Statics - TAM 211

Lecture 13

October 17, 2018

Announcements

- Upcoming deadlines:
- Friday (10/19)
  - Written Assignment 4
- Tuesday (10/23)
  - Prairie Learn HW5
- Quiz 2
  - Week of Oct 22

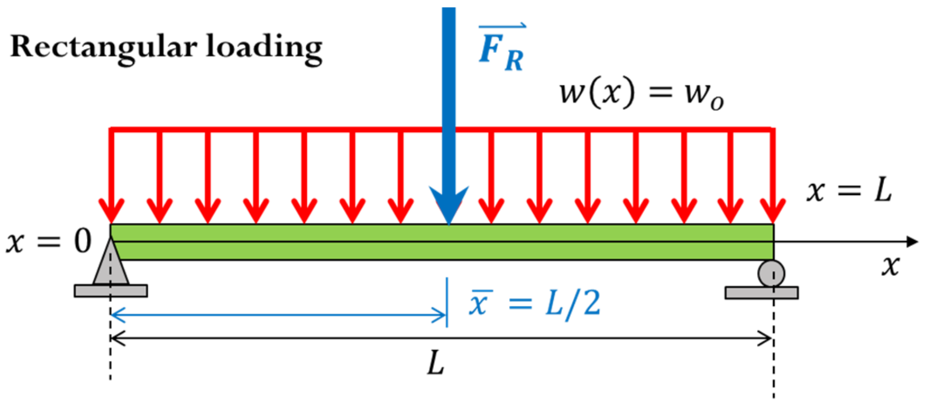
- Preparation for quiz:
- Practice PL HW on your own. Practice using a calculator.
- Monitor your time
- Read each question. Write givens, unknowns, draw FBD, write out equations
- HW reflections
  - What concepts did you struggle with?

Recap: Distributed loads

$$F_R = \int_0^L w(x)\,dx \qquad \bar{x} = \frac{\int_0^L x\ w(x)\,dx}{\int_0^L w(x)\,dx} \qquad M_o = \bar{x}\,F_R$$

Simple Shape Distributed loads

Rectangular loading

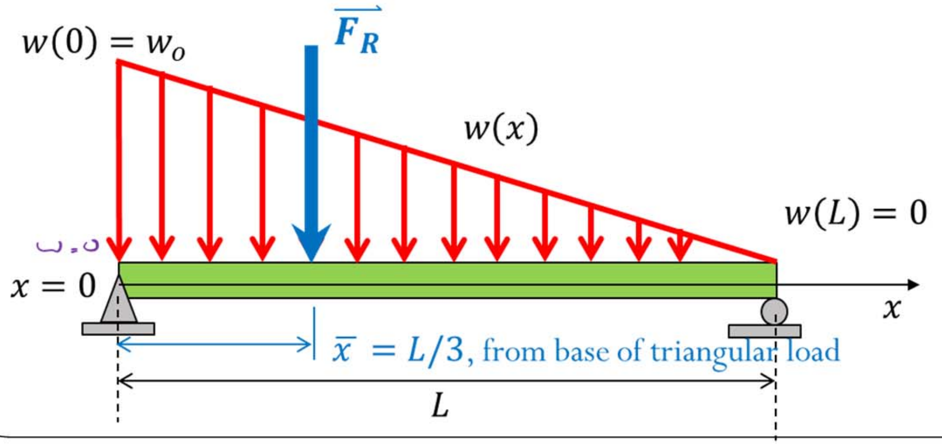


$$w(x) = w_o$$

$$\left|\overrightarrow{F_R}\right| = F_R = w_o L$$

$$\bar{x} = \frac{L}{2}$$

Triangular loading



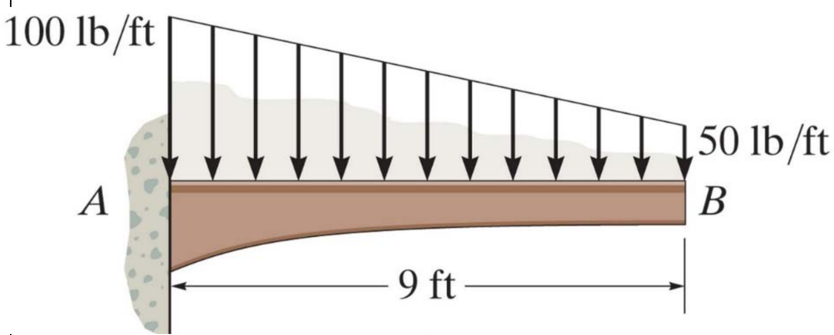
$$w(x) = w_o - \frac{w_o x}{L}$$

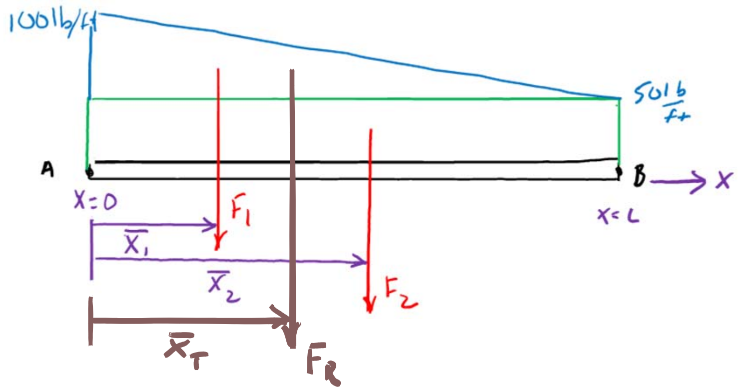
$$F_R = w_o \frac{L}{2}$$

$$\bar{x} = \frac{L}{3}$$

Recap: Superposition of simple shapes

- Divide complex distributed loads into multiple simple shapes of rectangles and/or triangles.
- Superimpose the resultant forces for each simple shape to determine the final composite resultant force.





Use

(1) Sum of vertical forces: $\boldsymbol{F_R} = \Sigma \boldsymbol{F_i}$

(2) Use sum of moments to find $\bar{x}_T$

$$\bar{x}_T = \frac{\Sigma \bar{x}_i \boldsymbol{F_i}}{\boldsymbol{F_R}}$$

Chapter 5: Equilibrium of Rigid Bodies

Goals and Objectives

- Introduce the free-body diagram for a rigid body
- Develop the equations of equilibrium for a 2D and 3D rigid body
- Solve rigid body equilibrium problems using the equations of equilibrium in 2D and 3D

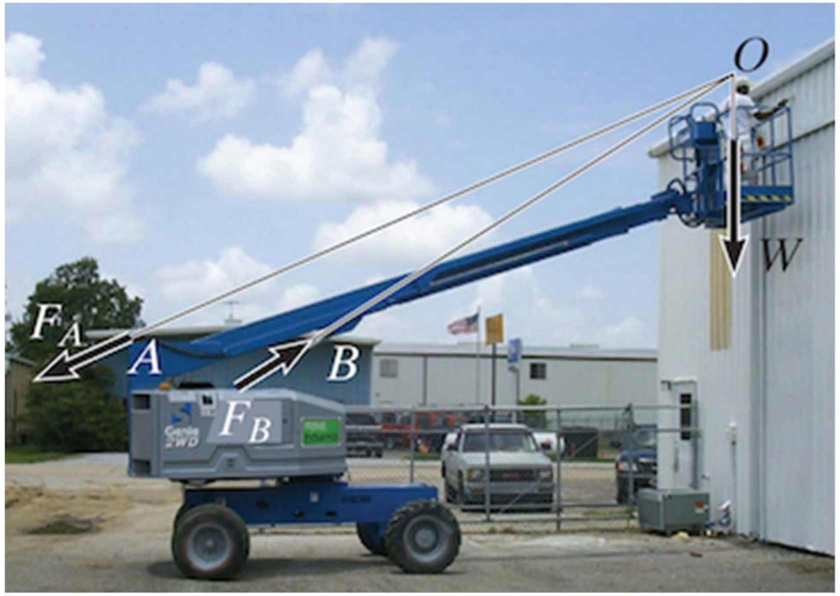
- Introduce concepts of
  - Support reactions for 2D and 3D bodies
  - Two- and three-force members
  - Constraints and statical determinacy

Equilibrium of a Rigid Body

Static equilibrium:

$\sum \vec{F} = \mathbf{0}$ (zero forces = no translation)

$\sum(\vec{M}) = \mathbf{0}$ (zero moment = no rotation)



Maintained by reaction forces and moments

Forces from supports / constraints are exactly enough to produce zero forces and moments

Assumption of rigid body

Shape and dimensions of body remain **unchanged** by application of forces.

More precisely:

All **deformations of bodies** are small enough to be ignored in analysis.

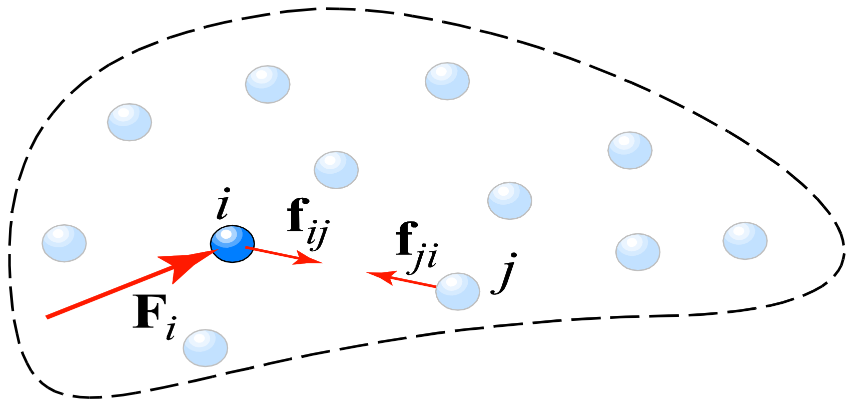
Equilibrium of a Rigid Body

Equilibrium of a rigid body is of central importance in statics. We regard a rigid body as a collection of particles.

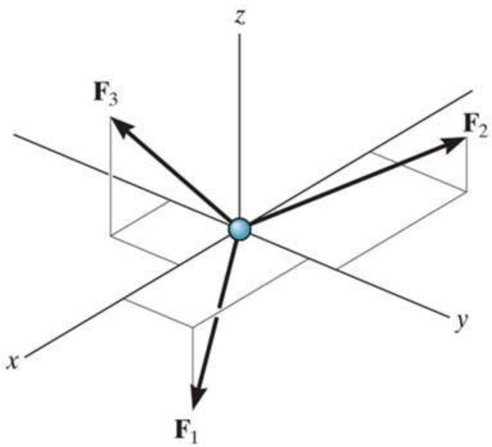
$\overrightarrow{F_i}$ = resultant external force on particle i

$\overrightarrow{f_{ij}}$ = internal force on particle i by particle j

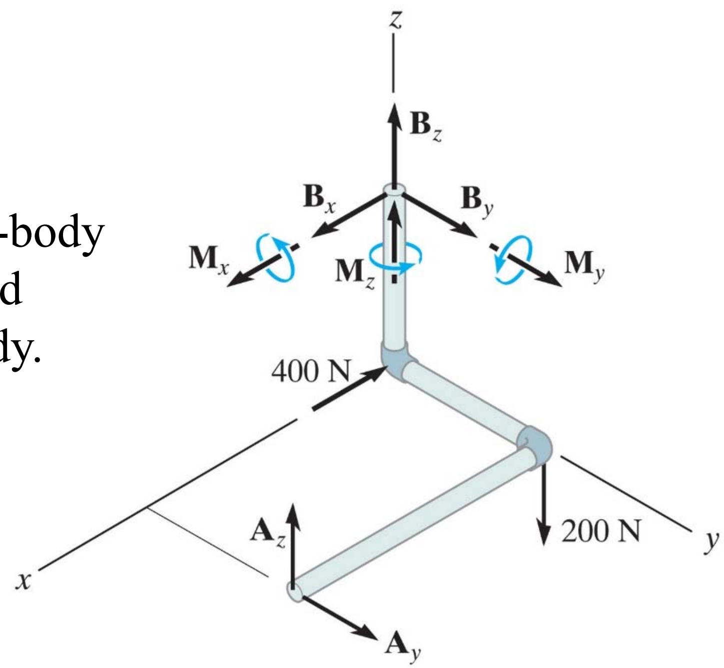
$\overrightarrow{f_{ij}}$ = internal force on particle j by particle i



Note that $\overrightarrow{f_{ij}} = \overrightarrow{f_{ij}}$ by Newton's third law. Therefore the internal forces will not appear in the equilibrium equations.

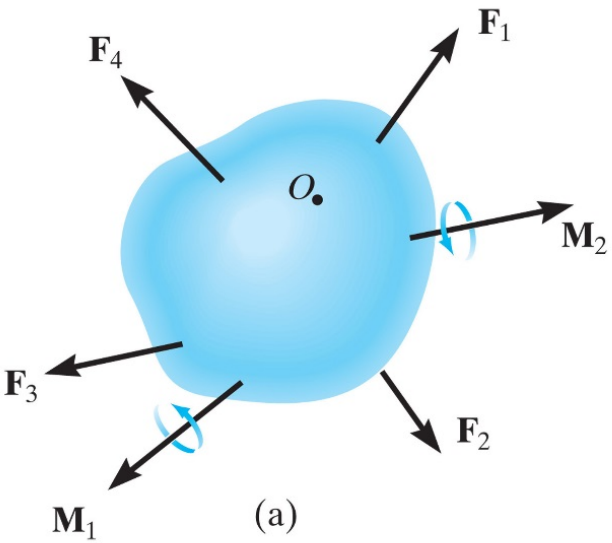


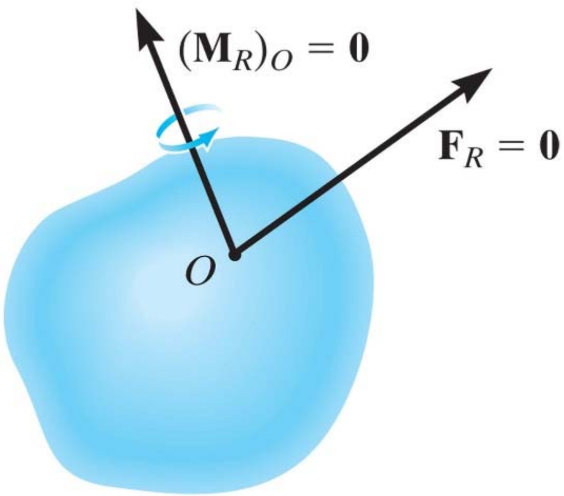
In contrast to the forces on a particle, the forces on a rigid-body are not usually concurrent and may cause rotation of the body.



Equilibrium of a Rigid Body

We can reduce the force and couple moment system acting on a body to an equivalent resultant force and a resultant couple moment at an arbitrary point O.

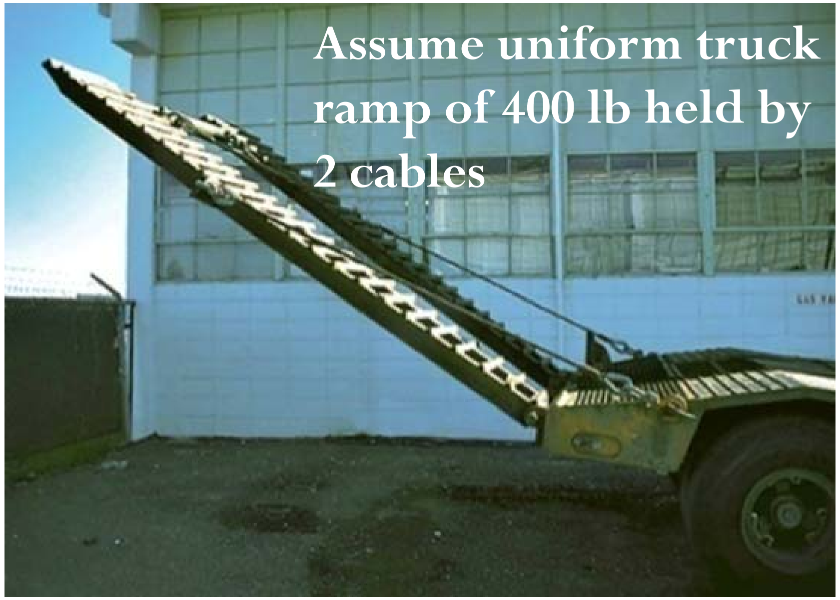




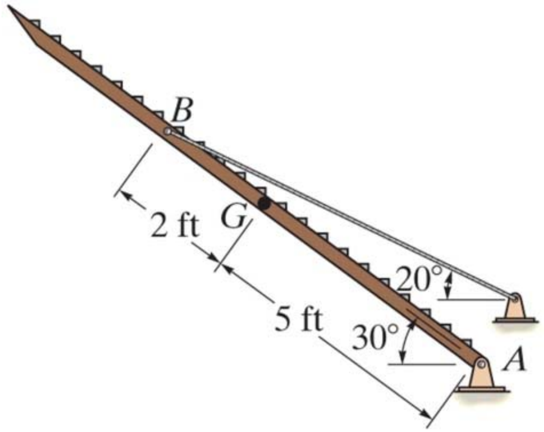
$$\overrightarrow{F_R} = \sum \vec{F} = 0$$

$$(\overrightarrow{M_R})_O = \sum \overrightarrow{M_O} = 0$$

Process of solving rigid body equilibrium problems



1. Create idealized model (model and assumptions)



2. Draw free body diagram showing ALL the external (applied loads and support reactions)

3. Apply equations of equilibrium

$$\overrightarrow{F_R} = \sum \vec{F} = 0$$

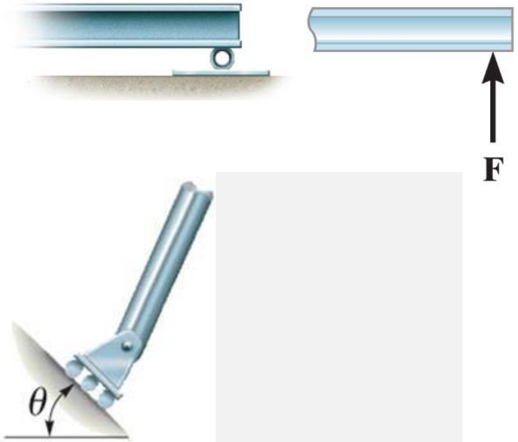
$$(\overrightarrow{M_R})_A = \sum \overrightarrow{M_A} = 0$$

In this case, let's sum moments about pt A

See Example 5.11 in text for full derivation

Equilibrium in two-dimensional bodies
(Support reactions)

Roller



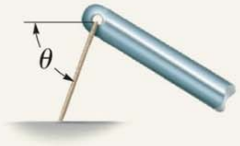
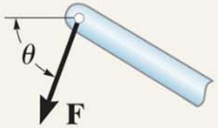
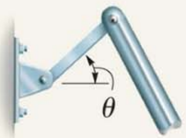
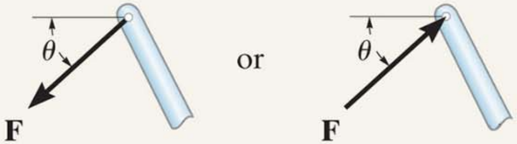
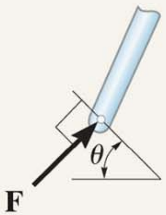
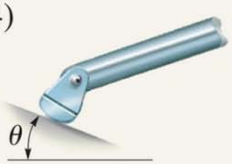
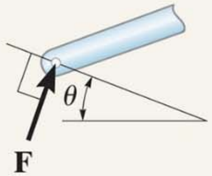
Smooth pin or hinge

Fixed support

- If a support prevents the translation of a body in a given direction, then a force is developed on the body on that direction
- If a rotation is prevented, a couple moment is exerted on the body

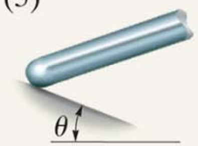
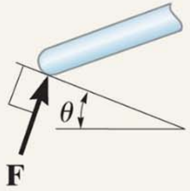
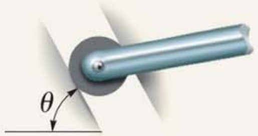
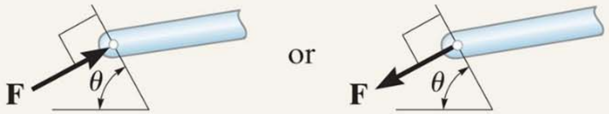
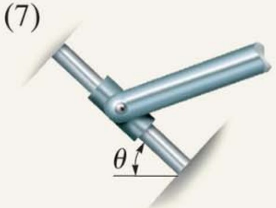
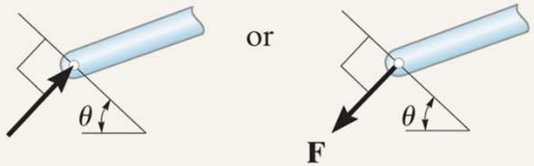
Types of connectors

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link		One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

Types of connectors

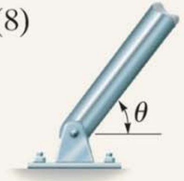
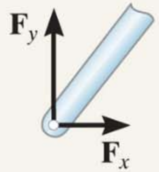
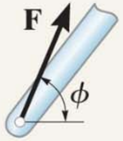
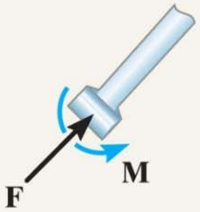
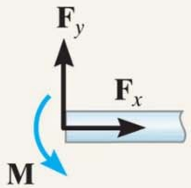
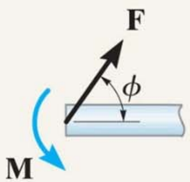
TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

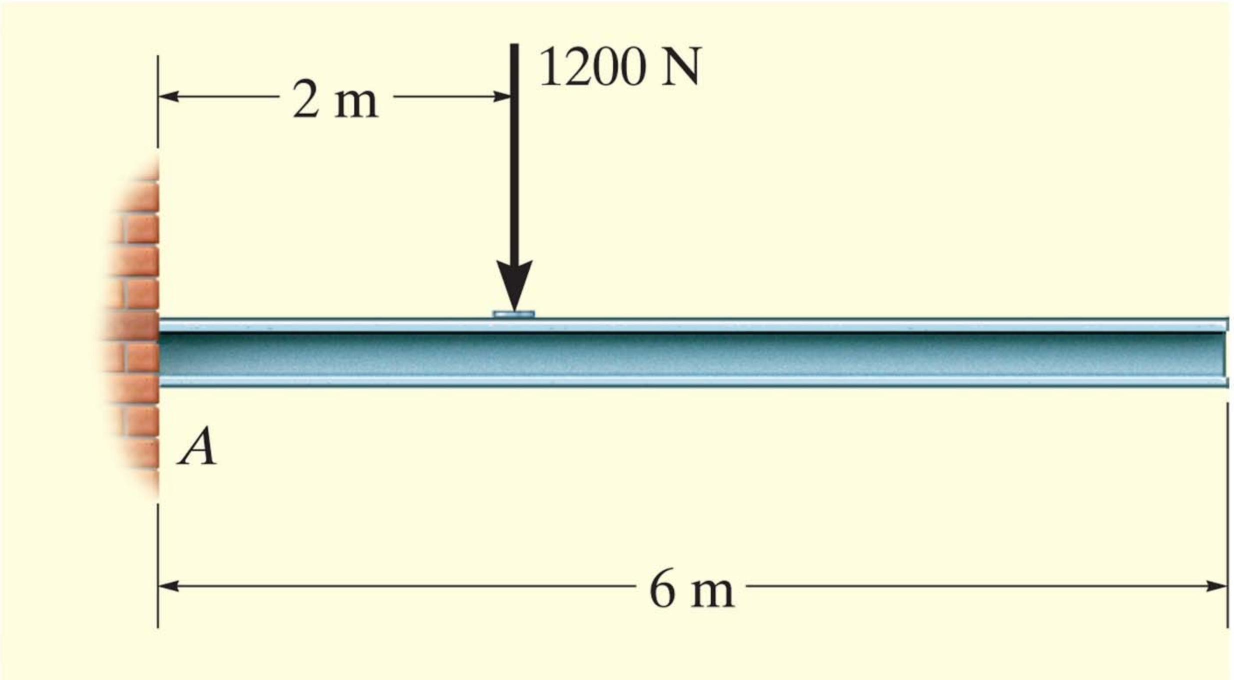
Types of Connection	Reaction	Number of Unknowns
(5)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  roller or pin in confined smooth slot		One unknown. The reaction is a force which acts perpendicular to the slot.
(7)  member pin connected to collar on smooth rod		One unknown. The reaction is a force which acts perpendicular to the rod.

continued

Types of connectors

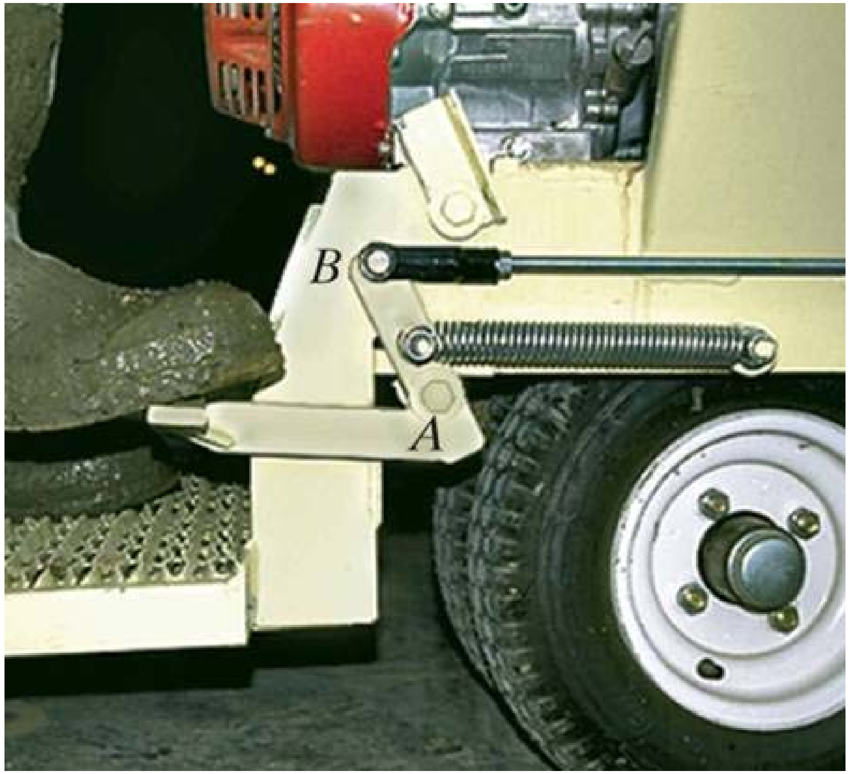
TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge	 or 	Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support	 or 	Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

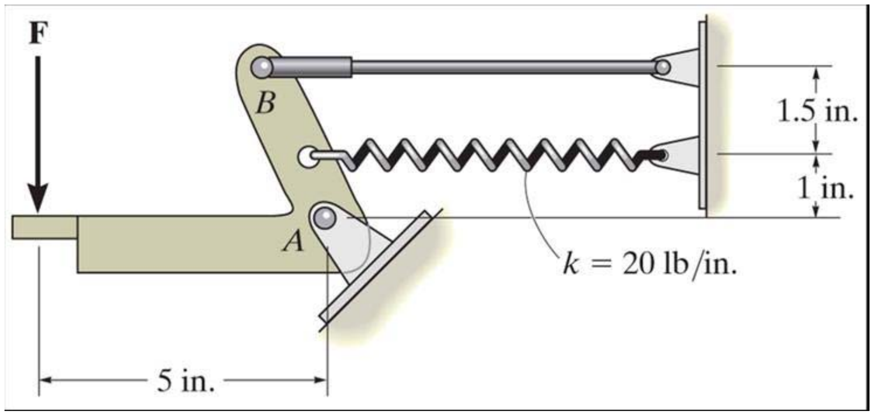


Beam has mass of 100 kg and experiences load of 1200 N. Identify support reaction type. Find support reactions at A.

See Example 5.1 in text



The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at B is 20 lb. Draw the FBD of the pedal



See Example 5.2 in text