Statics - TAM 211

Lecture 12

October 15, 2018

Announcements

- As announced during discussion section, you are encouraged and allowed to use your Casio calculator during PrairieLearn HWs and Quizzes.
  - You should learn to solve a system of equations by hand using a calculator
- PrairieLearn incorrect software issues:
  - Negative sign symbol (- vs. –)
  - Space between negative sign (-12 vs. - 12)
  - Solutions:
    - Always type in the negative sign symbol (-) into your PL answers for HW or Quiz.
    - Do not add space between negative symbol and number
  - All students with these errors were provided updated grades on Quiz 1. No credit for Quiz 2 and beyond.
- Upcoming deadlines:
- Tuesday (10/16)
  - Prairie Learn HW4
- Friday (10/19)
  - Written Assignment 4
- Quiz 2

Recap: General procedure for analysis

1. Read the problem carefully; write it down carefully.
2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.
3. Apply principles needed.
4. Solve problem symbolically. Make sure equations are dimensionally homogeneous
5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).
6. See if answer is reasonable.

Most effective way to learn engineering mechanics is to ***solve problems!***

Chapter 4: Force System Resultants

Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- How to find the moment about a specified axis
- Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading

Recap: Resultant or Equivalent Force and Moment Systems

Reducing a force system to a single resultant force $\vec{\boldsymbol{F}}_R$ and a single resultant couple moment about point O $\left(\vec{\boldsymbol{M}}_R\right)_o$:

$$\overrightarrow{\boldsymbol{F}_R} = \Sigma F_x \hat{\boldsymbol{\imath}} + \Sigma F_y \hat{\boldsymbol{\jmath}} + \Sigma F_z \hat{\boldsymbol{k}}$$

Magnitude: $|\overrightarrow{\boldsymbol{F}_R}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$

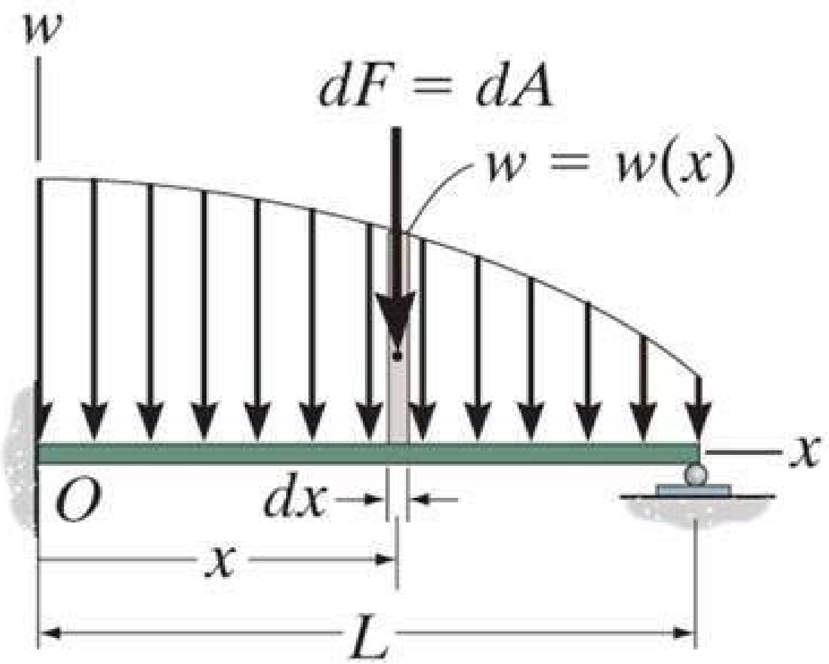
Orientation in Cartesian coordinate system: x-direction (F_x), y-direction (F_y), z-direction (F_z),

Orientation in Cylindrical coordinate system: $\theta = \tan^{-1}\frac{F_{opp}}{F_{adj}}$

$$\left(\vec{\boldsymbol{M}}_R\right)_o = \sum \boldsymbol{M}_o + \sum \boldsymbol{M}$$

Recap: Distributed loads

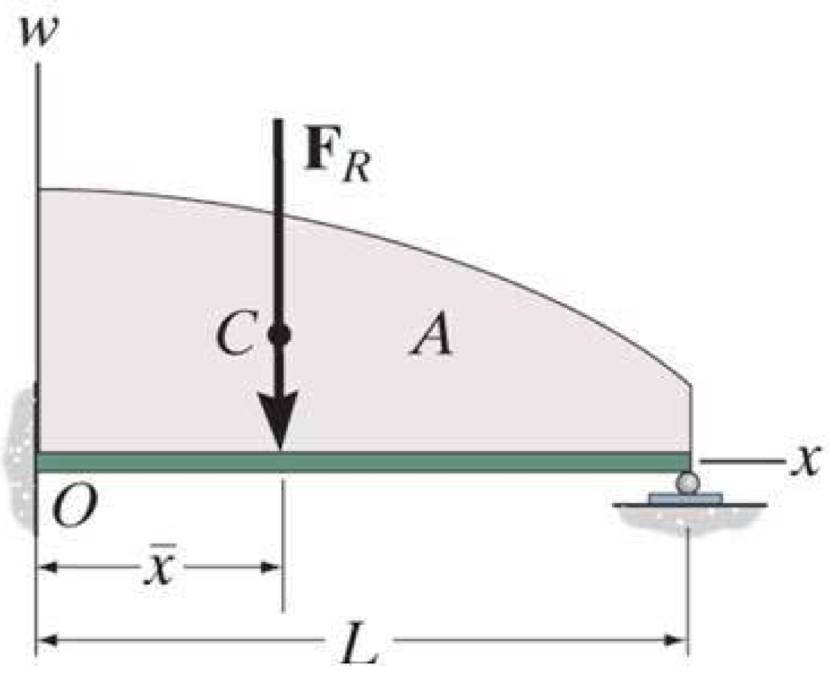
- Equivalent force system for distributed loading function $w(x)$ with units of $\frac{\text{force}}{\text{length}}$.
- Find magnitude F_R and location $\bar{x}$ of the equivalent resultant force for $\overrightarrow{\boldsymbol{F_R}}$



$$\left|\overrightarrow{\boldsymbol{F_R}}\right| = F_R = \int_0^L dF = \int_0^L w(x)\,dx = A$$

$$M_o = \int_0^L x\ w(x)\,dx = \bar{x}\,F_R$$

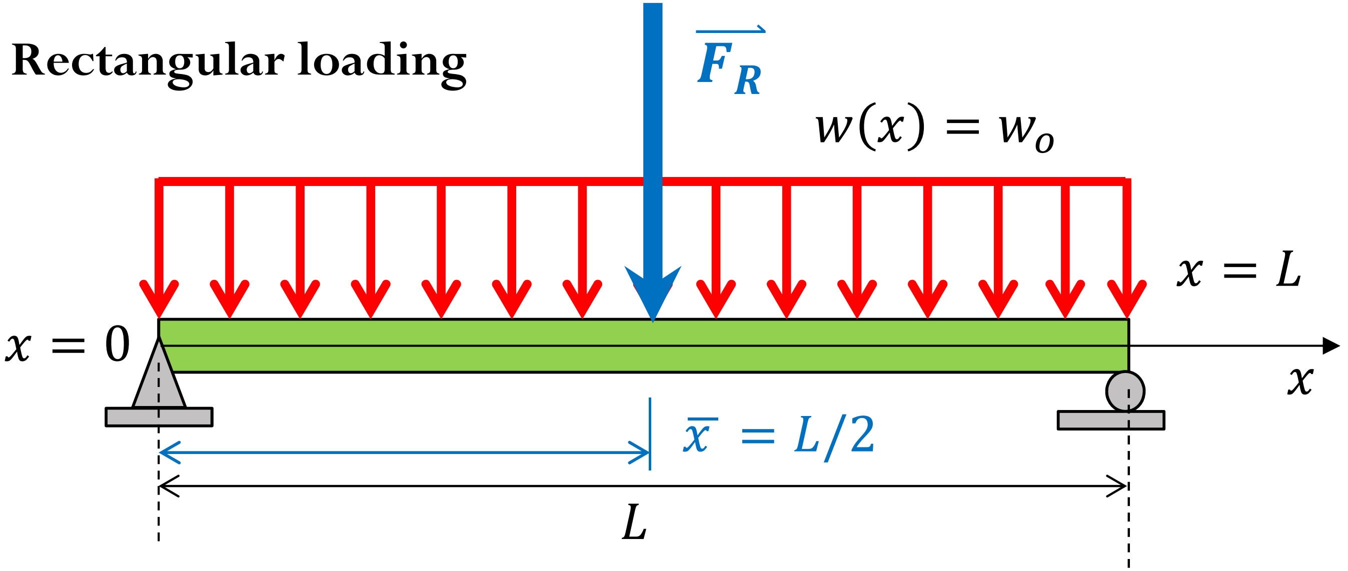
$$\bar{x} = \frac{M_o}{F_R} = \frac{\int_0^L x\ w(x)\,dx}{\int_0^L w(x)\,dx}$$



$\bar{x}$ = **geometric center or centroid** of area A under loading curve $w(x)$.

Recap: Simple Shape Distributed loads

Rectangular loading

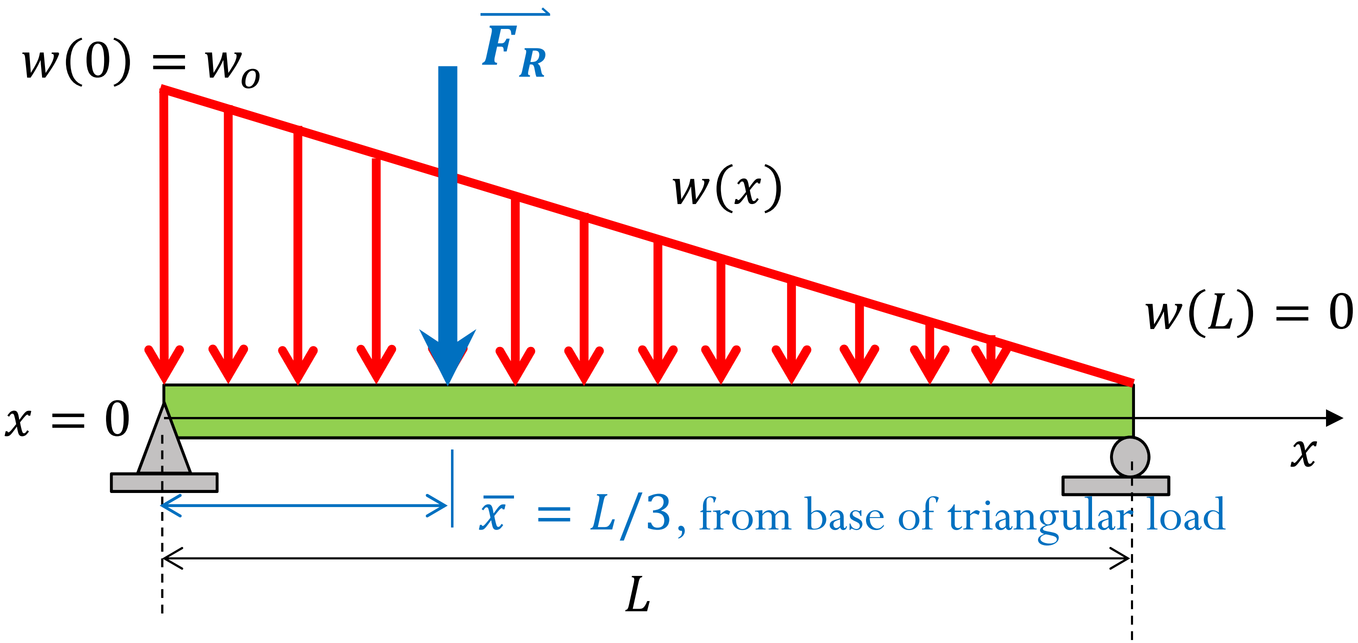


$$w(x) = w_o$$

$$\left|\overrightarrow{F_R}\right| = F_R = w_o L$$

$$\bar{x} = \frac{L}{2}$$

Triangular loading

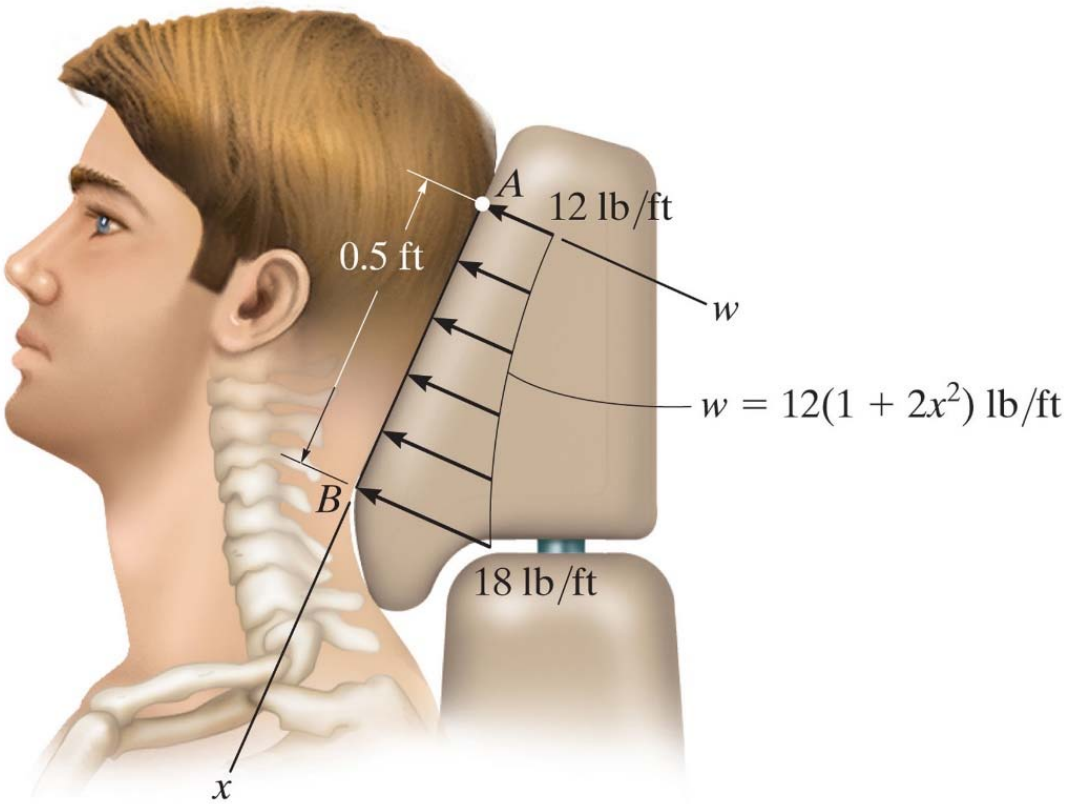


$$w(x) = w_o - \frac{w_o x}{L}$$

$$F_R = w_o \frac{L}{2}$$

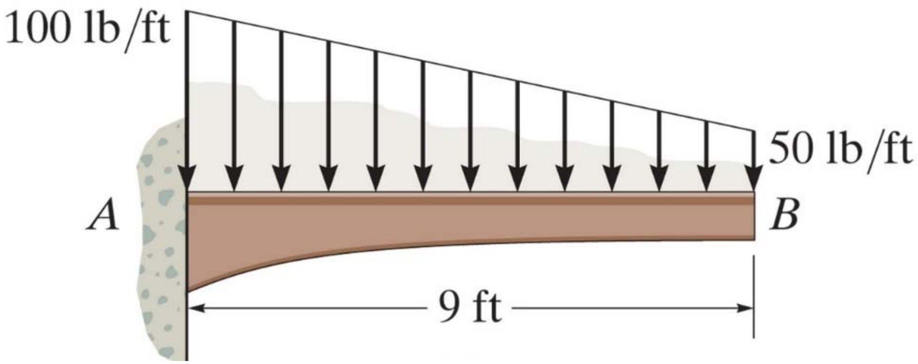
$$\bar{x} = \frac{L}{3}$$

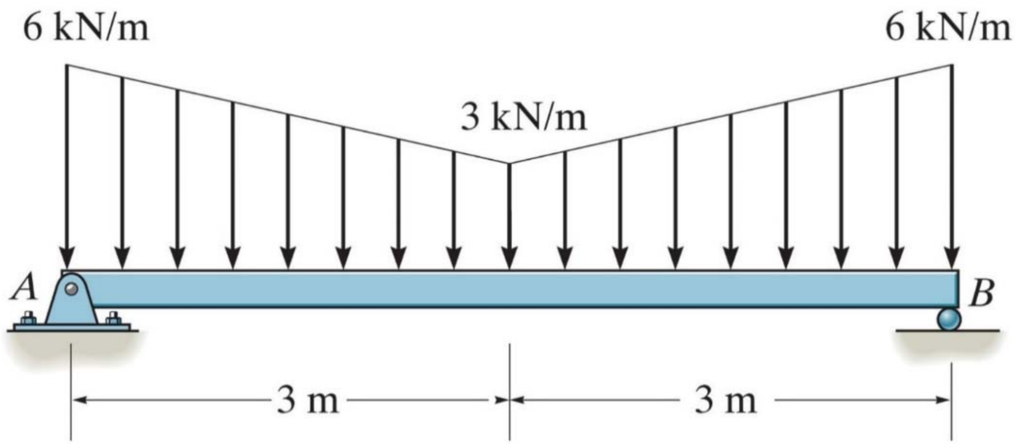
Find equivalent force and its location from point A for loading on headrest.

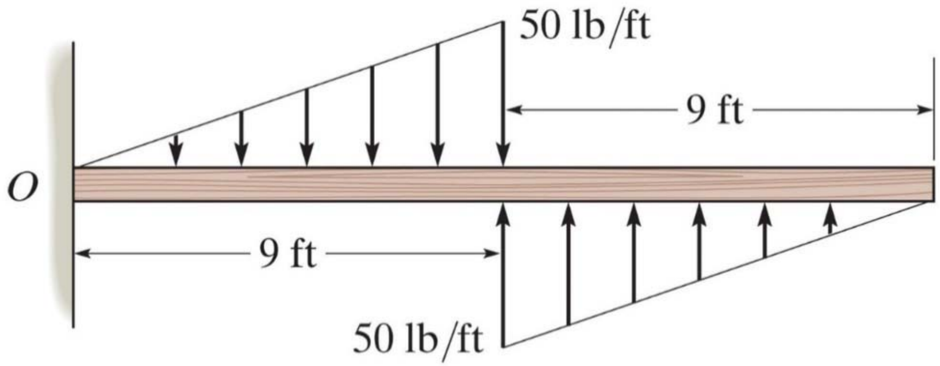


Superposition of simple shapes

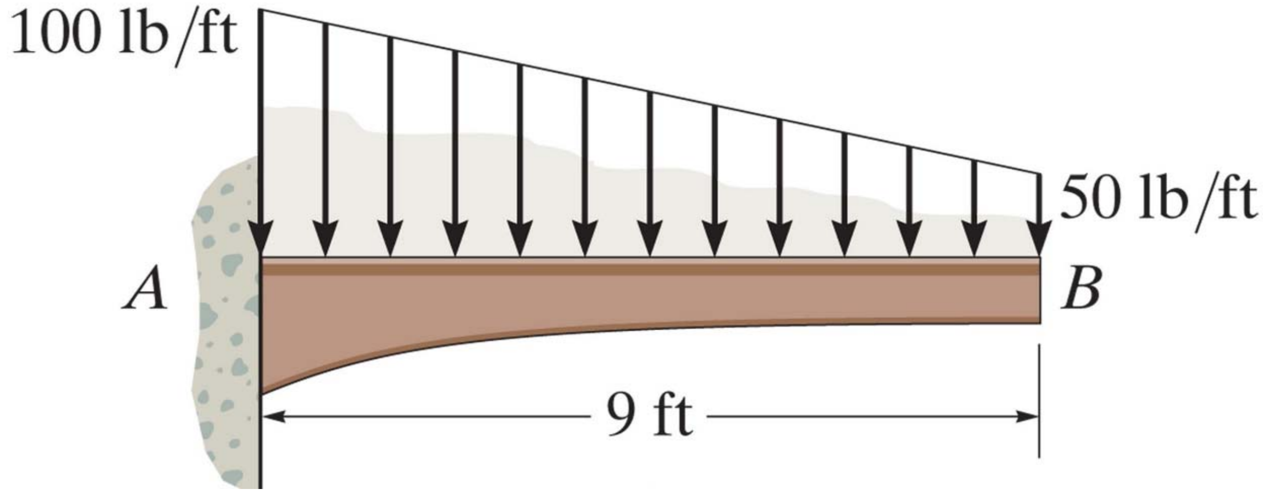
- Divide complex distributed loads into multiple simple shapes of rectangles and/or triangles.
- Superimpose the resultant forces for each simple shape to determine the final composite resultant force.





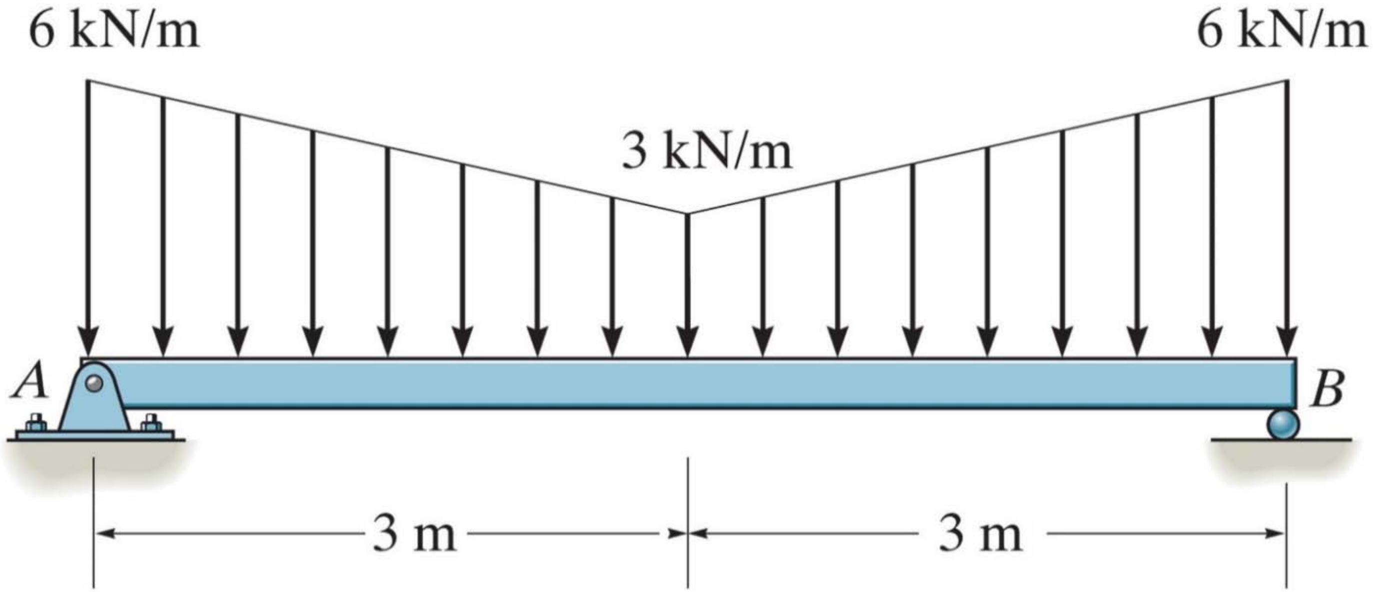


Determine the magnitude and location of the equivalent resultant of this load.

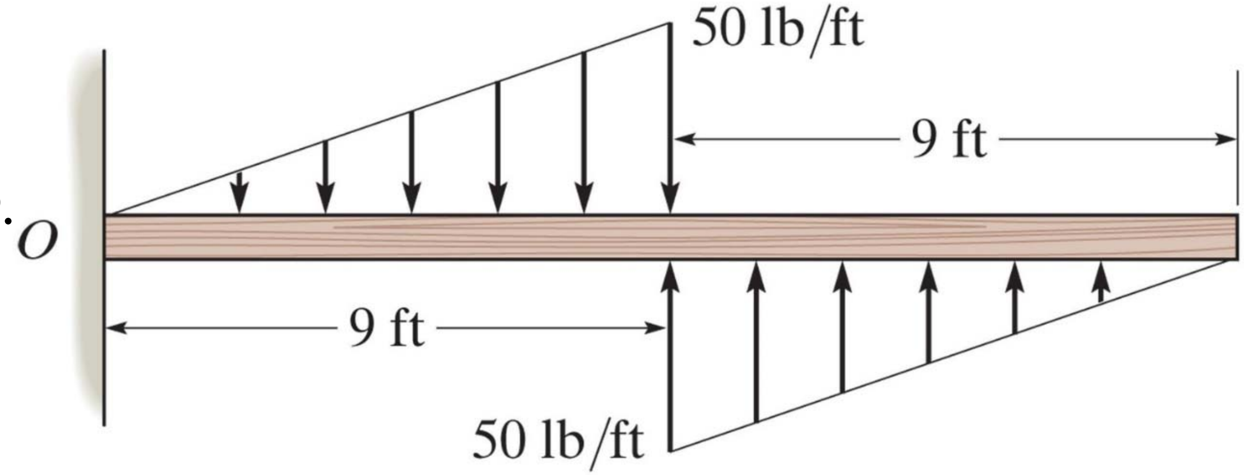


See Example 4.23 in text for full derivation

Replace the distributed loading by an equivalent resultant force and couple moment acting at point A.



Replace the loading by an equivalent resultant force and couple moment acting at point O.



Chapter 5: Equilibrium of Rigid Bodies

Goals and Objectives

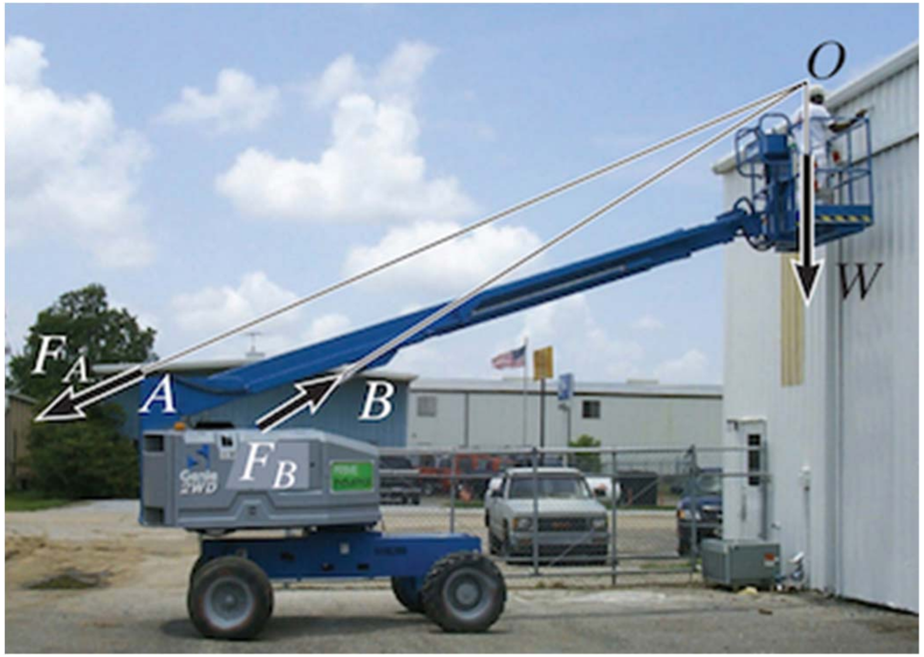
- Introduce the free-body diagram for a rigid body
- Develop the equations of equilibrium for a rigid body
- Solve rigid body equilibrium problems using the equations of equilibrium

Equilibrium of a Rigid Body

Static equilibrium:

$\sum \vec{\boldsymbol{F}} = \mathbf{0}$ (zero forces = no translation)

$\sum(\vec{\boldsymbol{M}}) = \mathbf{0}$ (zero moment = no rotation)



Maintained by reaction forces and moments

Forces from supports / constraints are exactly enough to produce zero forces and moments

Assumption of rigid body

Shape and dimensions of body remain **unchanged** by application of forces.

More precisely:
All **deformations of bodies** are small enough to be ignored in analysis.

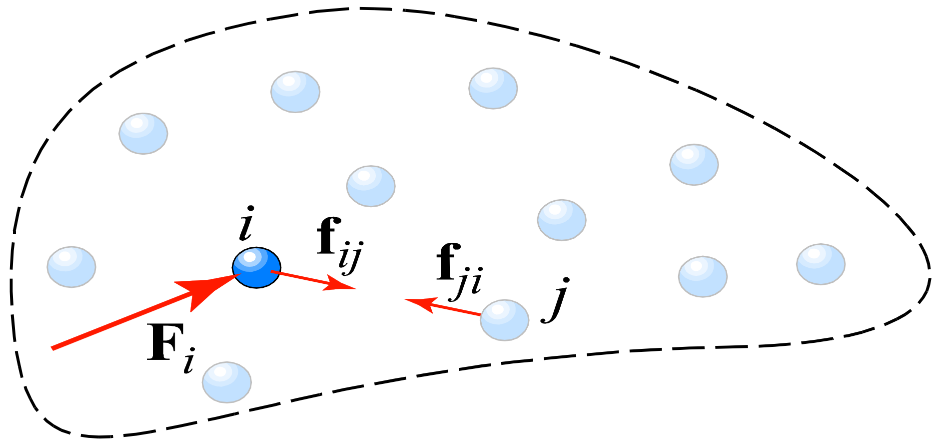
Equilibrium of a Rigid Body

Equilibrium of a rigid body is of central importance in statics. We regard a rigid body as a collection of particles.

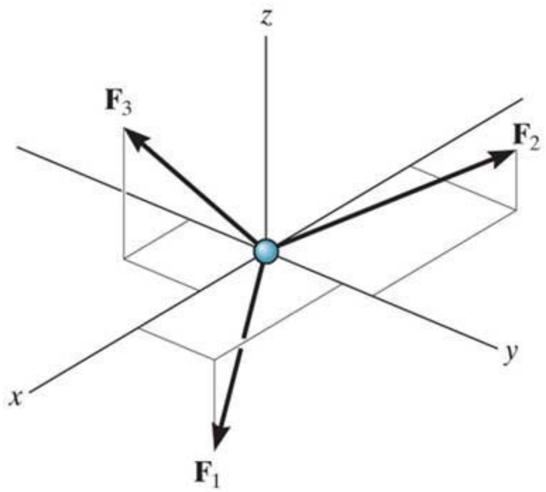
$\vec{F}_i$ = resultant external force on particle i

$\vec{f}_{ij}$ = internal force on particle i by particle j

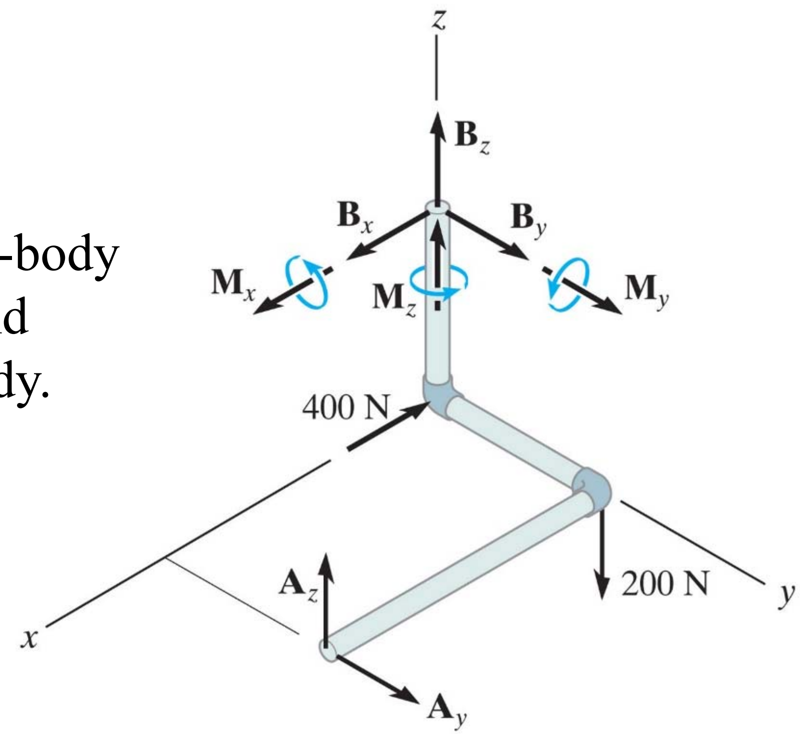
$\vec{f}_{ij}$ = internal force on particle j by particle i



Note that $\vec{f}_{ij} = \vec{f}_{ij}$ by Newton's third law. Therefore the internal forces will not appear in the equilibrium equations.

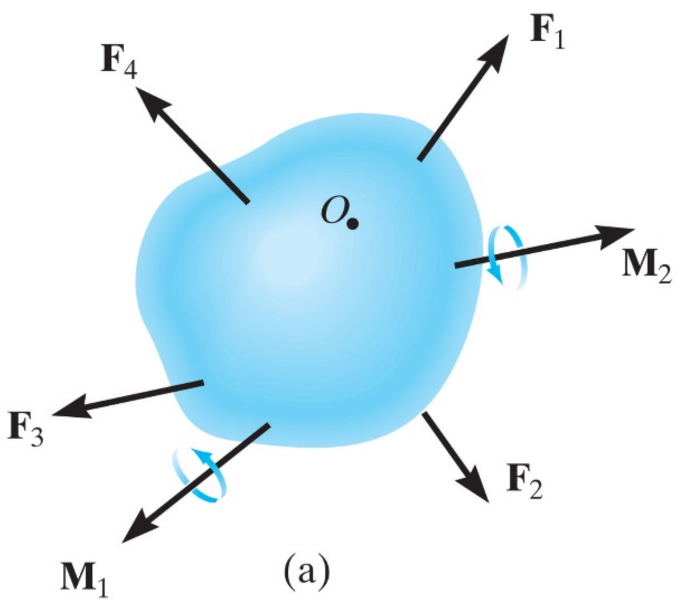


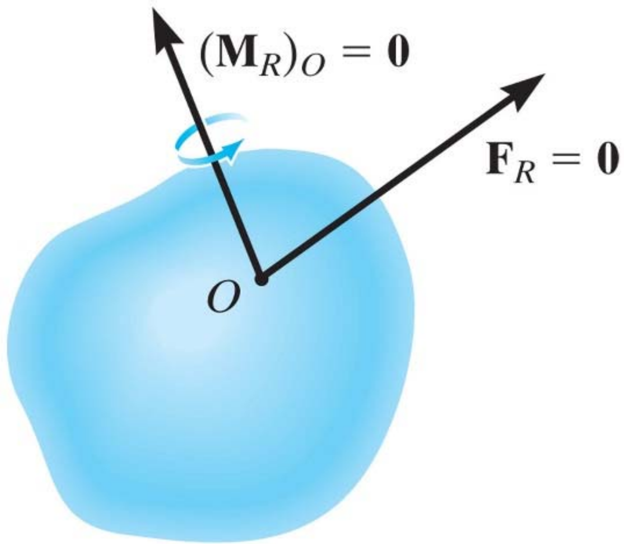
In contrast to the forces on a particle, the forces on a rigid-body are not usually concurrent and may cause rotation of the body.



Equilibrium of a Rigid Body

We can reduce the force and couple moment system acting on a body to an equivalent resultant force and a resultant couple moment at an arbitrary point O.

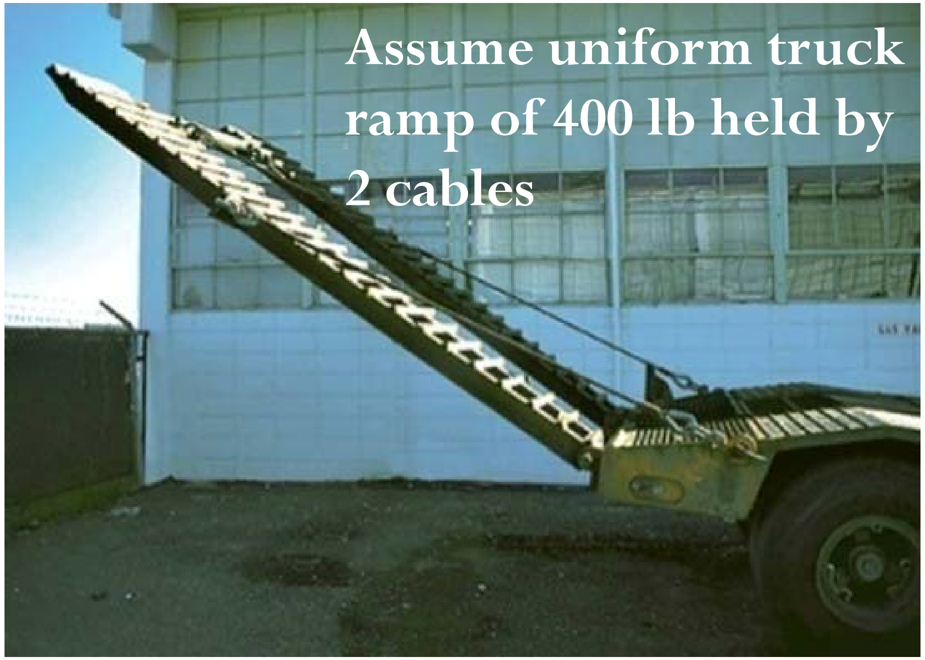




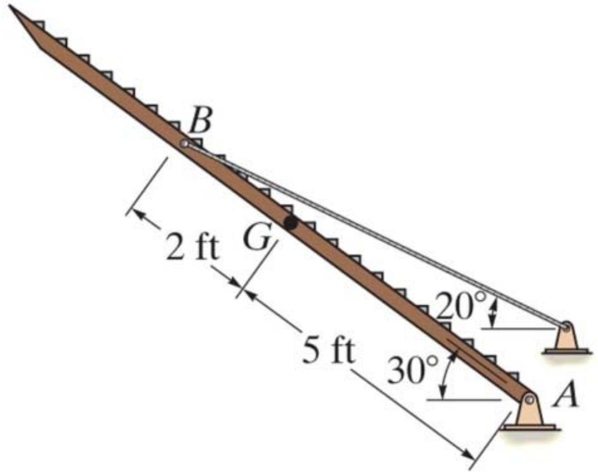
$$\overrightarrow{F_R} = \sum \vec{F} = 0$$

$$(\overrightarrow{M_R})_O = \sum \overrightarrow{M_O} = 0$$

Process of solving rigid body equilibrium problems



1. Create idealized model (modeling and assumptions)



2. Draw free body diagram showing ALL the external (applied loads and supports)

3. Apply equations of equilibrium

$$\overrightarrow{F_R} = \sum \vec{F} = 0$$

$$(\overrightarrow{M_R})_A = \sum \overrightarrow{M_A} = 0$$

In this case, let's sum moments about pt A

See Example 5.11 in text for full derivation