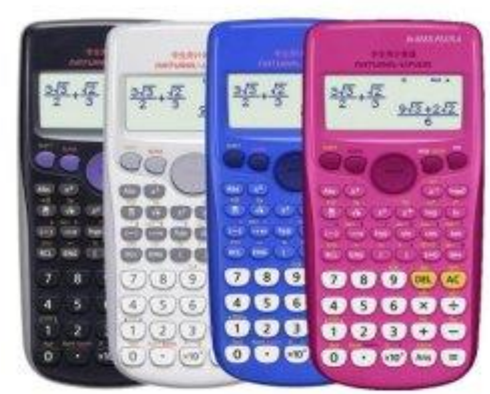
Statics - TAM 211

Lecture 12

October 15, 2018

Announcements

- As announced during discussion section, you are encouraged and allowed to use your Casio calculator during PrairieLearn HWs and Quizzes.
  - You should learn to solve a system of equations by hand using a calculator
- PrairieLearn incorrect software issues:
  - Negative sign symbol (- vs. –)
  - Space between negative sign (-12 vs. - 12)
  - Solutions:
    - Always type in the negative sign symbol (-) into your PL answers for HW or Quiz.
    - Do not add space between negative symbol and number
  - All students with these errors will be provided updated grades on Quiz 1. No credit for Quiz 2 and beyond.
- Upcoming deadlines:
- Tuesday (10/16)
  - Prairie Learn HW4
- Friday (10/19)
  - Written Assignment 4
- Quiz 2
  - Week of Oct 22



Recap: General procedure for analysis

1. Read the problem carefully; write it down carefully.
2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.
3. Apply principles needed.
4. Solve problem symbolically. Make sure equations are dimensionally homogeneous
5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).
6. See if answer is reasonable.

Most effective way to learn engineering mechanics is to ***solve problems!***

Chapter 4: Force System Resultants

Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- How to find the moment about a specified axis
- Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading

Recap: Resultant or Equivalent Force and Moment Systems

Reducing a force system to a single resultant force $\vec{\boldsymbol{F}}_R$ and a single resultant couple moment about point O $\left(\vec{\boldsymbol{M}}_R\right)_o$:

$$\overrightarrow{\boldsymbol{F}_{\boldsymbol{R}}} = \Sigma F_x \hat{\boldsymbol{i}} + \Sigma F_y \hat{\boldsymbol{j}} + \Sigma F_z \hat{\boldsymbol{k}}$$

Magnitude: $|\overrightarrow{\boldsymbol{F}_{\boldsymbol{R}}}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$

Orientation in Cartesian coordinate system: x-direction (F_x), y-direction (F_y), z-direction (F_z),

Orientation in Cylindrical coordinate system: $\theta = \tan^{-1} \frac{F_{opp}}{F_{adj}}$

$$\left(\overrightarrow{\boldsymbol{M}}_R\right)_o = \sum \boldsymbol{M}_o + \sum \boldsymbol{M}$$

$\vec{r}_{oi} \times \vec{F}_i$

Sum (couple moments)

Recap: Distributed loads

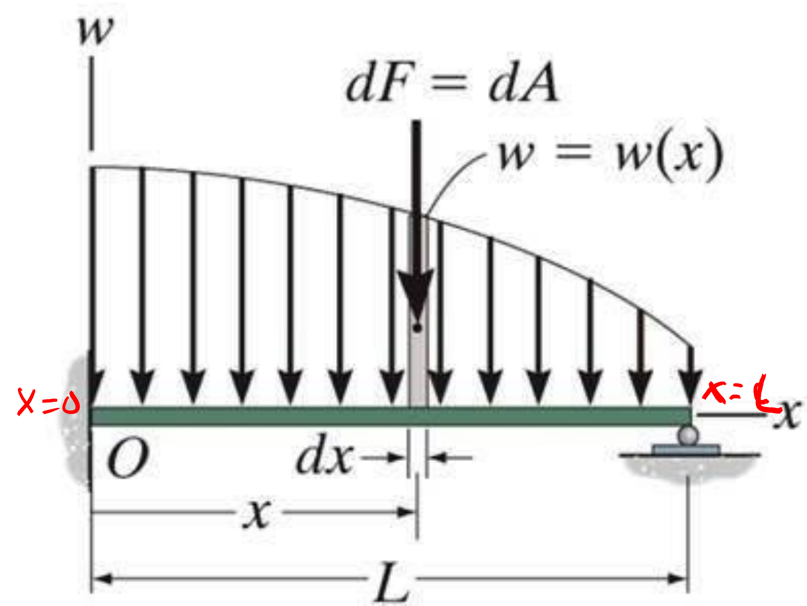
- Equivalent force system for distributed loading function $w(x)$ with units of $\frac{\text{force}}{\text{length}}$.
- Find magnitude F_R and location $\bar{x}$ of the equivalent resultant force for $\overrightarrow{\boldsymbol{F_R}}$

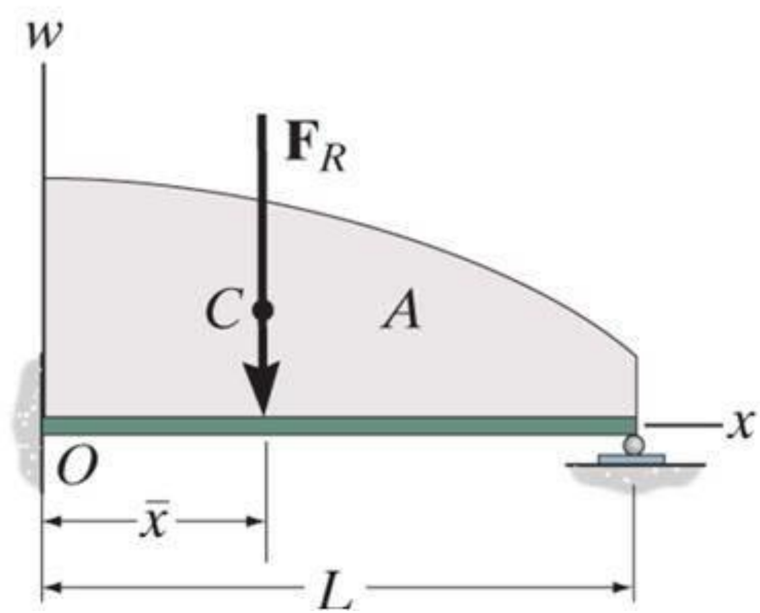
$$\left|\overrightarrow{\boldsymbol{F_R}}\right| = F_R = \int_0^L dF = \int_0^L w(x)\,dx = A$$

$$M_o = \int_0^L x\;w(x)\,dx = \bar{x}\,F_R$$

$$\bar{x} = \frac{M_o}{F_R} = \frac{\int_0^L x\;w(x)\,dx}{\int_0^L w(x)\,dx}$$

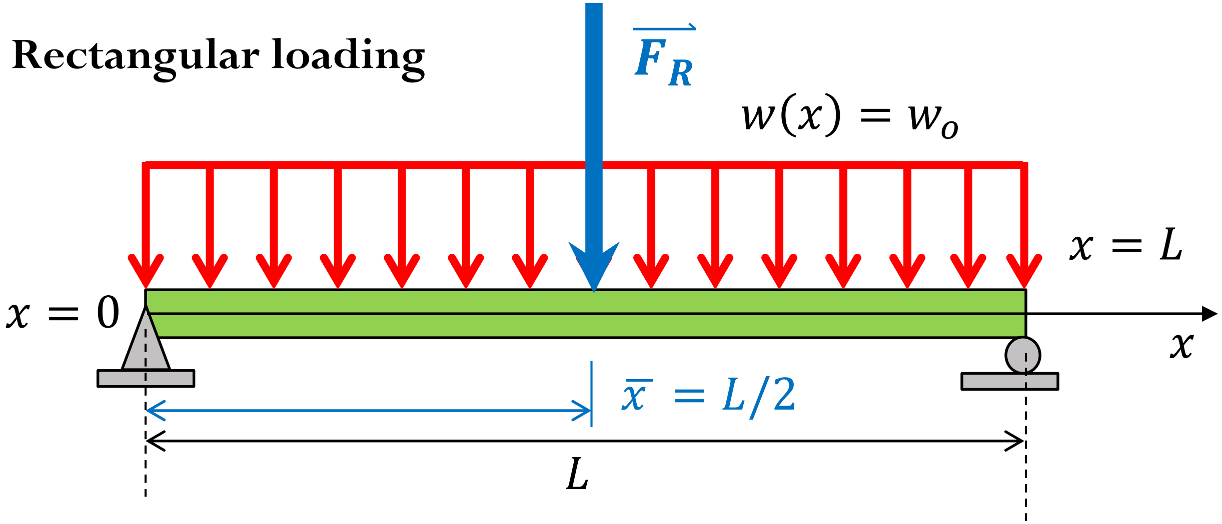
$\bar{x}$ = **geometric center or centroid** of area A under loading curve $w(x)$.





Recap: Simple Shape Distributed loads

Rectangular loading

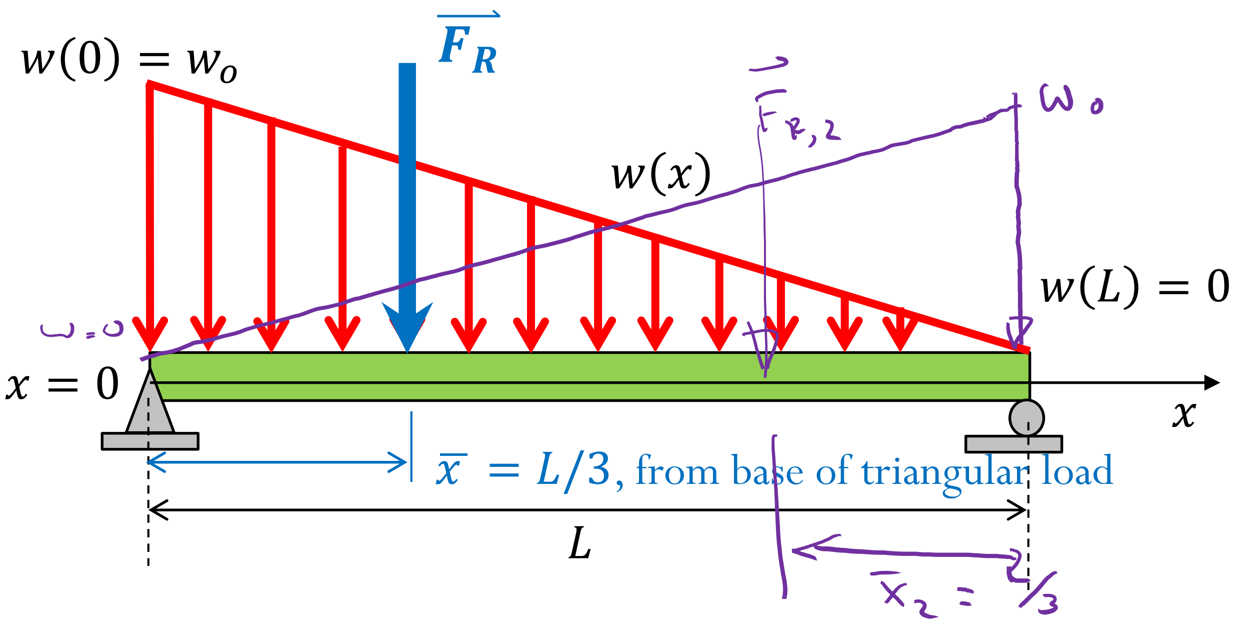


$$w(x) = w_o$$

$$\left|\overrightarrow{F_R}\right| = F_R = w_o L$$

$$\bar{x} = \frac{L}{2}$$

Triangular loading

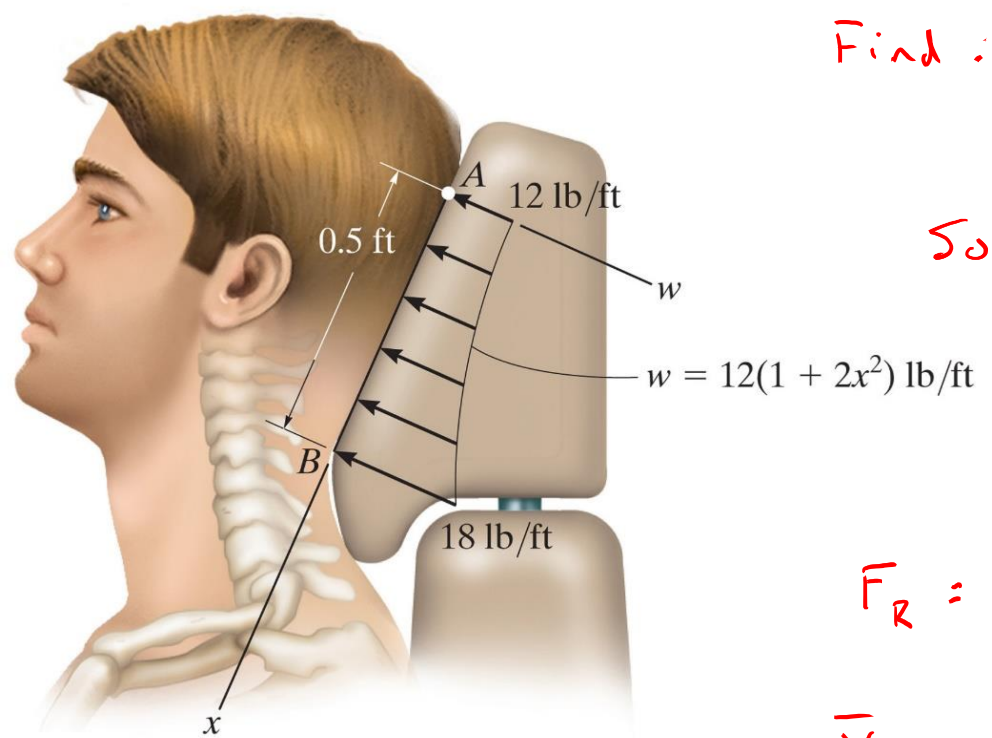


$$w(x) = w_o - \frac{w_o x}{L}$$

$$F_R = w_o \frac{L}{2}$$

$$\bar{x} = \frac{L}{3}$$

Find equivalent force and its location from point A for loading on headrest.



Find: $\vec{F}_R$ & $\bar{x}$ wrt pt. A

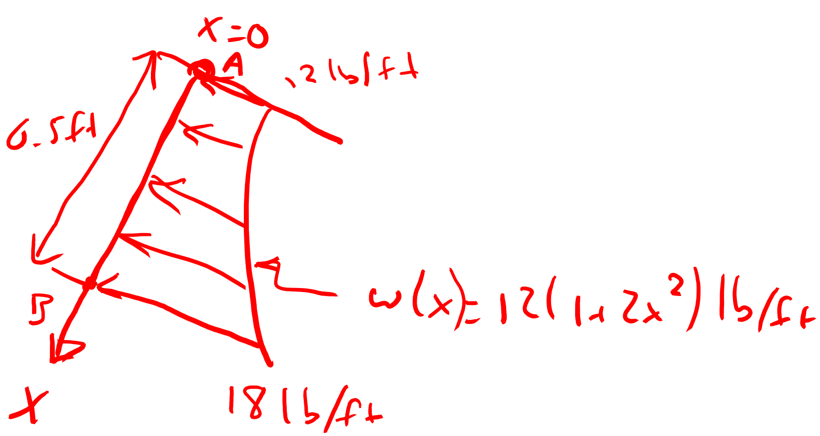
wrt: with respect to

Soln: $F_R = \int_0^L w(x)\,dx$

$$\bar{x} = \frac{M_o}{F_R} = \frac{\int_0^L x\,w(x)\,dx}{\int_0^L w(x)\,dx}$$

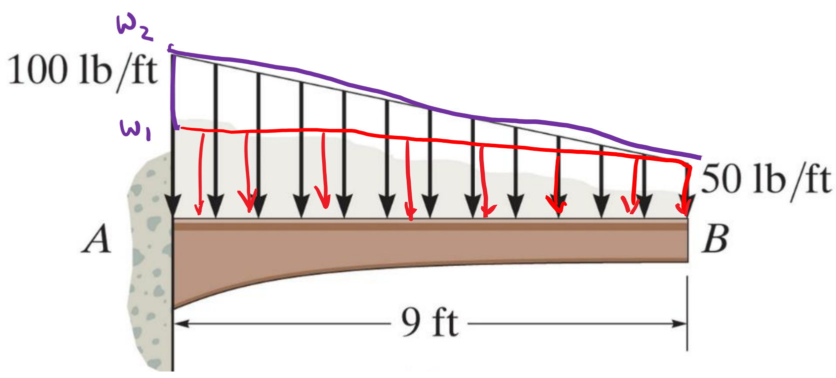
$$F_R = \int_0^{0.5\,ft} \left[12(1+2x^2)\ lb/ft\right] dx$$

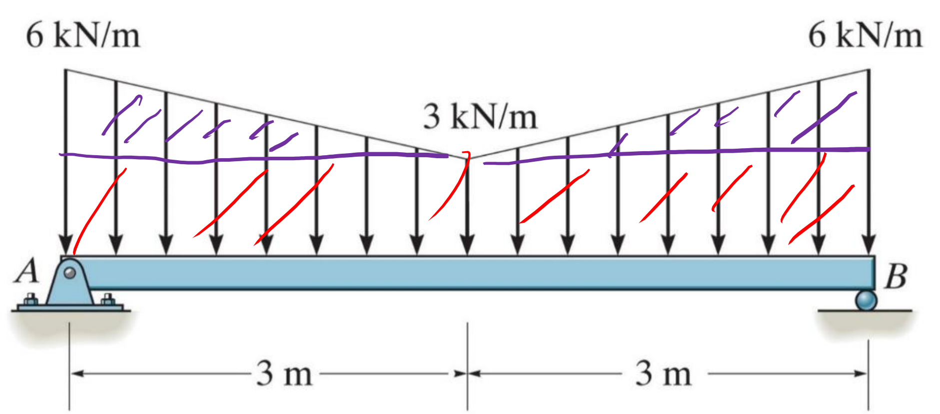
$$\bar{x} = \frac{\int_0^{0.5} x\left(12(1+2x^2)\right) dx}{\int_0^{0.5} \left[12(1+2x^2)\right] dx}$$

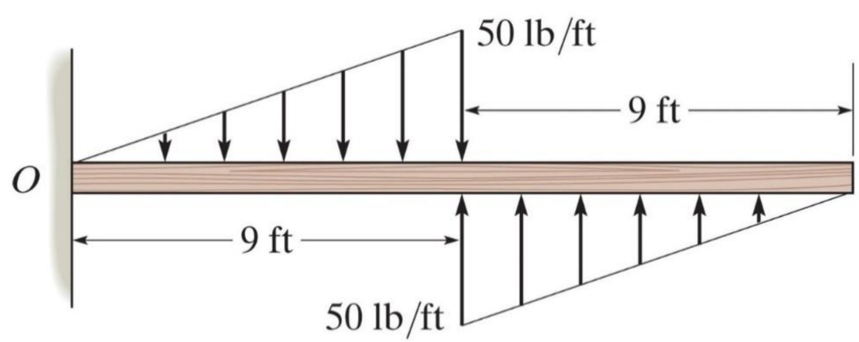


Superposition of simple shapes

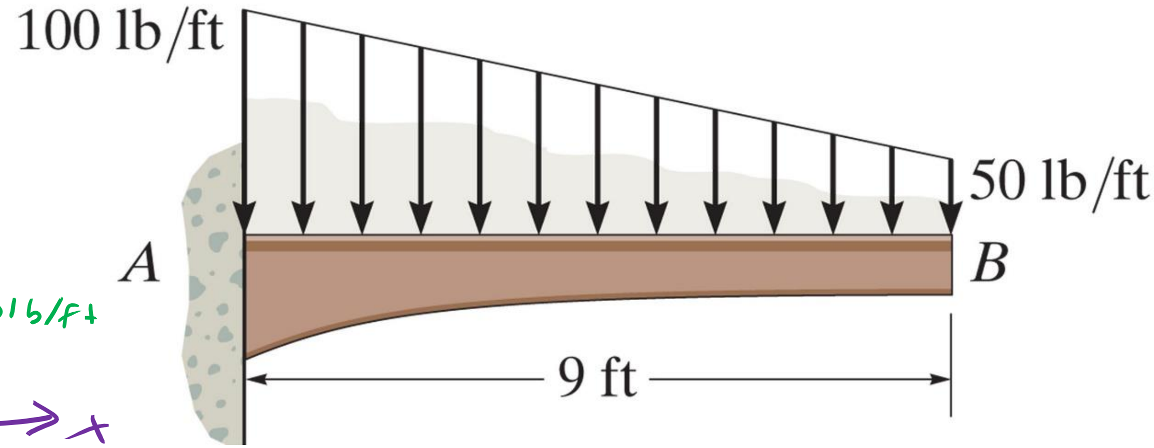
- Divide complex distributed loads into multiple simple shapes of rectangles and/or triangles.
- Superimpose the resultant forces for each simple shape to determine the final composite resultant force.

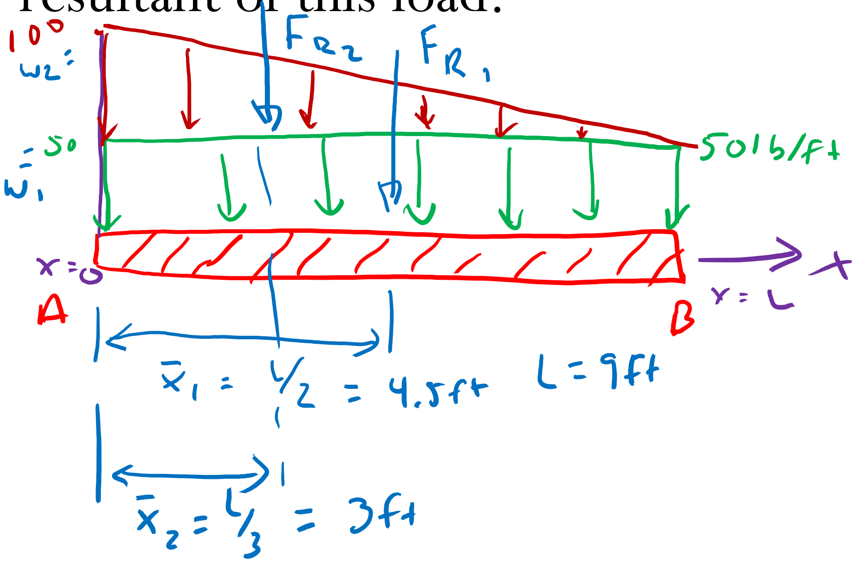


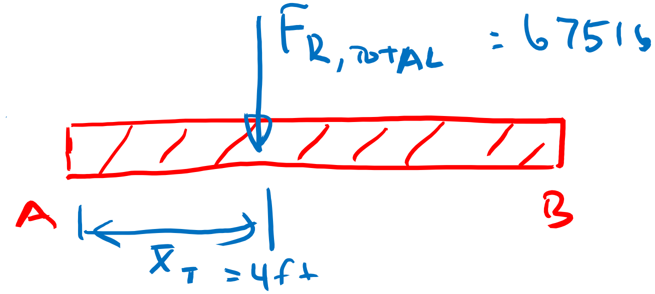




Determine the magnitude and location of the equivalent resultant of this load.







$$F_{R_1} = w_1 L = 50 L = 450 \text{ lb}$$

$$F_{R_2} = w_0 \frac{L}{2} = (100 - 50)\frac{L}{2} = 225 \text{ lb}$$

$$F_{R,T} = F_{R_1} + F_{R_2}$$

$$\boxed{F_{RT} = 675 \text{ lb} \downarrow}$$

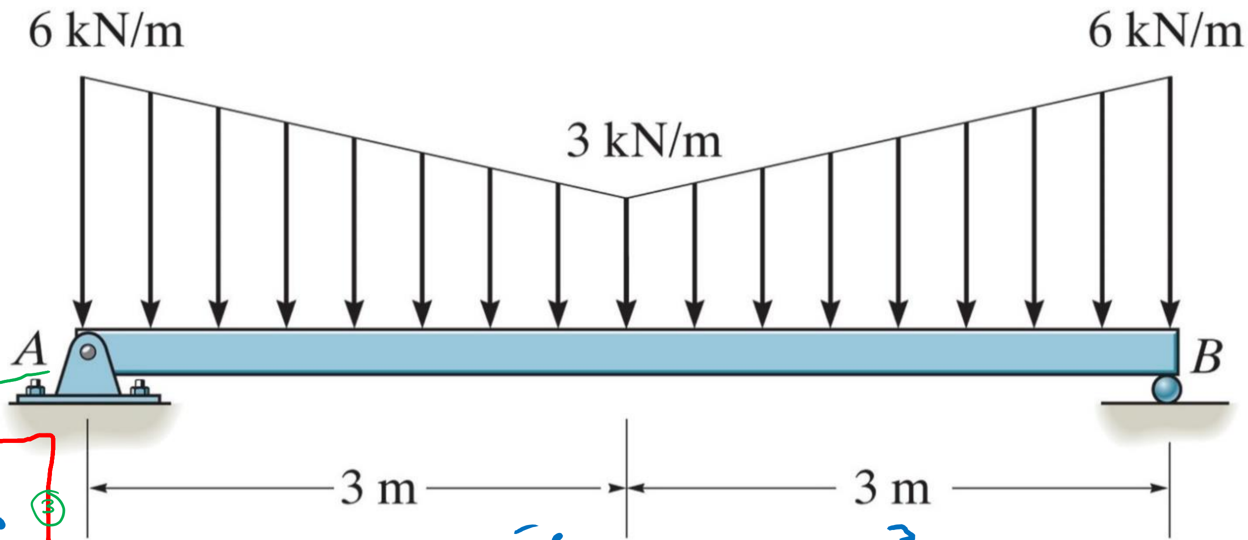
$\bar{x}_T = ?$

Sum Moments:

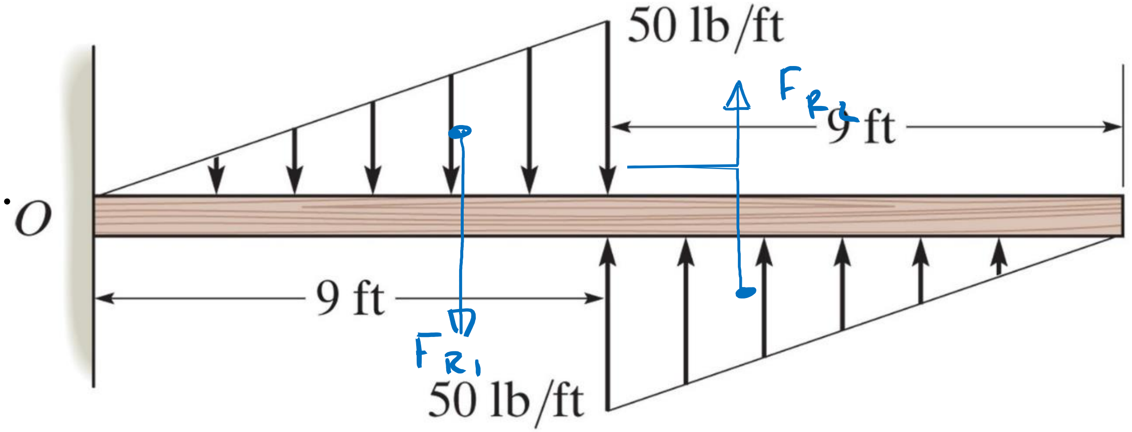
$$\bar{x}_1 F_{R_1} + \bar{x}_2 F_{R_2} = \bar{x}_T F_{RT} \Rightarrow \bar{x}_T = \frac{\bar{x}_1 F_{R_1} + \bar{x}_2 F_{R_2}}{F_{RT}} \Rightarrow \boxed{\bar{x}_T = 4 \text{ ft}}$$

See Example 4.23 in text for full derivation

Replace the distributed loading by an equivalent resultant force and couple moment acting at point A.



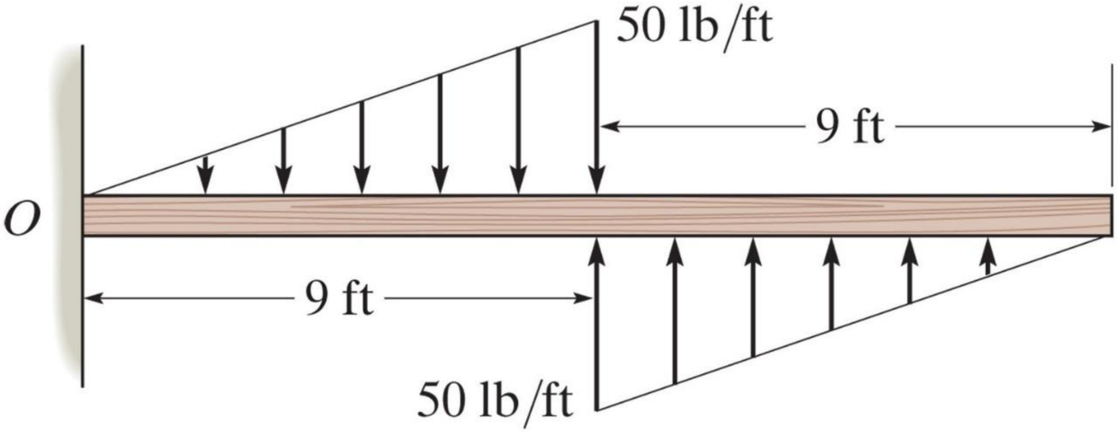
Replace the loading by an equivalent resultant force and couple moment acting at point O.



FIND : F_R & $\vec{M}_O$

SOLN: $\vec{F}_R = 0$

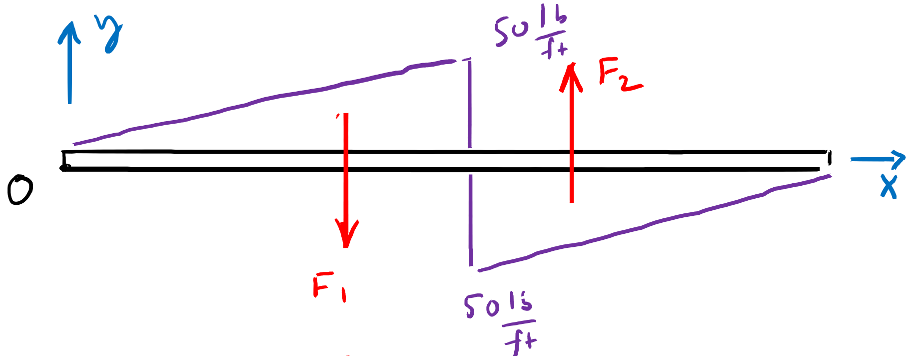
$\vec{M}_O = 1350 \text{ lbft } \hat{k}$ [+ ↺]

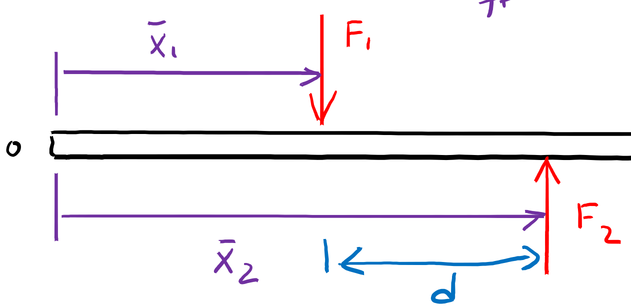


Replace the loading by an equivalent resultant force and couple moment acting at point O.

Find: F_R, $\vec{M}_o$

Soln: Draw FBD





Sum Forces:

$$\vec{F}_R = \sum F_y = F_1 + F_2$$

$$= -\frac{9\,ft}{2}\left(50\,\frac{lb}{ft}\right)\hat{j} + \frac{9\,ft}{2}\left(50\,\frac{lb}{ft}\right)\hat{j}$$

$\Rightarrow \vec{F}_R = 0$

$\Rightarrow \vec{F}_1 = -\vec{F}_2$

$$+\circlearrowleft\ \vec{M}_o = \vec{r} \times \vec{F} = -\bar{x}_1 F + \bar{x}_2 F_2$$

$$= -\left(\tfrac{2}{3}\,9\,ft\right)F_1 + \left(9\,ft + \tfrac{1}{3}\,9\,ft\right)F_2$$

$$= -(6\,ft)F_1 + (12\,ft)F_2$$

But $\vec{F}_1 = -\vec{F}_2$, $|\vec{F}_1| = |\vec{F}_2| = F$

$$= (6\,ft)F = (6\,ft)\left(\frac{9\,ft}{2}\right)\left(50\,\frac{lb}{ft}\right) = 1350\,lb\cdot ft\ (+\hat{k})$$

Or if note that $\vec{F}_1$ & $\vec{F}_2$ create a couple moment, then can say

$\sum \vec{M}_o = \vec{r} \times \vec{F} = dF$, where $F = F_1$ or F_2

$\therefore \vec{M}_o = (6\,ft)\left(\frac{9\,ft}{2}\right)\left(50\,\frac{lb}{ft}\right) = 1350\,lb\cdot ft\ (+\hat{k})$ ✓ same