Statics - TAM 211

Lecture 10

October 10, 2018

Announcements

- Upcoming deadlines:
- Friday (10/12)
  - Written Assignment 3
- Tuesday (10/16)
  - Prairie Learn HW4

Chapter 4: Force System Resultants

Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- How to find the moment about a specified axis
- Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading

Recap from lecture 9:

- **Moment of a force couple** ($\vec{\boldsymbol{F}}$ and $-\vec{\boldsymbol{F}}$)
  - $\overrightarrow{\boldsymbol{M_O}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}$, $\left|\overrightarrow{\boldsymbol{M_O}}\right| = Fd$ (where d ≈ ⊥ dist btw $\vec{\boldsymbol{F}}$ and $-\vec{\boldsymbol{F}}$)
  - Couple moment is a **free vector**, i.e. it is **independent** of the choice of location of O!

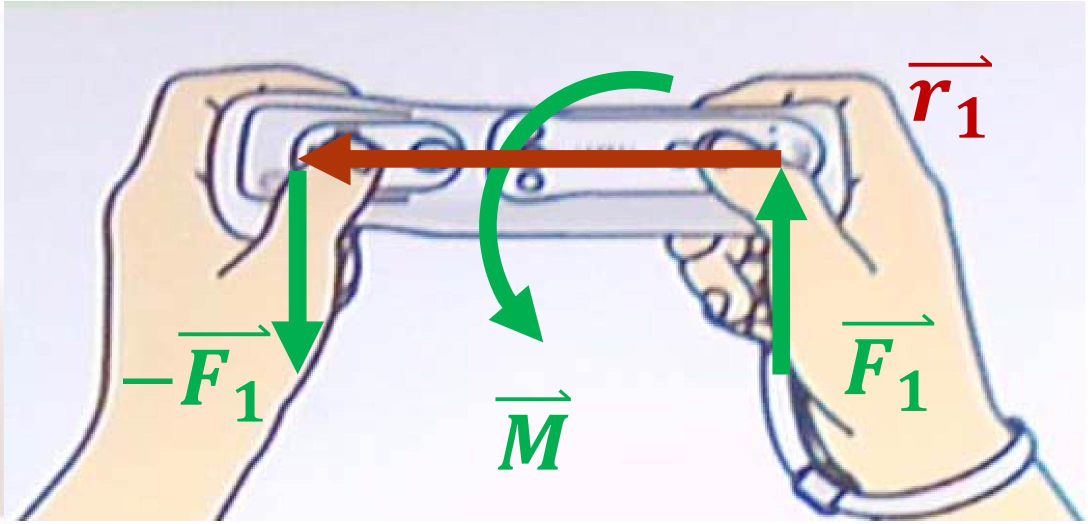
- **Equivalent couples**
  - $\boldsymbol{M_O = F_1\, d_1 = F_2\, d_2}$
  - For example, M_O = 10 Nm if F_1 = 5 N, d_1 = 2 m, or F_2 = 2.5 N, d_2 = 4 m

- **Resultant couple moment**
  - $\overrightarrow{\boldsymbol{M_R}} = \Sigma\overrightarrow{\boldsymbol{M_i}}$

Moment of a force

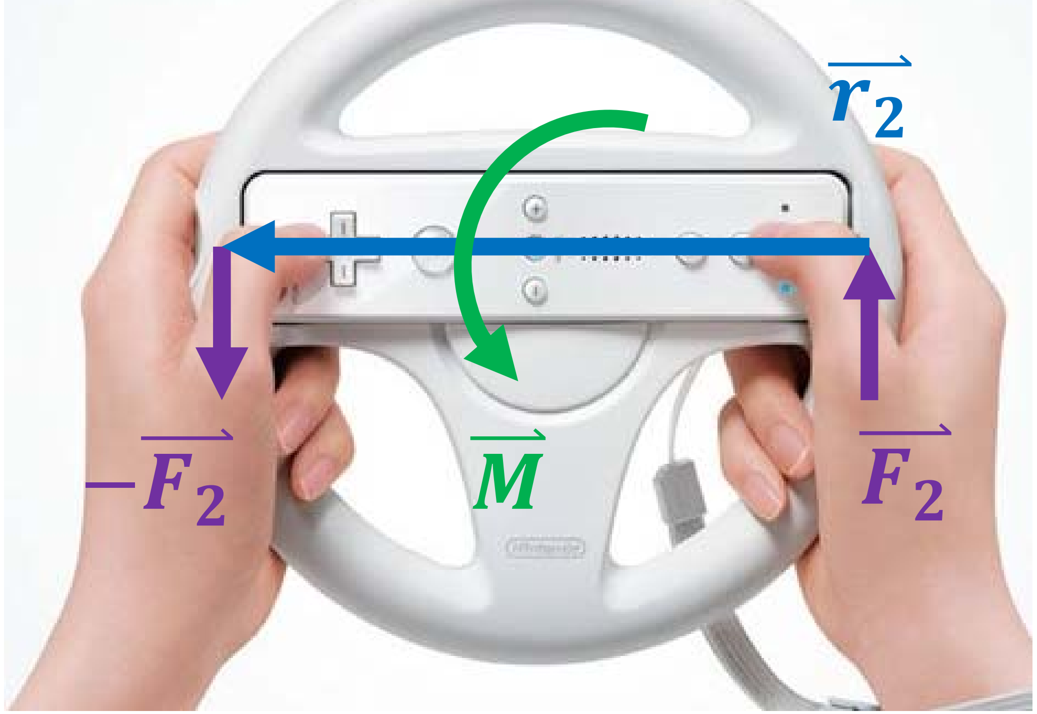
$\vec{M}$ is a free vector. It can be placed anywhere on the body, and still create a tendency for a rotation

Moment of a force couple and equivalent couples



$\vec{M}$ can be created with just one force or a force couple

The same $\vec{M}$ can be created with different pairings of force magnitude and distance between the forces



Equivalent (or equipollent) force systems

A force **system** is a collection of **forces** and **couples** applied to a body.

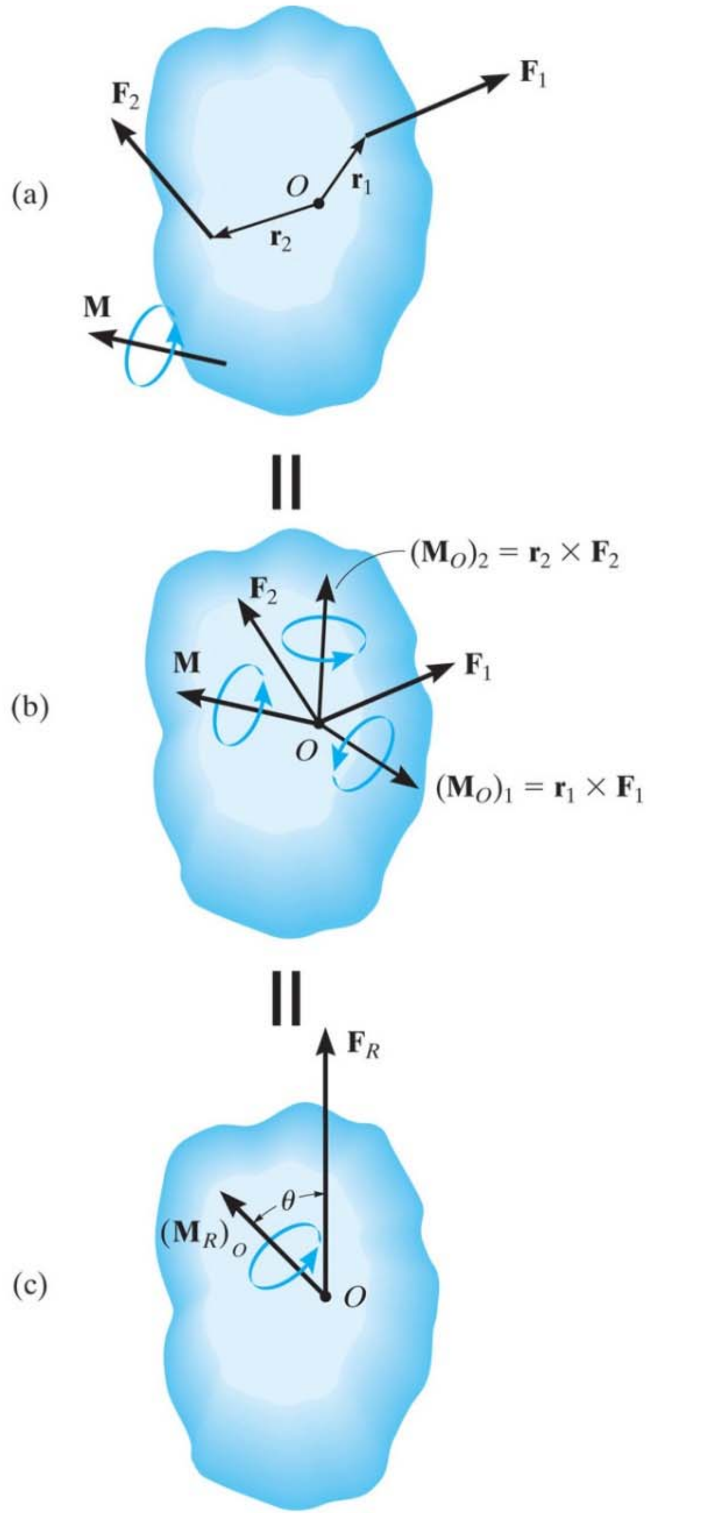
Two force systems are said to be **equipollent** (or equivalent) if they have the **same resultant force** AND the **same resultant moment** with respect to any point O.

Reducing a force system to a single resultant force $\boldsymbol{F}_R$ and a single resultant couple moment $(\boldsymbol{M}_R)_o$:

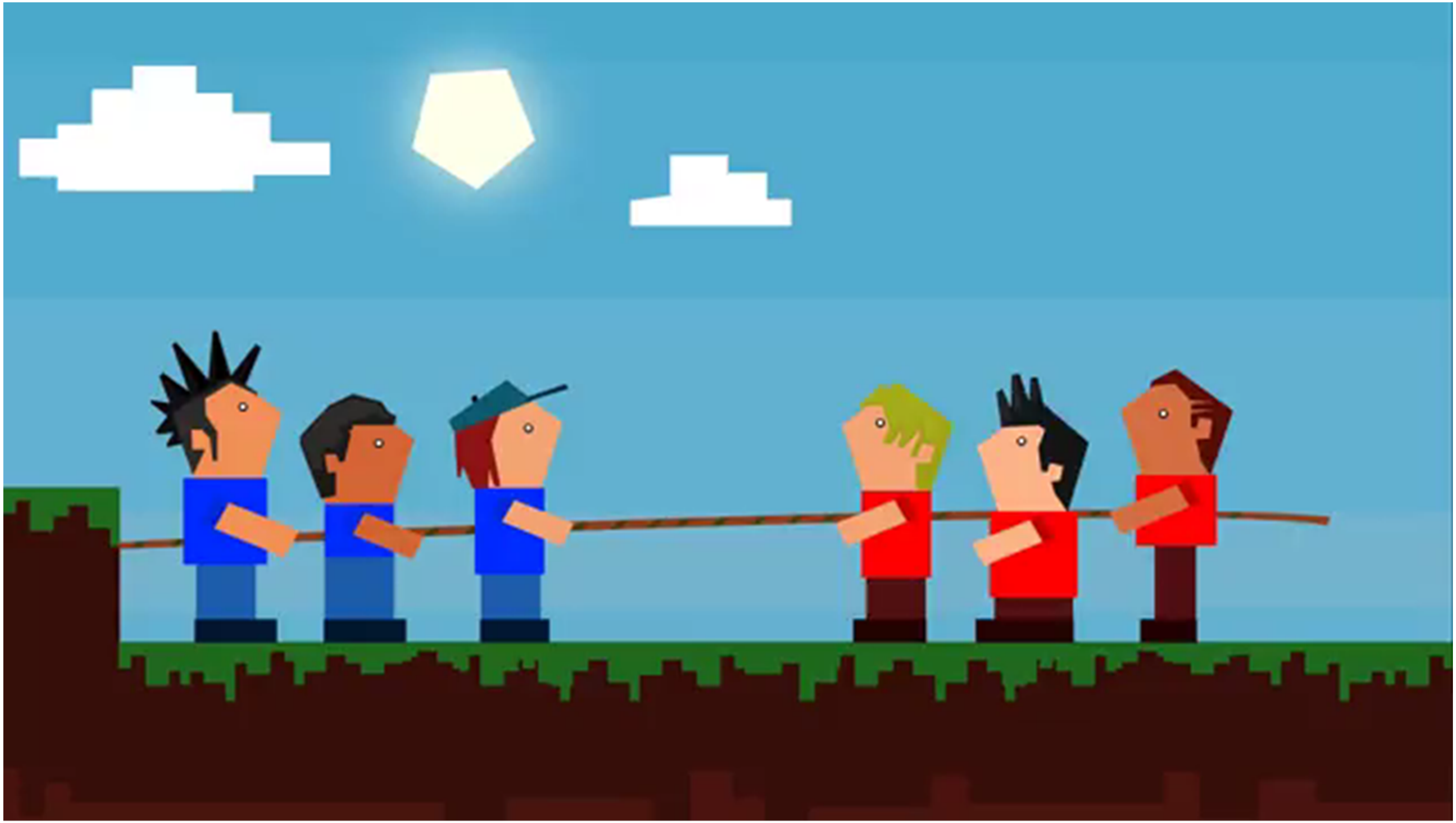
$$\overrightarrow{\boldsymbol{F}_R} = \Sigma F_x \hat{\boldsymbol{\imath}} + \Sigma F_y \hat{\boldsymbol{\jmath}} + \Sigma F_z \widehat{\boldsymbol{k}}$$

Magnitude: $|\overrightarrow{\boldsymbol{F}_R}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$ Orientation: $\theta = \tan^{-1} \frac{F_{opp}}{F_{adj}}$

$$(\boldsymbol{M}_R)_o = \sum \boldsymbol{M}_o + \sum \boldsymbol{M}$$

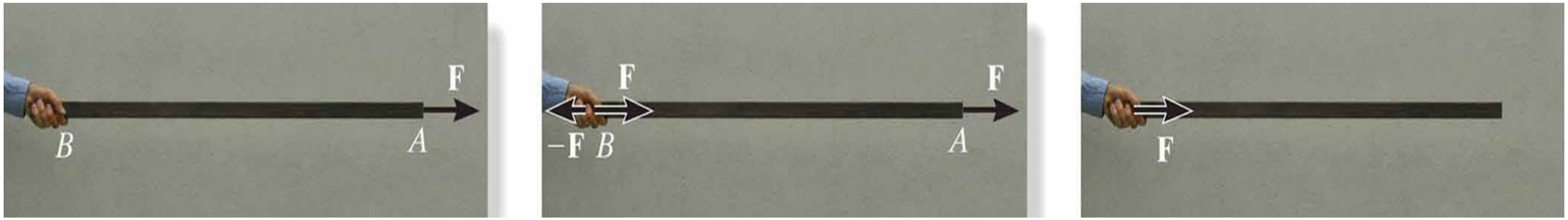


Moving a force on its line of action



https://www.wikihow.com/Win-at-Tug-of-War

Moving a force on its line of action (Equivalent forces)

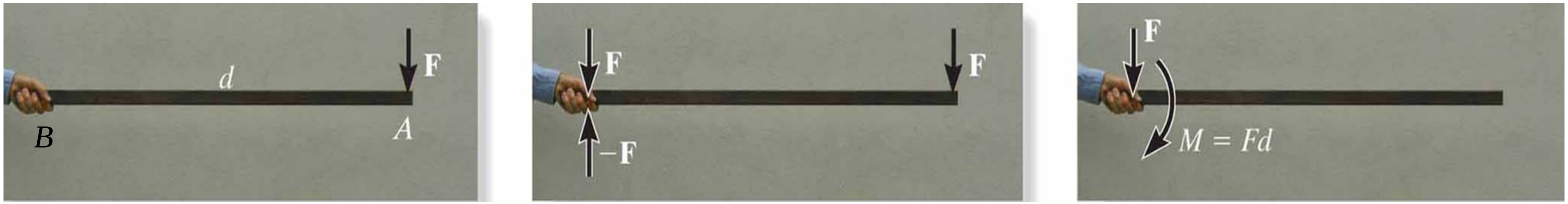


Moving a force from A to B, when both points are on the vector's line of action, does not change the **external effect**.

Hence, a force vector is called a **sliding vector**.

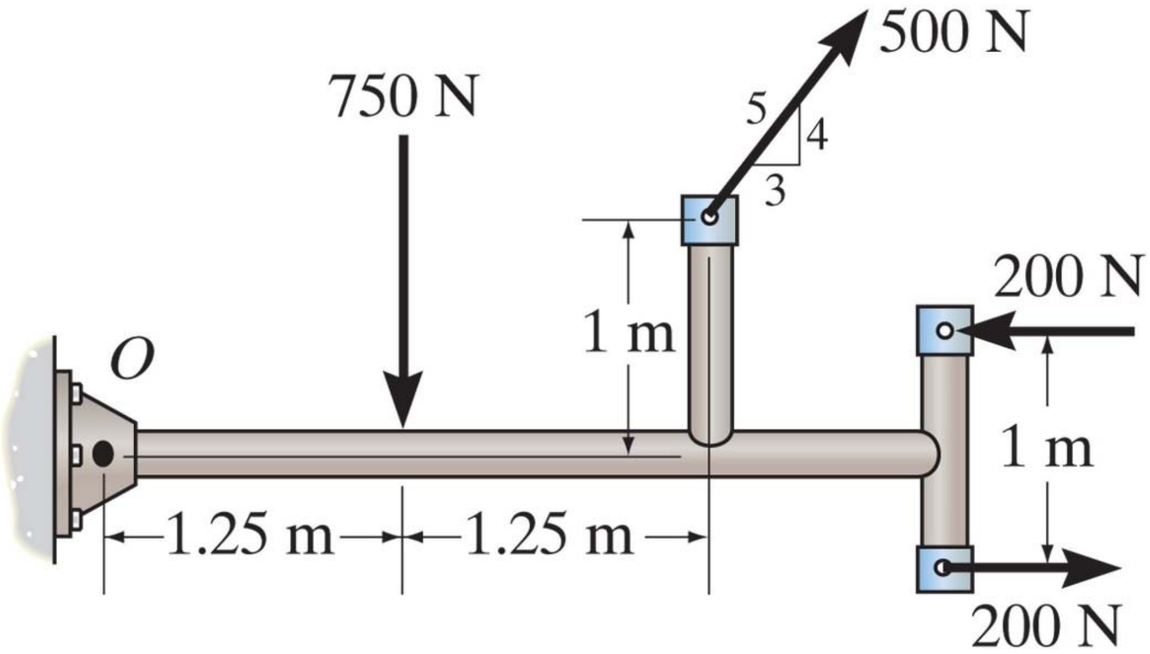
However, the **internal effect** of the force on the body does depend on where the force is applied.

Moving a force off of its line of action



The two force systems are equivalent (or equipollent) since the resultant force is the same in both systems, and the resultant moment with respect to any point P is the same in both systems.

So moving a force off its line of action means you have to "add" a new couple. Since this new couple moment is a **free vector**, it can be applied at any point on the body.



Replace the forces and couple system acting on the member by an equivalent force and couple moment acting at point O.