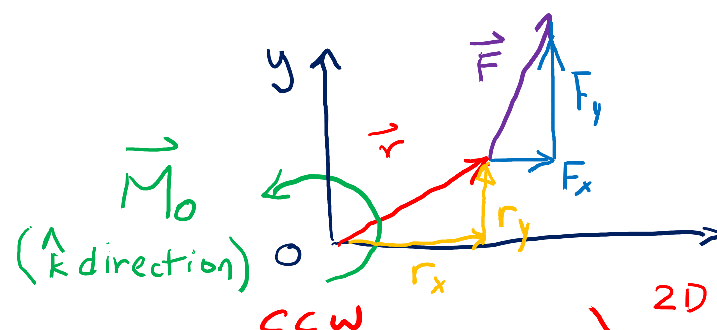
Magnitude: In a 2D case (where $\vec{F}$ is perpendicular to $\vec{d}$), the magnitude of the moment about point O is $M_O = F\, d$

Recap: Moment of a force





Vector Formulation



Use cross product: $\overrightarrow{M_O} = \vec{r} \times \vec{F}$

Direction: Defined by right hand rule.

$$\overrightarrow{M_O} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = (r_yF_z - r_zF_y)\hat{\imath} - (r_xF_z - r_zF_x)\hat{\jmath} + \overbrace{(r_xF_y - r_yF_x)}^{\text{2D (x,y) case}}\hat{k}$$

$= \vec{M}_o$ (x,y case)

Magnitude: recall $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$

$$M_O = \left|\overrightarrow{M_O}\right| = |\vec{r}|\left|\vec{F}\right| sin\theta = F(rsin\theta) = Fd$$

d is the perpendicular distance from O to $\vec{F}$

Example

Given: The angle $\theta = 30°$ and $x = 10$ m.

Find: The moment by $\vec{P}$ about point O. $\Rightarrow$ Find $\vec{M}_o$

$\vec{M}_o = \vec{r}_{OA} \times \vec{F}$

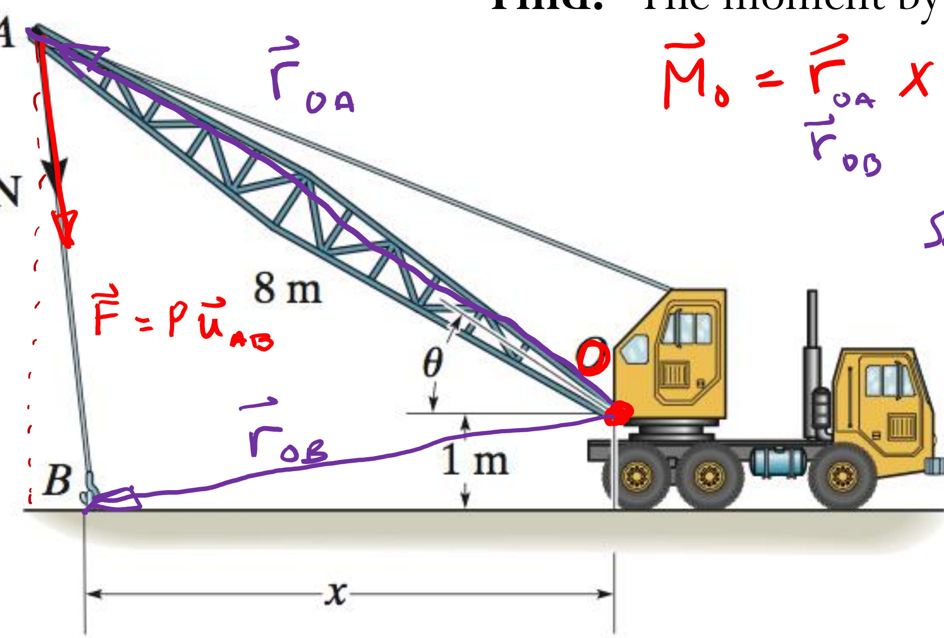
$\vec{r}_{OB}$

Soln: $M_o \cancel{=} 48$ kNm

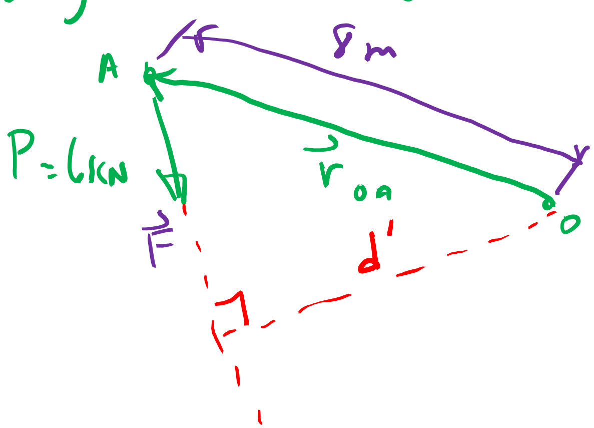
$\vec{M}_o$ in $+\hat{k}$ direction

$+\circlearrowleft$

Note: A is NOT over B



Why should $M_o < 48$ kNm?

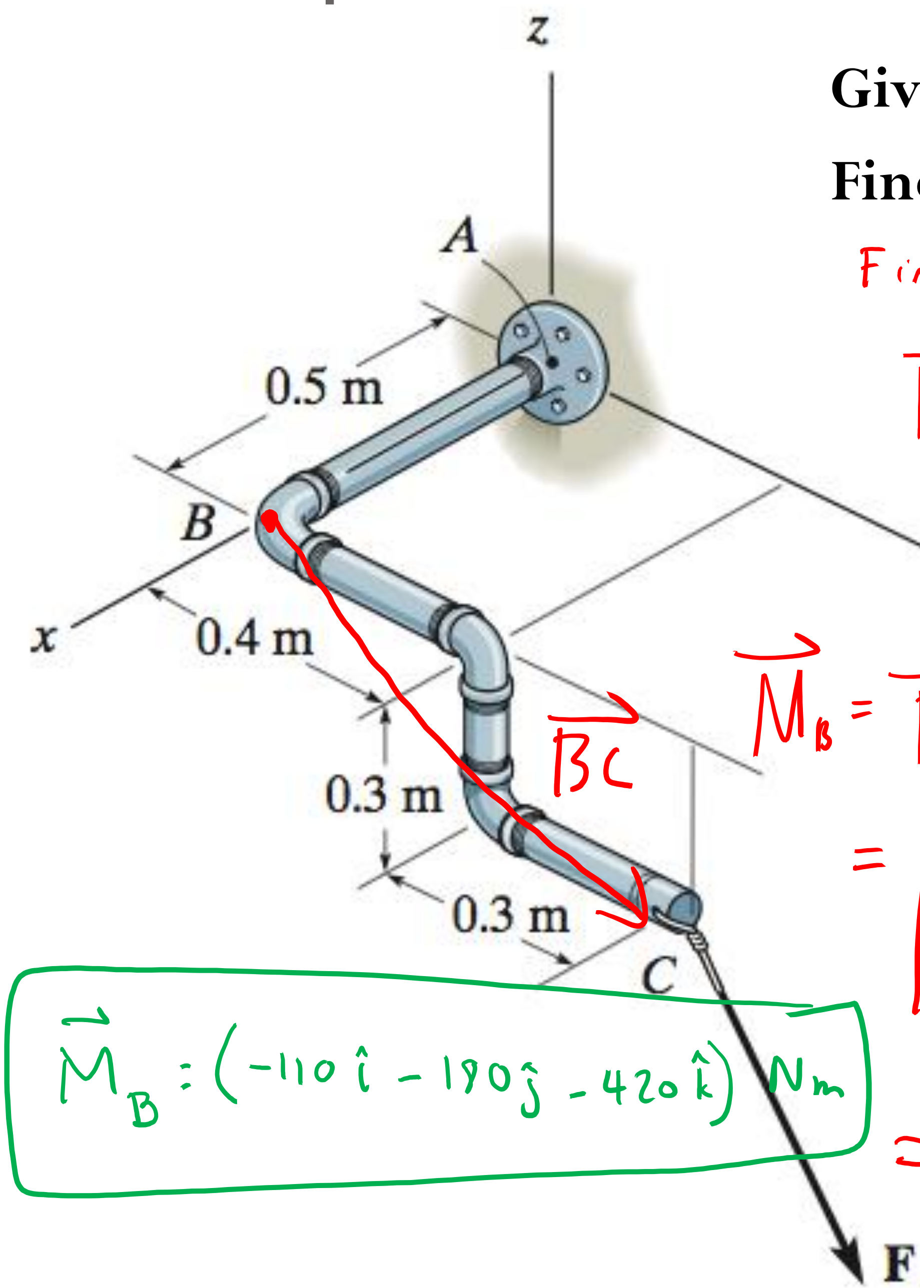


$\vec{M}_o = \vec{r}_{OA} \times \vec{F}$

$M_o = Fd'$

$d' < 8$m

Example – Vector Formulation



Given: **F** = {600**i** + 800**j** - 500**k**} N

Find: Moment of the force about point *B*.

Find: $\vec{M}_B$

$\vec{BC} = \langle 0, 0.7, -0.3 \rangle$

$\vec{F} = \langle 600, 800, -500 \rangle$

$\vec{M}_B = \vec{BC} \times \vec{F}$

$$= \begin{vmatrix} i & j & k \\ 0 & 0.7 & -0.3 \\ 600 & 800 & -500 \end{vmatrix}$$

$$= -500\times 0.7\, i - 0.3\times 600\, j + 0\, k - \left(800\times(-0.3)\, i + 0\, j + 600\times 0.7\, k\right)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.7 & -0.3 \\ 600 & 800 & -500 \end{vmatrix}$$

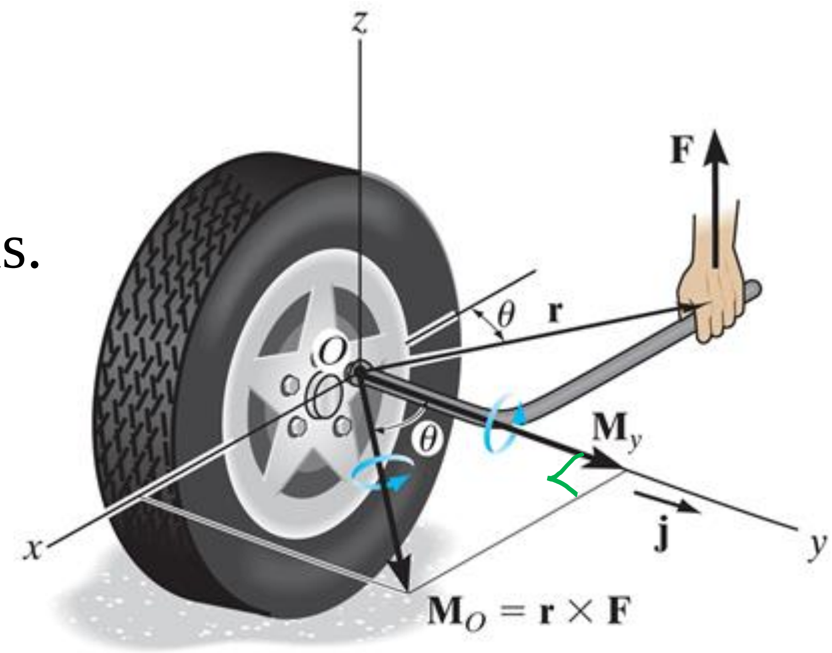
$\hat{i}\left(0.7(-500) - (-0.3\cdot 800)\right) + \dots$

$\vec{M}_B = (-110\,\hat{i} - 180\,\hat{j} - 420\,\hat{k})$ Nm

These notes in red or green/purple show two methods for calculating a determinant.

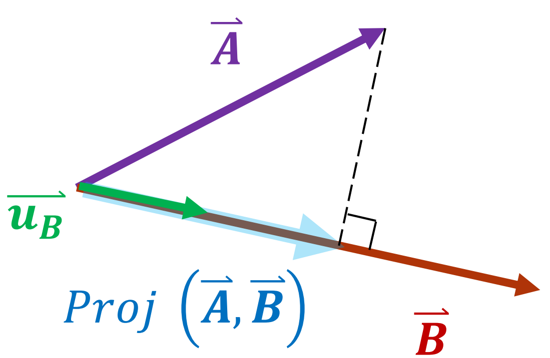
Moment of a force about a specified axis

A force is applied to the tool as shown. Find the magnitude of the moment of this force about the *y*-axis.



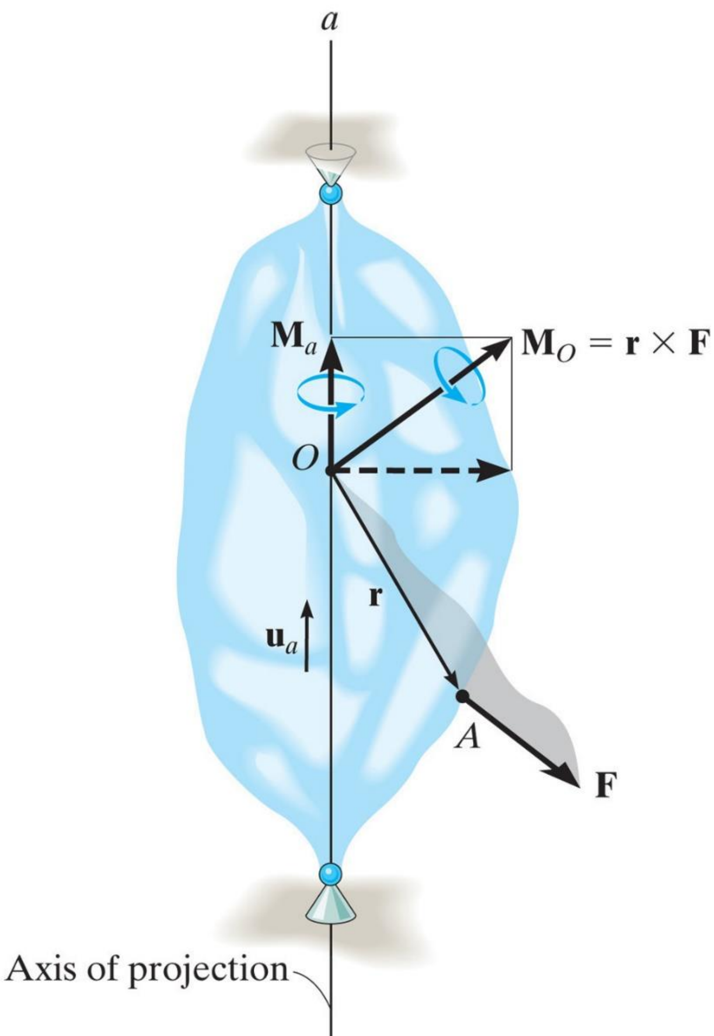
Recall: the projected component of a vector, $\vec{A}$, along the direction of another, $\vec{B}$, can be determined using the dot product.

$$Proj\left(\vec{A}, \vec{B}\right) = \left(\vec{A} \cdot \vec{u_B}\right)\vec{u_B}$$



Moment of a force about a specified axis (Scalar Triple Product)

The magnitude of the projected moment about any generic axis *a* can be computed using the scalar triple product:



$$\left|\overrightarrow{\boldsymbol{M_a}}\right| = \overrightarrow{\boldsymbol{M_o}} \cdot \overrightarrow{\boldsymbol{u_a}}$$

$$= \left(\vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}\right) \cdot \overrightarrow{\boldsymbol{u_a}}$$

$$= \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

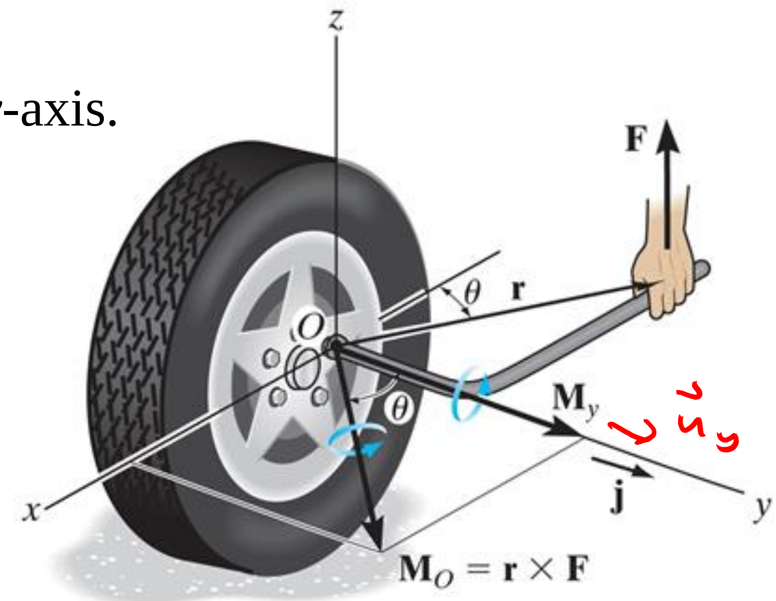
Scalar Triple Product

The direction of the projected moment about any generic axis *a* can be defined using :

$$\overrightarrow{\boldsymbol{M_a}} = \left|\overrightarrow{\boldsymbol{M_a}}\right| \overrightarrow{\boldsymbol{u_a}}$$

where $\overrightarrow{\boldsymbol{u_a}}$ is the unit vector along axis *a*

A force is applied to the tool as shown. Find the magnitude of the moment of this force about the y-axis.



Find: $|\vec{M}_y|$

$$M_y = (\vec{r} \times \vec{F}) \cdot \vec{u}_y$$

$\vec{F} = F\hat{k}$ $\quad\quad \vec{u}_y = \hat{j}$

$\vec{r} = -x\hat{i} + y\hat{j}$

$$\vec{m}_o = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -x & y & 0 \\ 0 & 0 & F \end{vmatrix} = yF\hat{i} - (-xF)\hat{j} + 0\hat{k}$$

$$M_y = (\vec{r} \times \vec{F}) \cdot \vec{u}_y = (yF\hat{i} + xF\hat{j}) \cdot \hat{j}$$

[dot product projection]

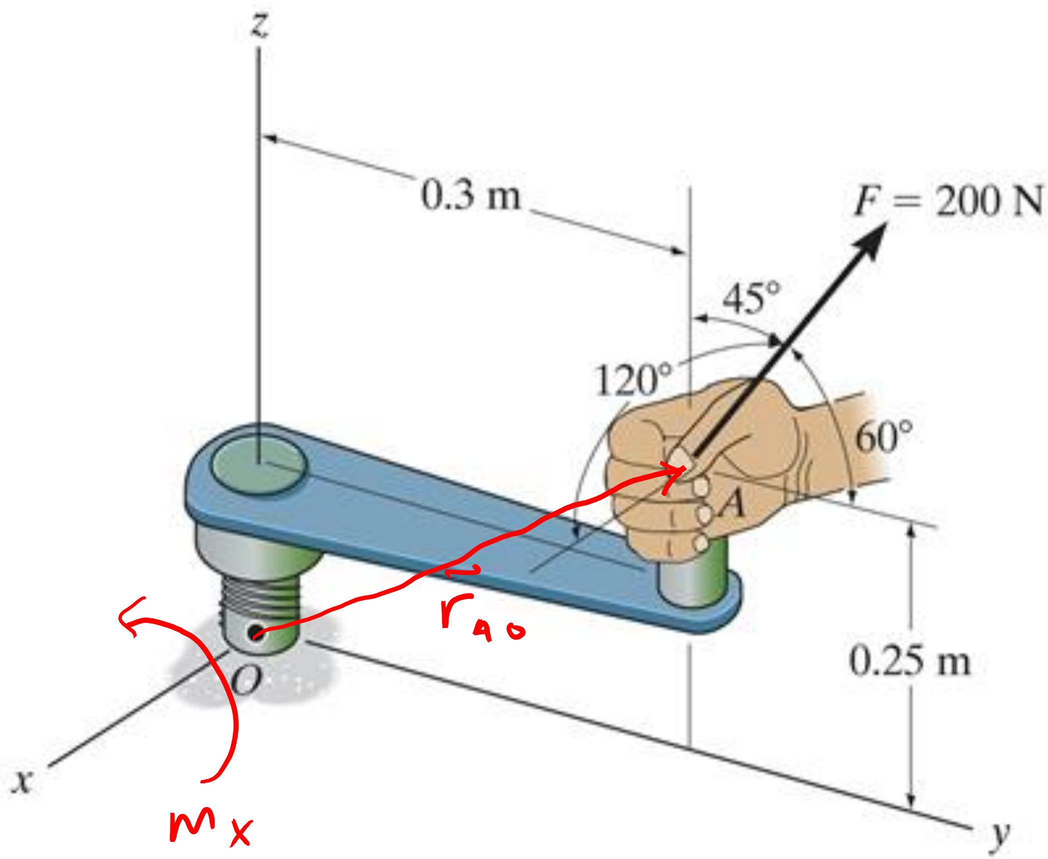
$$\boxed{M_y = xF \ \text{Nm}}$$

magnitude

Alternatively, use Scalar Triple Product

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ -x & y & 0 \\ 0 & 0 & F \end{vmatrix}$$

= same answer in one step!



A force is applied to the tool as shown. Find the magnitude of the moment of this force about the x axis.

Find: M_x

$$M_x = (\vec{r}_{AO} \times \vec{F}) \cdot \vec{u}_x$$

$$M_x = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.3 & 0.85 \\ -100 & 100 & 141.4 \end{vmatrix}$$

Note:

$$\vec{F} = 200\ N\,[\cos 120°, \cos 60°, \cos 45°]$$