Statics - TAM 211

Lecture 10

October 10, 2018

Announcements

- Upcoming deadlines:
- Friday (10/12)
  - Written Assignment 3
- Tuesday (10/16)
  - Prairie Learn HW4

Reminder:

Discussion Section

Absent > 5 minutes late

Chapter 4: Force System Resultants

Goals and Objectives

- ✓ Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- ✓ How to find the moment about a specified axis
- ✓ Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading

Recap from lecture 9:

- **Moment of a force couple** ($\vec{F}$ and $-\vec{F}$)
  - $\overrightarrow{M_O} = \vec{r} \times \vec{F}$, $\left|\overrightarrow{M_O}\right| = Fd$ (where d ≈ ⊥ dist btw $\vec{F}$ and $-\vec{F}$)
  - Couple moment is a **free vector**, i.e. it is **independent** of the choice of location of O!

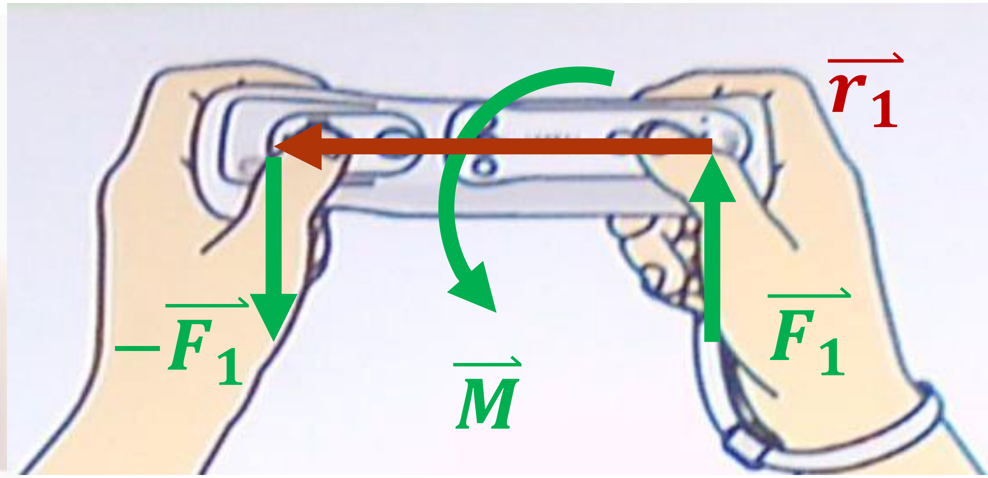
- **Equivalent couples**
  - $M_O = F_1 d_1 = F_2 d_2$
  - For example, $M_O = 10$ Nm if $F_1 = 5$ N, $d_1 = 2$ m, or $F_2 = 2.5$ N, $d_2 = 4$ m

- **Resultant couple moment**
  - $\overrightarrow{M_R} = \Sigma \overrightarrow{M_i}$

Moment of a force

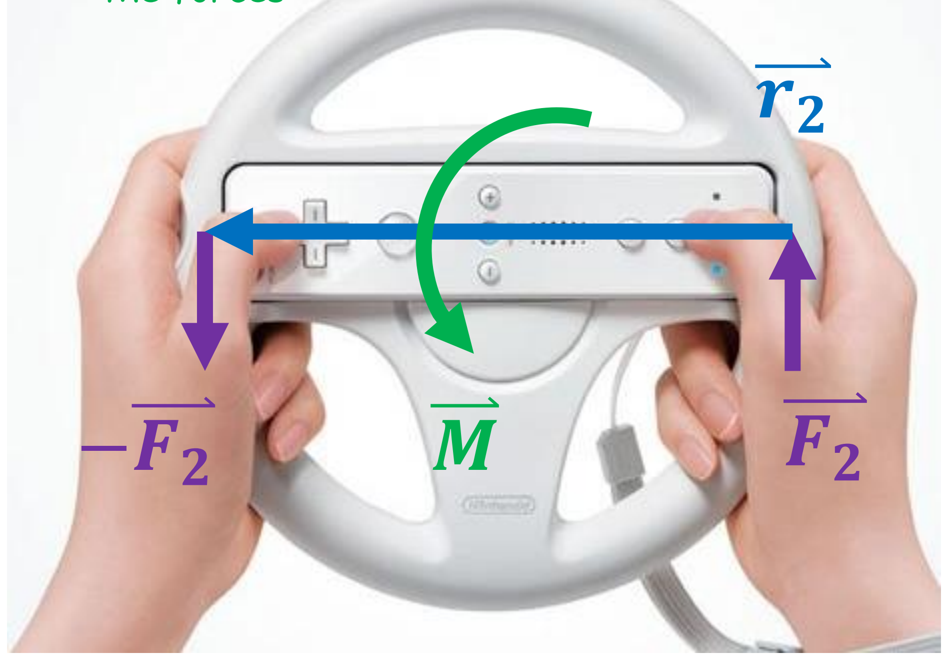
$\overrightarrow{M}$ is a free vector. It can be placed anywhere on the body, and still create a tendency for a rotation

Moment of a force couple and equivalent couples



$\vec{M}$ can be created with just one force or a force couple

The same $\vec{M}$ can be created with different pairings of force magnitude and distance between the forces

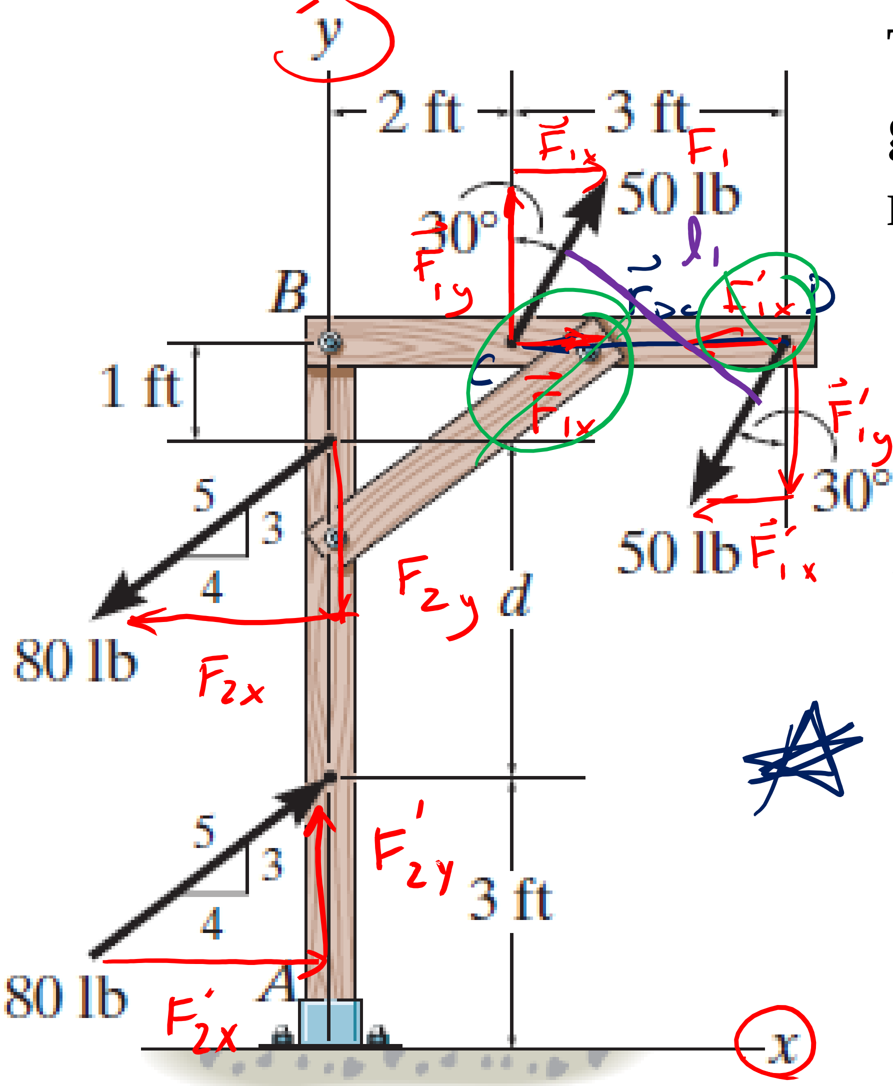


Recap from lecture 9:

- **Moment of a force couple** ($\vec{F}$ and $-\vec{F}$)
  - $\overrightarrow{M_O} = \vec{r} \times \vec{F}$, $\left|\overrightarrow{M_O}\right| = Fd$ (where d ≈ ⊥ dist btw $\vec{F}$ and $-\vec{F}$)
  - Couple moment is a **free vector**, i.e. it is **independent** of the choice of location of O!
  - Rotate your i>clicker/phone: apply equal & opposite (not co-linear) forces by each index finger, with **<u>same</u>** small force magnitude & gap between fingers, change locations along i>clicker/phone. Does it have the same rotation?

- **Equivalent couples**
  - Rotate your i>clicker/phone: increase (or decrease) force magnitude and increase (or decrease) gap to get the same rotation. $M_O = F\,d$
  - For example, $M_O = 10$ Nm if $F_1 = 5$ N, $d_1 = 2$ m, or $F_2 = 2.5$ N, $d_2 = 4$ m

- **Resultant couple moment**
  - $\overrightarrow{M_R} = \Sigma \overrightarrow{M_i}$



Two couples act on the beam with the geometry shown and d = 4 ft. Find the resultant couple

Want $\vec{M}_R = \Sigma \vec{M}_i$

Can any component of the applied forces be ignored when calculate couple moment ?

$+\circlearrowleft$ $\vec{M}_1 = \vec{r}_{DC} \times \vec{F}_1$

$\vec{r}_{DC} = 3ft\,(-\hat{i})$

$\vec{F}_1' = 50lb\,(\sin 30\,\hat{i} + \cos 30\,\hat{j})$

$\vec{M}_1 = -3\hat{i} \times (50\,(s30\,\hat{i} + c30\,\hat{j}))$

$= -130\ ft{\cdot}lb\ \hat{k}$

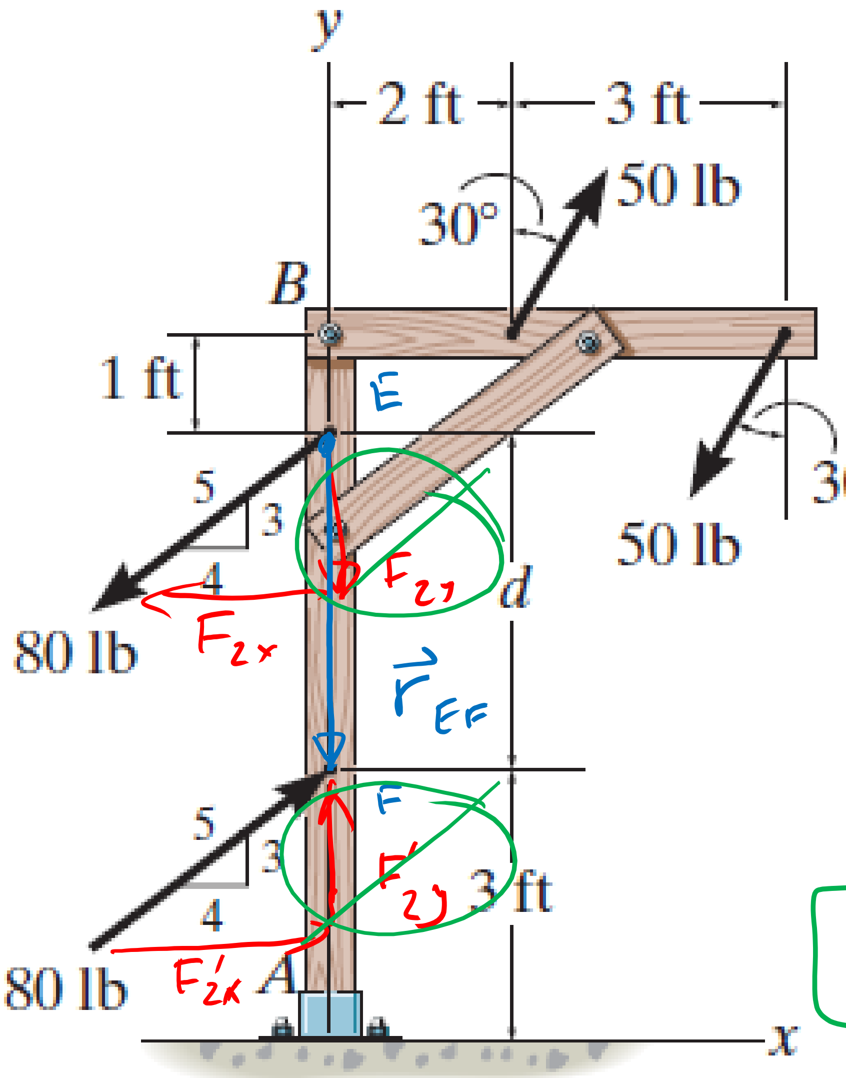
$\Rightarrow F_{1x}$ components do NOT contribute to couple moment

$\hat{i} \times \hat{i} = 0$

$\Rightarrow \vec{M}_1 = -3\hat{i} \times 50 \cdot \cos 30\,\hat{j}$

$M_1 = d_1 F_{1y}$

☆ $M_1 = l_1 F_1$ $\quad l_1 = |r_{DC}| \cos 30°$, $F_1 = 50\,lb$ $\}$ $M_1 = -130\,ft{\cdot}lb$



Lower beam: $\vec{M}_2 = ?$

$$\vec{M} = \vec{r}_{EF} \times \vec{F}_2$$

$$\vec{r}_{EF} = d(-\hat{j})$$

$$\vec{F}_2 = 80\left(\tfrac{4}{5}\right)\hat{i} + 80\left(\tfrac{3}{5}\right)\hat{j}$$

$+\circlearrowleft$ $\vec{M}_2 = 256 \text{ ft}\cdot\text{lb}\ \hat{k}$

Resultant Moment

$$\vec{M}_R = \vec{M}_1 + \vec{M}_2$$

$$= -130\hat{k} + 256\hat{k}$$

$$\boxed{\vec{M}_R = 126 \text{ ft}\cdot\text{lb}\ \hat{k}}$$

$+\circlearrowleft$ ccw