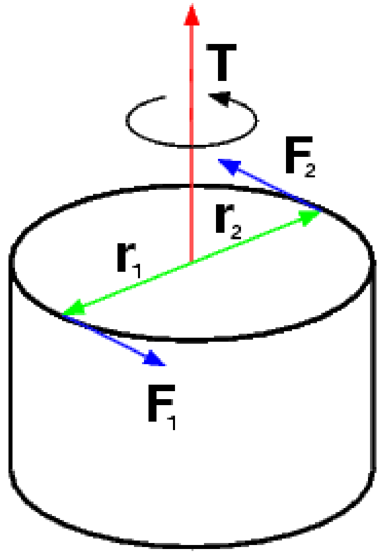
Statics - TAM 211

Lecture 9

October 8, 2018

Announcements

- Upcoming deadlines:
- Tuesday (10/9)
  - Prairie Learn HW3
- Friday (10/12)
  - Written Assignment 3



https://fr.wikipedia.org/wiki/Couple_(physique)

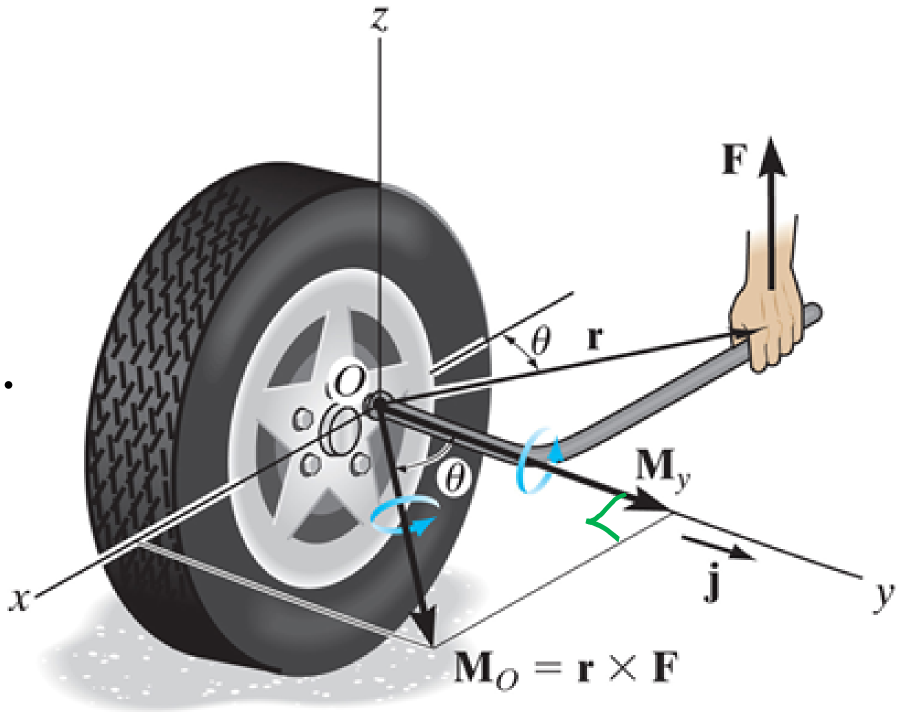
Chapter 4: Force System Resultants

Goals and Objectives

- Discuss the concept of the <u>moment of a force</u> and show how to calculate it in two and three dimensions
- How to find the <u>moment about a specified axis</u>
- Define the <u>moment of a couple</u>
- Finding <u>equivalence force and moment systems</u>
- Reduction of <u>distributed loading</u>

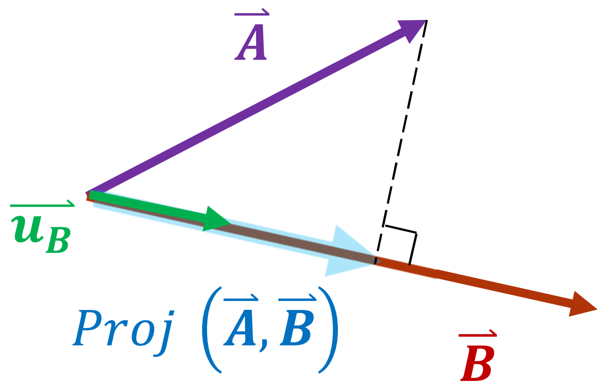
Recap: Moment of a force about a specified axis

A force is applied to the tool as shown. Find the magnitude of the moment of this force about the y-axis.



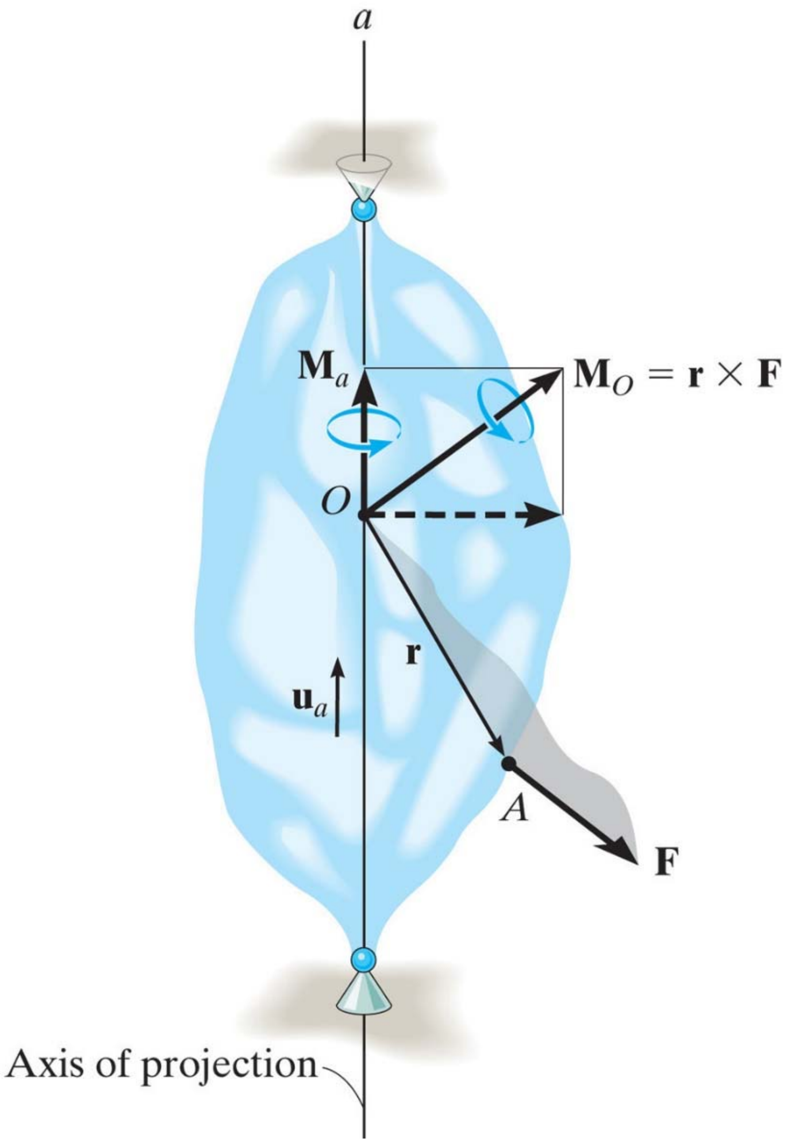
Recall: the projected component of a vector, $\vec{A}$, along the direction of another, $\vec{B}$, can be determined using the dot product.

$$Proj\left(\vec{\boldsymbol{A}}, \vec{\boldsymbol{B}}\right) = \left(\vec{\boldsymbol{A}} \cdot \overrightarrow{\boldsymbol{u_B}}\right)\overrightarrow{\boldsymbol{u_B}}$$



Recap: Moment of a force about a specified axis (Scalar Triple Product)

The magnitude of the projected moment about any generic axis *a* can be computed using the scalar triple product:



$$\left|\overrightarrow{\boldsymbol{M_a}}\right| = \overrightarrow{\boldsymbol{M_o}} \cdot \overrightarrow{\boldsymbol{u_a}}$$

$$= \overrightarrow{\boldsymbol{u_a}} \cdot \left(\vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}\right)$$

$$= \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

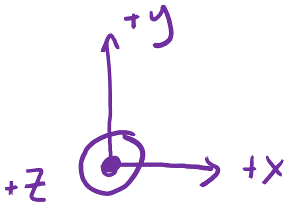
The direction of the projected moment about any generic axis *a* can be defined using :

$$\overrightarrow{\boldsymbol{M_a}} = \left|\overrightarrow{\boldsymbol{M_a}}\right| \overrightarrow{\boldsymbol{u_a}}$$

where $\overrightarrow{\boldsymbol{u_a}}$ is the unit vector along axis *a*

Equivalent couples

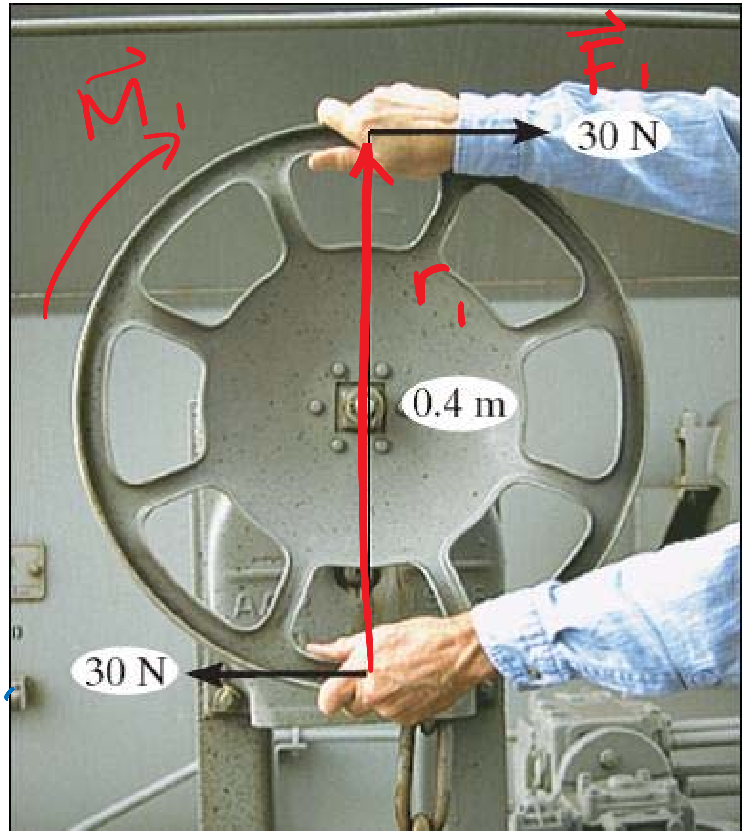
Define coordinate frame

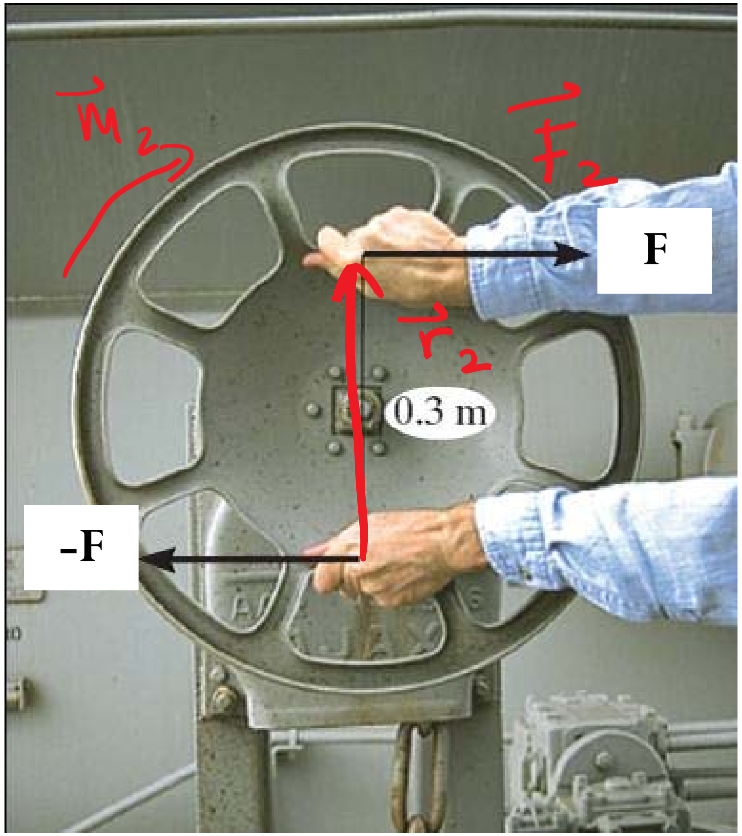


Notation for representing a vector pointing perpendicular (in or out of screen)

↖ Like arrow

tip tail





A torque or moment of 12 N·m is required to rotate the wheel. $M_1 = M_2 = 12\ Nm$

Would F be greater or less than 30 N?

$$\vec{M}_1 = \vec{r}_1 \times \vec{F}_1$$
$$= 0.4m\,\hat{j} \times 30N\,\hat{i}$$
$$= 12\ Nm\,(-\hat{k})$$

"caret" "hat" symbol

↑ since +z is counterclockwise ccw

$$\vec{M}_2 = \vec{r}_2 \times \vec{F}_2$$
$$= 0.3m\,\hat{j} \times FN\,\hat{i}$$
$$= 0.3F\ Nm\,(-\hat{k})$$
$$= -12\,Nm\,\hat{k}$$

$\therefore \boxed{F = 40N}$ greater than F_1

~~i>Clicker~~

$\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ form a couple. The moment of the couple is given by:

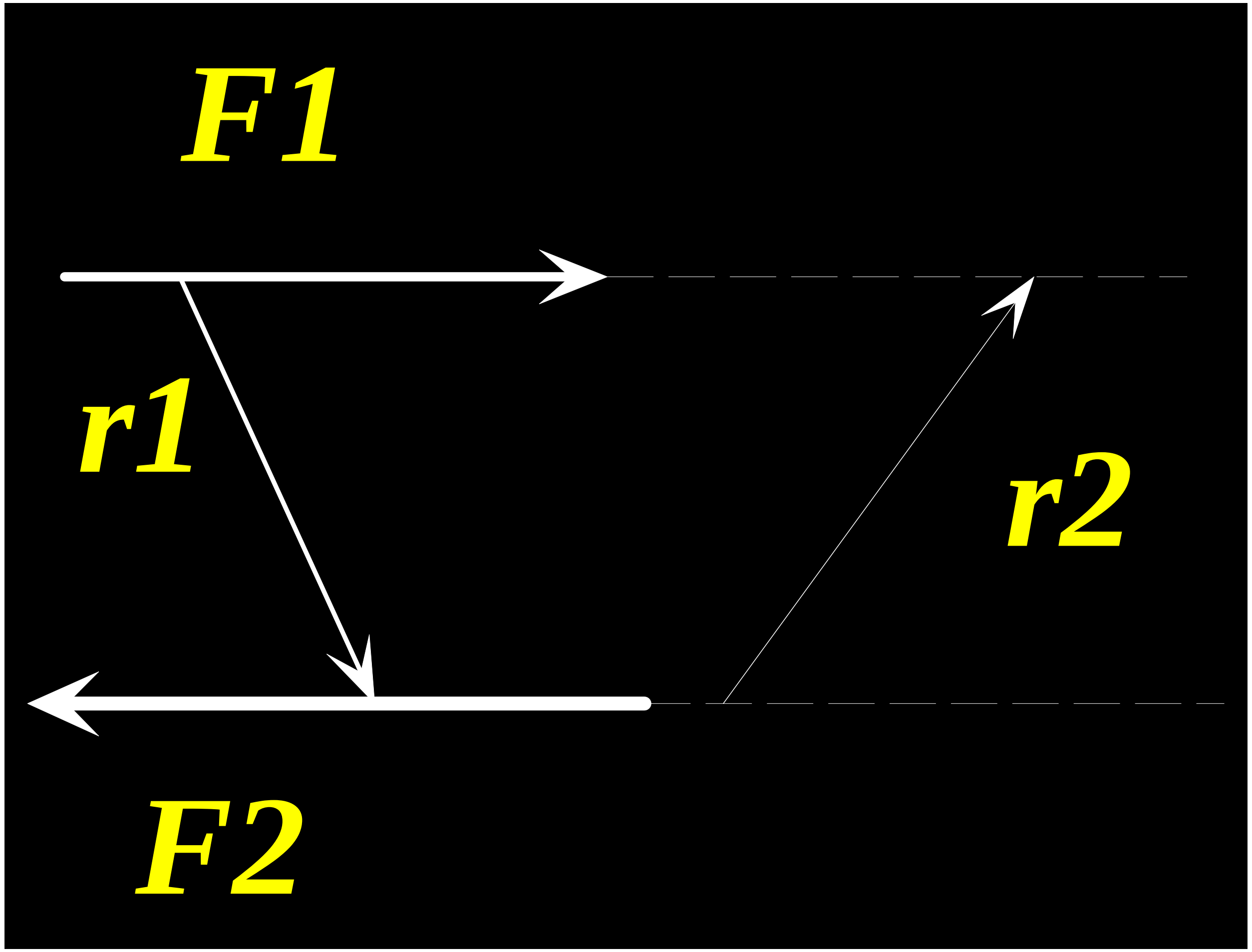
A) $\boldsymbol{r_1} \times \boldsymbol{F_1}$

Both correct

B) $\boldsymbol{r_2} \times \boldsymbol{F_1}$

C) $\boldsymbol{r_1} \times \boldsymbol{F_2}$

D) $\boldsymbol{r_2} \times \boldsymbol{F_2}$



Use right hand rule to determine direction of rotation.

Point figures in direction of $\vec{r}$, then curl fingers in direction of $\vec{F}$. Your thumb points in direction of moment vector $\vec{M}$.

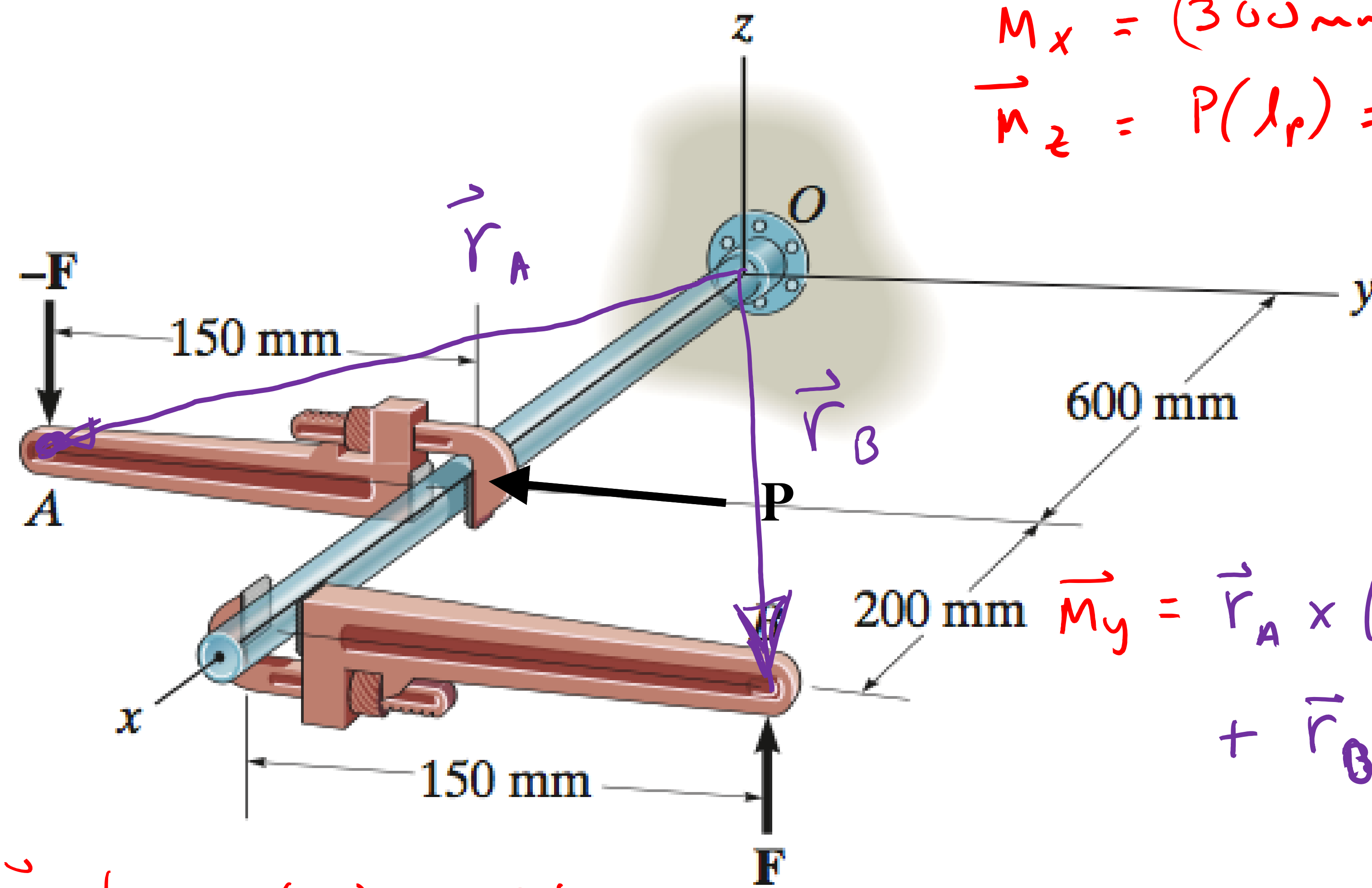
Important point: Moments are always defined as $\vec{M} = \vec{r} \times \vec{F}$.
If incorrectly write, $\vec{M} = \vec{F} \times \vec{r}$, then $\vec{M}$ points in wrong direction!

Find the moment about the support at *O*? F = 125 N, P = 100 N.

âbout the x-axis

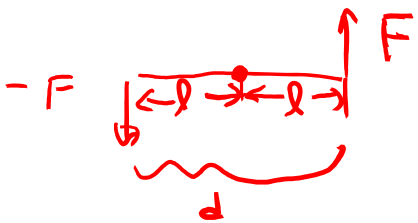
$\vec{M}_x = (300\,\text{mm})(125\,\text{N})\,\hat{\imath}$

$\vec{M}_z = P(l_p) = -(100\,\text{N})(600\,\text{mm})\,\hat{k}$



$\vec{M}_y = \vec{r}_A \times (-F)\,\hat{k} + \vec{r}_B \times (F)\,\hat{k}$

$|\vec{M}_F| = F(2\ell) = Fd$



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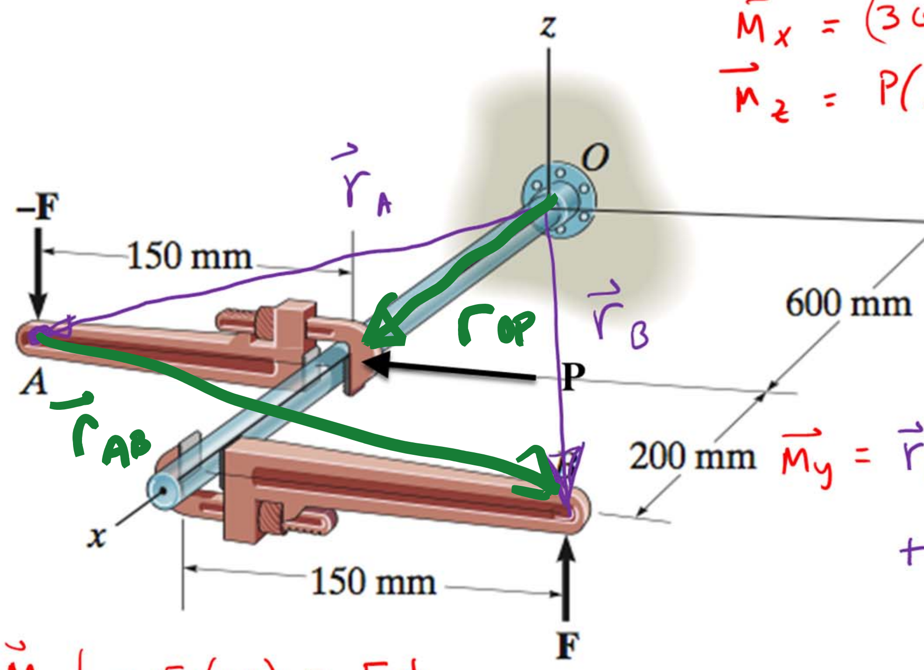

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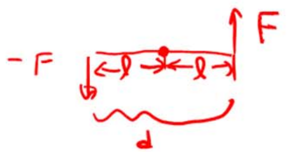
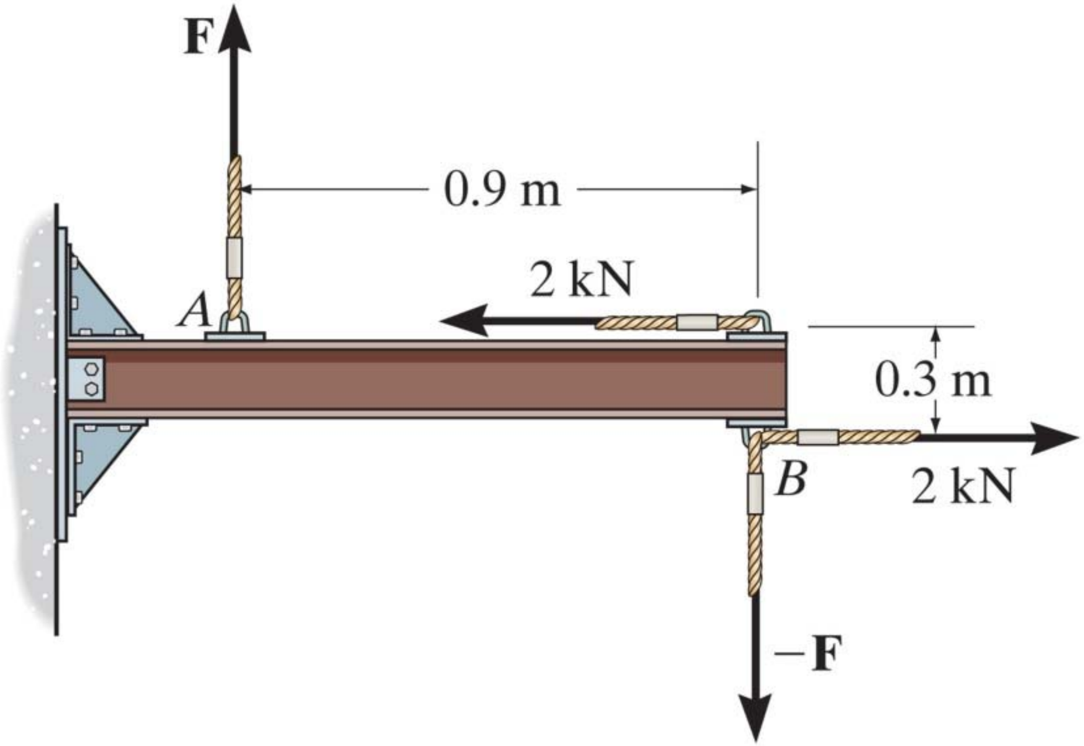


A couple of points that were brought up by students after class.

1) The two forces **F** cause vector components of a couple moment that are in both the x-axis and y-axis. Thus, to compute $\overrightarrow{\boldsymbol{M_y}}$ quickly one could just write $\overrightarrow{\boldsymbol{M_y}} = d_y F = (200\text{mm})(125\text{N})\hat{\imath}$.
2) It was not necessary to write each of the individual vector components. These could have been determined from a single equation that uses the principle of Resultant Moments, which is introduced in the next slides. Such that $\overrightarrow{\boldsymbol{M_R}} = \overrightarrow{\boldsymbol{M_1}} + \overrightarrow{\boldsymbol{M_2}}$, where $\overrightarrow{\boldsymbol{M_1}} = \overrightarrow{\boldsymbol{r_{AB}}} \times \overrightarrow{\boldsymbol{F}}$, and $\overrightarrow{\boldsymbol{M_2}} = \overrightarrow{\boldsymbol{r_{OP}}} \times \overrightarrow{\boldsymbol{P}}$

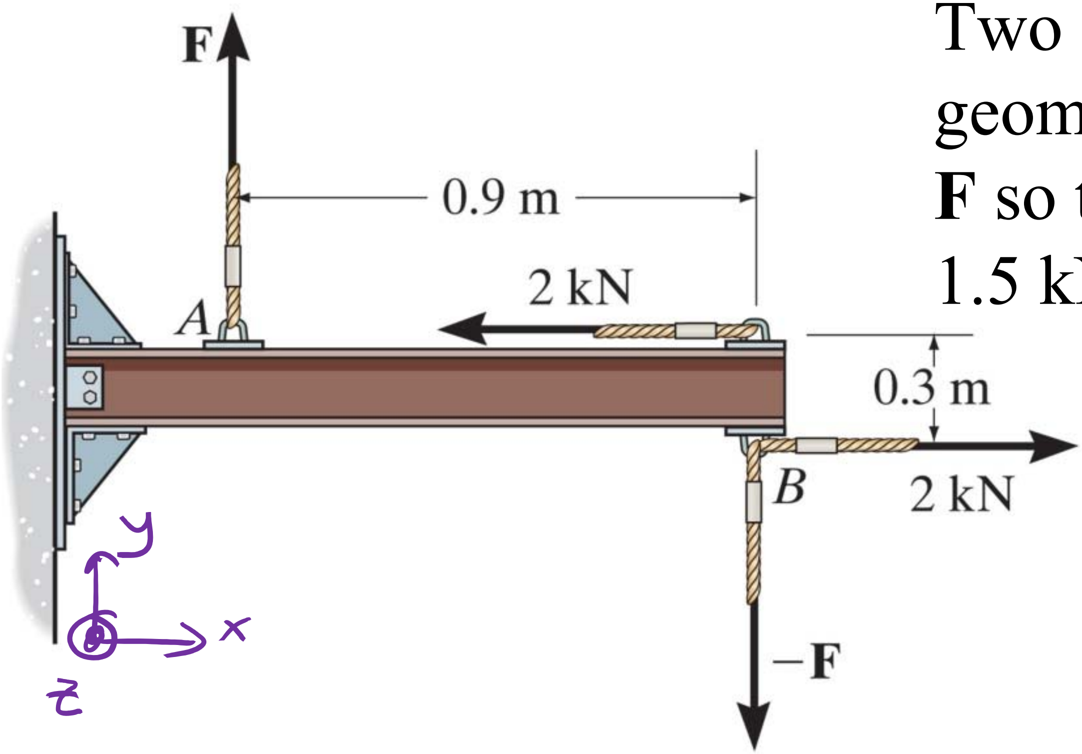
Resultant Couple Moment

Since couple moments are vectors, their resultant is due to vector addition:



$$\overrightarrow{M_R} = \overrightarrow{M_1} + \overrightarrow{M_2} + \cdots$$
$$= \Sigma \overrightarrow{M_i}$$
$$= \Sigma \left(\vec{r} \times \vec{F} \right)$$

Recall couple moments are free vectors
so these can be summed together anywhere on the body



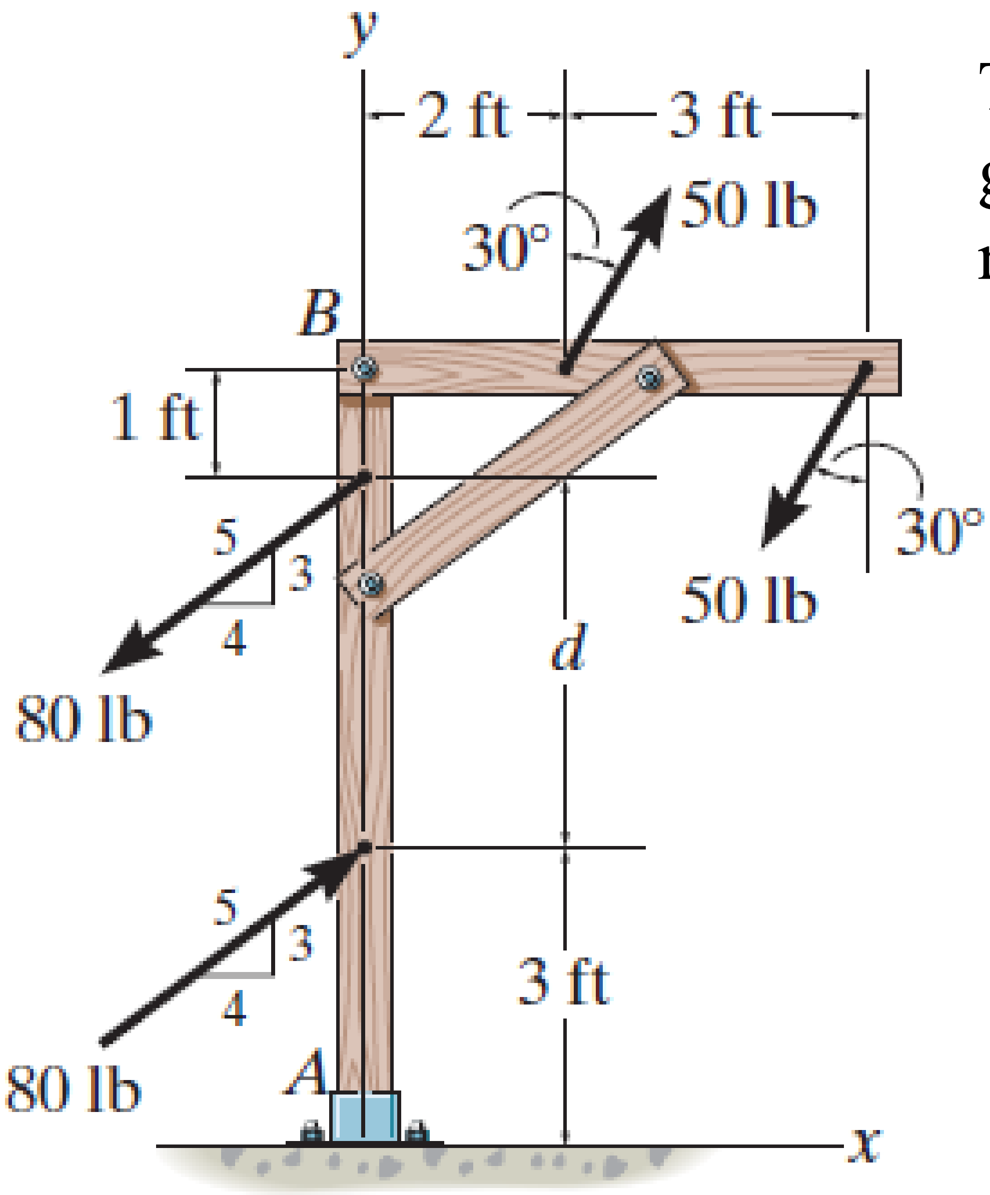
Two couples act on the beam with the geometry shown. Find the magnitude of **F** so that the resultant couple moment is 1.5 kN·m clockwise.

Want $\vec{M}_R = 1.5\ kNm\ (-\hat{k})$
cw

$= -1.5\ kNm\ \hat{k}$

$$(0.3)(2)\ kN\cdot m\ \hat{k} + F(0.9)\ kN\,m\ (-\hat{k}) = -1.5\ \hat{k}$$

$$\therefore \boxed{|\vec{F}| = 2.33\ kN}$$



Two couples act on the beam with the geometry shown and d = 4 ft. Find the resultant couple

Find: $\vec{M}_R$

Which components of each force contributes to rotation of the body?