Statics - TAM 211

Lecture 16

October 31, 2018

Announcements

- Upcoming deadlines:

- **Friday (11/2) all in Teaching Building A418-420**
  - 8:00 am: Quiz 3, Chapter 5. On paper.
  - 9:00 am: Lecture 17
  - 10:00 am: Discussion section for ALL students
- Friday (11/2)
  - Written Assignment 6
- Tuesday (11/6)
  - Prairie Learn HW7

2D rigidbodies [3 eqns]

Quiz 4:
3D bodies [6 eqns]

Happy Halloween

http://freepngimages.com/wp-content/uploads/2015/09/halloween-pumpkins.png

Chapter 5: Equilibrium of Rigid Bodies

Goals and Objectives

- Introduce the free-body diagram for a rigid body
- Develop the equations of equilibrium for a 2D and 3D rigid body
- Solve rigid body equilibrium problems using the equations of equilibrium in 2D and 3D

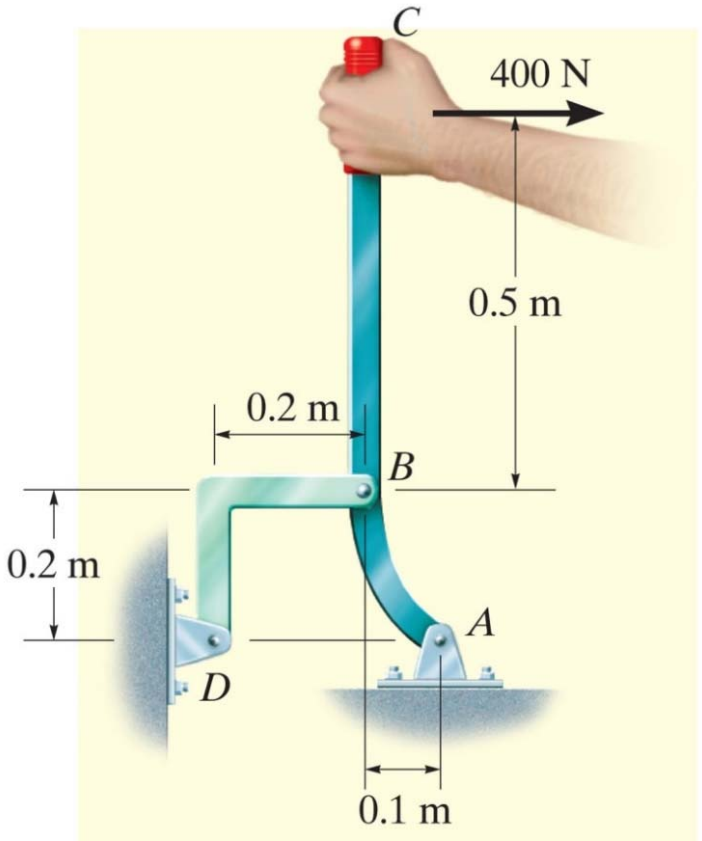
- Introduce concepts of
  - Support reactions for 2D and 3D bodies
  - Two- and three-force members
  - Constraints and statical determinacy

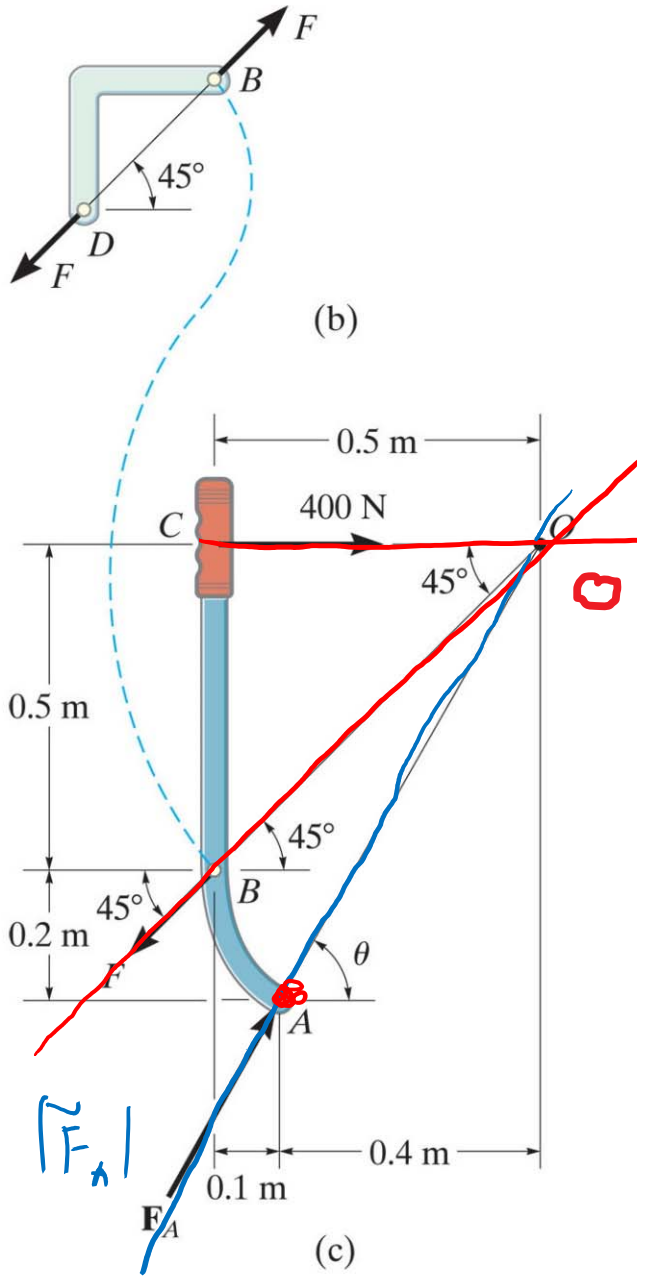
Two-force and three-force members

One can use these concepts to quickly identify the direction of an unknown force.

Two-force member: (2FM)
the two forces at ends are equal, opposite, collinear

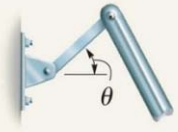
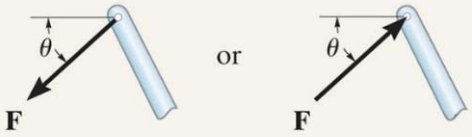
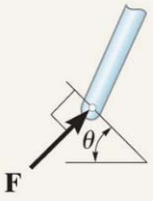
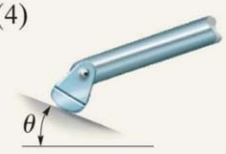
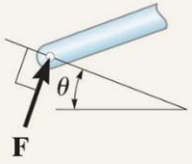
Three-force member: (3FM) a force system where the three forces
1. meet at the same point (point O), or
2. are parallel

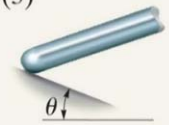
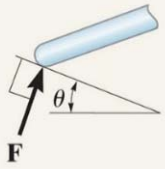
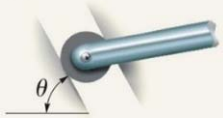
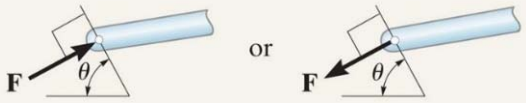
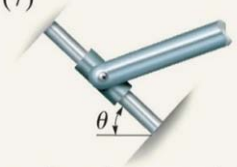
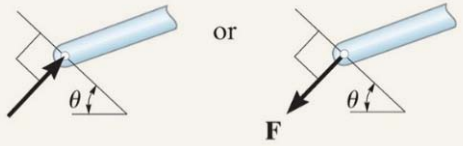
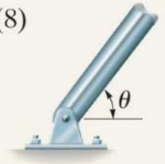
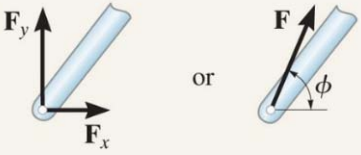
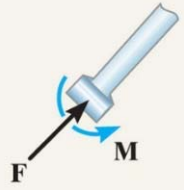
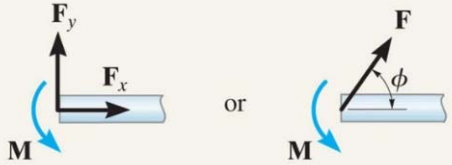




See Example 5.13 in text

Types of 2D connectors

Types of Connection	Reaction
(1)  cable	
(2)  weightless link	
(3)  roller	
(4)  rocker	

Types of Connection	Reaction
(5)  smooth contacting surface	
(6)  roller or pin in confined smooth slot	
(7)  member pin connected to collar on smooth rod	
(8)  smooth pin or hinge	
(9)  member fixed connected to collar on smooth rod	
(10)  fixed support	

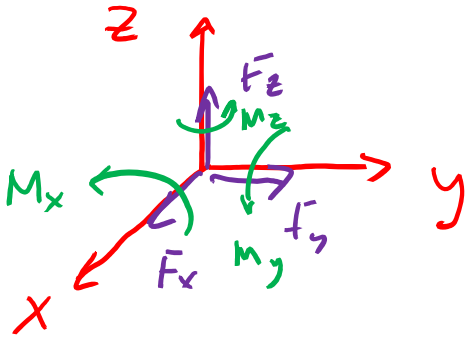
Equilibrium of a rigid body

Now we add the z-axis to the coordinate system!

Equilibrium of a rigid body

Now we add the z-axis to the coordinate system!

How many Equations of Equilibriums?



Six equations!

2D Problems

Translation: $\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$

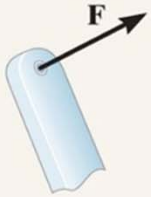
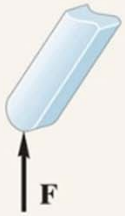
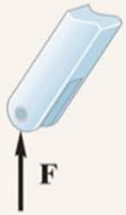
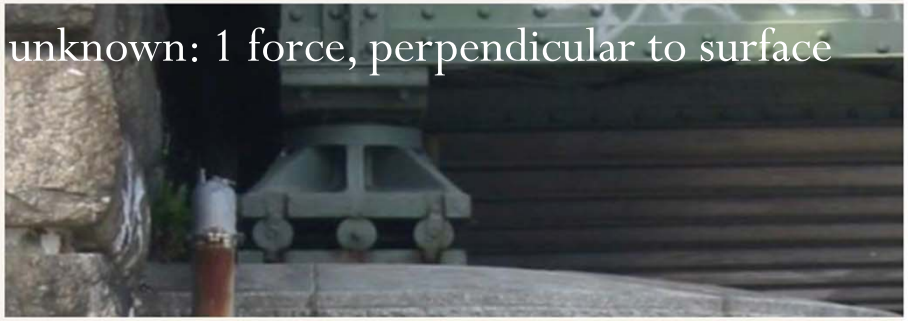
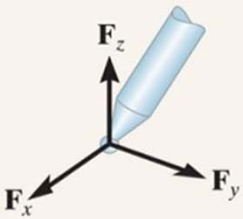
Rotation: $\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$

6 Eqns

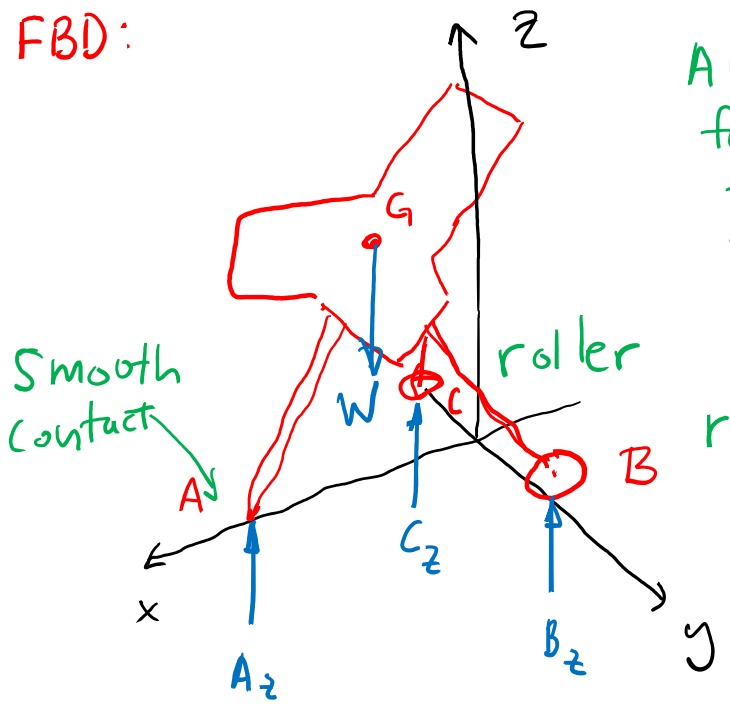
⇒ 6 unknowns can be solved!

For one FBD

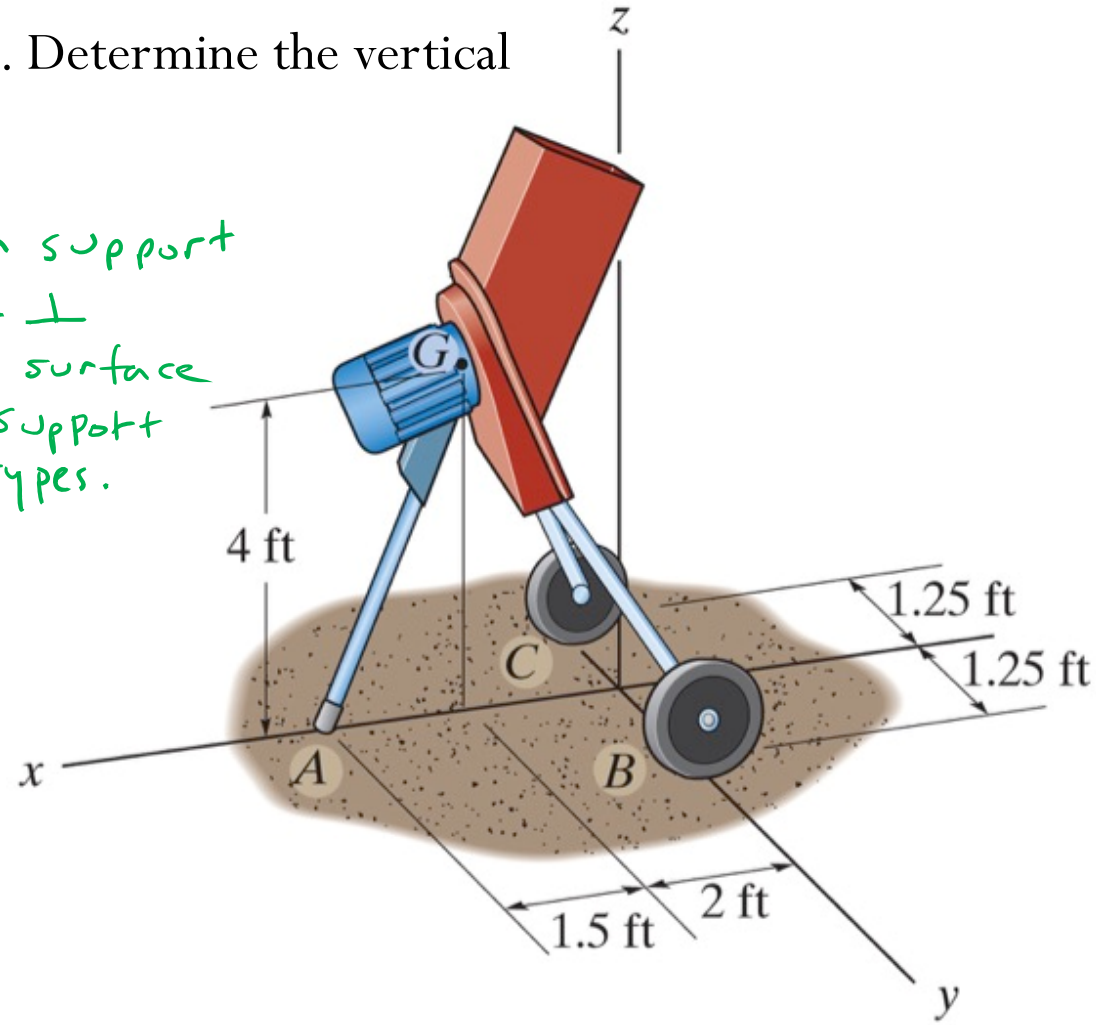
TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		1 unknown: 1 force, along cable 
(2)  smooth surface support		1 unknown: 1 force, perpendicular to surface 
(3)  roller		1 unknown: 1 force, perpendicular to surface 
(4)  ball and socket		3 unknowns: 3 forces, prevent translation 

The 50-lb mulching has a center of gravity at G. Determine the vertical reactions at the **smooth contact** point A.



All reaction support forces are $\perp$ to contact surface for these support types.



Find: A_z

3 UNKNOWNS. $A_z, B_z, C_z \Rightarrow$ Need 3 eqns.

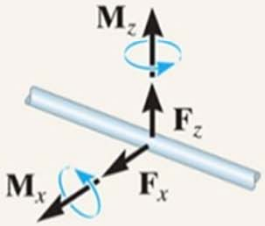
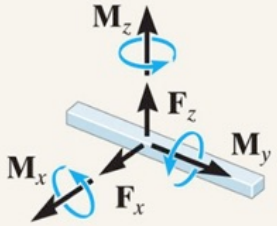
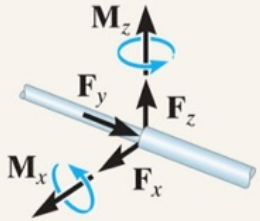
Since the problem statement only asks for reaction force at point A, then only need to find A_z.

Quickest solution is to use $\curvearrowleft \Sigma M_y = 0$: $\Sigma M_y = r_G W - r_A A_z = 0$

$(2\text{ft})(50\text{lb}) - (3.5\text{ft})A_z = 0$

$\Rightarrow \boxed{A_z = \frac{100}{3.5}\text{lb}}$

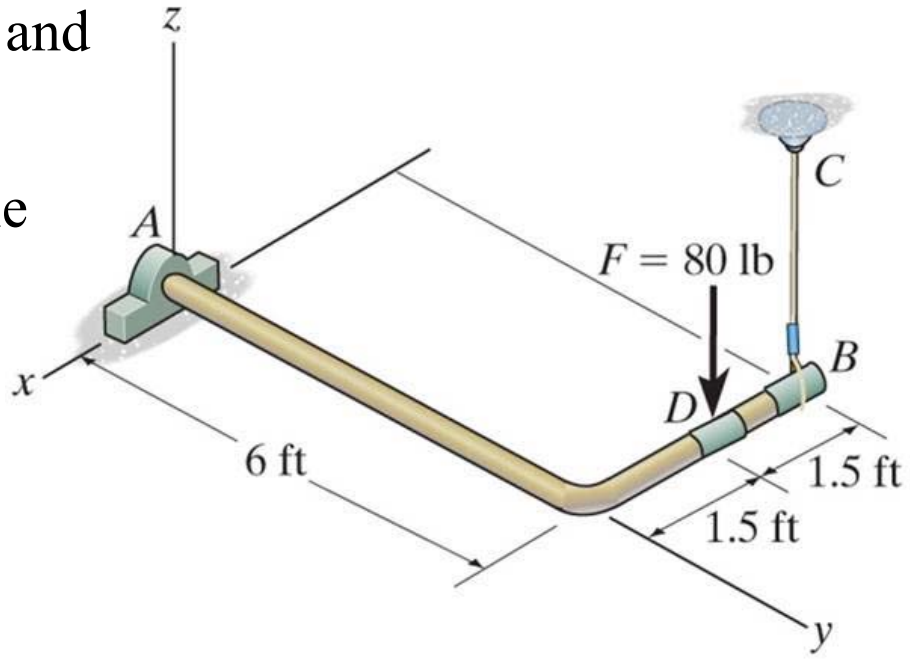
TABLE 5–2 Continued

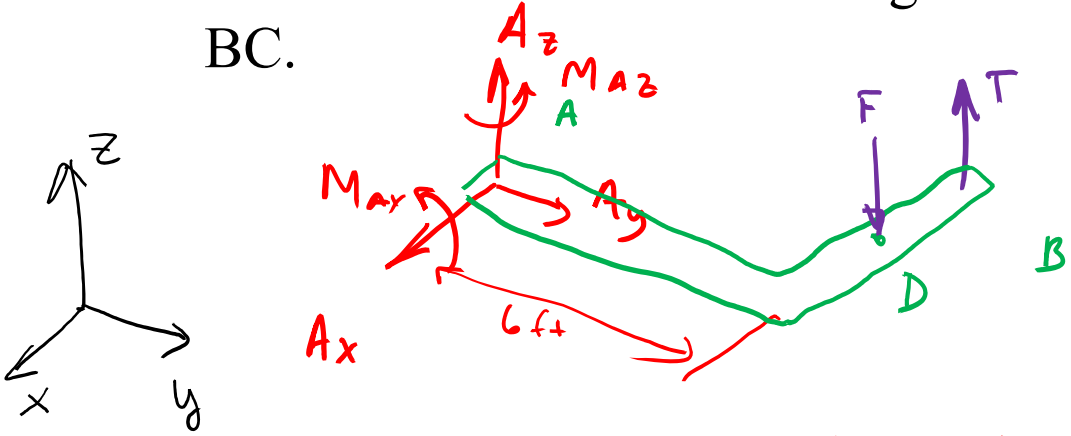
Types of Connection	Reaction	Number of Unknowns
(5)  single journal bearing	 Shaft rotates and slides along axis	 4 unknowns: 2 forces, 2 couple-moments*
(6)  single journal bearing with square shaft	 Shaft ~~rotates and~~ slides along axis	5 unknowns: 2 forces, 3 couple-moments* 
(7)  single thrust bearing	 Shaft rotates ~~and slides~~ along axis	5 unknowns: 3 forces, 2 couple-moments* 

* Couple-moments are not applied to FBD if the body is supported elsewhere by additional bearings, pins or hinges that are **properly aligned** to prevent rotation in one or more axes).

Given: The rod, supported by thrust bearing at A and cable BC, is subjected to an 80 lb force.

Find: Reactions at the thrust bearing A and cable BC.





6 unknowns: $T, A_x, A_y, A_z, M_{Ax}, M_{Az}$ ⇒ use all 6 eqns

$\Sigma F_x = 0$: $\boxed{A_x = 0}$

$\Sigma F_y = 0$: $\boxed{A_y = 0}$

$\Sigma F_z = 0$: $A_z - T - 80\,lb = 0 \Rightarrow A_z = 80 - T$

$\Sigma M_x = 0$: $M_{Ax} + (6\,ft)T - (6\,ft)\,80\,lb = 0 \Rightarrow M_{Ax} = 480 - 6T$

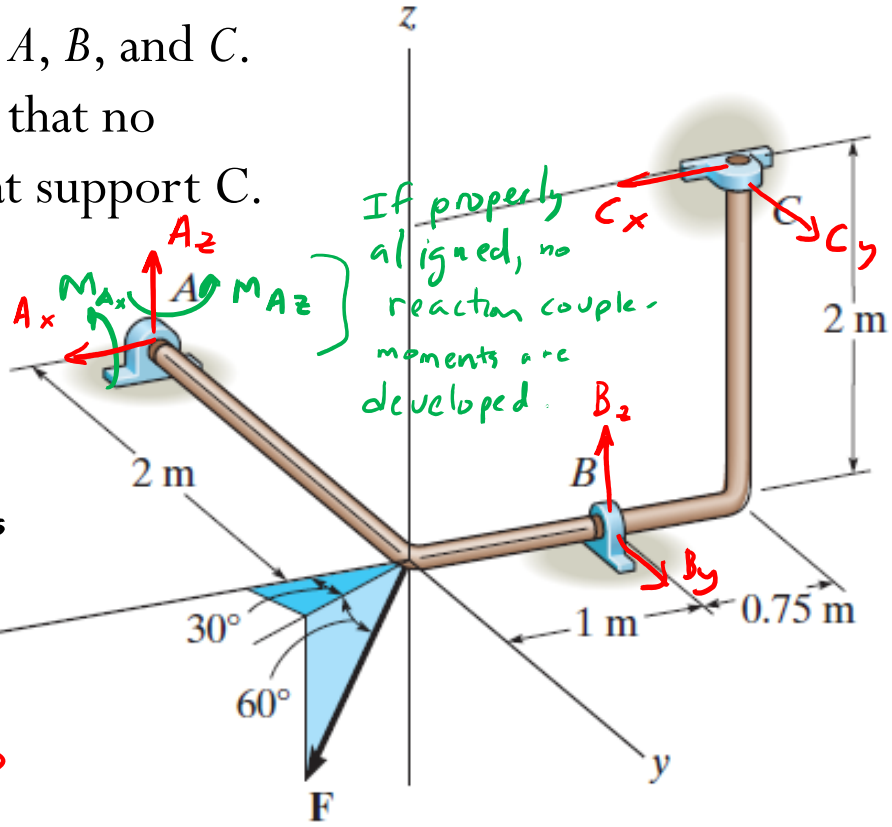
$\Sigma M_y = 0$: $(3\,ft)T - (1.5\,ft)\,80\,lb = 0 \Rightarrow \boxed{T = 40\,lb}\uparrow \therefore \boxed{A_z = 40\,lb}\uparrow$

$\Sigma M_z = 0$: $\boxed{M_{Az} = 0}$

$\boxed{M_{Ax} = 240\ ft\cdot lb\ \curvearrowright}$

Answers: $A_x = 0$, $A_y = 0$, $A_z = 40\,lb\,\hat{k}\uparrow$, $T = 40\,lb\,\hat{k}\uparrow$, $M_{Ax} = 240\,ft\cdot lb\,\hat{i}$, $M_{Az} = 0$

A bent rod is supported by smooth journal bearings at A, B, and C. $F = 800$ N. The supports are **properly aligned** such that no moment support is present. Determine the reactions at support C.



"Properly aligned" means that the supports are perfectly collinear or orthogonal.

⇒ Properly aligned

⇒ only 6 unknowns

$A_x, A_z, B_y, B_z, C_x, C_y$

∴ can solve for with 6 equations

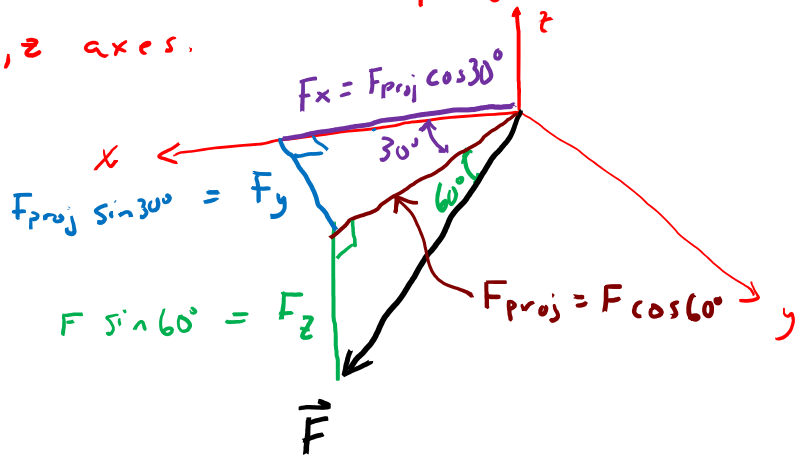
Pointers for this problem:

1) Bearings are properly aligned

∴ No couple-moments at bearings in FBD

Since JOURNAL bearings, only have reaction forces in axes ⊥ to shaft axis.

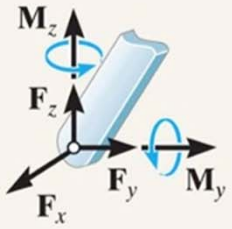
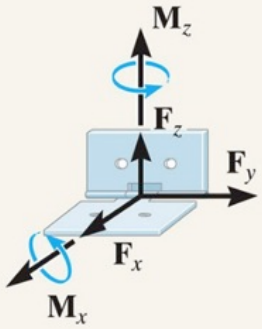
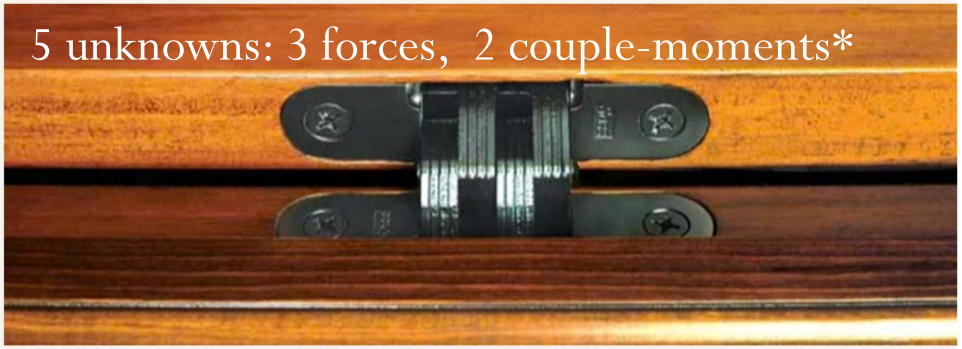
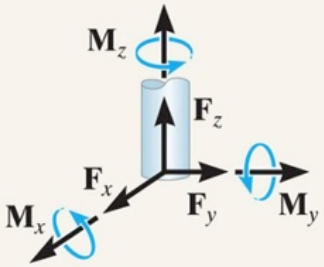
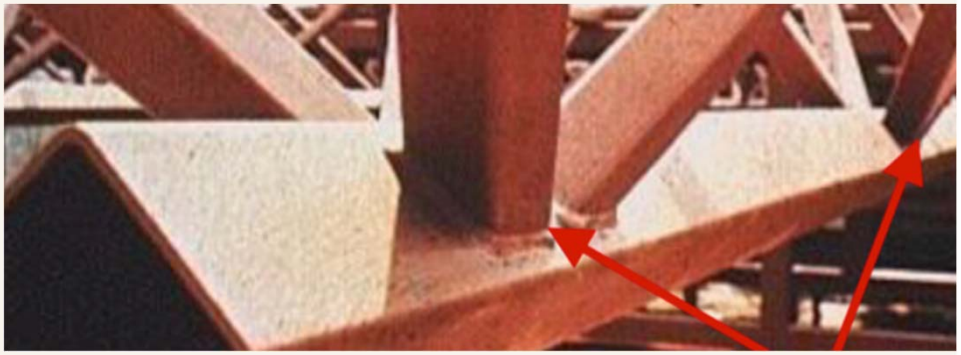
2) For applied force $\vec{F}$, need to consider how $\vec{F}$ will project onto x, y, z axes.



$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

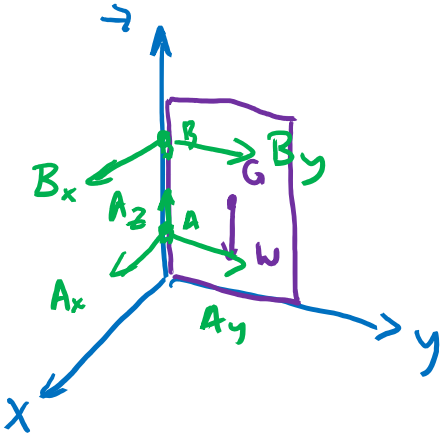
$$\vec{F} = (F\cos 60°)\cos 30°\,\hat{i} + (F\cos 60°)\sin 30°\,\hat{j} - F\sin 60°\,\hat{k}$$

TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  single smooth pin		
(9)  single hinge		
(10)  fixed support		 6 unknowns: 3 forces, 3 couple-moments

Note: for fixed supports, must always apply couple-moments to FBD

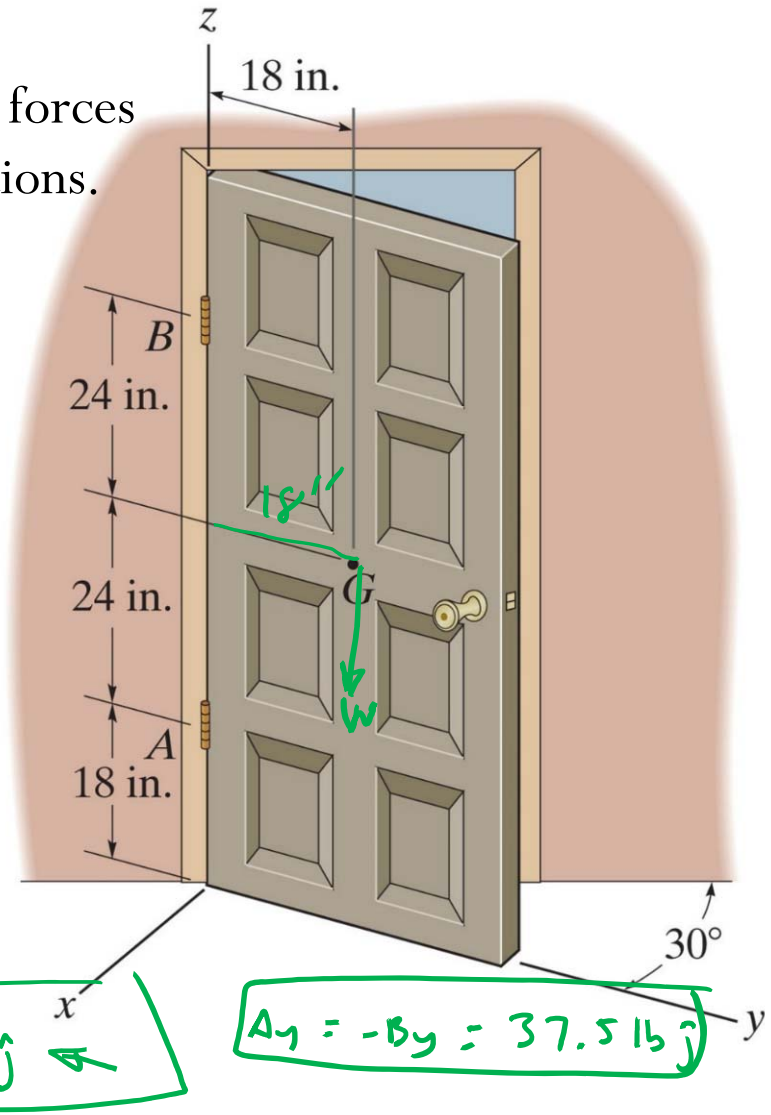
The 100 lb door has its center of gravity at *G*. Determine the components of reaction at hinges *A* and *B* if hinge B resists only forces in the x and y directions and A resists forces in the x, y, z directions.



Assume properly aligned
⇒ No Moments on FBD

Assume hinge A is resisting translation in z-axis, but not hinge B.

Unknowns: 5



SOLN:

$\Sigma F_x: \quad A_x + B_x = 0 \quad A_x = -B_x$

$\Sigma F_y: \quad A_y + B_y = 0 \quad A_y = -B_y$

$\Sigma F_z: \quad A_z - W = 0$

$\boxed{A_z = W = 100 \text{ lb } \hat{k} \uparrow}$

$+\circlearrowleft \Sigma M_x: \quad -18''A_y - 66''B_y - 18''W = 0$

$-18''(-B_y) - 66''B_y - 18''W = 0$

$\boxed{B_y = -37.5 \text{ lb } \hat{j}}$

$\boxed{A_y = -B_y = 37.5 \text{ lb } \hat{j}}$

$+\circlearrowright \Sigma M_y: \quad 18''A_x + 66''B_x = 0$

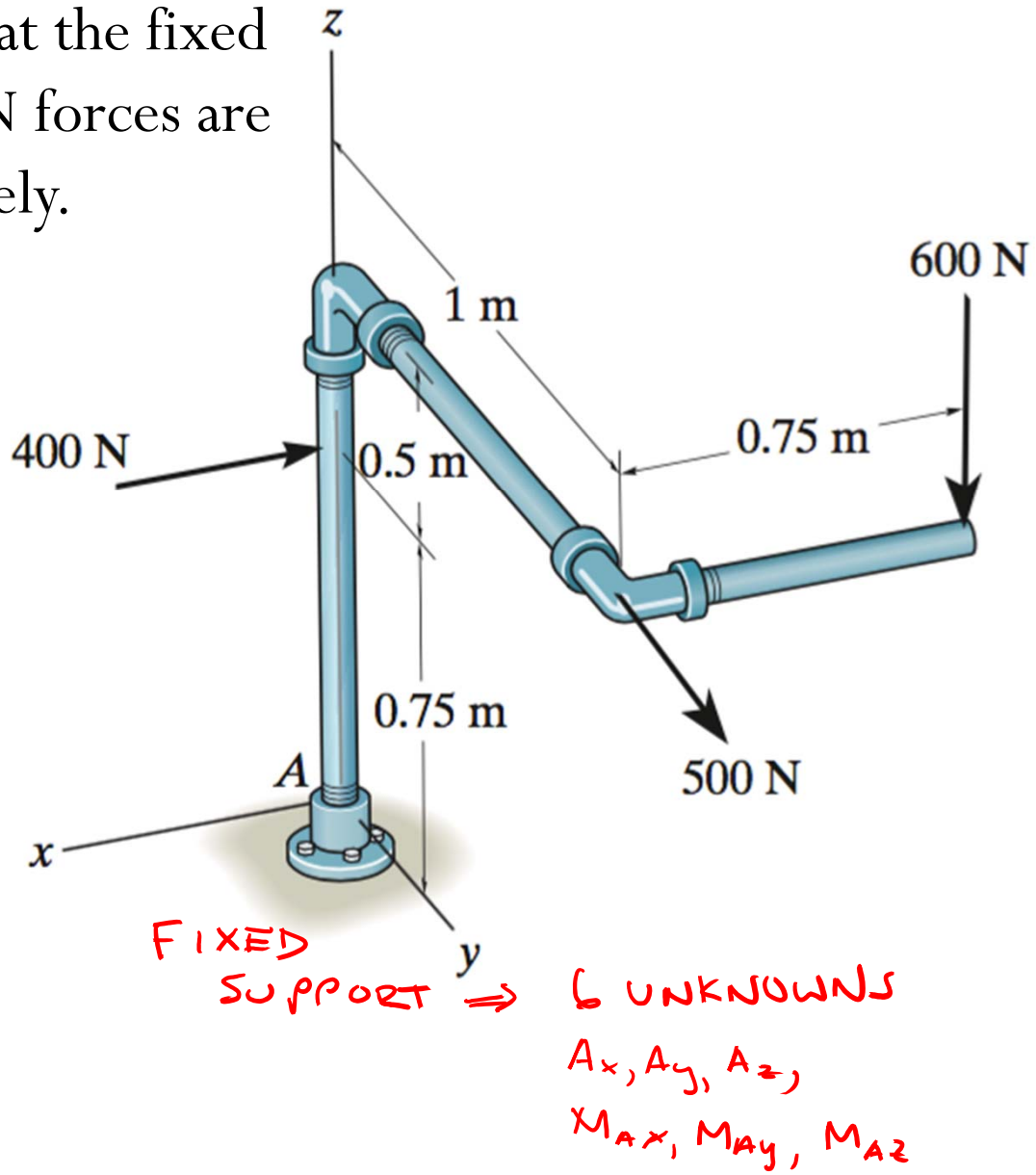
$18(-B_x) + 66B_x = 0$

$48B_x = 0$

$\boxed{B_x = 0}$ $\boxed{A_x = 0}$

Multiple door hinges are ideally "properly aligned" and are perfectly collinear on the same axis. Otherwise, the door will "bind" or have difficulty opening.

Determine the components of reaction at the fixed support A. The 400 N, 500 N, and 600 N forces are parallel to the x, y, and z axes, respectively.



Draw FBD for blue structure.
How many unknowns? ⇒ 6

Check your solution:

$A_x : F_1 = 400\,N\,\hat{i}$

$A_y = -500\,N\,\hat{j}$

$A_z = 600\,N\,\hat{k}$

$M_{Ax} = 1225\,N\,m + \circlearrowleft\,\hat{i}$

$M_{Ay} = 750\,N\,m\,\hat{j}$

$M_{Az} = 0$

FIXED SUPPORT ⇒ 6 UNKNOWNS

$A_x, A_y, A_z,$

M_{Ax}, M_{Ay}, M_{Az}

Calculate the reaction forces and moments at the support D at the base of the structure.

Draw FBD for blue structure.
How many unknowns?

D is fixed support
∴ 6 unknowns : $D_x, D_y, D_z, M_{Dx}, M_{Dy}, M_{Dz}$