Statics - TAM 211

Lecture 15

October 22, 2018

Announcements

- Students are encouraged to practice drawing FBDs, writing out equilibrium equations, and solving these by hand using your calculator.
- Upcoming deadlines:
- Tuesday (10/23)
  - Prairie Learn HW5
- **Quiz 2, Wednesday (10/24)**
  - During class time (9:00 am)
  - Computer Lab (D211 for ME, D331 for CEE)
  - Chapter 4
- Friday (10/26)
  - Written Assignment 5
- **Friday (11/2) all in Teaching Building A418-420**
  - 8:00 am: Quiz 3, Chapter 5. On paper.
  - 9:00 am: Lecture 17
  - 10:00 am: Discussion section for ALL students
- No class
  - Friday October 26 (Sports Meeting day)
  - Monday October 29

- PrairieLearn incorrect software issues:
  - Negative sign symbol (- vs. –)
  - Space between negative sign (-12 vs. - 12)
  - Solutions:
    - Always type in the negative sign symbol (-) into your PL answers for HW or Quiz.
    - Do not add space between negative symbol and number
  - All students with these errors will be provided updated grades on Quiz 1. No credit for Quiz 2 and beyond.

Chapter 5: Equilibrium of Rigid Bodies

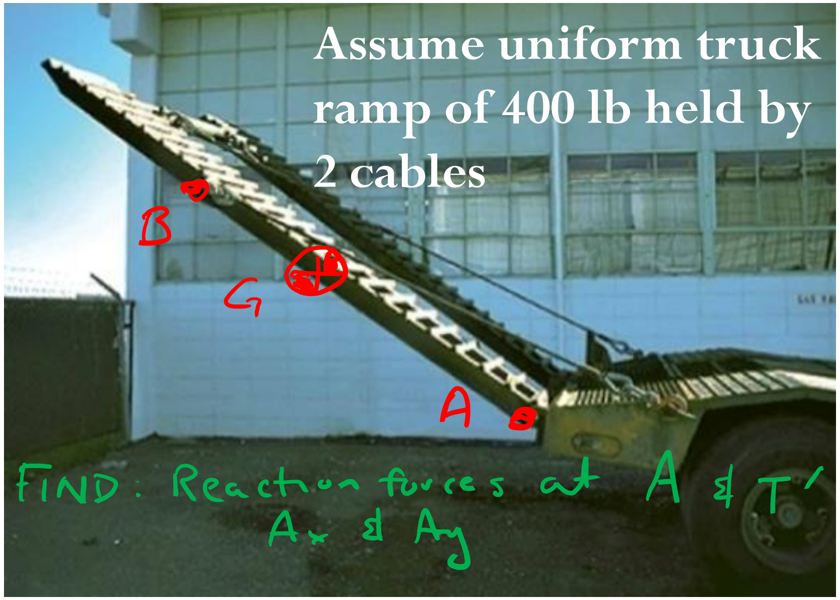
Goals and Objectives

- Introduce the free-body diagram for a rigid body
- Develop the equations of equilibrium for a 2D and 3D rigid body
- Solve rigid body equilibrium problems using the equations of equilibrium in 2D and 3D

- Introduce concepts of
  - Support reactions for 2D and 3D bodies
  - Two- and three-force members
  - Constraints and statical determinacy

Process of solving rigid body equilibrium problems

See Example 5.11 in text for full derivation

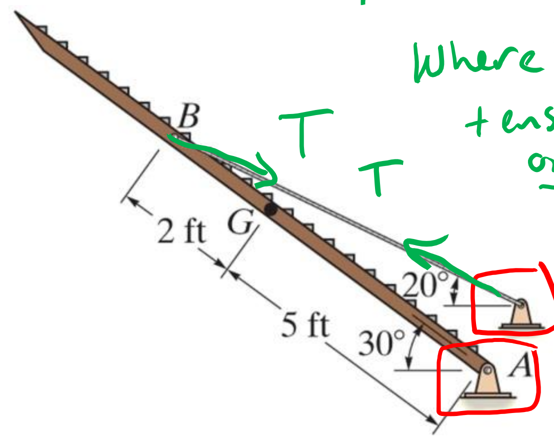


Center of Gravity or Center of mass

1. Create idealized model (model and assumptions)

Note: $T = 2T'$

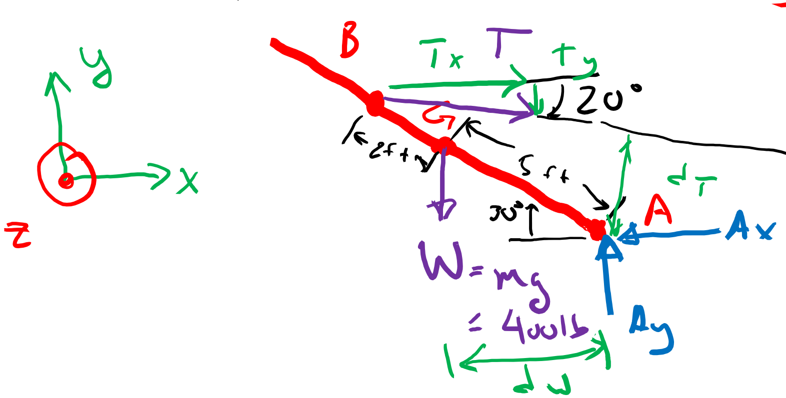
Where T' = tension on one cable



supports "pin joints"

2. Draw free body diagram showing ALL the external (applied loads and support reactions)

FBD of RAMP only



3. Apply equations of equilibrium

$$\overrightarrow{F_R} = \sum \vec{F} = 0$$

$\xrightarrow{+}$ $\Sigma F_x: -A_x + T\cos 20° = 0$ (1)

$+\uparrow$ $\Sigma F_y: A_y - W - T\sin 20° = 0$ (2)

$$(\overrightarrow{M_R})_A = \sum \overrightarrow{M_A} = 0$$

Let's sum moments about pt A. **Pick pt to sum moments that eliminates as many unknowns as possible.**

$+\circlearrowleft$ $\Sigma M_A: +W(d_w) - T(d_T) = 0$ (3)

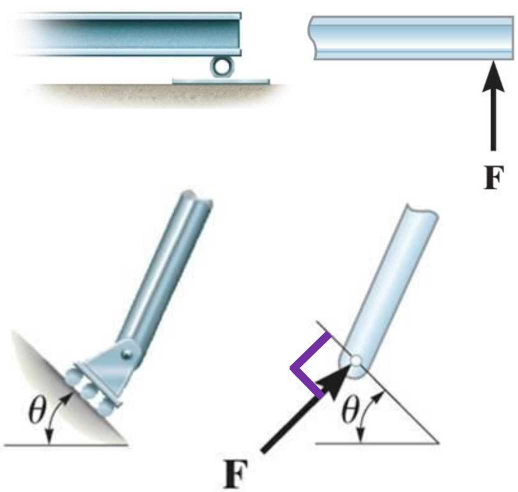
3 Unknowns (A_x, A_y, T'), 3 equations (1-3)

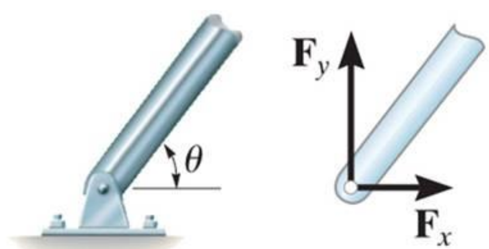
$\Rightarrow$ Determinate system ∴ Can solve.

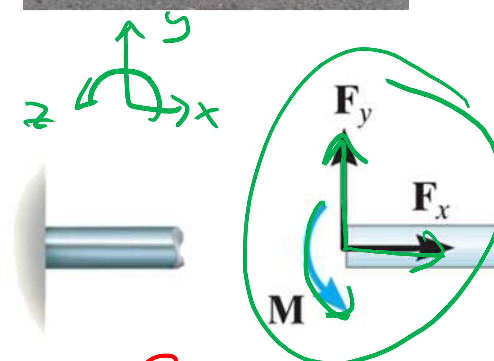
This slide presents the basic approach for problem solving for this course (previous slide). Understand how to do this approach!

Recap: Equilibrium in two-dimensional bodies
(Support reactions)

Roller	Smooth pin or hinge	Fixed support
		







Reaction Forces

on the body

Reaction Moment

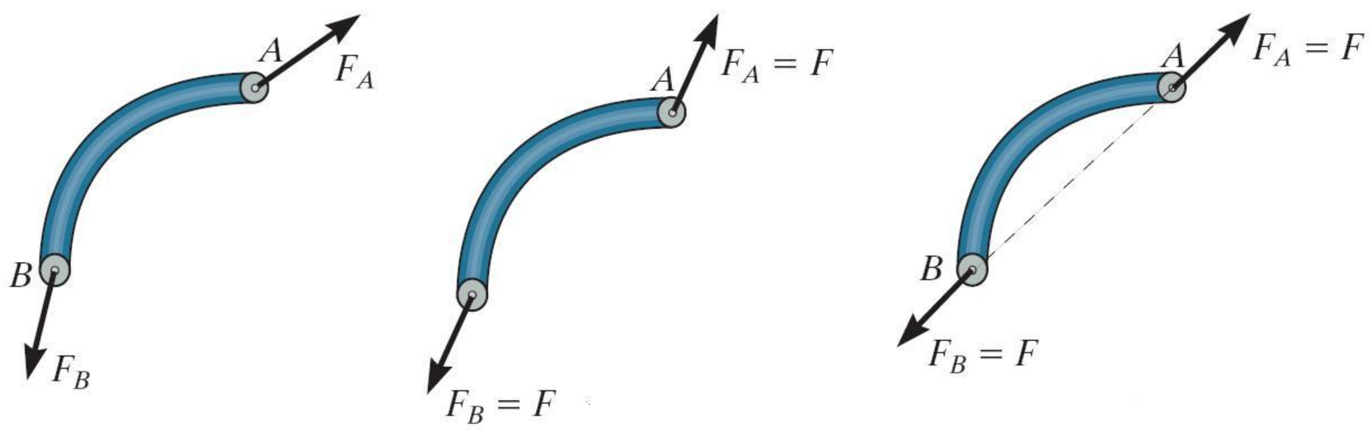
- If a support prevents the translation of a body in a given direction, then a force is developed on the body on that direction
- If a rotation is prevented, a couple moment is exerted on the body

Two-force members ("2FM")

As the name implies, two-force members have forces applied at only two points.

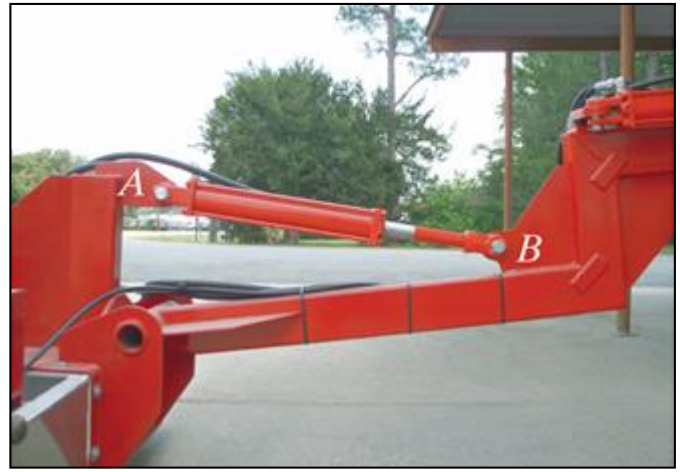
If we apply the equations of equilibrium to such members, we can quickly determine that **the <u>resultant forces</u> at A and B must be equal in magnitude and act in the opposite directions along the line joining points A and B.**

$\Sigma \vec{F} = 0 \rightarrow \Sigma \vec{F} = \vec{F}_A + \vec{F}_B = 0 \rightarrow \vec{F}_B = -\vec{F}_A$ (opposite directions) $\rightarrow |\vec{F}_A| = |\vec{F}_B| = F$ (equal magnitudes)

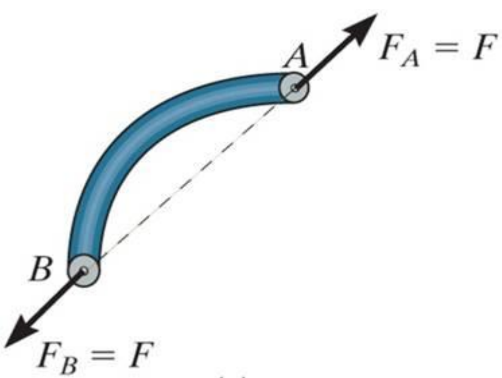


Two-force member: the two forces at ends are equal, opposite, collinear

Examples of two-force members



In the cases above, members AB can be considered as two-force members, provided that their weight is neglected.

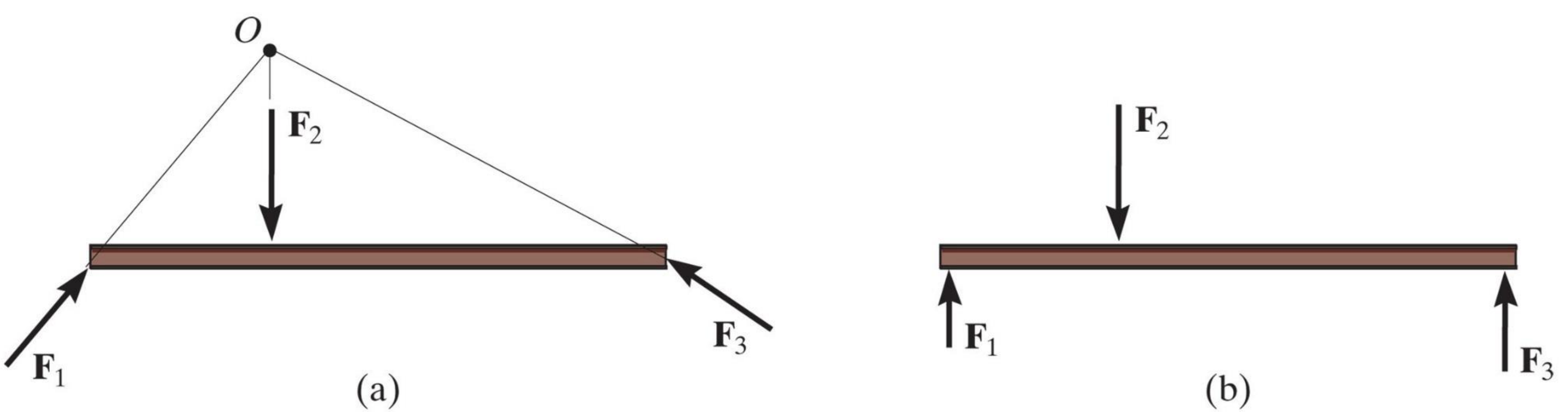


Two-force members **simplify** the equilibrium analysis of some rigid bodies since the **directions of the resultant forces at A and B are thus known** (along the line joining points A and B).

Three-force members ("3FM")

As the name implies, three-force members have forces applied at only three points.

Moment equilibrium can be satisfied only if the three forces are concurrent or parallel force system

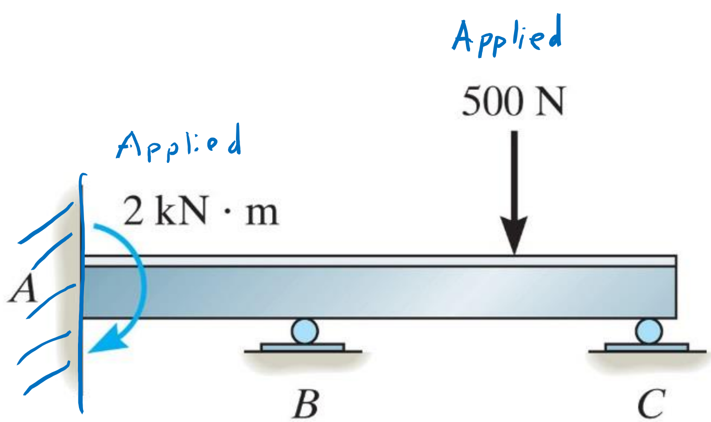


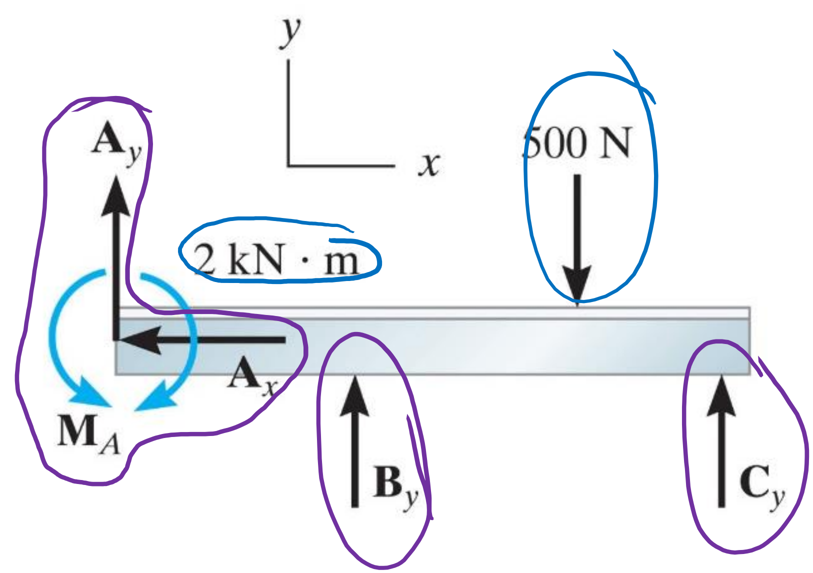
Three-force member: a force system where the three forces
(a) meet at the same point (point O), or
(b) are parallel

Constraints

To ensure equilibrium of a rigid body, it is not only necessary to satisfy equations of equilibrium, but the body must also be properly constrained by its supports

- **Redundant constraints**: the body has more supports than necessary to hold it in equilibrium; the problem is STATICALLY INDERTERMINATE and cannot be solved with statics alone. Too many unknowns, not enough equations



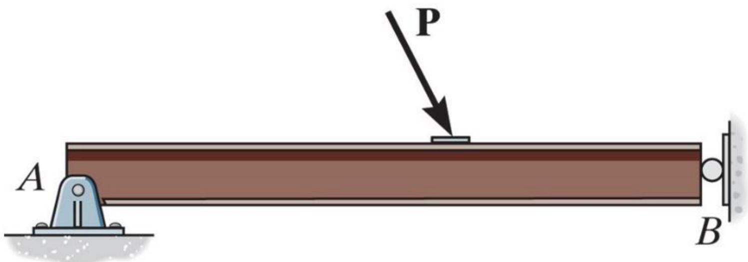


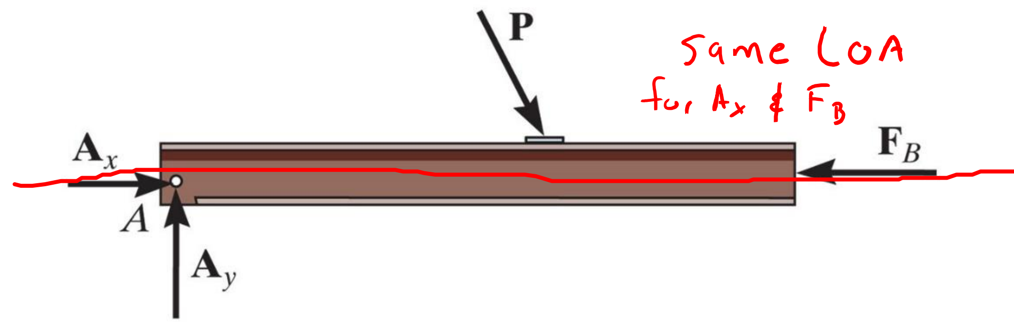
5 unknowns: A_x, A_y, B_y, C_y, M_A

3 eqns $\Sigma F_x, \Sigma F_y, \Sigma M$

Constraints

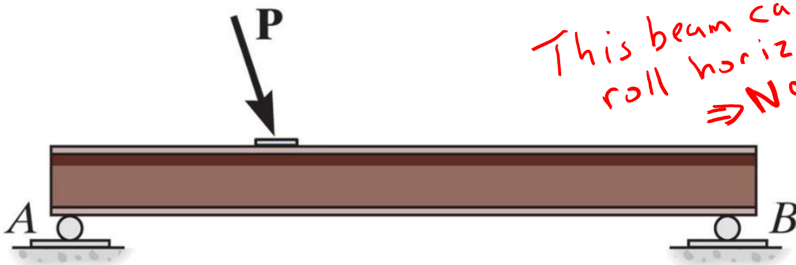
- **Improper constraints:** In some cases, there may be as many unknown reactions as there are equations of equilibrium (statically determinate). However, if the supports are not properly constrained, the body may become unstable for some loading cases.
  - BAD: Reactive forces are concurrent at same point (point A) or line of action



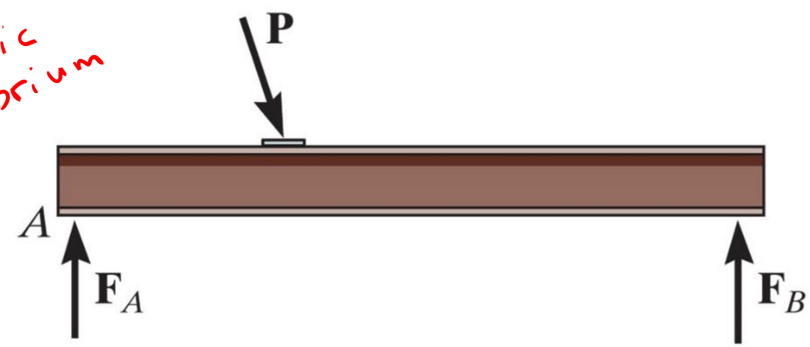


Same LOA for A_x & F_B

- BAD: Reactive forces are parallel



This beam can roll horizontally
⇒ Not static equilibrium



Stable body: lines of action of reactive forces do not intersect at common axis, and are not parallel

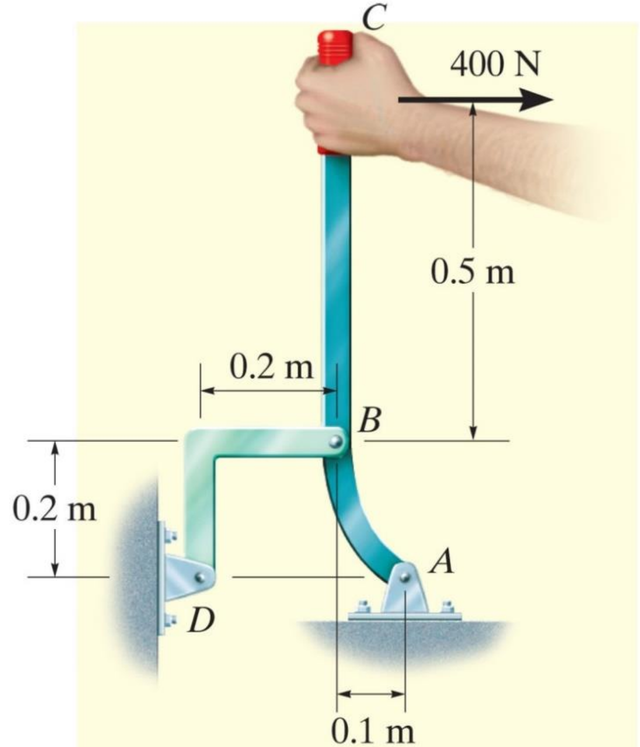
Two-force and three-force members

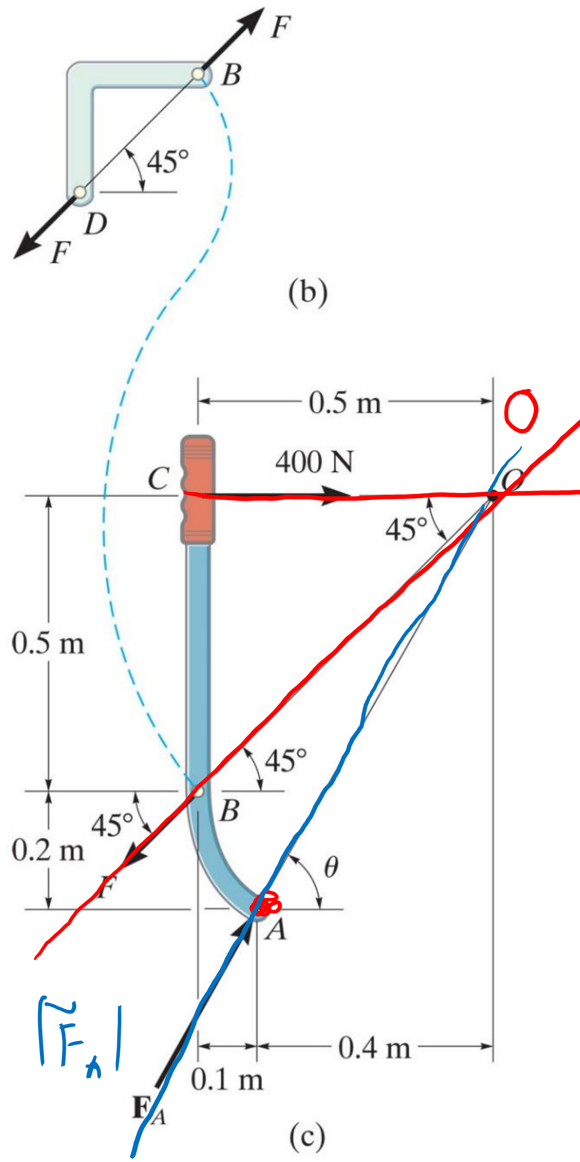
One can use these concepts to quickly identify the direction of an unknown force.

Two-force member: (2FM)
the two forces at ends are equal, opposite, collinear

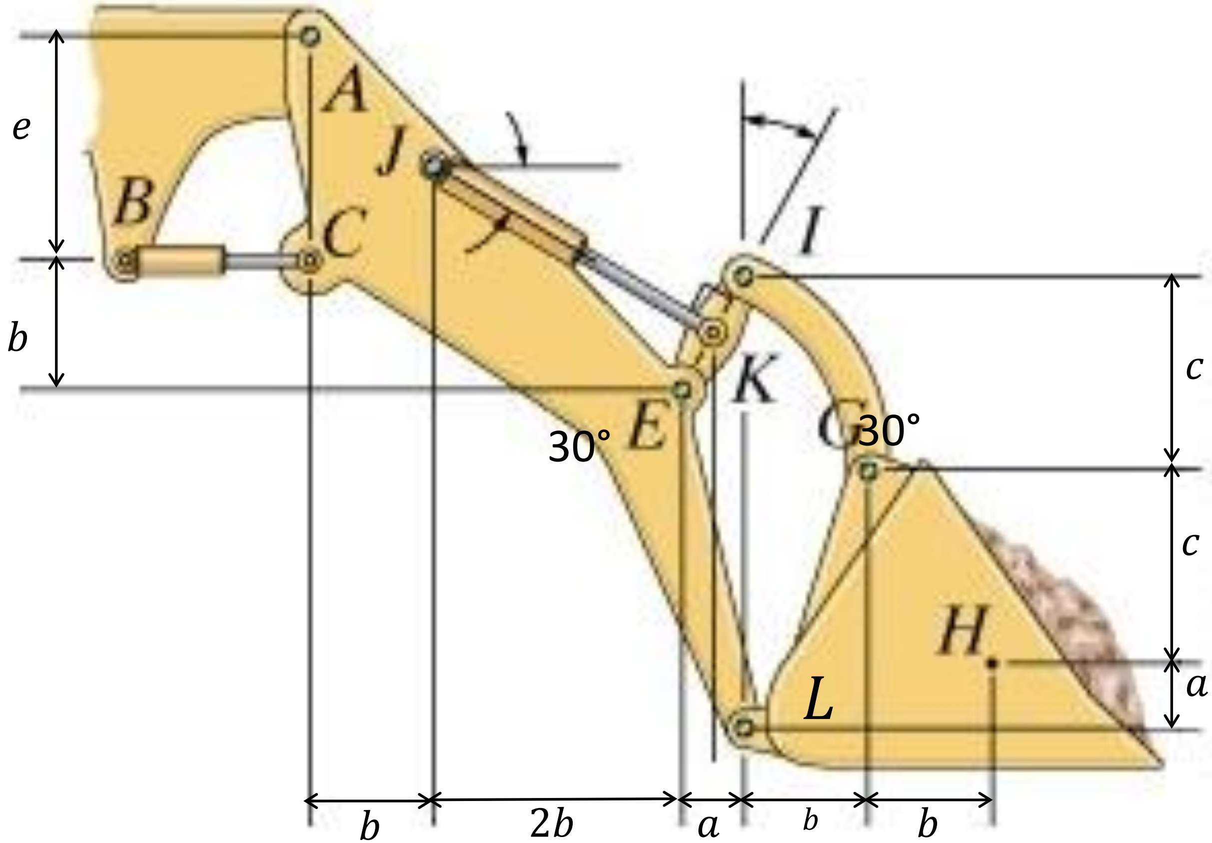
(3FM)
Three-force member: a force system where the three forces

1. meet at the same point (point O), or
2. are parallel





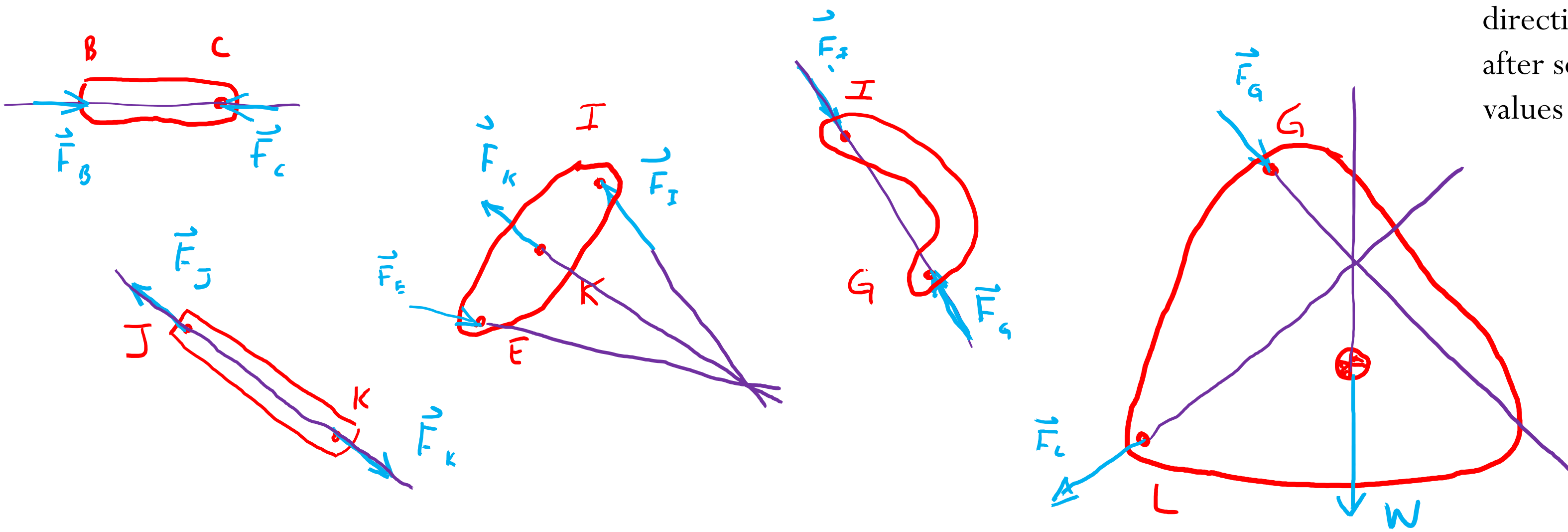
See Example 5.13 in text



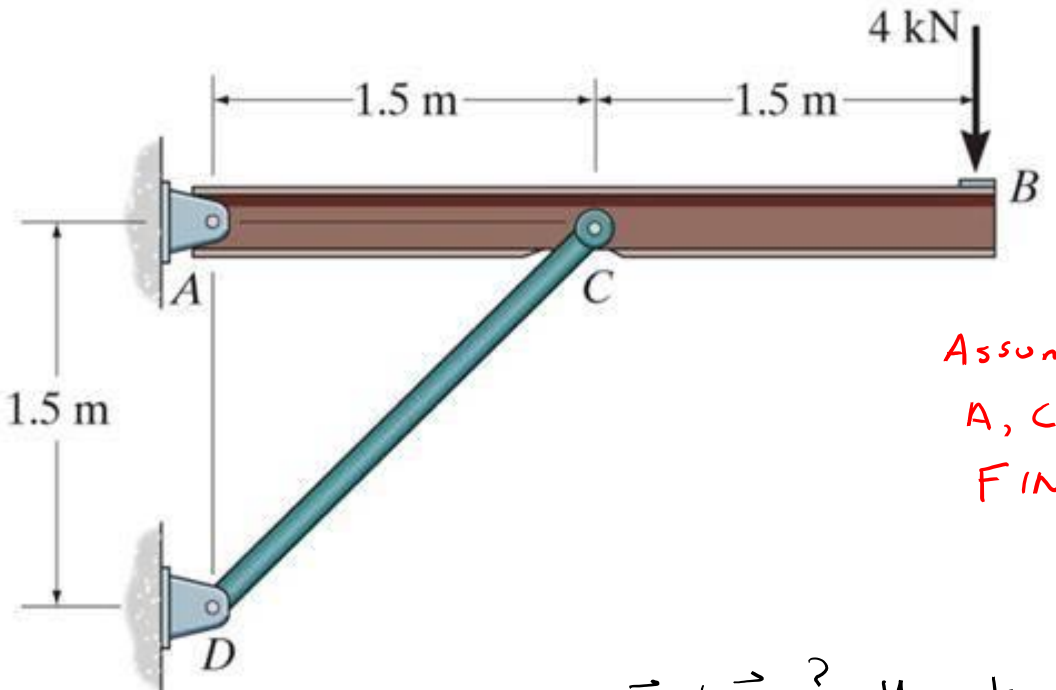
Draw FBDs for each two or three force member (BC, JK, IE, I,G, Bucket).

Ignore weight of each link.

Include dirt weight in bucket.



Directions of arrows of unknown forces/moments are arbitrary on FBD. Actual direction will be determined after solving for unknown values



Given the 4kN load at B of the beam is supported by pins at A and C. Find the support reactions at A and C.

Assume beams AB & CD are massless.

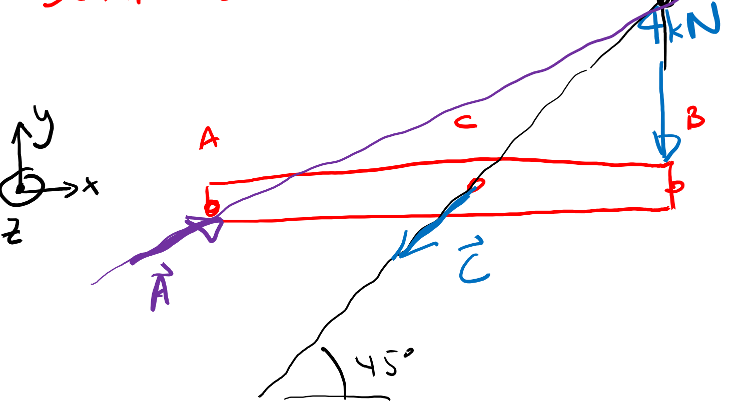
A, C, D are pinned joints.

FIND: $\vec{A}$ & $\vec{C}$ (reaction forces)

(A_x, A_y) (C_x, C_y)

How to know directions of $\vec{A}$ & $\vec{C}$? Use knowledge of 2FM & 3FM.

Beam AB: 3FM

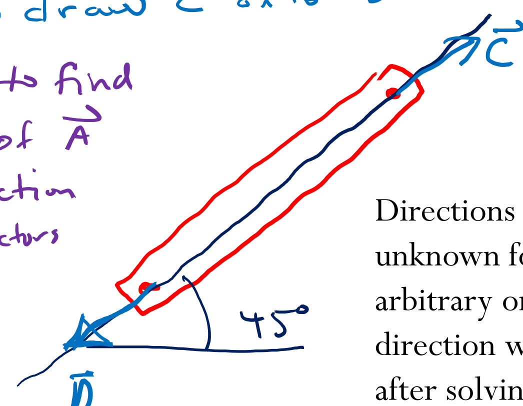


Beam CD: 2FM

① Find direction of $\vec{C}$ from 2FM

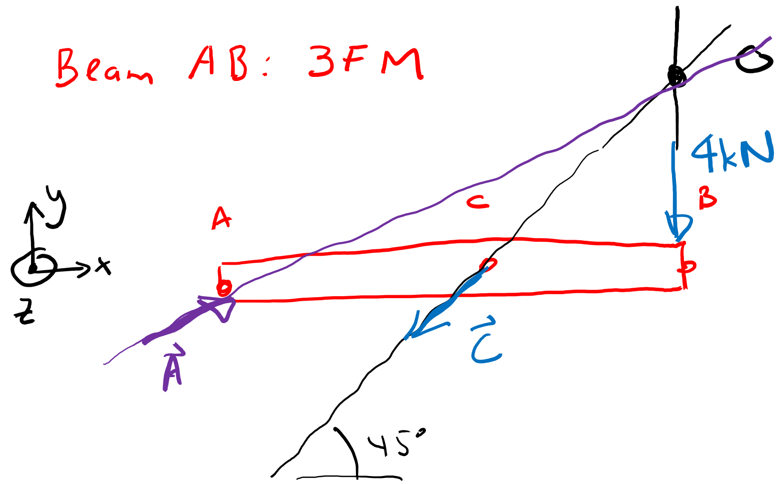
② draw $\vec{C}$ onto beam AB as equal & opposite of force on beam CD

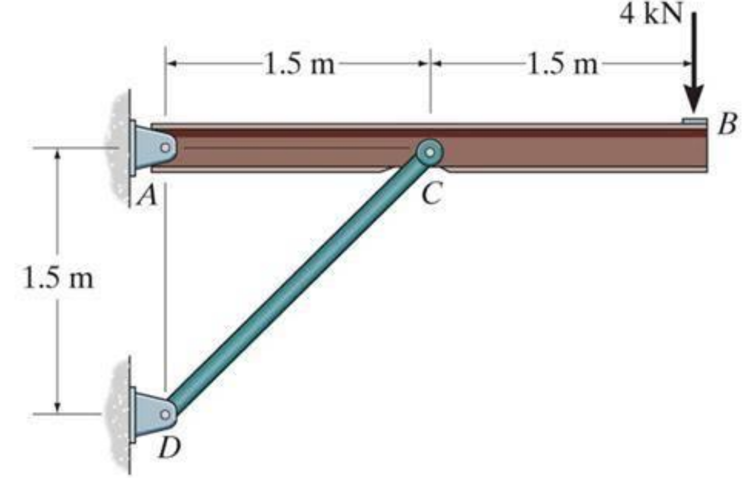
③ use 3FM to find direction of $\vec{A}$ as intersection of force vectors



Directions of arrows of unknown forces/moments are arbitrary on FBD. Actual direction will be determined after solving for unknown values

Beam AB: 3FM





On Beam AB:

4 unknowns: $\vec{A}(A_x, A_y)$ & $\vec{C}(C_x, C_y)$ ⇒ only 3 EoE ⇒ #UNK > #EQNS

$\Sigma F_x = 0$ ①

$\Sigma F_y = 0$ ②

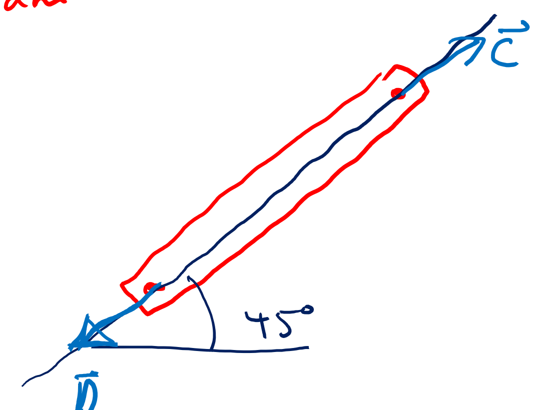
$\Sigma M_? = 0$ ③

∴ Need more eqns!

Get more eqns from looking at another body

← pick ΣM_A so do not need to solve for $\vec{A}$

Beam CD: 2FM



$\Sigma F_x = 0$ ④

$\Sigma F_y = 0$ ⑤

$\Sigma M_D = 0$ ⑥

⇒ 6 UNK $(A_x, A_y, C_x, C_y, D_x, D_y)$ & 6 EQNS ∴ can solve

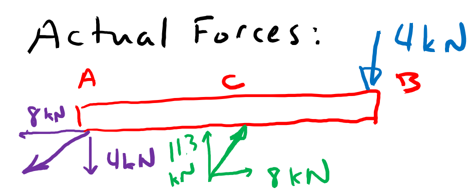
Final solution:

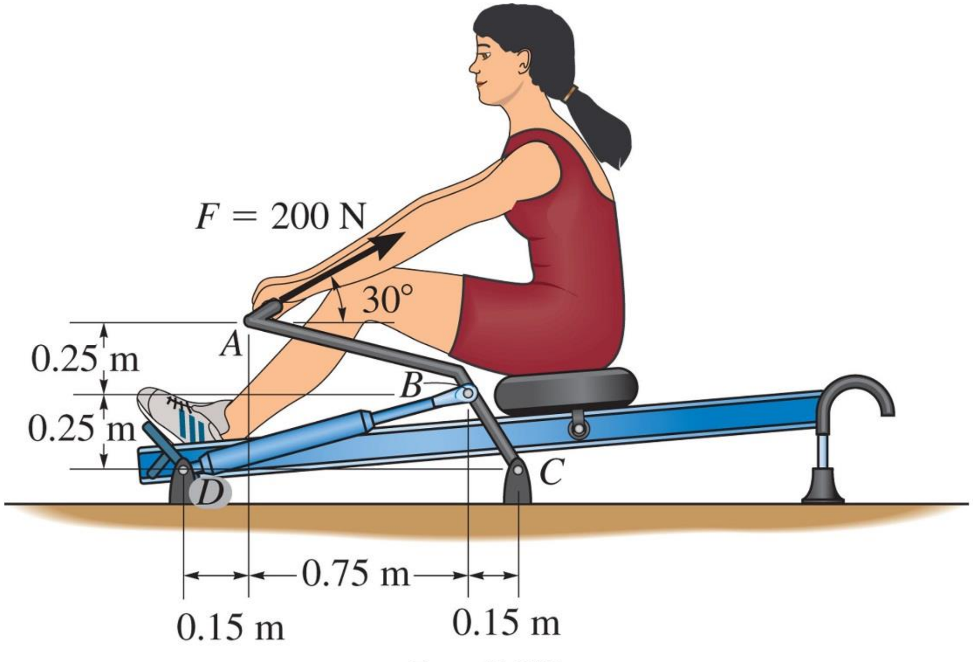
$A_x = -8\text{kN}(\hat{i})$ $A_y = -4\text{kN}(\hat{j})$

$C_x = 8\text{kN}(\hat{i})$ $C_y = 11.3\text{kN}(\hat{j})$

$D_x = -8\text{kN}(\hat{i})$ $D_y = -11.3\text{kN}(\hat{j})$

Actual Forces:





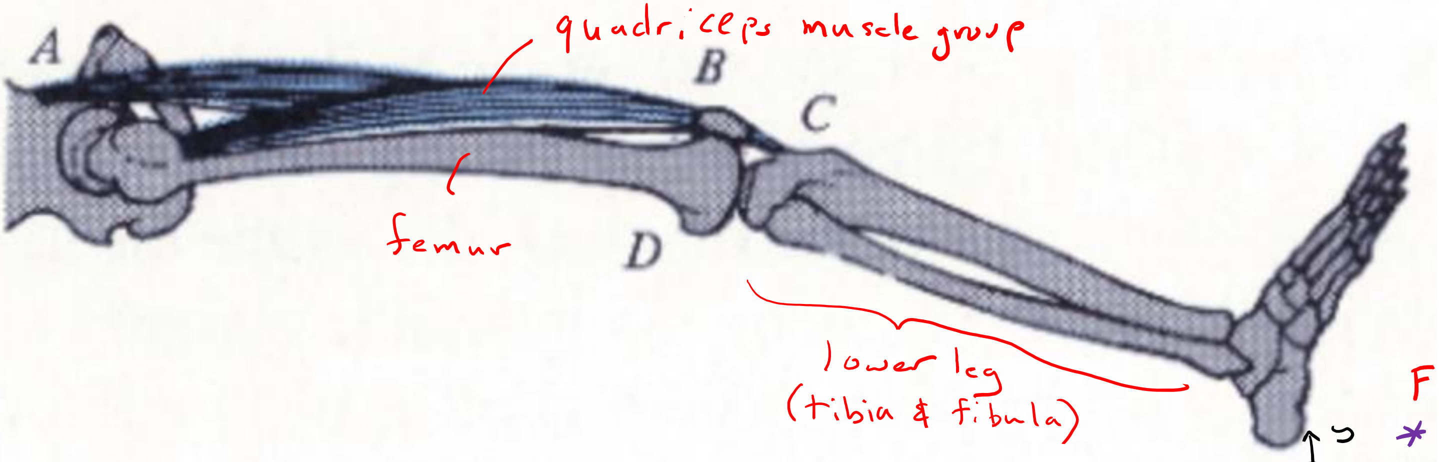
The woman exercises on the rowing machine. If she exerts a holding force of F = 200 N on the handle ABC, determine the reaction force at pin C and the force developed along the hydraulic cylinder BD on the handle.

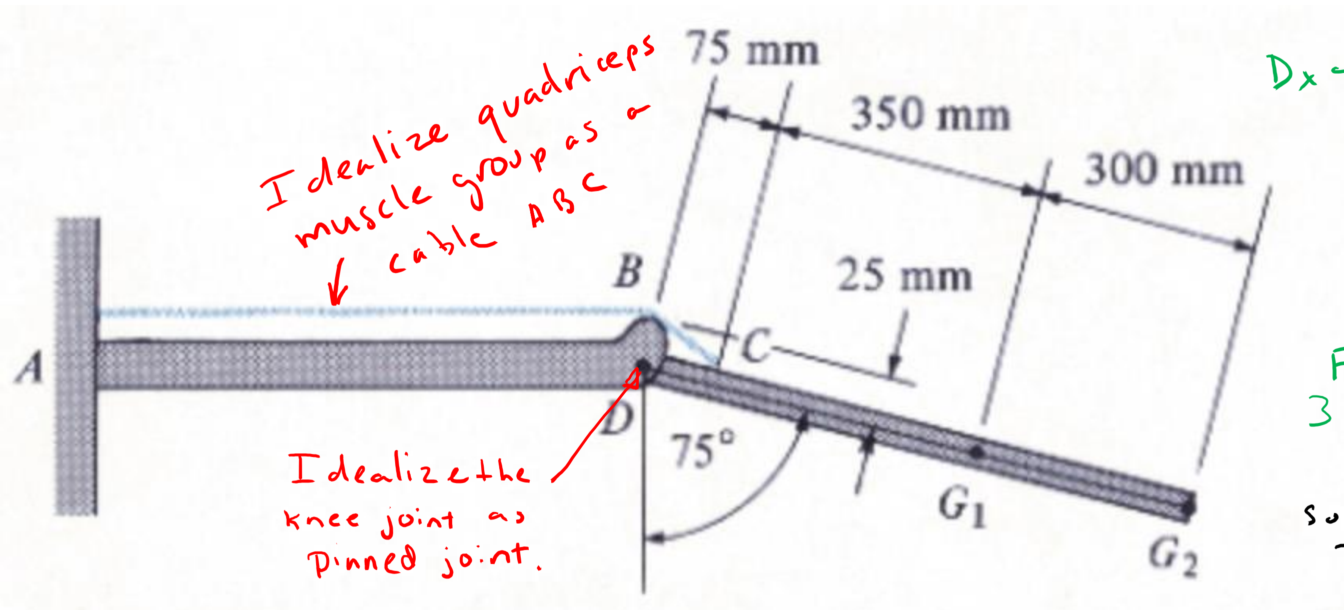
Practice with this problem.

BD : 2FM , ABC : 3FM

Find: $\vec{C}$, $\vec{B}$

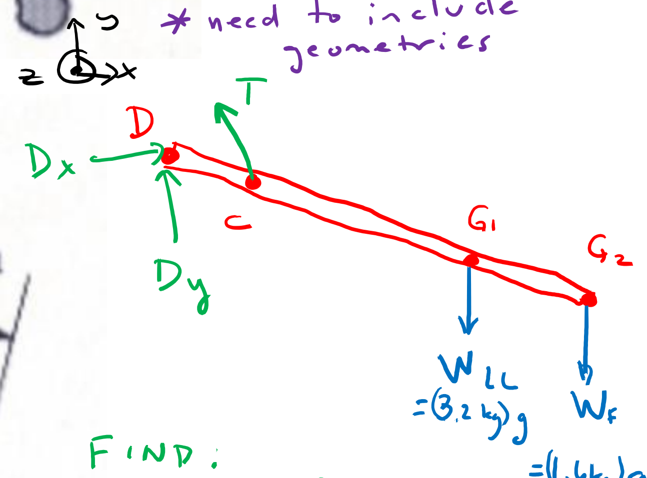
A skeletal diagram of the lower leg is shown. Model the lower leg and determine the tension T in the quadriceps and the magnitude of the resultant force at the femur (pin) at D in order to hold the lower leg in the position shown. The lower leg has a mass of 3.2 kg and the foot has a mass of 1.6 kg.





FBD of lower leg:

* need to include geometries

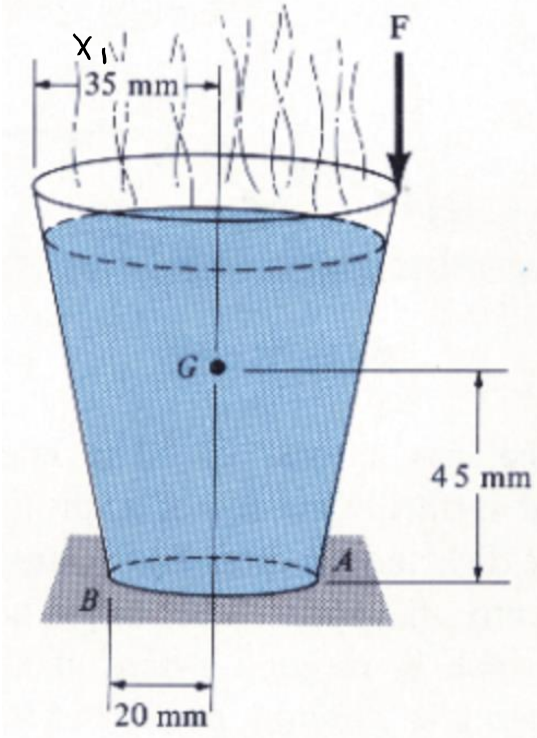


FIND:

3 unknown forces: D_x, D_y, T

Solutions:

$T = 1007\text{ N}$, $D_x = 982\text{N}$, $D_y = 508\text{N}$

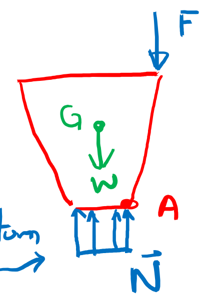


x_2

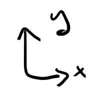
The cup is filled with 125 g of liquid. The mass center is located at G. If a vertical force F is applied to the rim of the cup, determine its magnitude so the cup is on the verge of tipping over.

"On the verge of tipping" means that concentrated load $\vec{N}$ acts at a specific point.

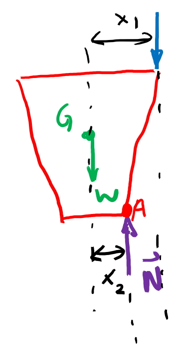
FBD



Distributed load on bottom of cup if cup is level



FBD if on verge of tipping



$\xrightarrow{+} \Sigma F_x : 0$

$+\uparrow \Sigma F_y : N - G - F = 0$

$+\circlearrowleft \Sigma M_A : W(x_2) - F(x_1 - x_2) = 0$

"verge of tipping" = almost, but does not rotate

Solution:

$F = 1635\ N$

"verge": edge, limit