Statics - TAM 211

Lecture 18

November 5, 2018

Announcements

- Upcoming deadlines:
- Tuesday (11/6)
  - Prairie Learn HW 7
- Friday (11/9)
  - Written Assignment 7

Chapter 6: Structural Analysis

Goals and Objectives

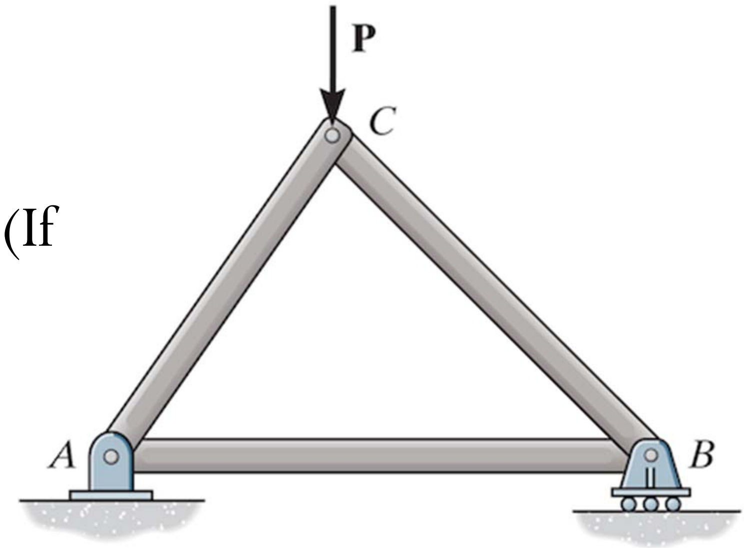
- Determine the forces in members of a truss using the method of joints

- Determine zero-force members

- Determine the forces in members of a truss using the method of sections

- Determine the forces and moments in members of a frame or machine

Recap: Truss Analysis



Assumption of trusses

- Loading applied at joints, with negligible weight (If weight included, vertical and split at joints)
- Members joined by smooth pins
- Pins in equilibrium: $\sum F_x = 0$ and $\sum F_y = 0$

Zero-force members

Two situations:

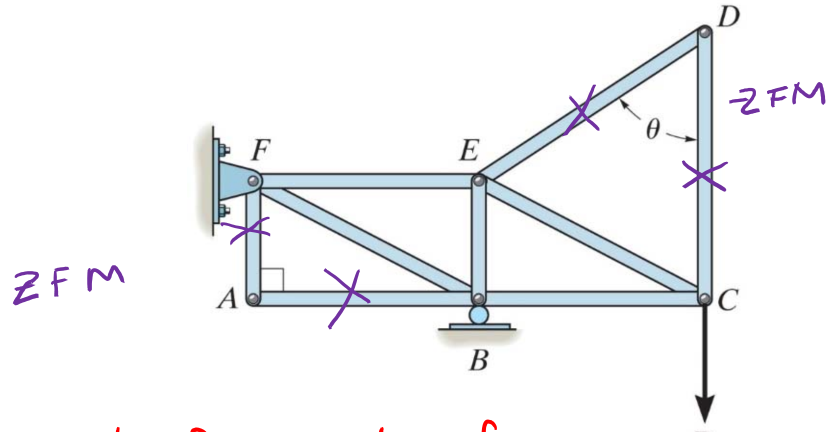
- Two non-collinear members , no external or support at jt → **Both members are ZFM**
- Two collinear member, plus third non-collinear, no loads on third member → **Non-collinear member is ZFM**.

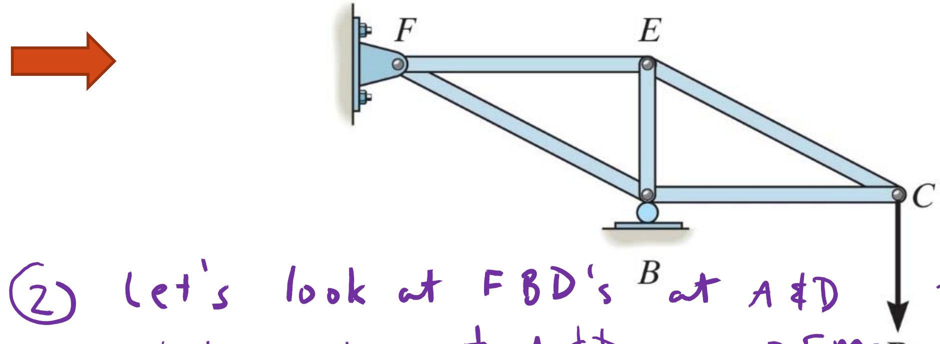
Method of joints

Procedure for analysis to find forces within links:

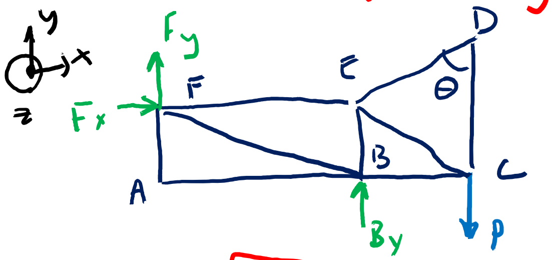
- Determine external support reactions
- Free-body diagram for each joint
- Start with joints with at least 1 known force and 1-2 unknown forces
- Assume the unknown force members to be in *tension*

RECAP: Use Method of joints to prove that members attached to A and D should be FZM





① Solve for reaction forces by considering the entire structure as one single rigid body.



$\Sigma F_x = 0$: $\boxed{F_x = 0}$

$\Sigma F_y = 0$: $F_y + B_y - P = 0$

$\Sigma M_F = 0$: $(r_{FE})B_y - (r_{AC})P = 0$

$+\circlearrowleft$

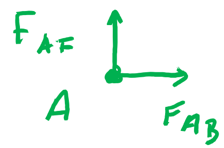
$$\boxed{B_y = \left(\frac{r_{AC}}{r_{FE}}\right)P}$$

$$\boxed{F_y = \left(1 - \frac{r_{AC}}{r_{FE}}\right)P}$$

② Let's look at FBD's at A & D to prove that members at A & D are ZFMs

Assume that there are forces pointing in tension and aligned in direction of truss link members. Label forces as F_{ij} where i is tail and j is arrow head

@ Jt A:

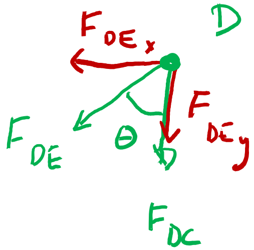


$\Sigma F_x = 0$: $F_{AB} = 0$

$\Sigma F_y = 0$: $F_{AF} = 0$

✓ ZFMs

@ Jt D: $\Sigma F_x = 0$: $F_{DE_x} = 0$



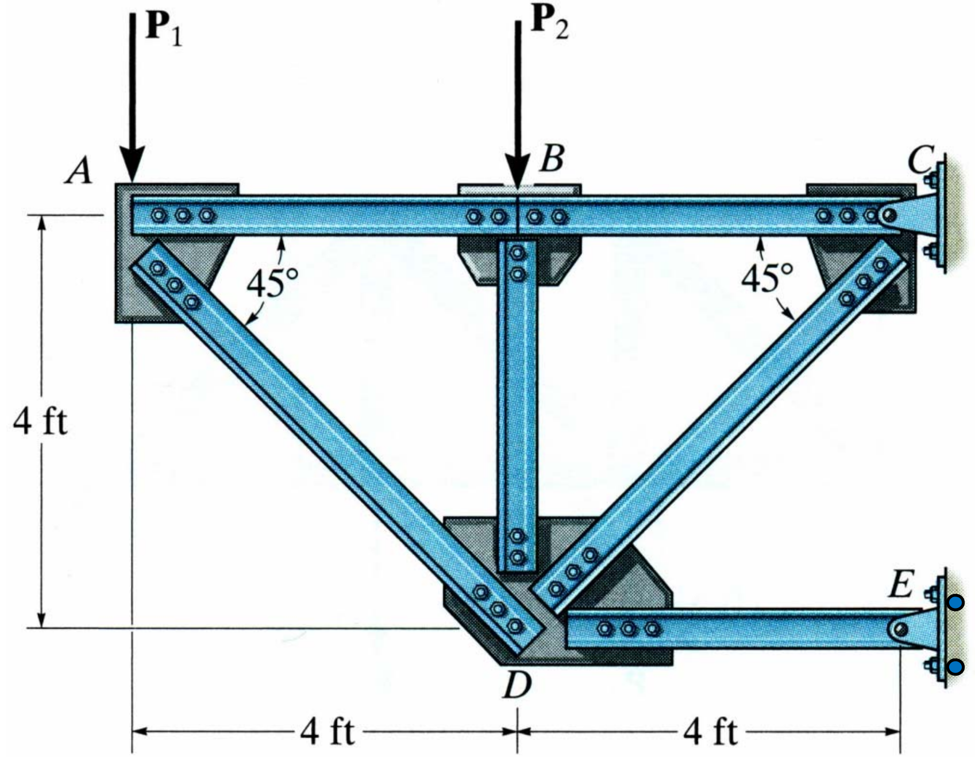
$\Sigma F_y = 0$: $F_{DE_y} + F_{DC} = 0$

Note: $F_{DE_x} = F_{DE} \sin\theta = 0$

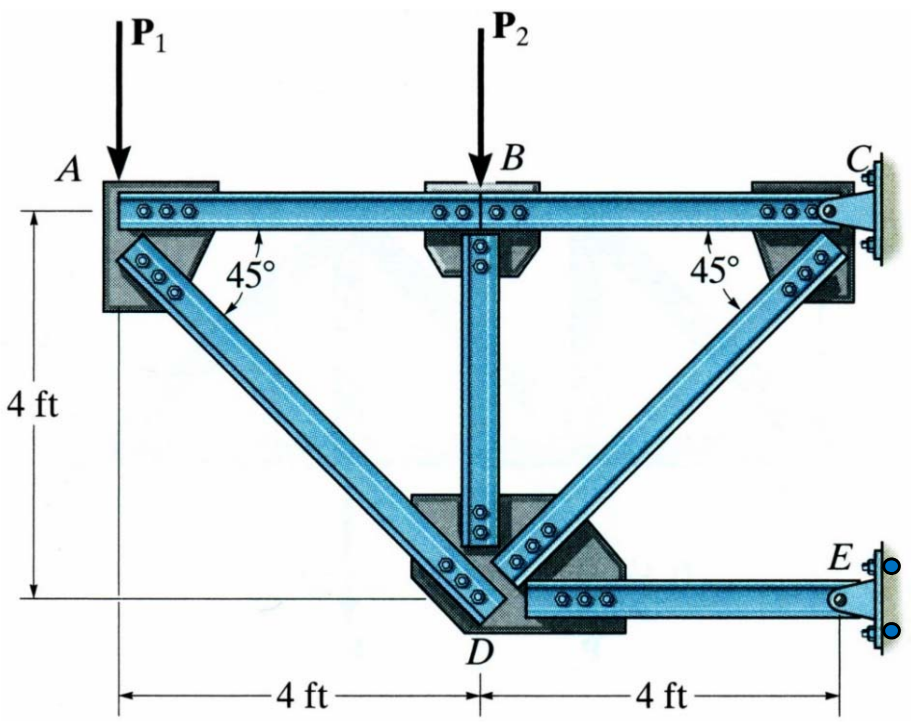
since $\theta \neq 0$, ∴ $F_{DE} = 0$ ✓ ZFM

$\Rightarrow F_{DE_y} + F_{DC} = F_{DE}\cos\theta + F_{DC} = 0$

∴ $F_{DC} = 0$ ✓ ZFM



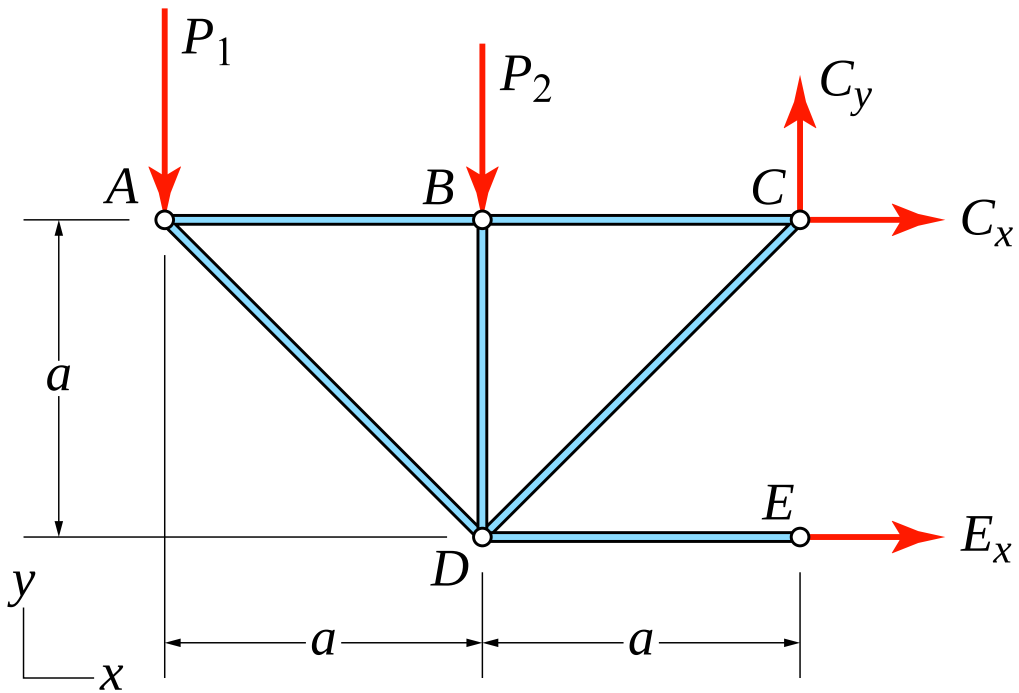
The truss, used to support a balcony, is subjected to the loading shown. Approximate each joint as a pin and determine the force in each member. State whether the members are in tension or compression.



The truss, used to support a balcony, is subjected to the loading shown. Approximate each joint as a pin and determine the force in each member. State whether the members are in tension or compression.

Solution:

Start by setting the entire structure into **external** equilibrium. Draw the FBD.



Equilibrium requires $\sum \boldsymbol{F} = \boldsymbol{0}$ and $(\sum \boldsymbol{M})_C = \boldsymbol{0}$

$$\Sigma F_x = 0: \qquad C_x + E_x = 0\,,$$

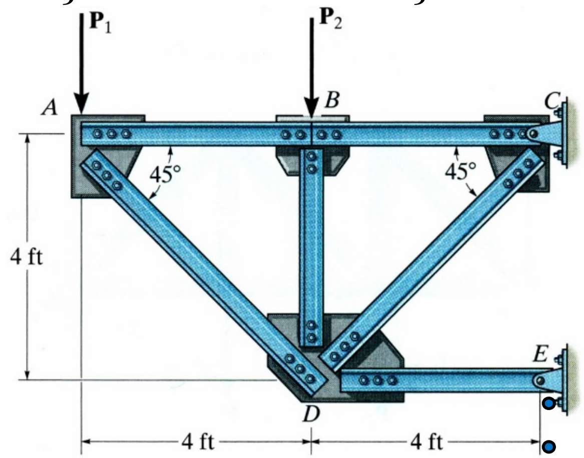
$$\Sigma F_y = 0: \qquad C_y - P_1 - P_2 = 0\,,$$

$$\Sigma M_C = 0: \quad 2aP_1 + aP_2 + aE_x = 0\,.$$

Solving these equations gives the *external* reactions

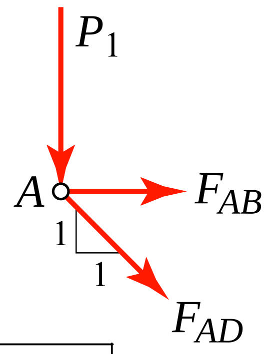
$$\boxed{C_x = 2P_1 + P_2, \quad C_y = P_1 + P_2, \quad E_x = -(2P_1 + P_2)\,.}$$

Next, start with a joint, draw the FBD, set it into *force* equilibrium *only*, and move to the next joint. Start with joints with at least 1 known force and 1-2 unknown forces.



$$C_x = 2P_1 + P_2, \quad C_y = P_1 + P_2, \quad E_x = -(2P_1 + P_2).$$

For example, start with **joint *A*:**

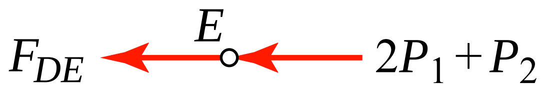


$$\Sigma F_x = 0: \quad F_{AB} + \tfrac{1}{\sqrt{2}} F_{AD} = 0,$$

$$\Sigma F_y = 0: \quad -P_1 - \tfrac{1}{\sqrt{2}} F_{AD} = 0.$$

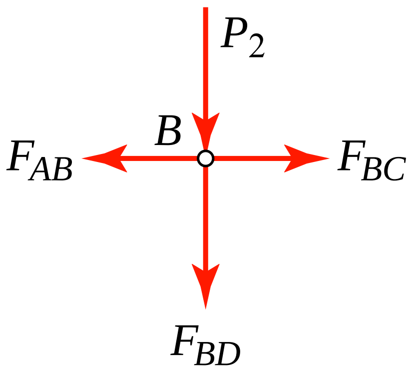
$$F_{AD} = -\sqrt{2}P_1, \quad F_{AB} = -\tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) = +P_1.$$

Joint E:



$$F_{DE} = E_x = -(P_1 + P_2)$$

Joint *B*:

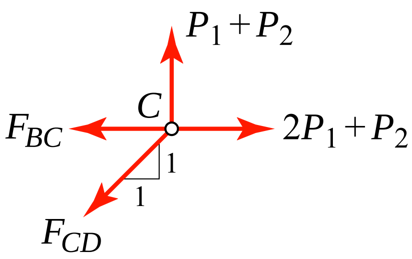


$$\Sigma F_x = 0: \quad -F_{AB} + F_{BC} = 0,$$

$$\Sigma F_y = 0: \quad -P_2 - F_{BD} = 0.$$

$$F_{BC} = F_{AB} = +P_1, \quad F_{BD} = -P_2.$$

Joint C:

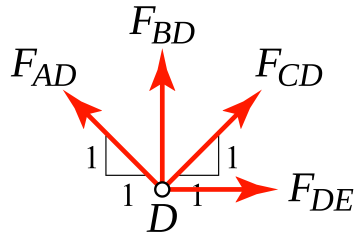


$$\Sigma F_x = 0: \quad -F_{BC} - \tfrac{1}{\sqrt{2}} F_{CD} + 2P_1 + P_2 = 0,$$

$$\Sigma F_y = 0: \quad -\tfrac{1}{\sqrt{2}} F_{CD} + P_1 + P_2 = 0.$$

$$F_{CD} = \sqrt{2}(2P_1 + P_2 - P_1) = \sqrt{2}(P_1 + P_2),$$
$$F_{CD} = \sqrt{2}(P_1 + P_2) \quad \text{(check)}.$$

Joint D: *only needed for check*



$$\Sigma F_x = 0: \quad -\tfrac{1}{\sqrt{2}} F_{AD} + \tfrac{1}{\sqrt{2}} F_{CD} + F_{DE} = 0,$$

$$\Sigma F_y = 0: \quad \tfrac{1}{\sqrt{2}} F_{AD} + F_{BD} + \tfrac{1}{\sqrt{2}} F_{CD} = 0.$$

$$F_{DE} = \tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) - \tfrac{1}{\sqrt{2}}\sqrt{2}(P_1 + P_2) = -(2P_1 + P_2),$$
$$\tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) - P_2 + \tfrac{1}{\sqrt{2}}\sqrt{2}(P_1 + P_2) = 0 \quad \text{(check)}.$$

Note: The checks would not have been satisfied if the external reactions had been calculated incorrectly.

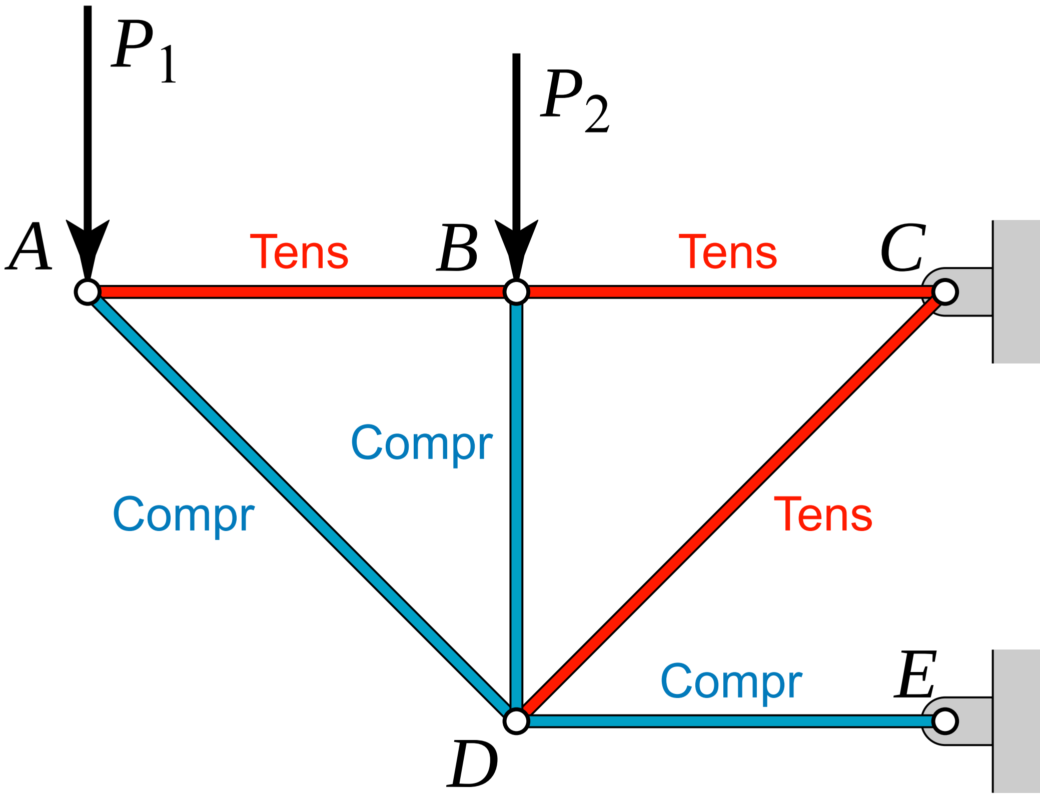
Note: The order in which the joints are set in equilibrium is usually arbitrary. Sometimes not all member loads are requested.

If provided numerical values:

$P_1 = 800\ \text{lb}$

$P_2 = 0$

$$F_{AB} = P_1 = 800\ \text{lb (T)}$$
$$F_{BC} = P_1 = 800\ \text{lb (T)}$$
$$F_{AD} = -\sqrt{2}P_1 = -1130\ \text{lb (C)}$$
$$F_{BD} = -P_2 = 0$$
$$F_{CD} = \sqrt{2}(P_1 + P_2) = 1130\ \text{lb (T)}$$
$$F_{DE} = -(2P_1 + P_2) = -1600\ \text{lb (C)}$$

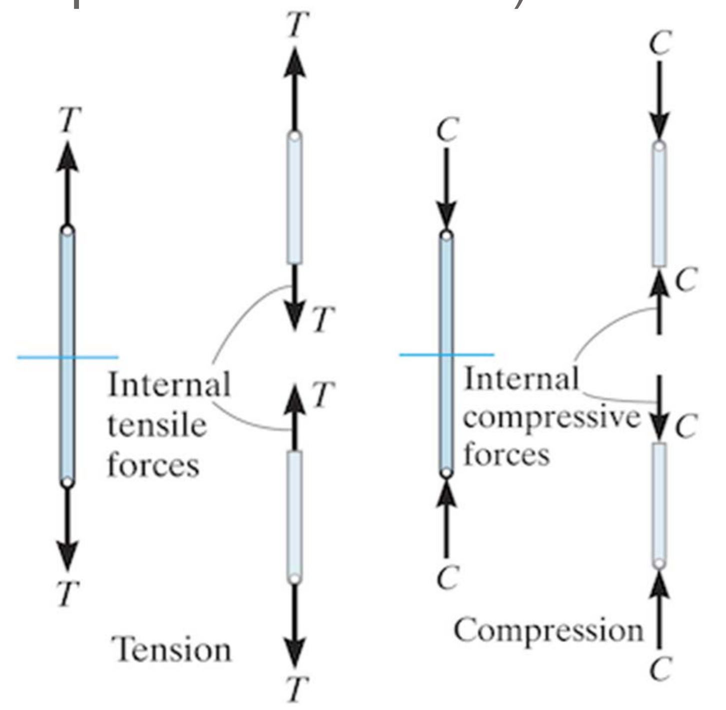


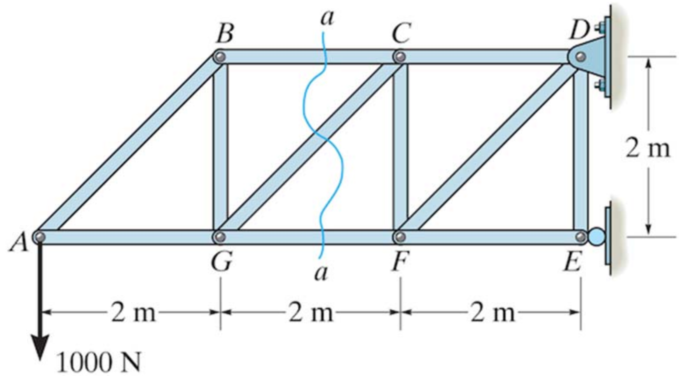
Note that, in the absence of P_2, member BD is a zero-force member

Note: Seven scalar equations of equilibrium were needed to obtain this answer. Might there be a shorter way?

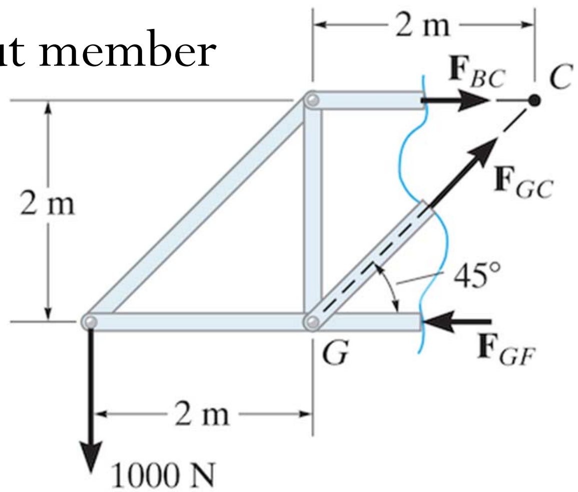
Method of sections (Use to solve for specific link force)

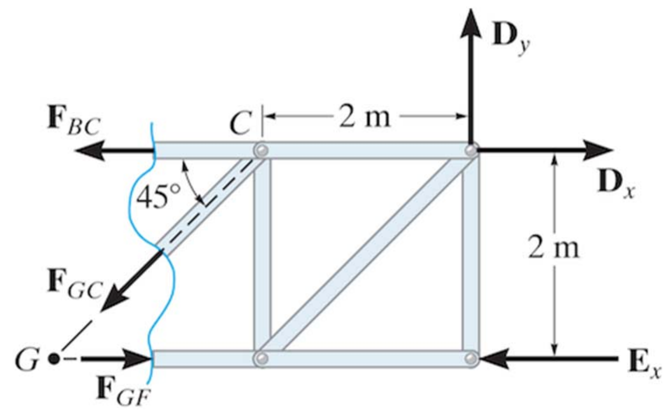
- Determine external support reactions (if necessary)
- "Cut" the structure at a section of interest into two separate pieces and set either part into force and moment equilibrium (your cut should be such that you have **no more than** three unknowns)





- Extend lines at cut to find point of intersection
- Draw unknown truss forces in cut member

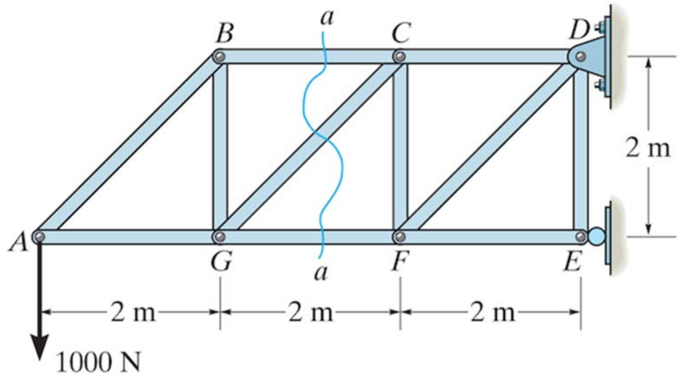


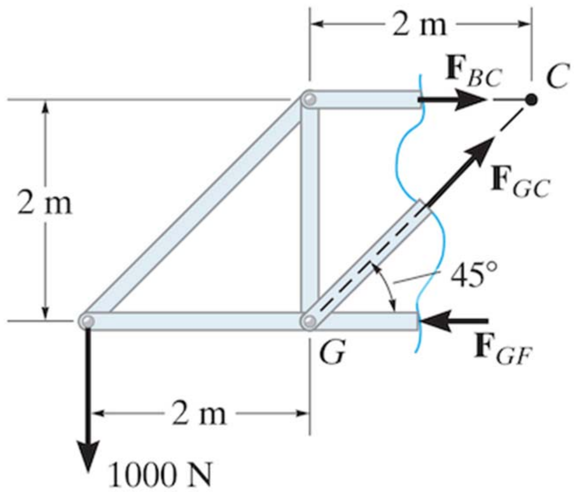


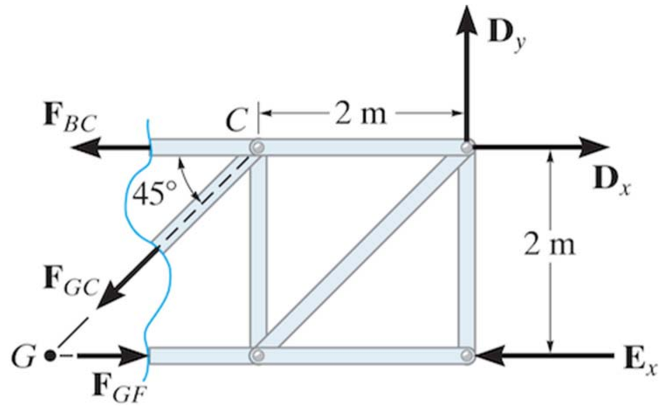
- Determine equilibrium equations (e.g., moment around point of intersection of two lines)
- Assume all internal loads are tensile.

Method of sections

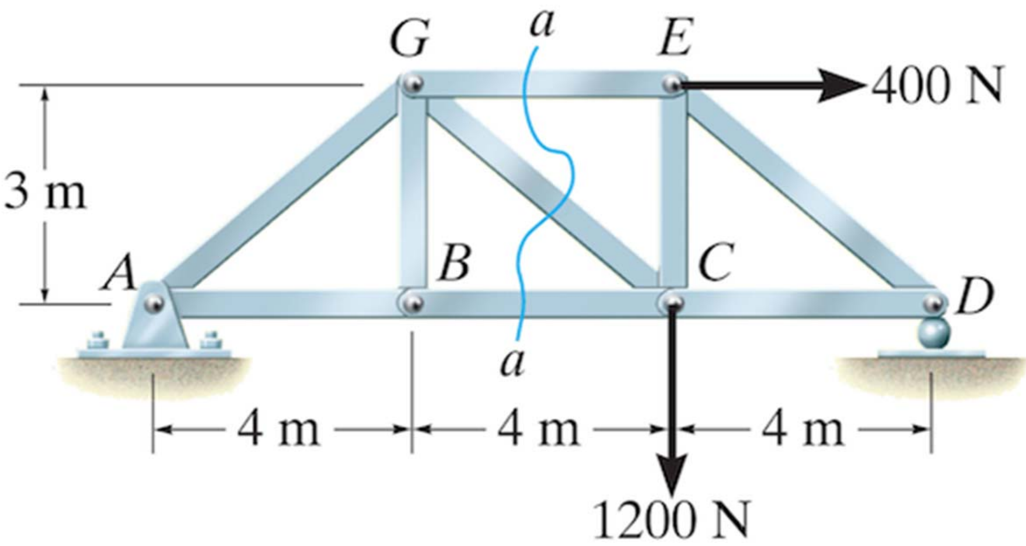
- Determine equilibrium equations (e.g., moment around point of intersection of two lines)
- Assume all internal loads are tensile.

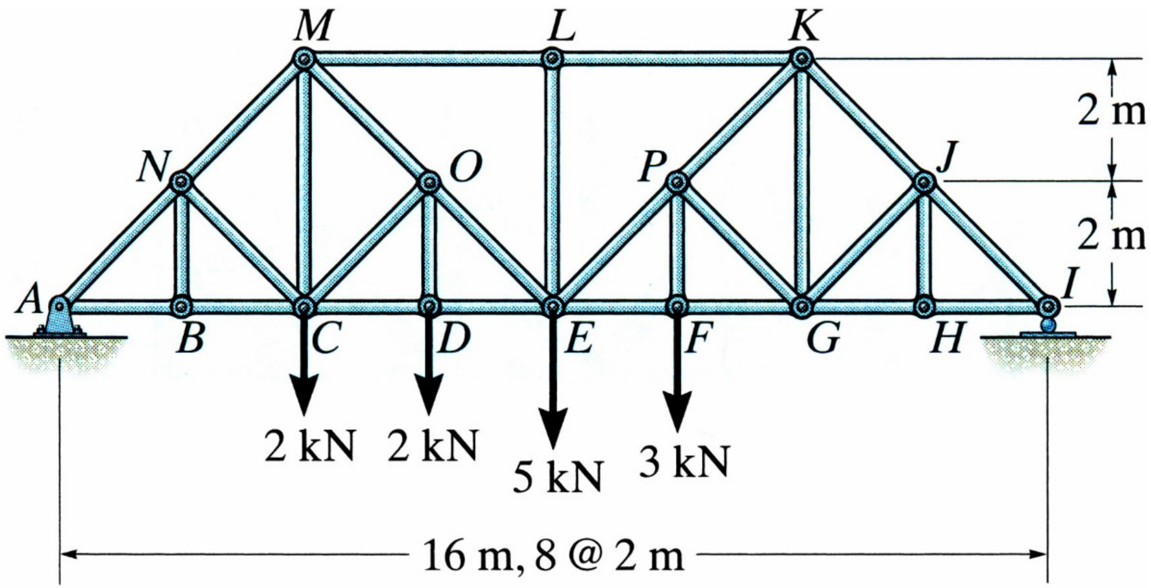




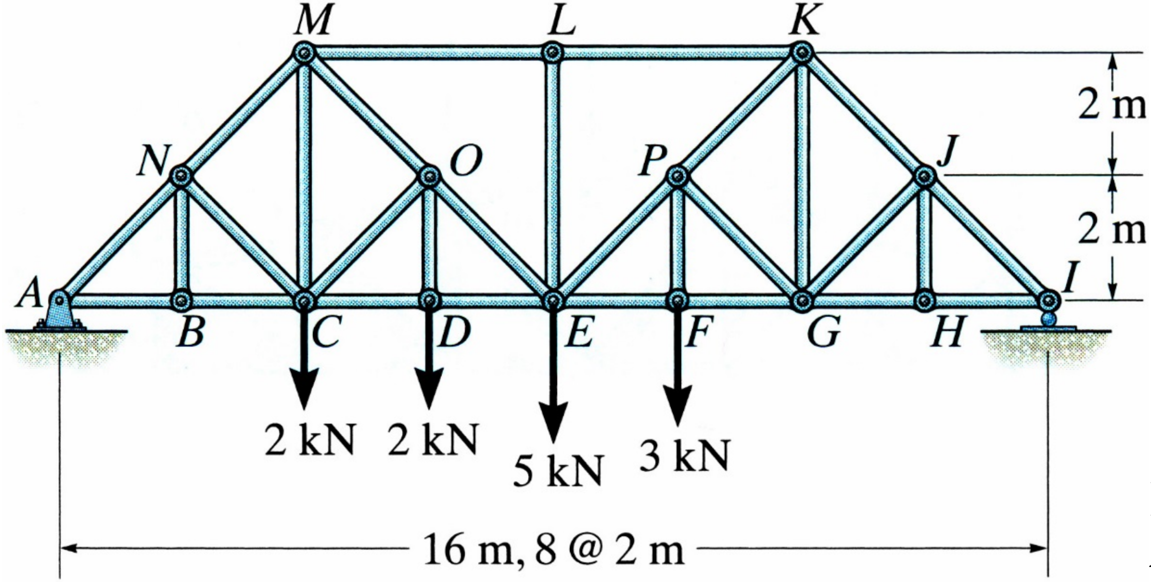


Determine the force in member BC of the truss and state if the member is in tension or compression.





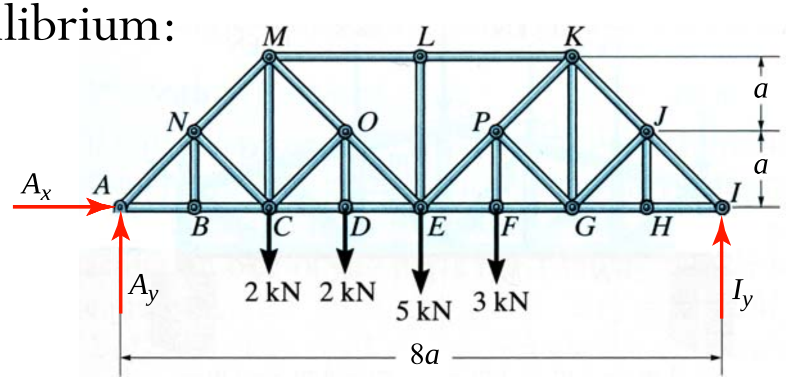
Determine the force in members OE, LE, LK of the Baltimore truss and state if the member is in tension or compression.



Determine the force in members OE, LE, LK of the Baltimore truss and state if the member is in tension or compression.

Solution:

(1) Draw free-body diagram of entire structure, and set into external equilibrium:



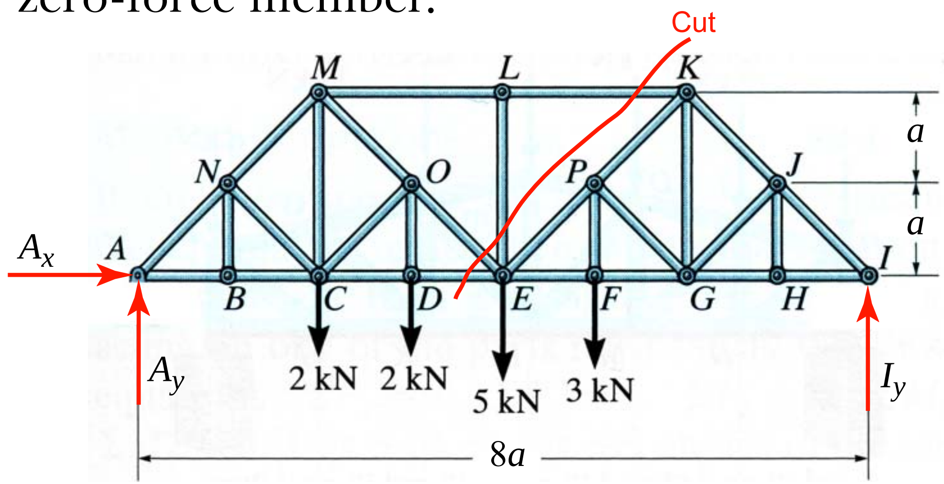
$$\Sigma F_x = 0: \qquad A_x = 0\,,$$

$$\Sigma F_y = 0: \qquad A_y + I_y - 2 - 2 - 5 - 3 = 0\,,$$

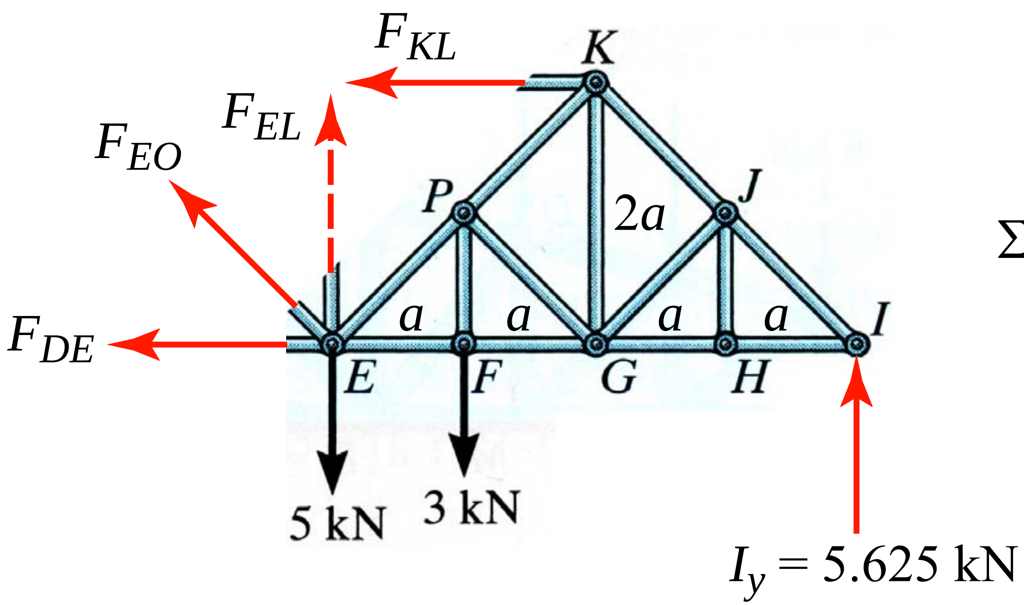
$$\Sigma M_A = 0: \quad -2a(2) - 3a(2) - 4a(5) - 5a(3) + 8aI_y = 0\,.$$

$$\boxed{A_x = 0, \quad A_y = 6.375\text{ kN}, \quad I_y = 5.625\text{ kN}.}$$

(2) Use method of sections, since cutting LK, LE, OE, and DE will separate the truss into two pieces. Note that LE is a zero-force member.



Normally, introducing four unknowns would make the problem intractable. However, *LE* is a *zero-force* member. Set *either* remaining section into equilibrium. Here, there is no real preference, but the right half will be fine



$$\Sigma F_x = 0: \quad -F_{DE} - \tfrac{1}{\sqrt{2}} F_{EO} - F_{KL} = 0\,,$$

$$\Sigma F_y = 0: \quad \tfrac{1}{\sqrt{2}} F_{EO} + 0 - 5 - 3 + I_y = 0\,,$$

$$\Sigma M_E = 0: \quad 0(5) + a(-3) + 4aI_y + 2aF_{KL} = 0\,.$$

$F_{DE} = +7.38$ kN (T),	$F_{EL} = 0$ (zero-force),
$F_{EO} = +3.36$ kN (T),	$F_{KL} = -9.75$ kN (C).