Statics - TAM 211

Lecture 18

November 5, 2018

Announcements

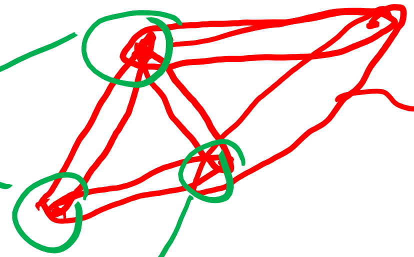
- Upcoming deadlines:
- Tuesday (11/6)
  - Prairie Learn HW 7
- Friday (11/9)
  - Written Assignment 7

Note that in Chapter 5, a rigid body could have more than two forces and more than two pin joints. For TRUSS structures in Chapter 6, these structures are made of ONLY links, which are 2FMs with only 2 pin joints.

Chapter 6: Structural Analysis

2D Bodies

Pin Joints



links, members,
truss member

LINK: 2FM
2 pin joints

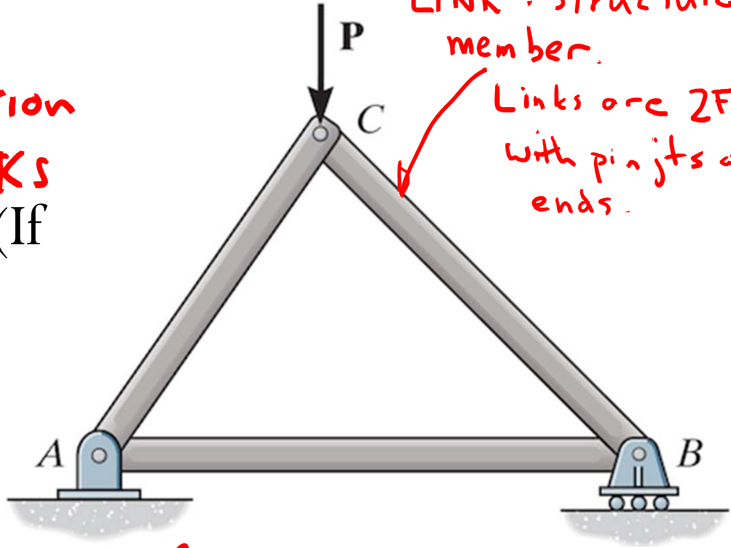
Goals and Objectives

- Determine the forces in members of a truss using the method of joints
- Determine zero-force members
- Determine the forces in members of a truss using the method of sections
- Determine the forces and moments in members of a frame or machine

Recap: Truss Analysis

"TRUSS" ≡ combination of many LINKS

"LINK": structural member. Links are 2FM with pin jts at ends.



Assumption of trusses

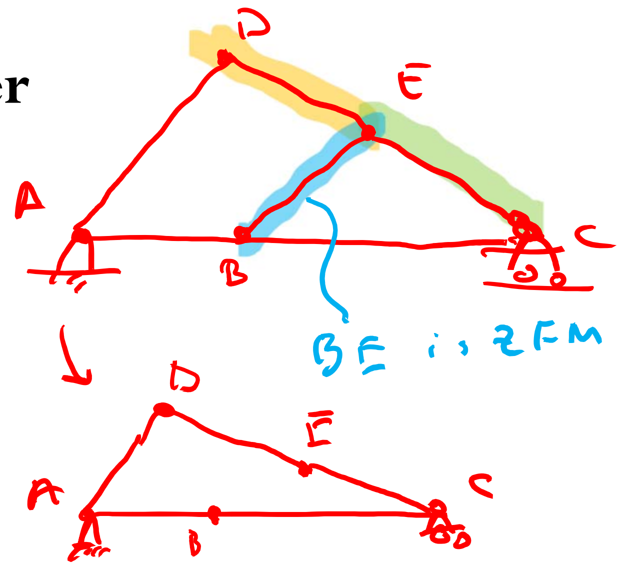
- Loading applied at joints, with negligible weight (If weight included, vertical and split at joints)
- Members joined by smooth pins
- Pins in equilibrium: $\boxed{\sum F_x = 0 \text{ and } \sum F_y = 0}$

Force determining forces within a link.

Zero-force members

Two situations:

- Two non-collinear members , no external or support at jt → **Both members are ZFM**
- Two collinear member, plus third non-collinear, no loads on third member → **Non-collinear member is ZFM**.

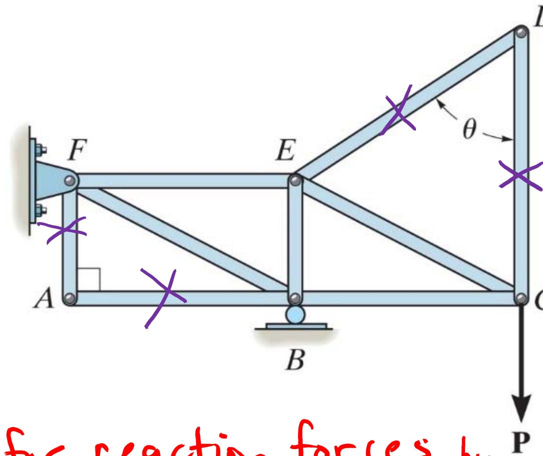


Method of joints

Procedure for analysis to find forces within links:

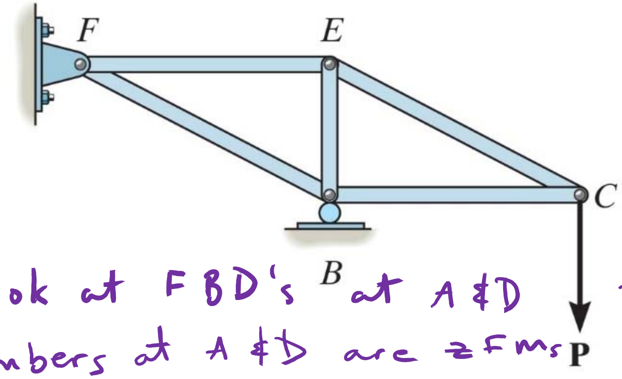
- Determine external support reactions
- Free-body diagram for each joint
- Start with joints with at least 1 known force and 1-2 unknown forces
- Assume the unknown force members to be in *tension*

RECAP: Use Method of joints to prove that members attached to A and D should be FZM

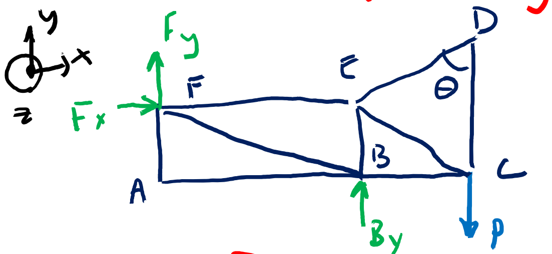


ZFM

ZFM



① Solve for reaction forces by considering the entire structure as one single rigid body.



$\Sigma F_x = 0$: $\boxed{F_x = 0}$

$\Sigma F_y = 0$: $F_y + B_y - P = 0$

$\Sigma M_F = 0$: $(r_{FE}) B_y - (r_{AC}) P = 0$

$+\circlearrowleft$

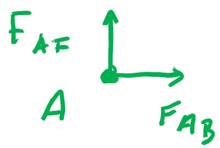
$$\boxed{B_y = \left(\frac{r_{AC}}{r_{FE}}\right) P}$$

$$\boxed{F_y = \left(1 - \frac{r_{AC}}{r_{FE}}\right) P}$$

② Let's look at FBD's at A & D to prove that members at A & D are ZFMs

Assume that there are forces pointing in tension and aligned in direction of truss link members. Label forces as F_{ij} where i is tail and j is arrow head

@ Jt A :

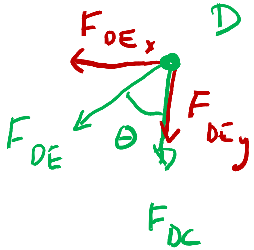


$\Sigma F_x = 0$: $F_{AB} = 0$

$\Sigma F_y = 0$: $F_{AF} = 0$

✓ ZFMs

@ Jt D :



$\Sigma F_x = 0$: $F_{DE_x} = 0$

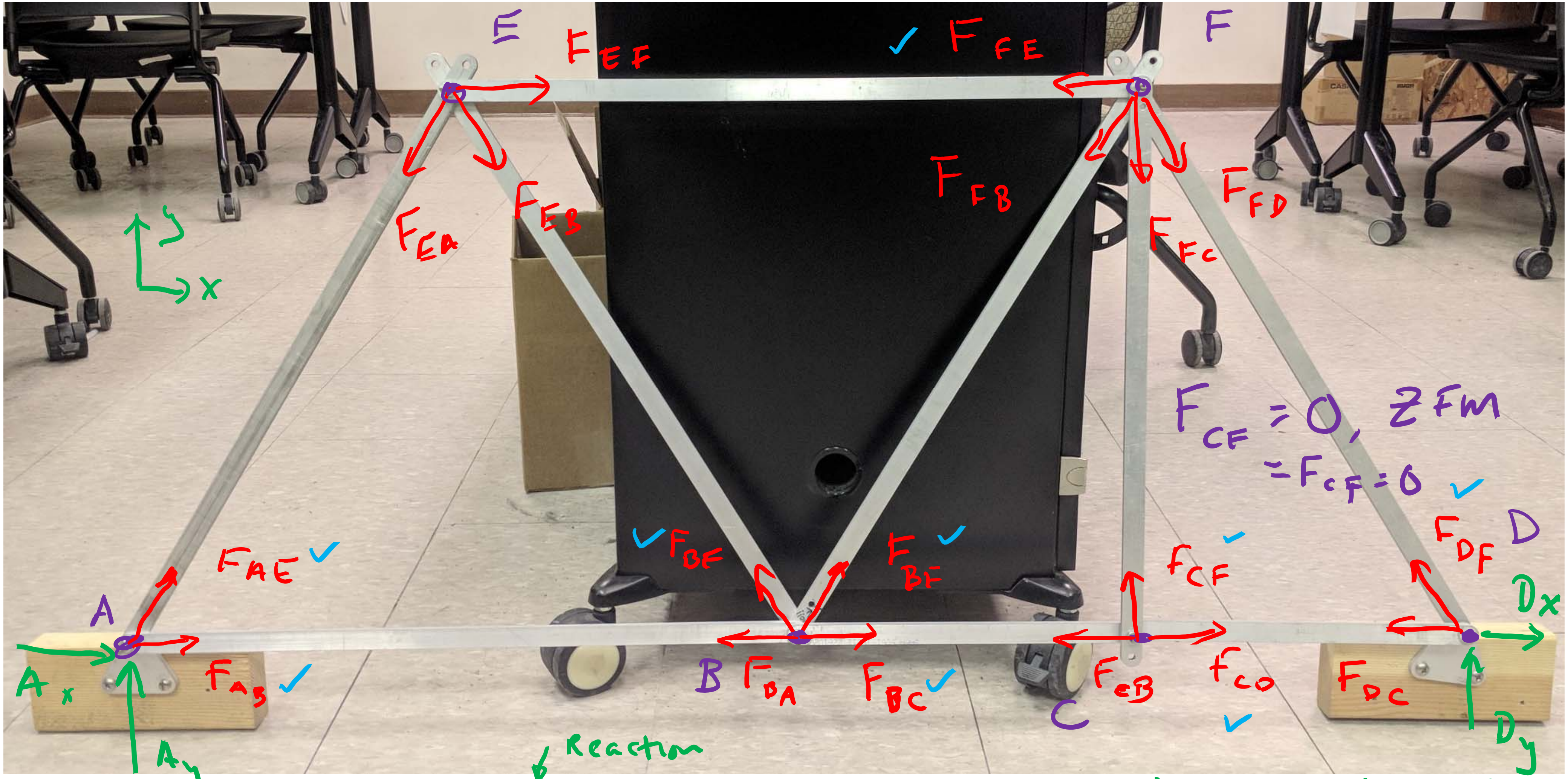
$\Sigma F_y = 0$: $F_{DE_y} + F_{DC} = 0$

Note: $F_{DE_x} = F_{DE} \sin\theta = 0$

since $\theta \neq 0$, $\therefore F_{DE} = 0$ ✓ ZFM

$\Rightarrow F_{DE_y} + F_{DC} = F_{DE}\cos\theta + F_{DC} = 0$

$\therefore F_{DC} = 0$ ✓ ZFM



① Solve for Support Rxn Forces using entire single rigid body (Chap 5)

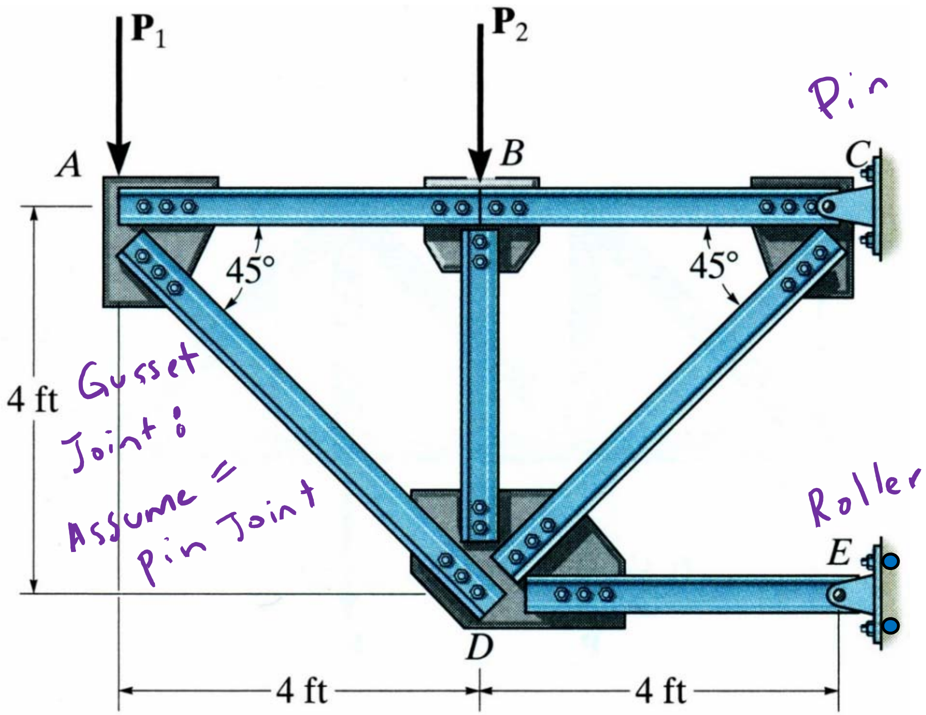
② Method of Joints: @ each jt. Draw FBD

Assume unknown forces are in tension.

9 unks → 9 Eqns

1 ZFM → 8 unks → 8 eqns

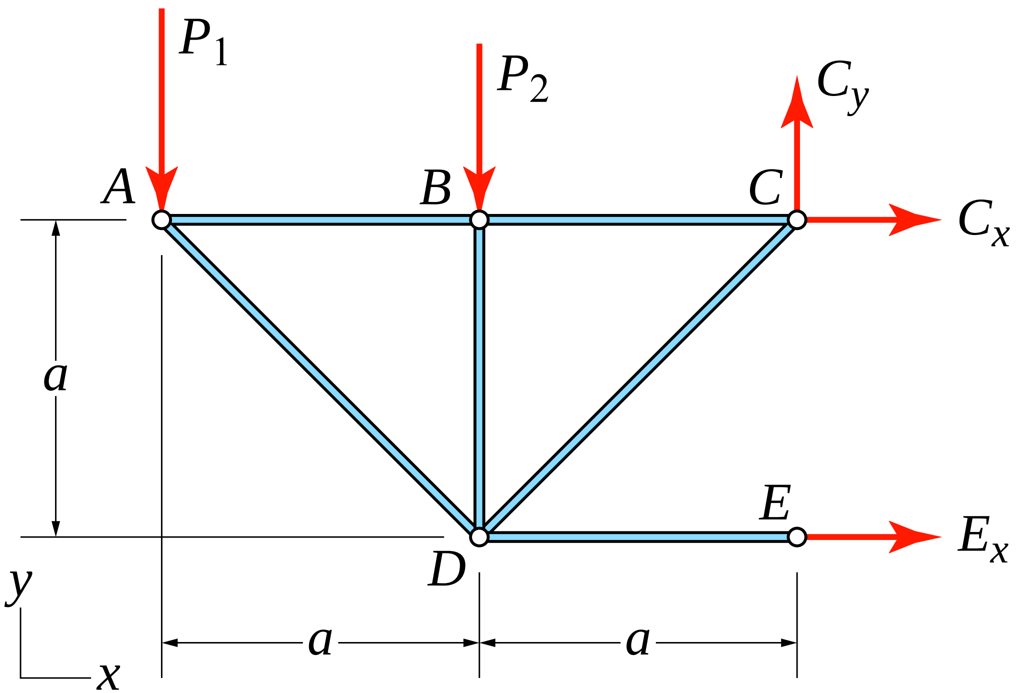
* ⇒ NOTE: ZFM condition is ONLY for current loading condition



The truss, used to support a balcony, is subjected to the loading shown. Approximate each joint as a pin and determine the force in each member. State whether the members are in tension or compression.

Solution:

Start by setting the entire structure into **external** equilibrium. Draw the FBD.



Equilibrium requires $\sum \boldsymbol{F} = \mathbf{0}$ and $\left(\sum \boldsymbol{M}\right)_C = \mathbf{0}$

$$\Sigma F_x = 0: \qquad C_x + E_x = 0\,,$$

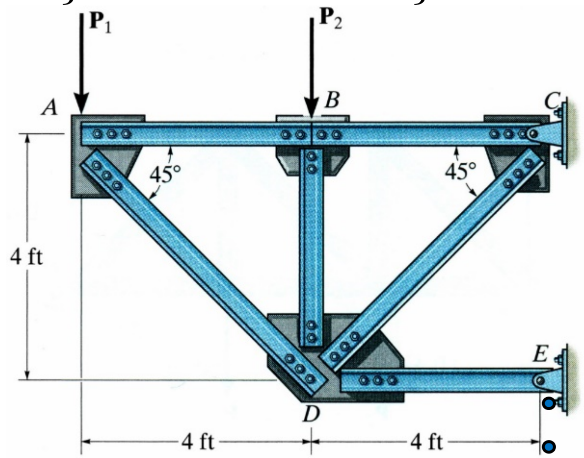
$$\Sigma F_y = 0: \qquad C_y - P_1 - P_2 = 0\,,$$

$$\Sigma M_C = 0: \quad 2aP_1 + aP_2 + aE_x = 0\,.$$

Solving these equations gives the *external* reactions

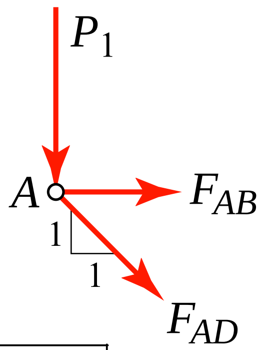
$$\boxed{C_x = 2P_1 + P_2, \quad C_y = P_1 + P_2, \quad E_x = -(2P_1 + P_2)\,.}$$

Next, start with a joint, draw the FBD, set it into *force* equilibrium *only*, and move to the next joint. Start with joints with at least 1 known force and 1-2 unknown forces.



$$\boxed{C_x = 2P_1 + P_2, \quad C_y = P_1 + P_2, \quad E_x = -(2P_1 + P_2).}$$

For example, start with **joint *A***:

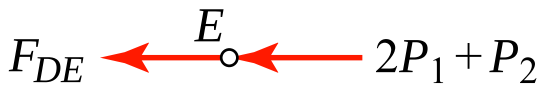


$$\Sigma F_x = 0: \quad F_{AB} + \tfrac{1}{\sqrt{2}} F_{AD} = 0,$$

$$\Sigma F_y = 0: \quad -P_1 - \tfrac{1}{\sqrt{2}} F_{AD} = 0.$$

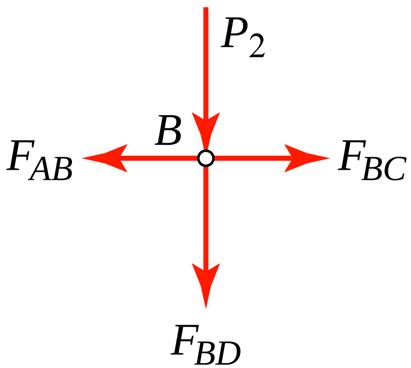
$$\boxed{F_{AD} = -\sqrt{2}P_1, \quad F_{AB} = -\tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) = +P_1.}$$

Joint E:



$$\boxed{F_{DE} = E_x = -(P_1 + P_2)}$$

Joint *B*:

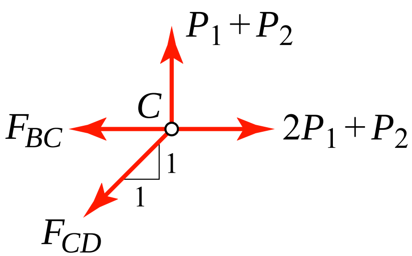


$$\Sigma F_x = 0: \quad -F_{AB} + F_{BC} = 0,$$

$$\Sigma F_y = 0: \quad -P_2 - F_{BD} = 0.$$

$$\boxed{F_{BC} = F_{AB} = +P_1, \quad F_{BD} = -P_2.}$$

Joint C:

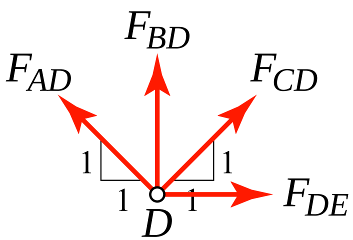


$$\Sigma F_x = 0: \quad -F_{BC} - \tfrac{1}{\sqrt{2}} F_{CD} + 2P_1 + P_2 = 0,$$

$$\Sigma F_y = 0: \quad -\tfrac{1}{\sqrt{2}} F_{CD} + P_1 + P_2 = 0.$$

$$\boxed{\begin{aligned} F_{CD} &= \sqrt{2}(2P_1 + P_2 - P_1) = \sqrt{2}(P_1 + P_2), \\ F_{CD} &= \sqrt{2}(P_1 + P_2) \quad \text{(check)}. \end{aligned}}$$

Joint D: *only needed for check*



$$\Sigma F_x = 0: \quad -\tfrac{1}{\sqrt{2}} F_{AD} + \tfrac{1}{\sqrt{2}} F_{CD} + F_{DE} = 0,$$

$$\Sigma F_y = 0: \quad \tfrac{1}{\sqrt{2}} F_{AD} + F_{BD} + \tfrac{1}{\sqrt{2}} F_{CD} = 0.$$

$$\boxed{\begin{aligned} F_{DE} &= \tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) - \tfrac{1}{\sqrt{2}}\sqrt{2}(P_1 + P_2) = -(2P_1 + P_2), \\ &\tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) - P_2 + \tfrac{1}{\sqrt{2}}\sqrt{2}(P_1 + P_2) = 0 \quad \text{(check)}. \end{aligned}}$$

Note: The checks would not have been satisfied if the external reactions had been calculated incorrectly.

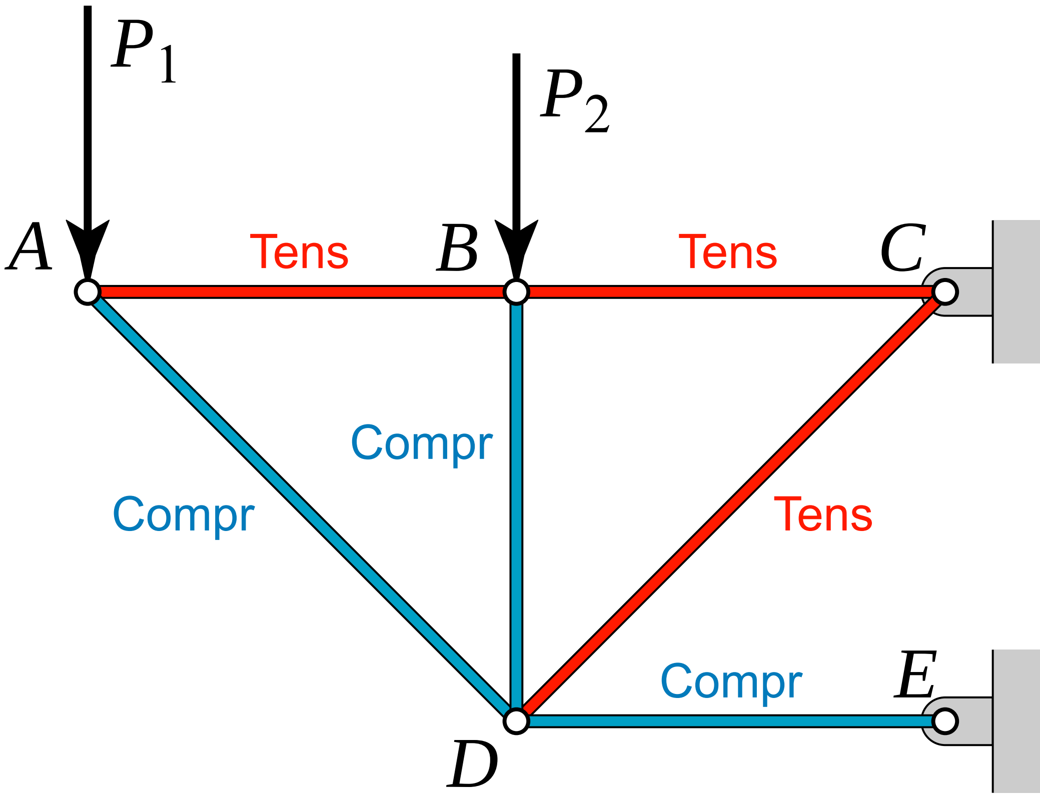
Note: The order in which the joints are set in equilibrium is usually arbitrary. Sometimes not all member loads are requested.

If provided numerical values:

$P_1 = 800 \text{ lb}$

$P_2 = 0$

$$
\boxed{
\begin{aligned}
F_{AB} &= P_1 = 800 \text{ lb (T)} \\
F_{BC} &= P_1 = 800 \text{ lb (T)} \\
F_{AD} &= -\sqrt{2}P_1 = -1130 \text{ lb (C)} \\
F_{BD} &= -P_2 = 0 \\
F_{CD} &= \sqrt{2}(P_1 + P_2) = 1130 \text{ lb (T)} \\
F_{DE} &= -(2P_1 + P_2) = -1600 \text{ lb (C)}
\end{aligned}
}
$$

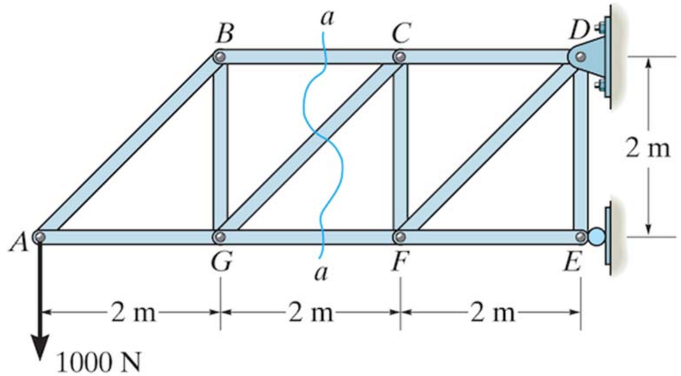


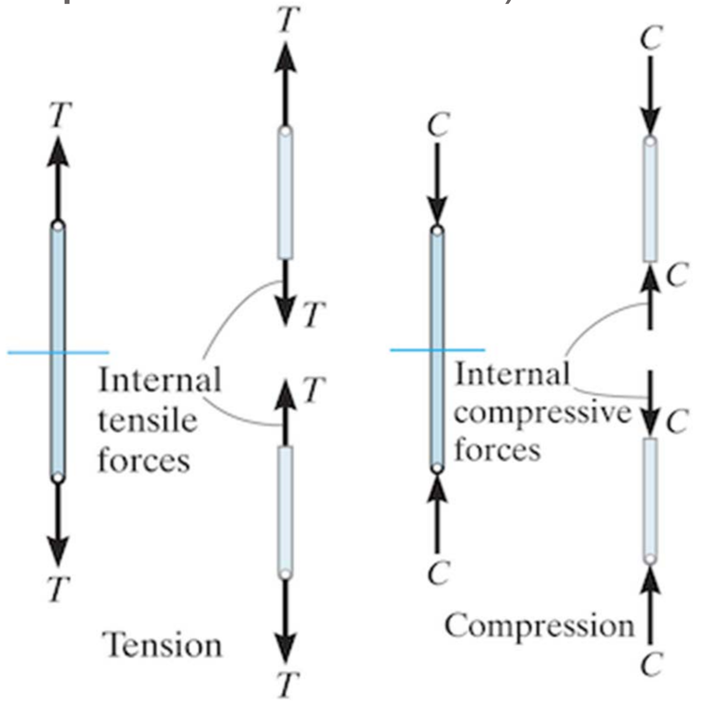
Note that, in the absence of P_2, member BD is a zero-force member

Note: Seven scalar equations of equilibrium were needed to obtain this answer. Might there be a shorter way?

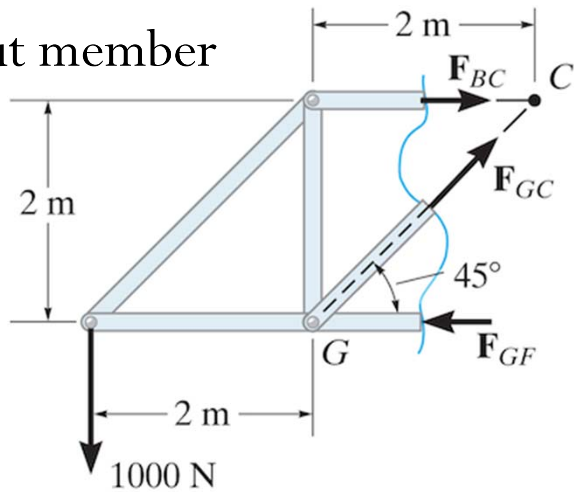
Method of sections (Use to solve for specific link force)

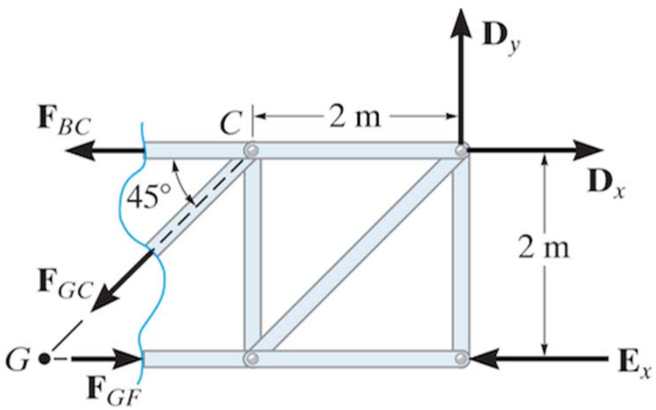
- Determine external support reactions (if necessary)
- "Cut" the structure at a section of interest into two separate pieces and set either part into force and moment equilibrium (your cut should be such that you have **no more than** three unknowns)





- Extend lines at cut to find point of intersection
- Draw unknown truss forces in cut member

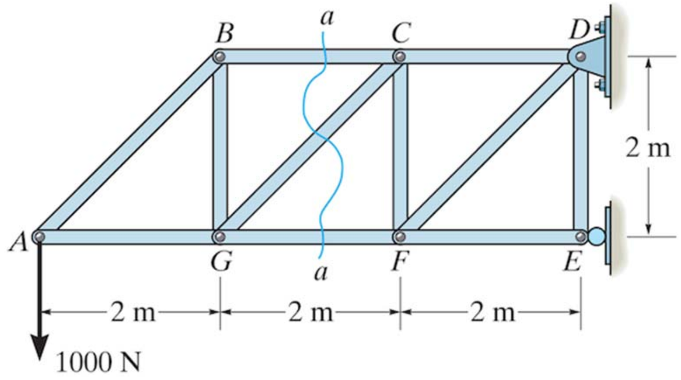


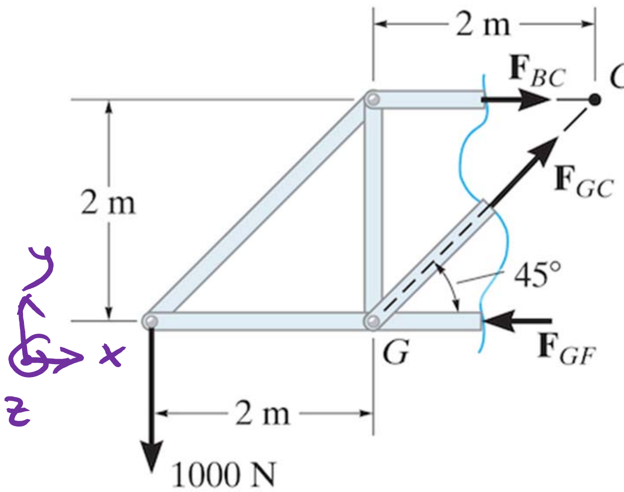


- Determine equilibrium equations (e.g., <u>moment around point of intersection of two lines</u>)
- Assume all internal loads are tensile.

Method of sections

- Determine equilibrium equations (e.g., moment around point of intersection of two lines)
- Assume all internal loads are tensile.

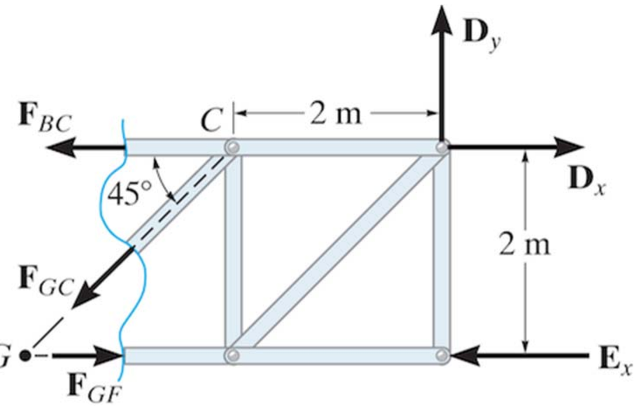




3 UNK

⇒ Pick this side

6 UNK



Need 6 eqns to solve for 6 unknowns

⇒ More work

$$\Sigma F_x: \; F_{BC} + F_{GC}\cos 45^\circ - F_{GF} = 0$$

$$\Sigma F_y: \; F_{GC}\sin 45^\circ - 1000\,N = 0$$

$$\boxed{F_{GC} = 1414\,N}$$

$$\Sigma M_C: \; (-2m)(F_{GF}) + (4m)(1000N) = 0$$

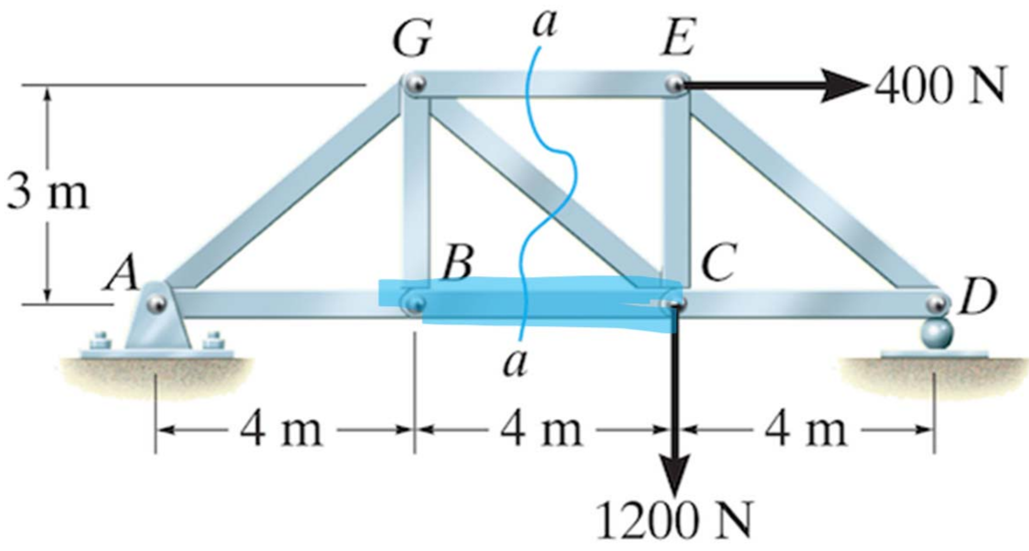
$$\boxed{F_{GF} = 2000\,N}$$

$$F_{BC} = 2000 - 1414 = \boxed{586\,N = F_{BC}}$$

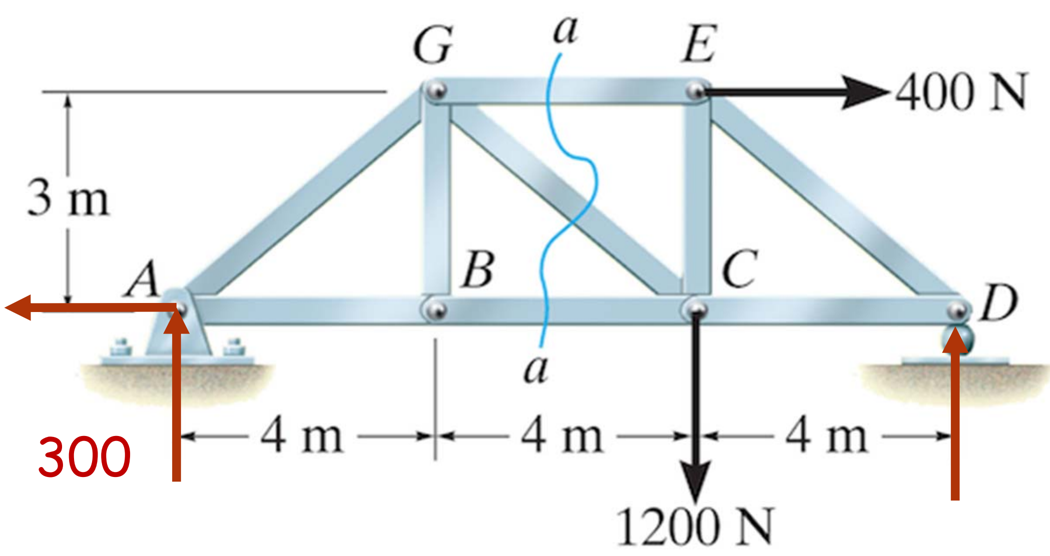
Determine the force in member BC of the truss and state if the member is in tension or compression.

① Solve for support rxn forces at A & D. (Chap 5)

Use this space to practice.

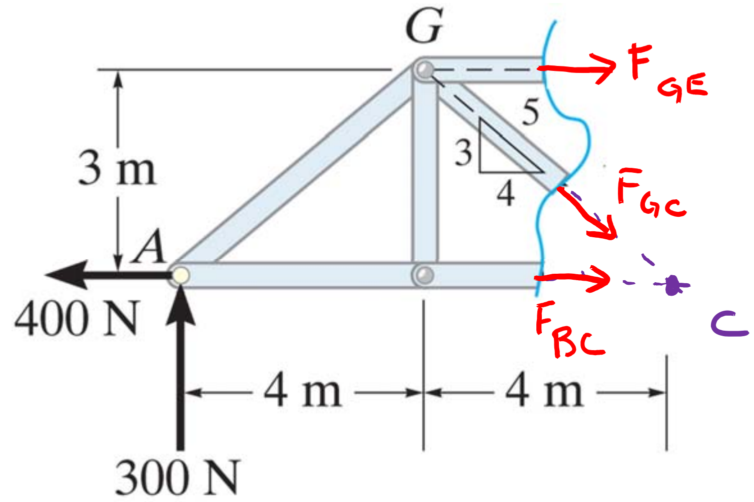


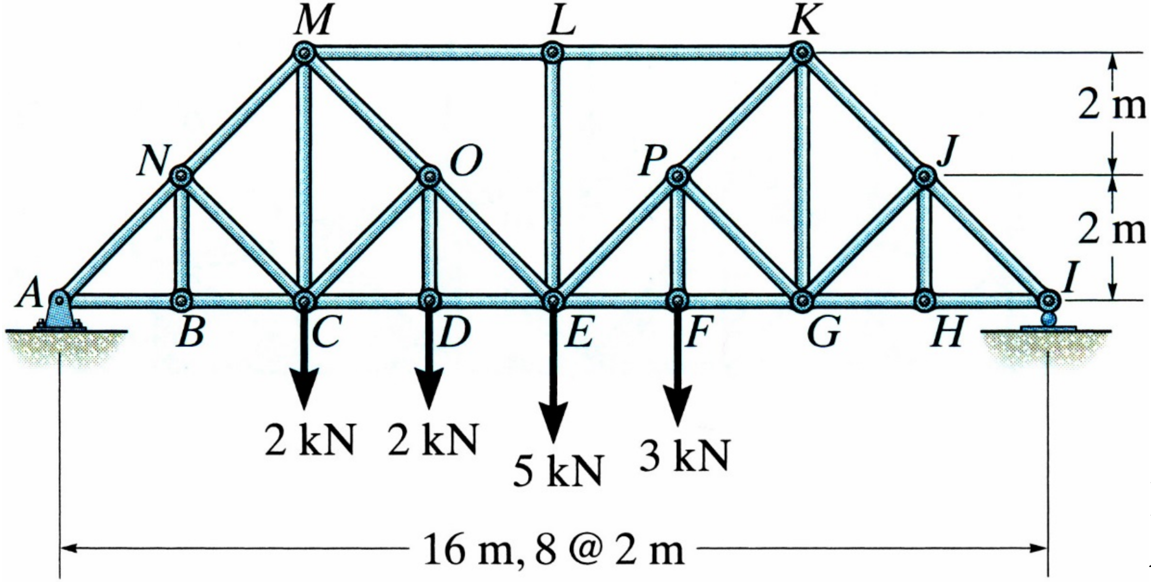
Determine the force in member BC of the truss and state if the member is in tension or compression.



② After solving for A_x, A_y, D_y, use Method of Sections to solve for F_{BC}.

Solve problem on your own. Show that $F_{BC} = 800N$ ∴ link BC is in tension.

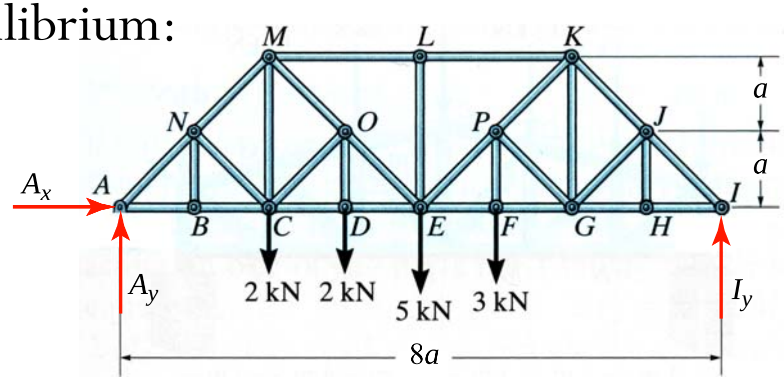




Determine the force in members OE, LE, LK of the Baltimore truss and state if the member is in tension or compression.

Solution:

(1) Draw free-body diagram of entire structure, and set into external equilibrium:



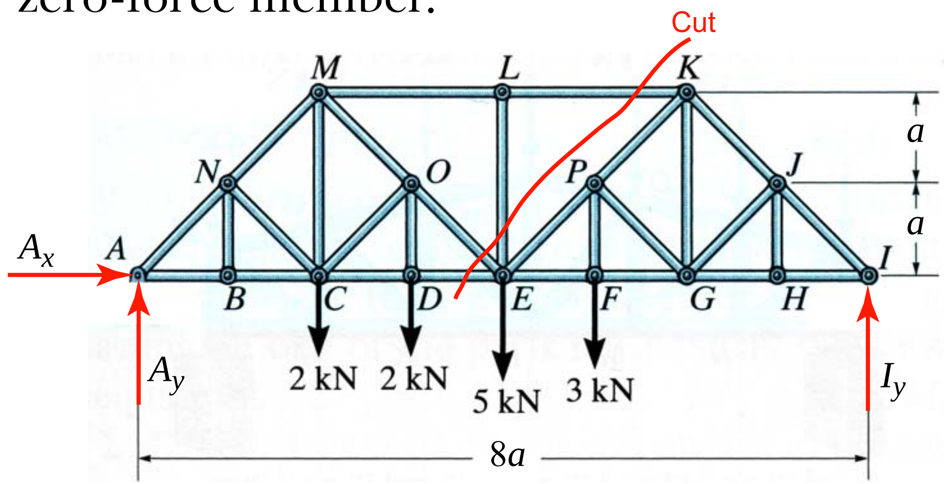
$$\Sigma F_x = 0: \qquad A_x = 0,$$

$$\Sigma F_y = 0: \qquad A_y + I_y - 2 - 2 - 5 - 3 = 0,$$

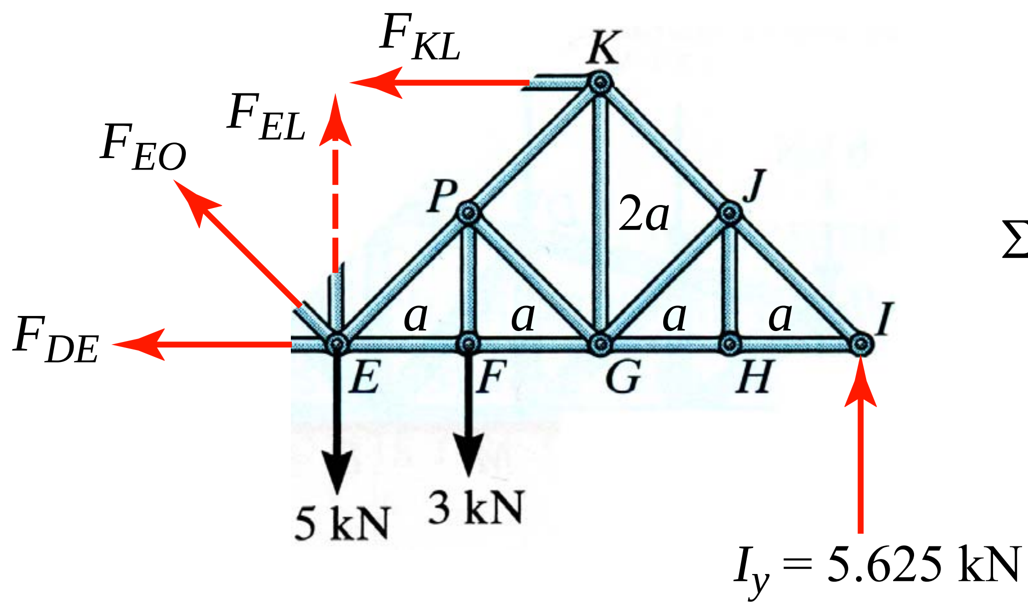
$$\Sigma M_A = 0: \quad -2a(2) - 3a(2) - 4a(5) - 5a(3) + 8aI_y = 0.$$

$$\boxed{A_x = 0, \quad A_y = 6.375\text{ kN}, \quad I_y = 5.625\text{ kN}.}$$

(2) Use method of sections, since cutting LK, LE, OE, and DE will separate the truss into two pieces. Note that LE is a zero-force member.



Normally, introducing four unknowns would make the problem intractable. However, *LE* is a *zero-force* member. Set *either* remaining section into equilibrium. Here, there is no real preference, but the right half will be fine

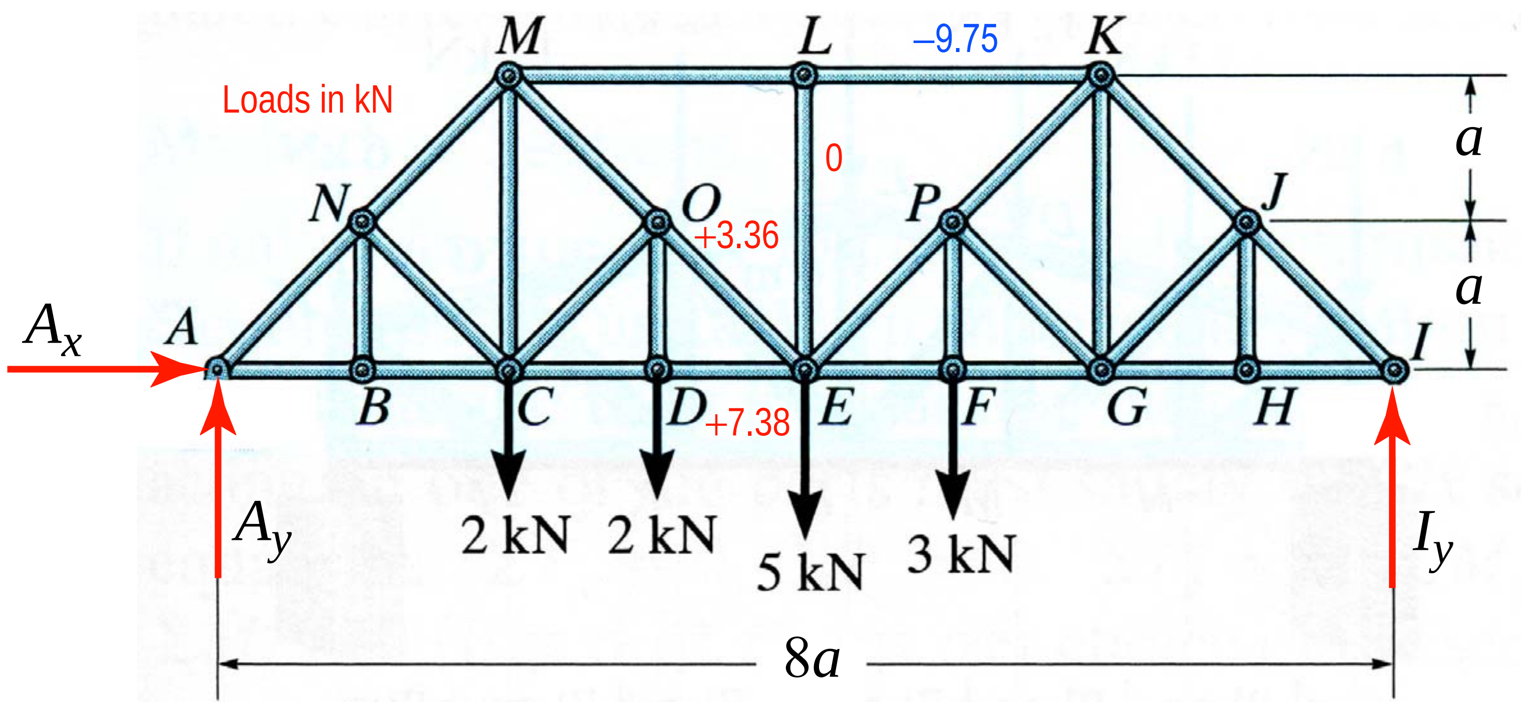


$$\Sigma F_x = 0: \quad -F_{DE} - \tfrac{1}{\sqrt{2}} F_{EO} - F_{KL} = 0,$$

$$\Sigma F_y = 0: \quad \tfrac{1}{\sqrt{2}} F_{EO} + 0 - 5 - 3 + I_y = 0,$$

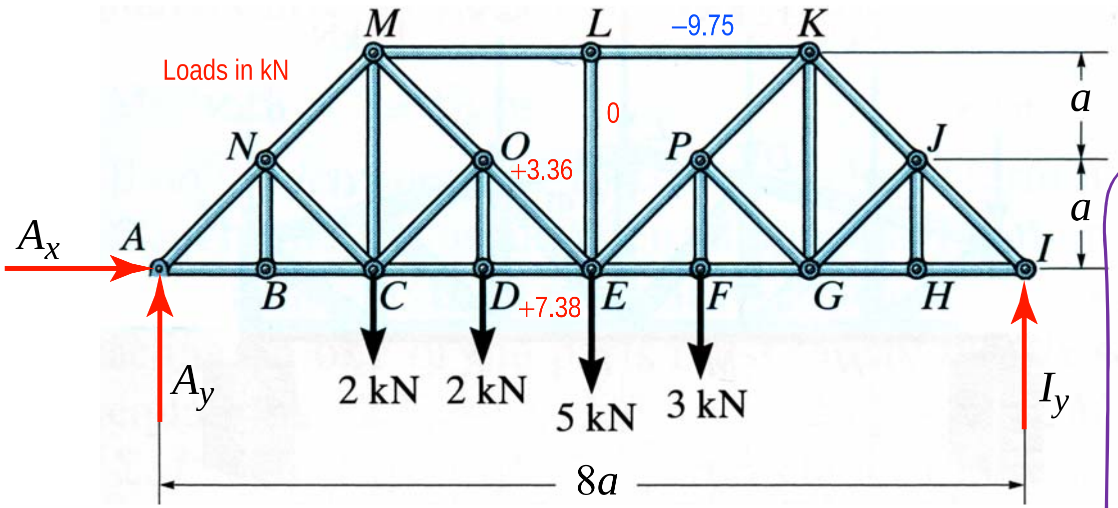
$$\Sigma M_E = 0: \quad 0(5) + a(-3) + 4aI_y + 2aF_{KL} = 0.$$

$$\boxed{\begin{array}{ll} F_{DE} = +7.38 \text{ kN (T)}, & F_{EL} = 0 \text{ (zero-force)}, \\ F_{EO} = +3.36 \text{ kN (T)}, & F_{KL} = -9.75 \text{ kN (C)}. \end{array}}$$



Notes added after class:

At the end of lecture today, students asked why is link LE a ZFM since joint E has an applied load of 5kN?



The answer can be determined by examining the situation definitions of a ZFM, which were given in Lecture 17. These situations allow us to find ZFMs by inspection (i.e., by looking without calculations).

Note that these definitions are with respect to the forces and links at a <u>**specific joint**</u>.

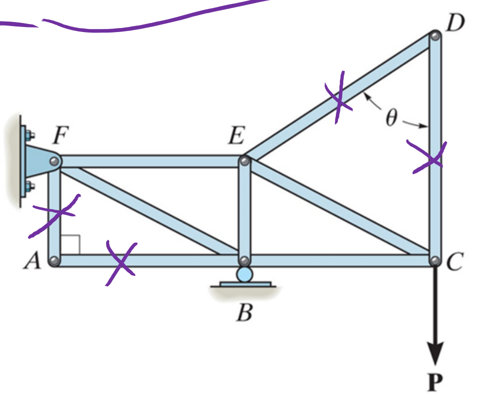
Therefore for the structure to the above-left, there are no external or support reaction forces on joint L; thus LE is a ZFM. We see the same for joints C & D in bottom-middle structure (links DA & CA are ZFM), and joint E in bottom-right structure (link BE is ZFM).

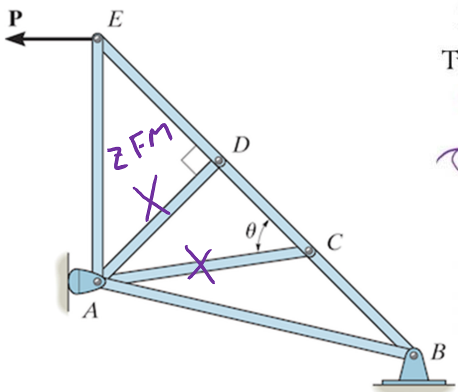
Zero-force members

- Particular members in a structure may experience no force for certain loads.
- Zero-force members are used to increase stability
- Identifying members with zero-force can expedite analysis.

Two situations:

- Joint with two non-collinear members, no external or support reaction applied to the joint → **Both members are zero-force members**.
- Joint with two collinear member, plus third non-collinear, no external or support reaction applied to non-collinear member → **Non-collinear member is a zero-force member**.





Zero-force members

Two situations:

- Two non-collinear members , no external or support at jt → **Both members are ZFM**
- Two collinear member, plus third non-collinear, no loads on third member → **Non-collinear member is ZFM**.