Statics - TAM 211

Lecture 11

October 12, 2018

Announcements

❑ Upcoming deadlines:

- Friday (10/12)
  - Written Assignment 3
- Tuesday (10/16)
  - Prairie Learn HW4
- Quiz 2
  - Next week, maybe
    following

15 Oct
22 Oct

Chapter 4: Force System Resultants

Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- How to find the moment about a specified axis
- Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading

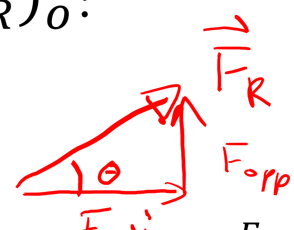
Equivalent (or equipollent) force systems

A force **system** is a collection of **forces** and **couples** applied to a body.

Two force systems are said to be **equipollent** (or equivalent) if they have the **same resultant force** AND the **same resultant moment** with respect to any point O.

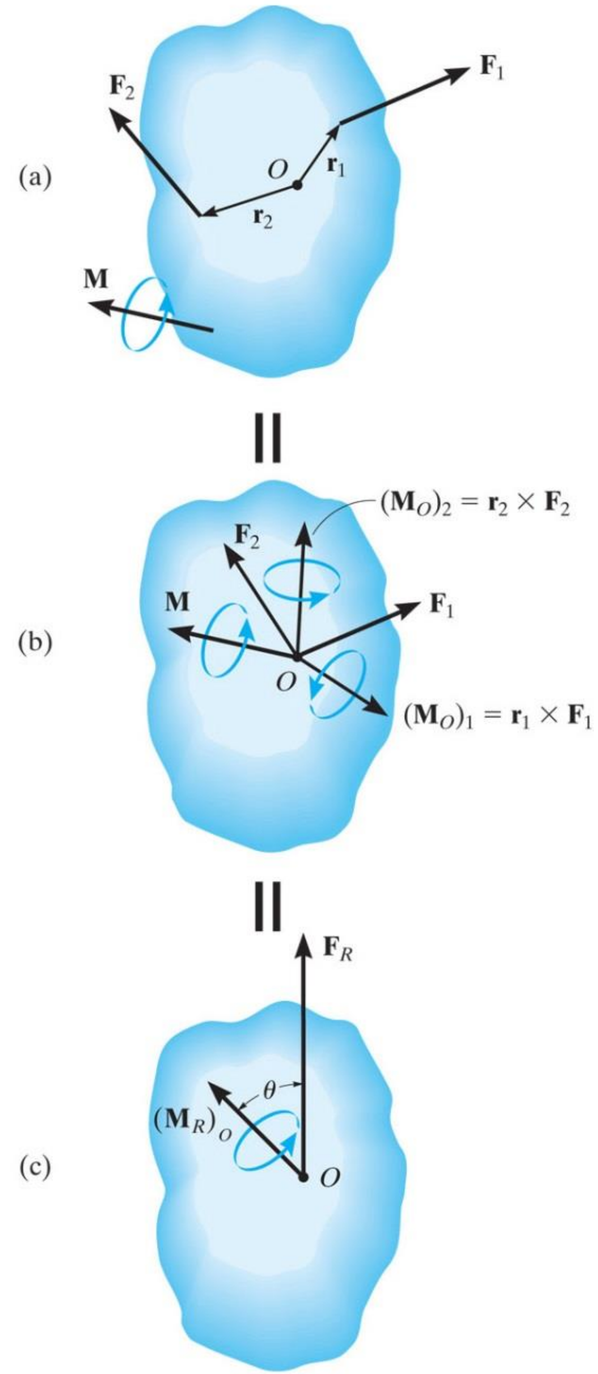
Reducing a force system to a single resultant force $\boldsymbol{F}_R$ and a single resultant couple moment $(\boldsymbol{M}_R)_o$:

$$\overrightarrow{\boldsymbol{F}_R} = \Sigma F_x \hat{\boldsymbol{\imath}} + \Sigma F_y \hat{\boldsymbol{\jmath}} + \Sigma F_z \widehat{\boldsymbol{k}}$$

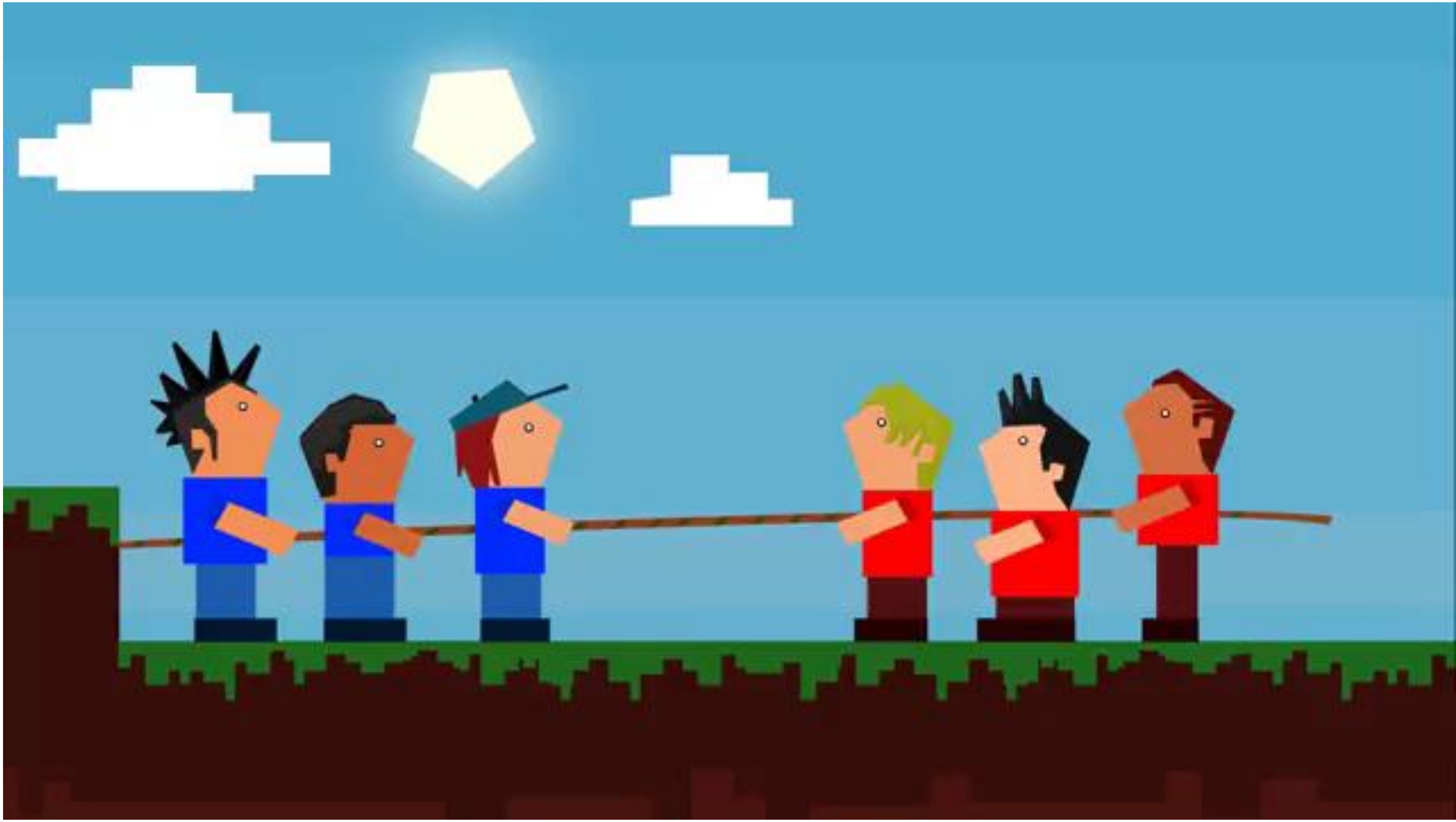


Magnitude: $|\overrightarrow{\boldsymbol{F}_R}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$ Orientation: $\theta = \tan^{-1}\frac{F_{opp}}{F_{adj}}$

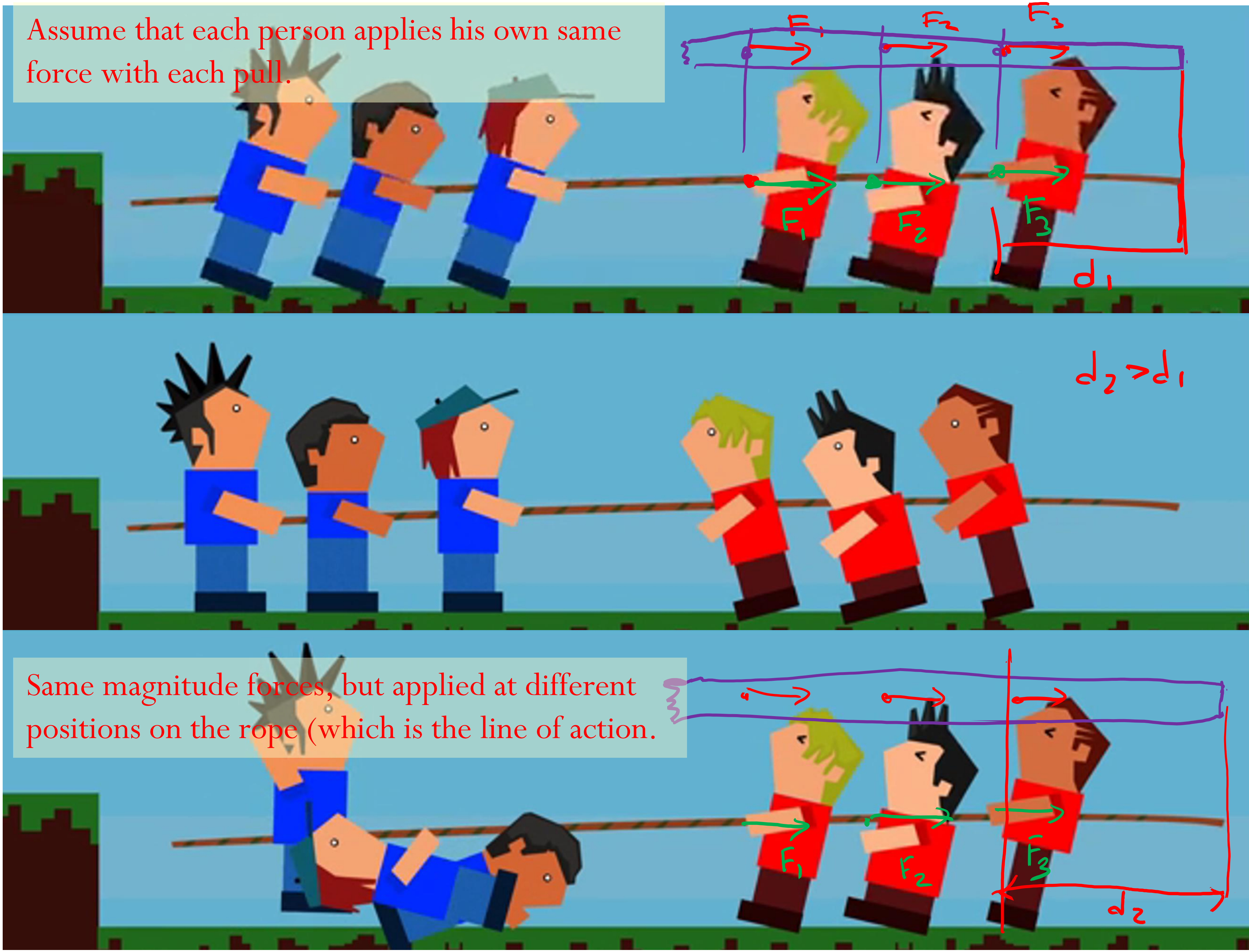
$$(\boldsymbol{M}_R)_o = \sum \boldsymbol{M}_o + \sum \boldsymbol{M}$$



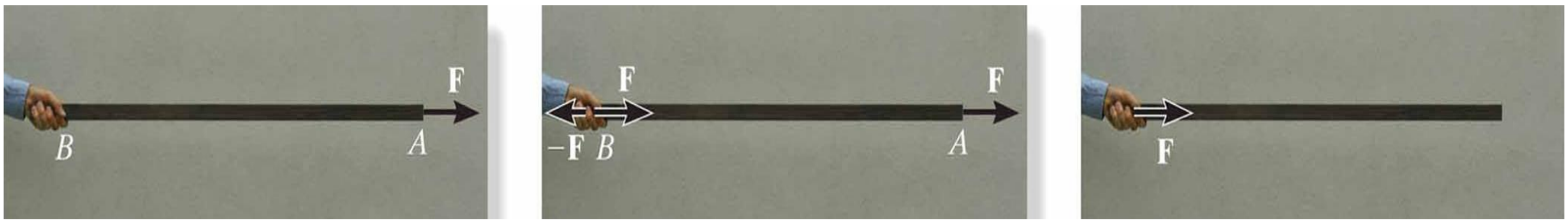
Moving a force on its line of action



https://www.wikihow.com/Win-at-Tug-of-War


Assume that each person applies his own same force with each pull.
F1
F2
F3
d1
d2>d1
Same magnitude forces, but applied at different positions on the rope (which is the line of action.
F1
F2
F3
d2

Moving a force <u>on</u> its line of action (Equivalent forces)



Moving a force from A to B, when both points are on the vector's line of action, does not change the **external effect**.

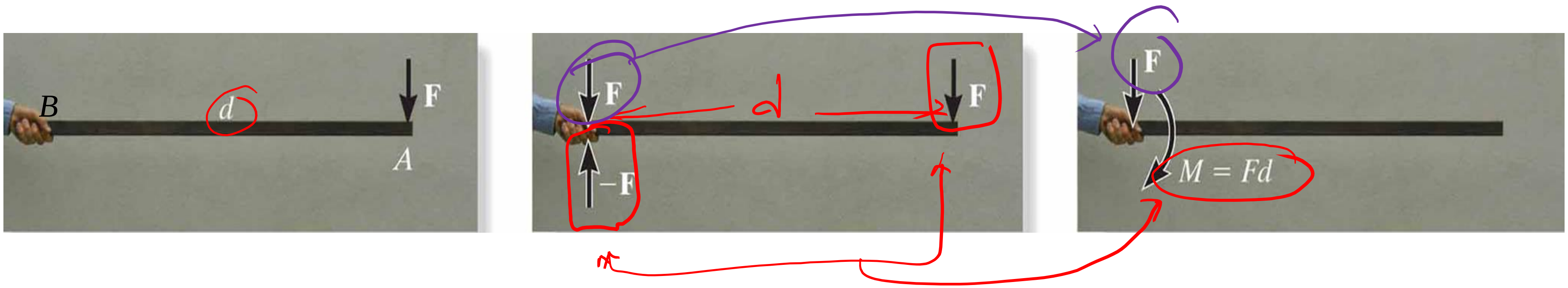
Hence, a force vector is called a **sliding vector**.

However, the **internal effect** of the force on the body does depend on where the force is applied.

Moving a force **off** of its line of action

Old: $\vec{F}$

New: $\vec{F}$, but also need $\vec{M} = Fd = \vec{d} \times \vec{F}$ due to moving $\vec{F}$ off of its line of action

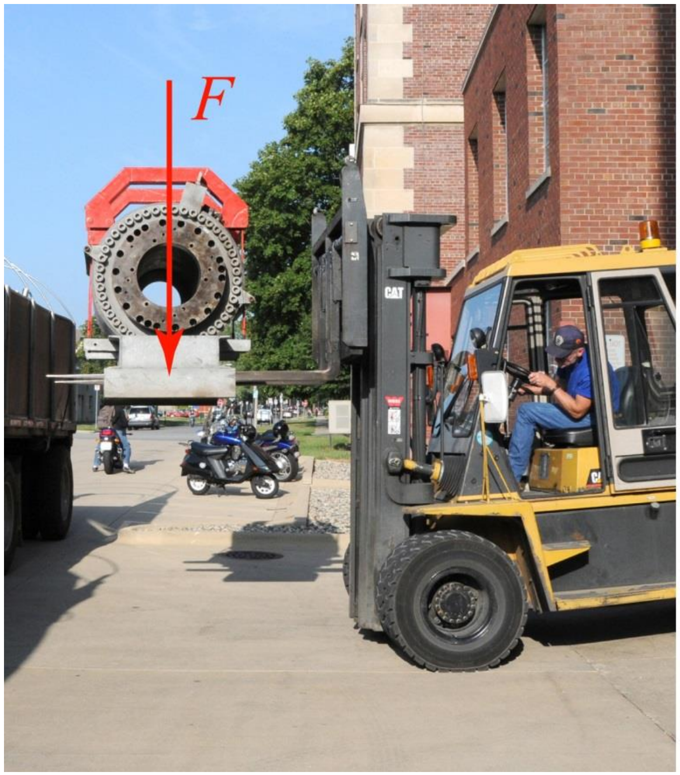


The two force systems are equivalent (or equipollent) since the resultant force is the same in both systems, and the resultant moment with respect to any point P is the same in both systems.

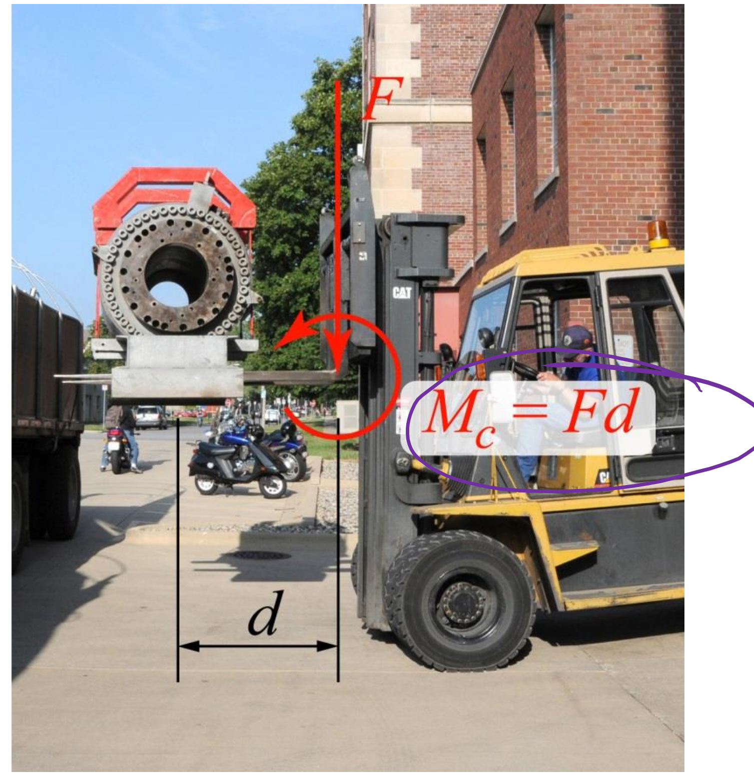
$\Sigma F_{old} = F(-\hat{j})$ $\quad \Sigma F_{new} = 2F(-\hat{j}) + F(\hat{j}) = F(-\hat{j})$ ✓same

So moving a force off its line of action means you have to "add" a new couple. Since this new couple moment is a **free vector**, it can be applied at any point on the body.

Are these systems the same?



Force system I

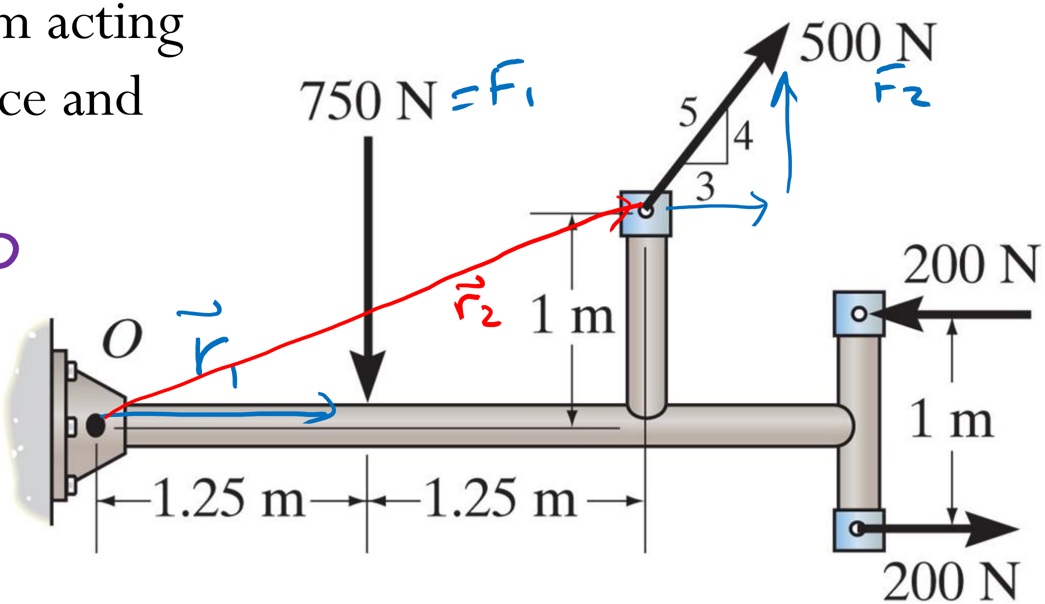


Force system II

A – YES

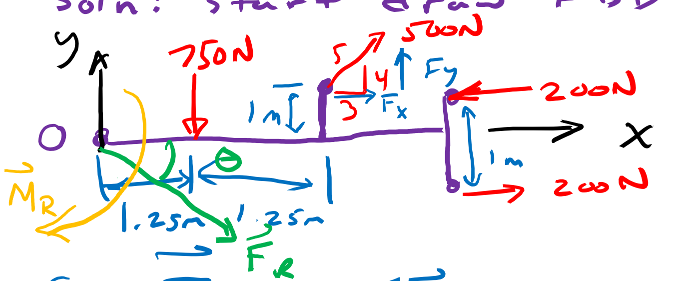
B – NO

Replace the forces and couple system acting on the member by an equivalent force and couple moment acting at point O.



Find: $\vec{F}_R$, $\vec{M}_R$ at point O

Soln: ① start draw FBD



② $\vec{F}_R = \sum \vec{F}$

$F_x: +500N\left(\frac{3}{5}\right) - 200N + 200N = 300N(+\hat{i}) = F_{Rx}$

$F_y: +500N\left(\frac{4}{5}\right) - 750N = -350N(\hat{j}) = F_{Ry}$

$|\vec{F}_R| = \sqrt{F_x^2 + F_y^2} = 461\text{ N}$, $\theta = \tan^{-1}\left(\frac{F_{opp}}{F_{adj}}\right) = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = 49.4^\circ$

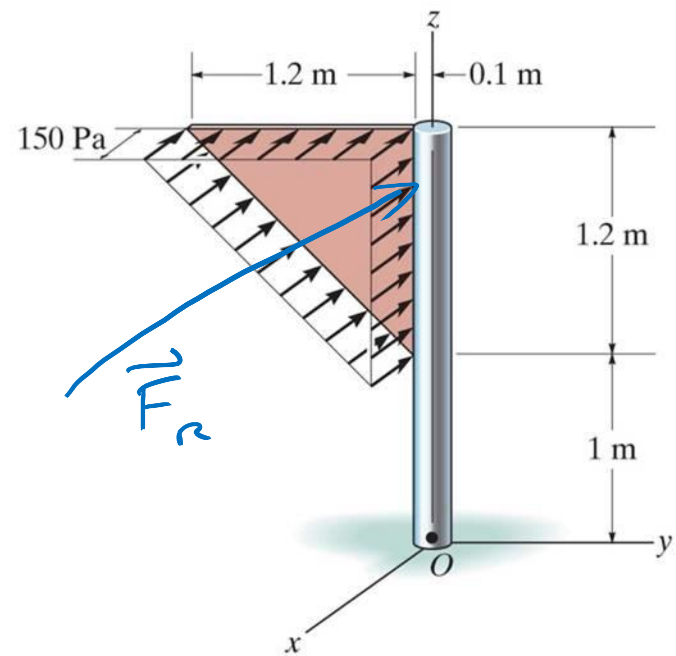
$\vec{M}_R = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{M}_{couple,\ 200N}$

$200N(1m)$ ccw ↺ $+\hat{k}$

$\vec{M}_R = -(1.25m)(750N)\hat{k} + (1m\,\hat{j} + 2.5m\,\hat{i}) \times \left(500\left(\tfrac{3}{5}\right)N\,\hat{i} + 500\left(\tfrac{4}{5}\right)N\,\hat{j}\right) + 200N{\cdot}m\,\hat{k}$

$\vec{M}_R = -37.5\,N{\cdot}m\,\hat{k}$

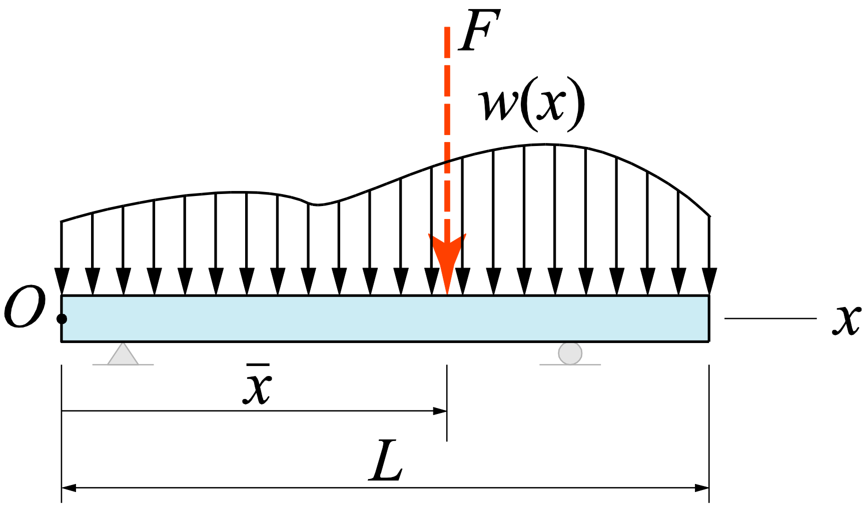
Reduction to a simple distributed load



The lumber places a distributed load (due to the weight of the wood) on the beams. To analyze the load's effect on the steel beams, it is often helpful to reduce this distributed load to a single force. How would you do this?

To be able to design the joint between the sign and the sign post, we need to determine a single equivalent resultant force and its location.

Reduction to a simple distributed load



In structural analysis, often presented with **distributed load** $w(x)$ (force/unit length) and need to find equivalent loading F.

Ex: winds, fluids, weight on body's surface.

By equipollence, we require that $\sum F$ be the same in both systems, i.e.,

$$\left|\vec{\boldsymbol{F}}\right| = \int_0^L w(x)\,dx = A$$

Assume uniform beam of constant width b

and $\sum M_P$ with respect to any point P be the same in both systems, i.e.,

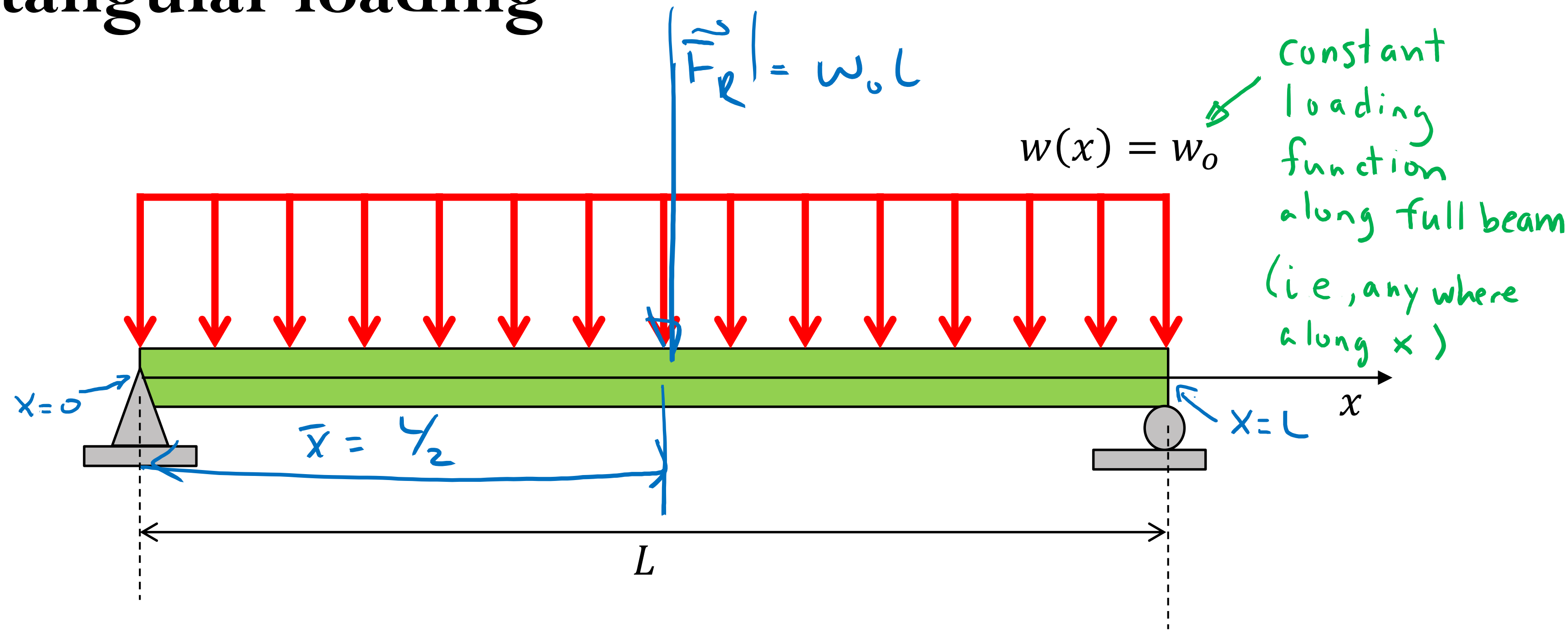
$$\int_0^L w(x)\,x\,dx = \bar{x}\,F$$

Combining both equations gives:

$$\bar{x} = \frac{\int_0^L w(x)x\,dx}{\int_0^L w(x)dx}$$

$\bar{x}$ = **geometric center or centroid** of area A under loading curve $w(x)$.

Rectangular loading

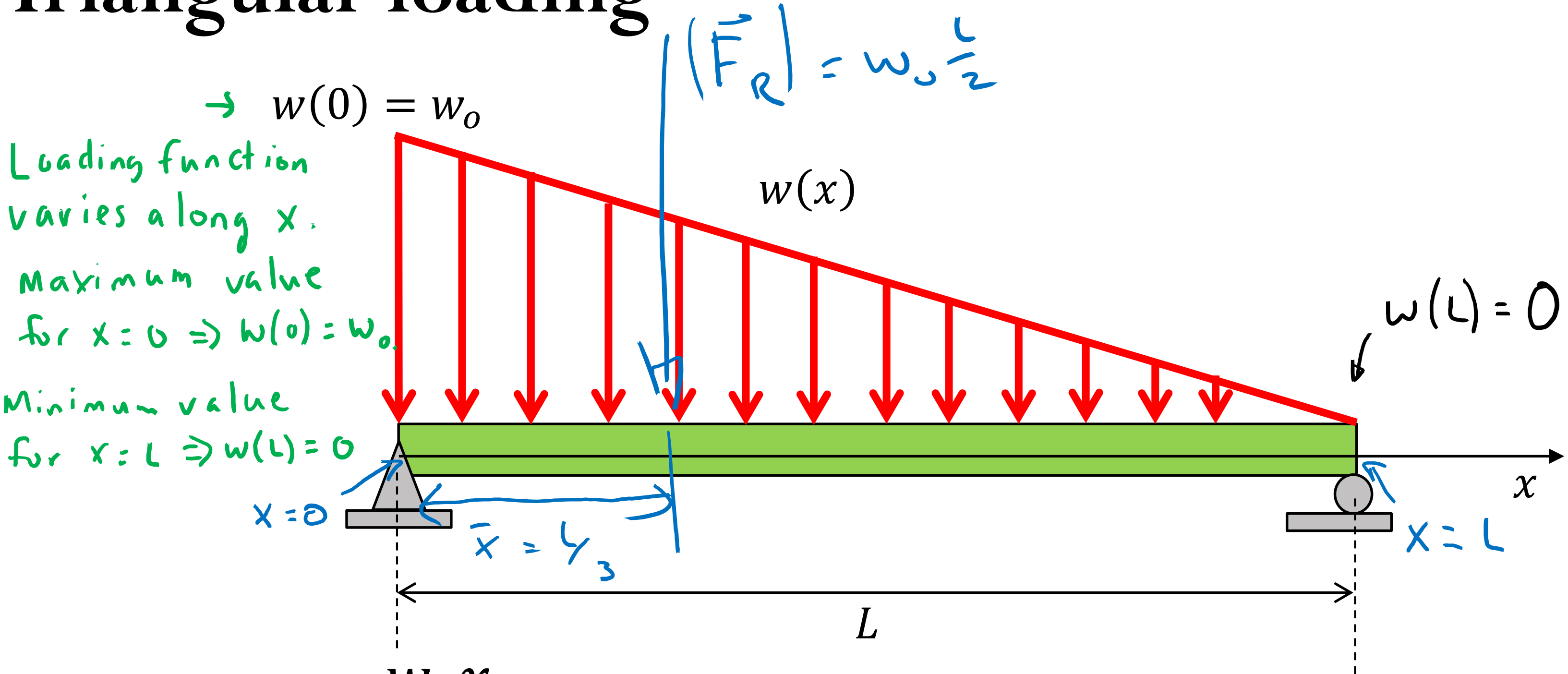


$$w(x) = w_o \qquad \boldsymbol{F} = \int_0^L w(x)\,dx \qquad \bar{x} = \frac{\int_0^L w(x)\,x\,dx}{\int_0^L w(x)\,dx}$$

$$|\vec{\boldsymbol{F}}_R| = \int_0^L w_o\,dx = w_o L$$

$$\bar{x} = \frac{\int_0^L w_o x\,dx}{\int_0^L w_o\,dx} = \frac{w_o \frac{L^2}{2}}{w_o L} = \frac{L}{2}$$

Triangular loading



$$w(x) = w_o - \frac{w_o x}{L}$$

$$\left|\vec{F}_R\right| = \int_0^L \left(w_o - \frac{w_o x}{L}\right) dx = w_o L - w_o \frac{L}{2} = w_o \frac{L}{2}$$

$$\bar{x} = \frac{\int_0^L \left(w_o - \frac{w_o x}{L}\right) x \, dx}{\int_0^L \left(w_o - \frac{w_o x}{L}\right) dx} = \frac{w_o \frac{L^2}{2} - \frac{w_o}{L} \frac{L^3}{3}}{w_o \frac{L}{2}} = \frac{\frac{w_o L^2}{6}}{\frac{w_o L}{2}} = \frac{L}{3}$$