Statics - TAM 211

Lecture 11

October 12, 2018

Announcements

❑ Upcoming deadlines:

- Friday (10/12)
  - Written Assignment 3
- Tuesday (10/16)
  - Prairie Learn HW4
- Quiz 2
  - Next week, maybe

Chapter 4: Force System Resultants

Goals and Objectives

- Discuss the concept of the <u>moment of a force</u> and show how to calculate it in two and three dimensions

- How to find the <u>moment about a specified axis</u>

- Define the <u>moment of a couple</u>

- Finding <u>equivalence force and moment systems</u>

- Reduction of <u>distributed loading</u>

Equivalent (or equipollent) force systems

A force **system** is a collection of **forces** and **couples** applied to a body.

Two force systems are said to be **equipollent** (or equivalent) if they have the **same resultant force** AND the **same resultant moment** with respect to any point O.

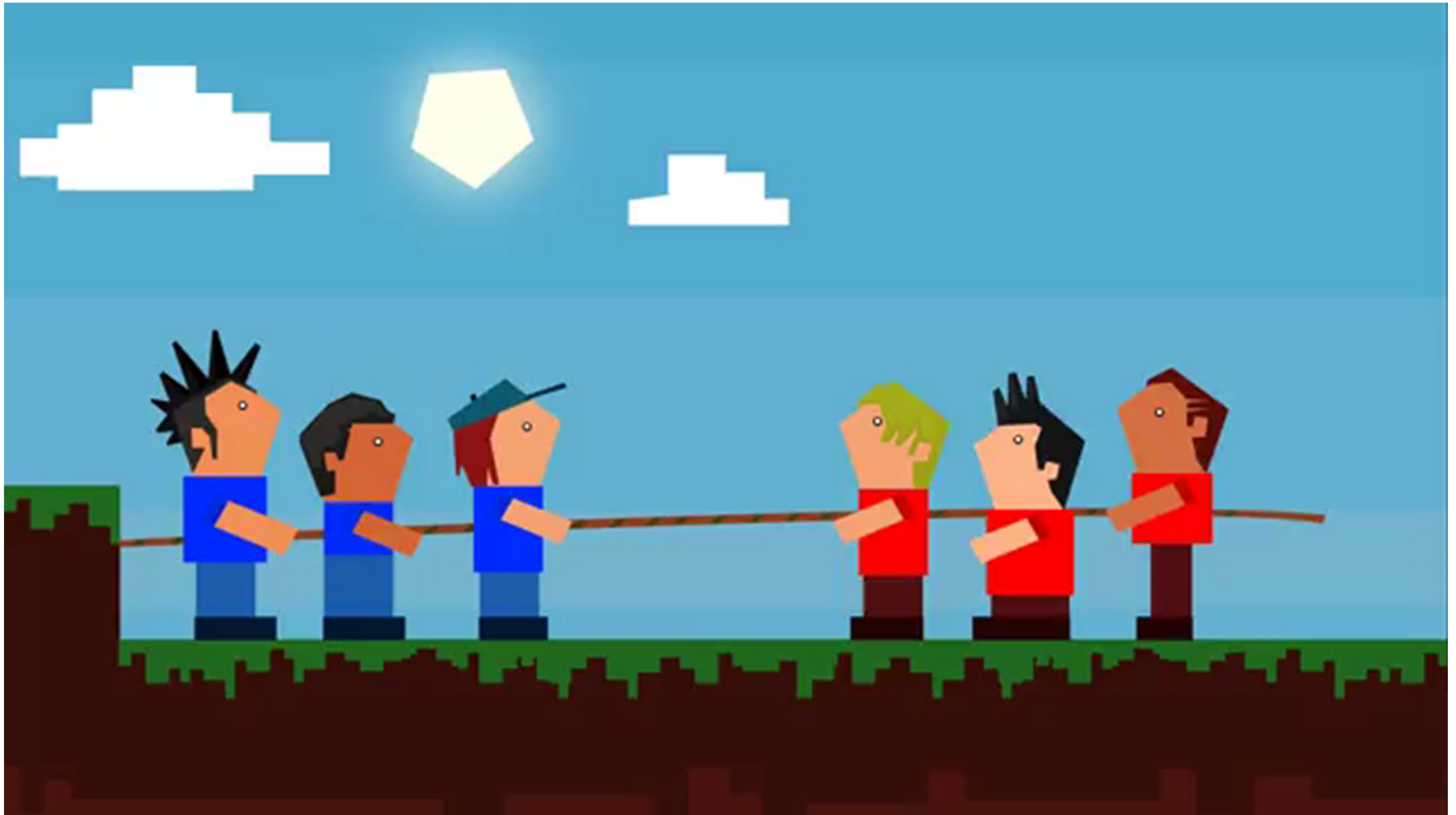Reducing a force system to a single resultant force $\boldsymbol{F}_R$ and a single resultant couple moment $(\boldsymbol{M}_R)_o$:

$$\overrightarrow{F_R} = \Sigma F_x \hat{\boldsymbol{i}} + \Sigma F_y \hat{\boldsymbol{j}} + \Sigma F_z \widehat{\boldsymbol{k}}$$

Magnitude: $|\overrightarrow{F_R}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$ Orientation: $\theta = \tan^{-1} \dfrac{F_{opp}}{F_{adj}}$

$$(M_R)_o = \Sigma M_o + \Sigma M$$

(a)

(b)

$(M_O)_2 = r_2 \times F_2$

$(M_O)_1 = r_1 \times F_1$

(c)
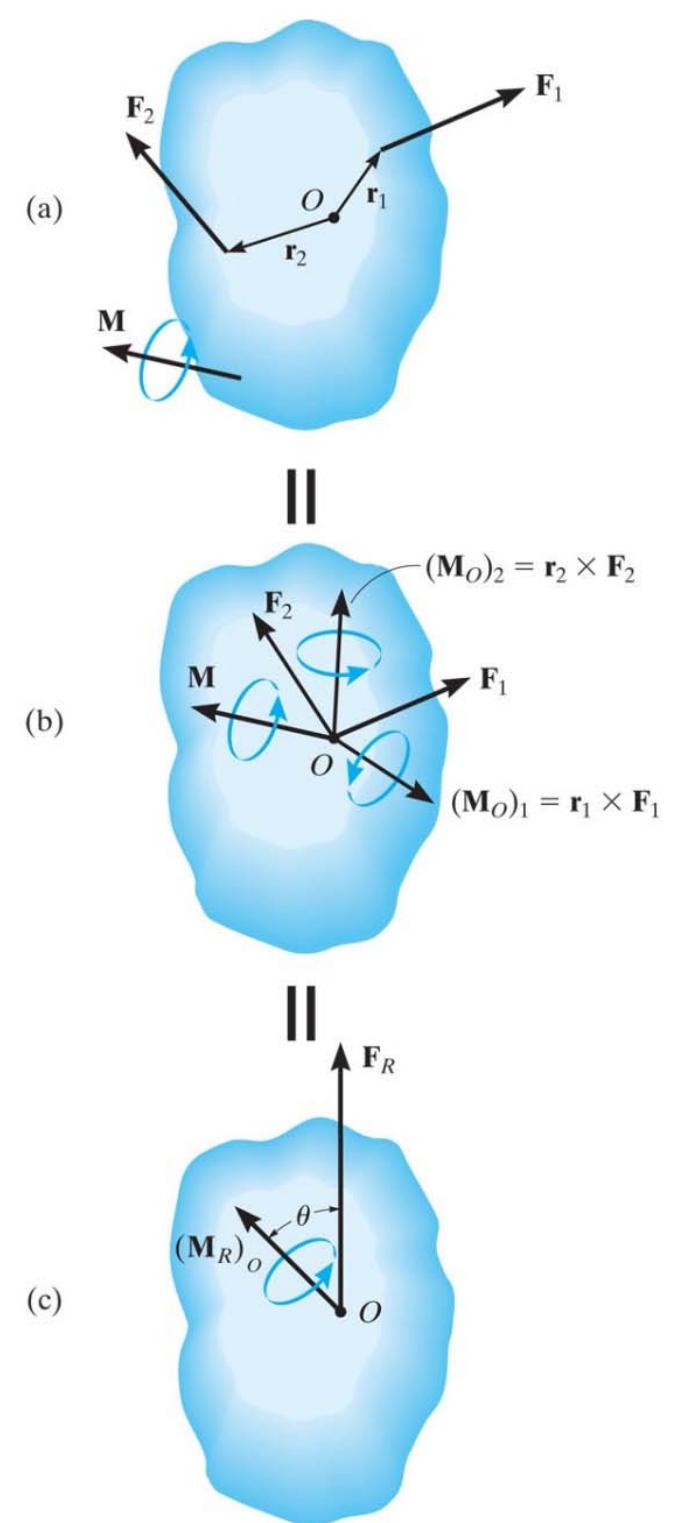
Moving a force on its line of action

Moving a force <u>on</u> its line of action (Equivalent forces)

Moving a force from A to B, when both points are on the vector's line of action, does not change the **external effect**.

Hence, a force vector is called a **sliding vector**.

However, the **internal effect** of the force on the body does depend on where the force is applied.

Moving a force **off** of its line of action

The two force systems are equivalent (or equipollent) since the resultant force is the same in both systems, and the resultant moment with respect to any point P is the same in both systems.
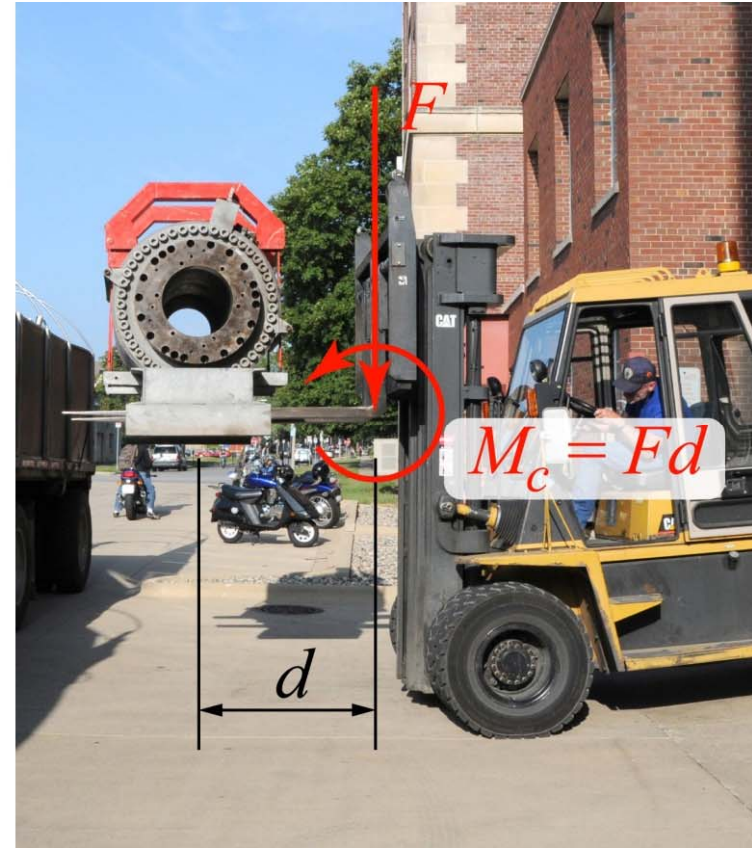
So moving a force off its line of action means you have to "add" a new couple. Since this new couple moment is a **free vector**, it can be applied at any point on the body.
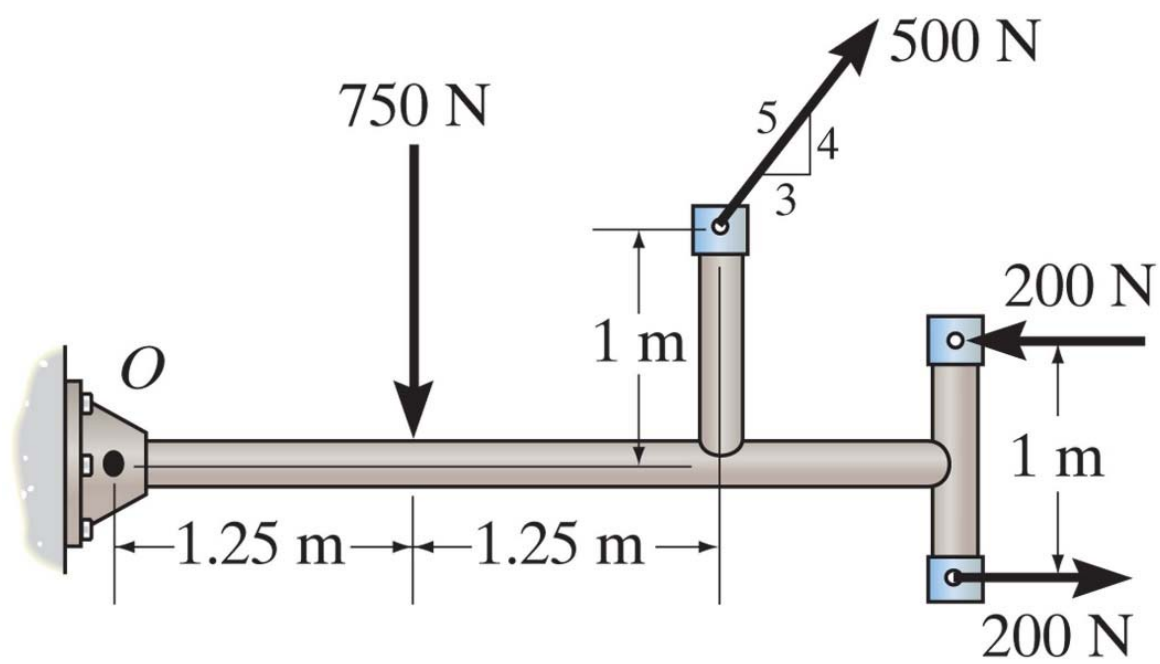
Are these systems the same?
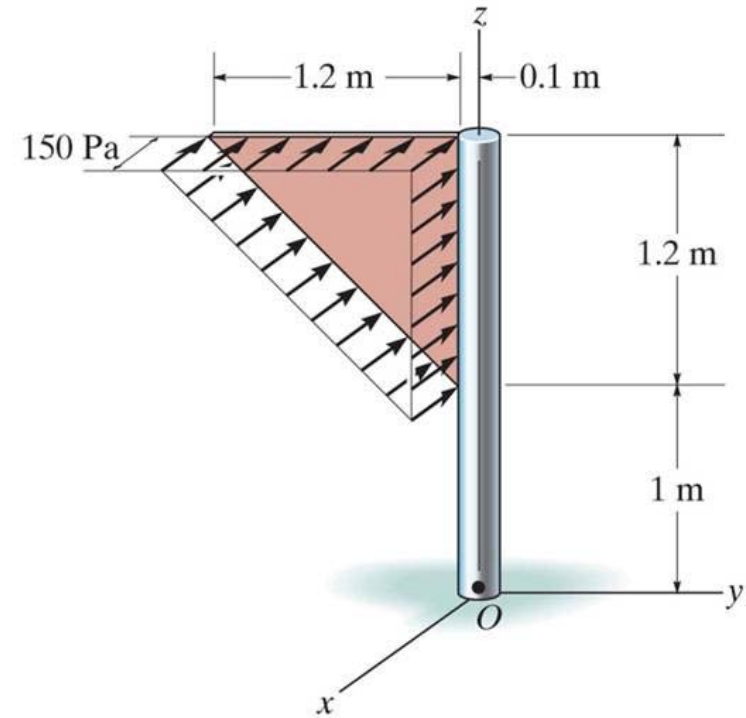

Force system I

Force system II

A – YES

B – NO

Replace the forces and couple system acting on the member by an equivalent force and couple moment acting at point O.

500 N

750 N

5
4
3

200 N

1 m

O

1 m

−1.25 m−−1.25 m−

200 N

Reduction to a simple distributed load

The lumber places a distributed load (due to the weight of the wood) on the beams. To analyze the load's effect on the steel beams, it is often helpful to reduce this distributed load to a single force. How would you do this?

To be able to design the joint between the sign and the sign post, we need to determine a single equivalent resultant force and its location.

Reduction to a simple distributed load
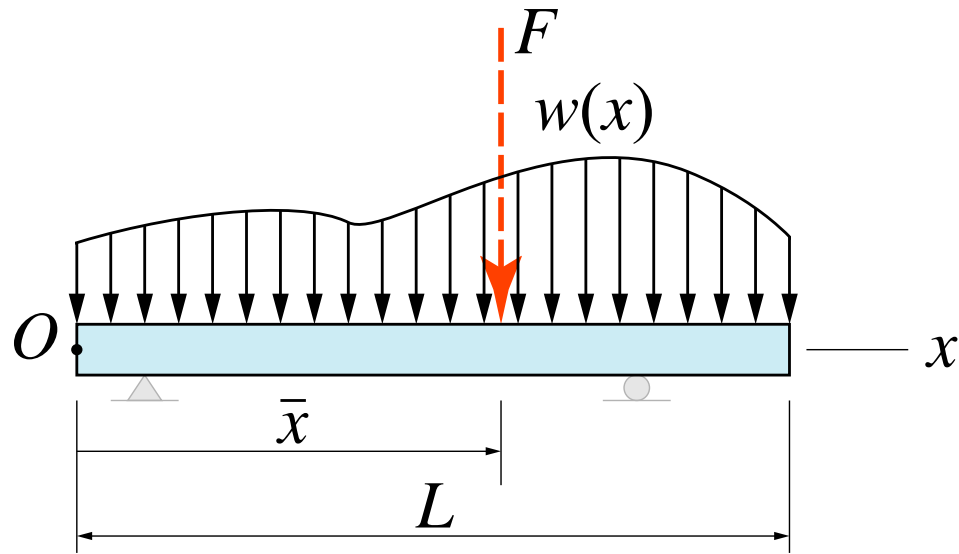
In structural analysis, often presented with **distributed load** $w(x)$ (force/unit length) and need to find equivalent loading $\boldsymbol{F}$.

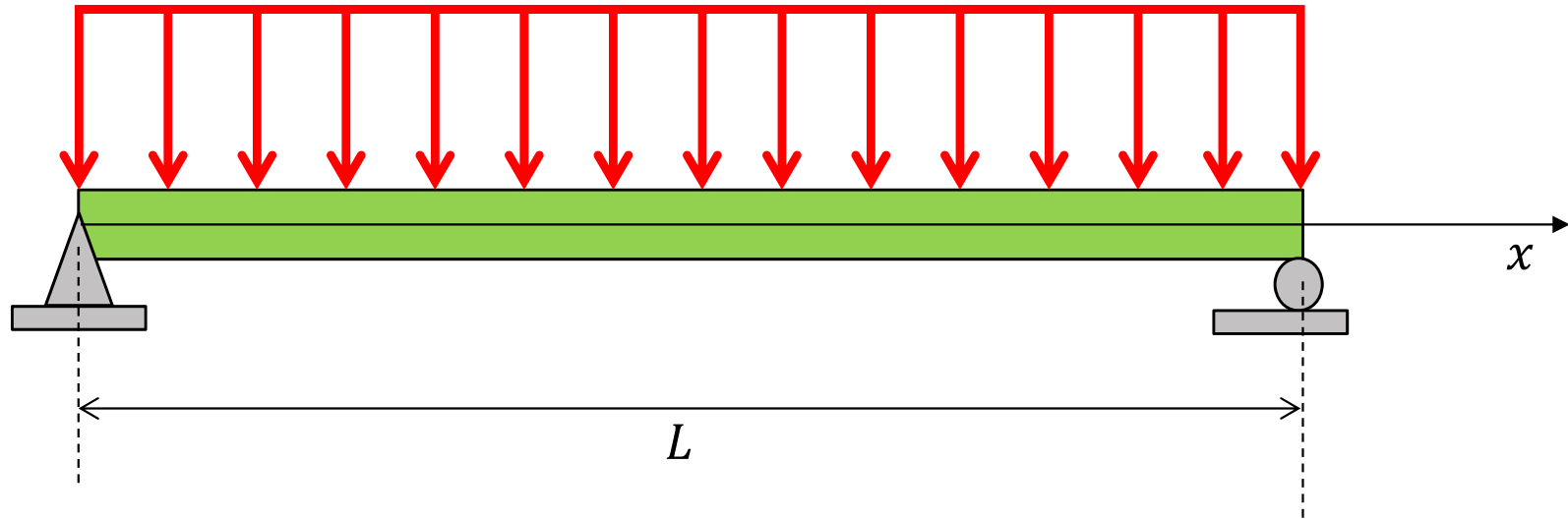Ex: winds, fluids, weight on body's surface.

By equipollence, we require that $\sum \boldsymbol{F}$ be the same in both systems, i.e.,

and $\sum \boldsymbol{M}_P$ with respect to any point P be the same in both systems, i.e.,

Combining both equations gives:

Rectangular loading

$$w(x) = w_o$$

Triangular loading

$w(0) = w_o$

$w(x)$

x

L