Statics - TAM 211

Lecture 19

November 7, 2018

Announcements

- Upcoming deadlines:
- Friday (11/9)
  - Written Assignment 7
- Tuesday (11/12)
  - Prairie Learn HW 8

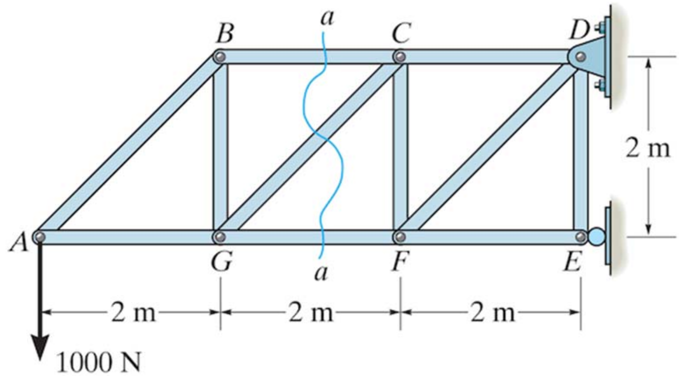
Chapter 6: Structural Analysis

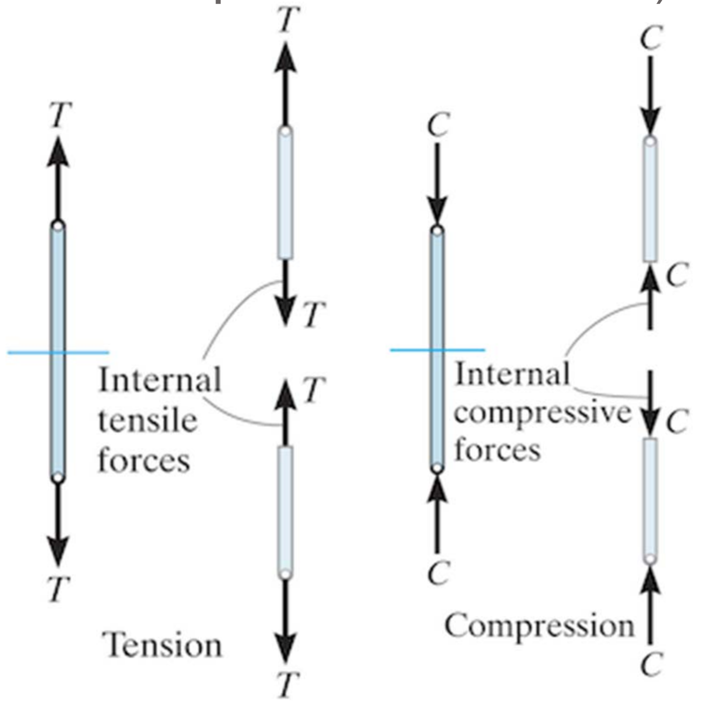
Goals and Objectives

- Determine the forces in members of a truss using the method of joints
- Determine zero-force members
- Determine the forces in members of a truss using the method of sections
- Determine the forces and moments in members of a frame or machine

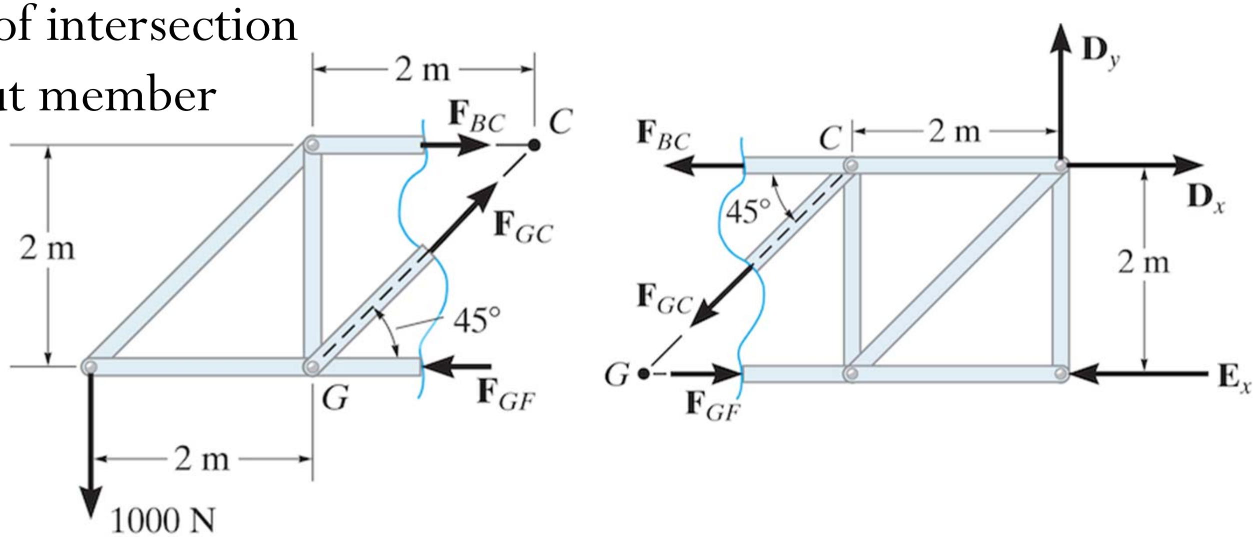
Recap: Method of sections (Solve for specific link force)

- Determine external support reactions (if necessary)
- "Cut" the structure at a section of interest into two separate pieces and set either part into force and moment equilibrium (your cut should be such that you have **<u>no more than</u>** three unknowns)



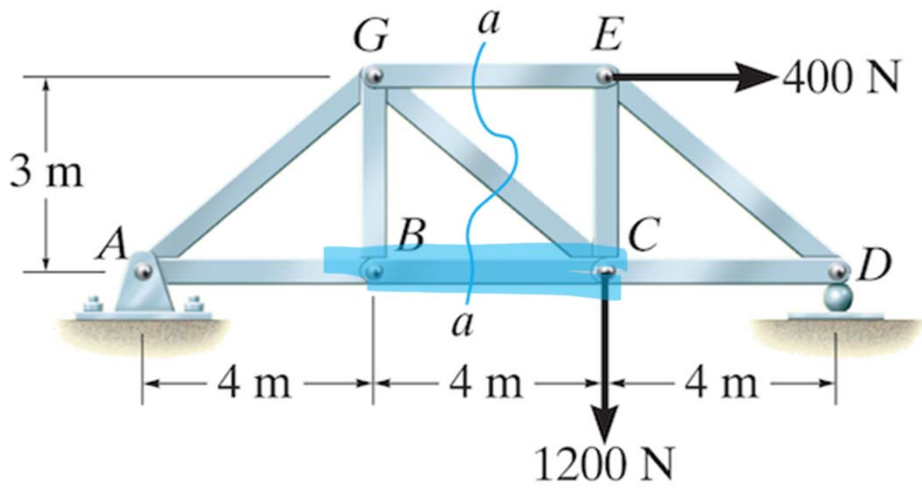


- Extend lines at cut to find point of intersection
- Draw unknown truss forces in cut member

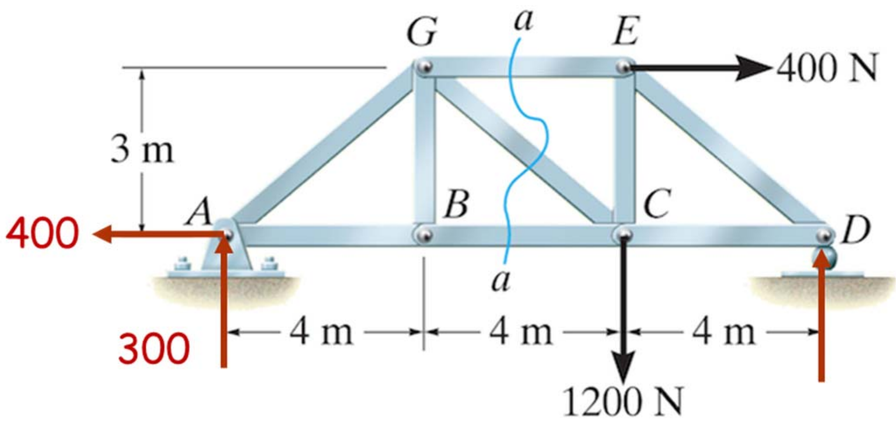


- Determine equilibrium equations (e.g., <u>moment around point of intersection of two lines</u>)
- Assume all internal loads are tensile.

Determine the force in member BC of the truss and state if the member is in tension or compression.



Solve for support rxn forces at A & D (Chap 5)

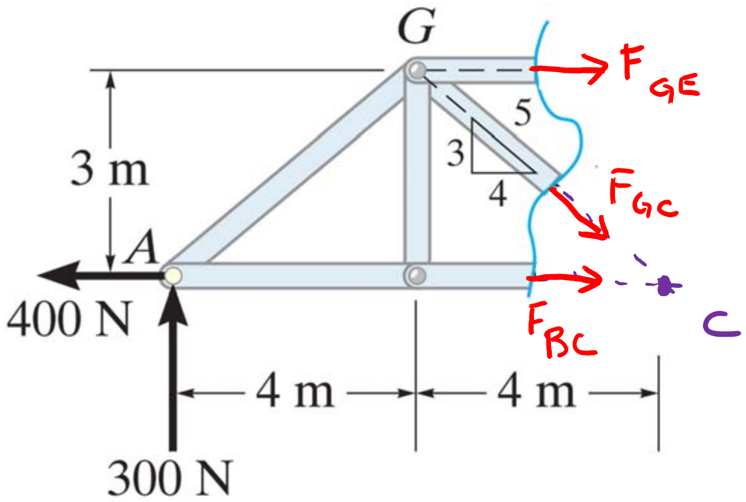


② After solving for A_x, A_y, D_y, Use Method of Sections to solve for F_{BC}.

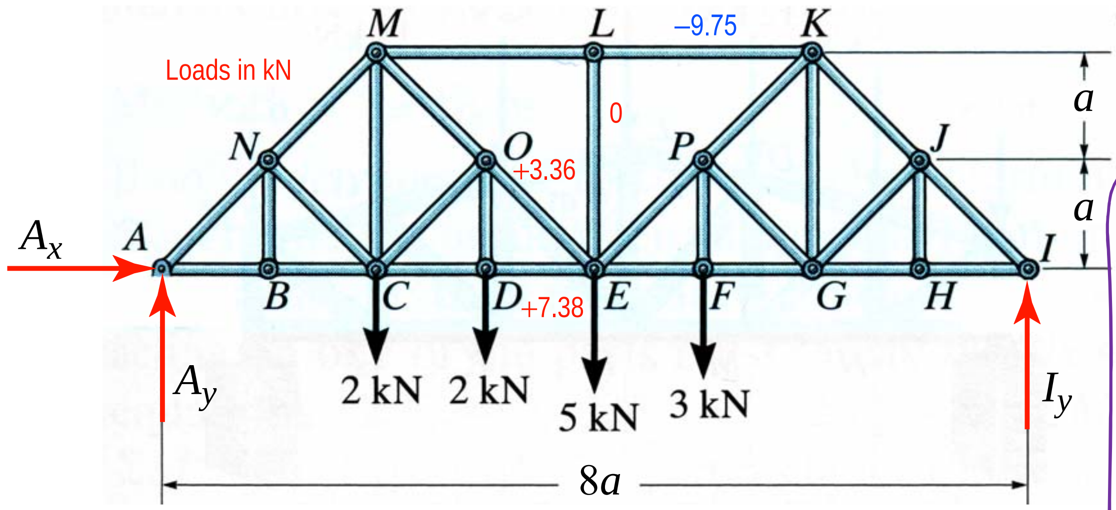
$\Sigma F_x = 0$ $\Sigma M_C = 0$

$\Sigma F_y = 0$

Solve problem on your own. Show that $F_{BC} = 800\text{N}$ ∴ link BC is in tension



Notes added after class:
At the end of lecture today, students asked why is link LE a ZFM since joint E has an applied load of 5kN?



The answer can be determined by examining the situation definitions of a ZFM, which were given in Lecture 17. These situations allow us to find ZFMs by inspection (i.e., by looking without calculations).
Note that these definitions are with respect to the forces and links at a **specific joint**.

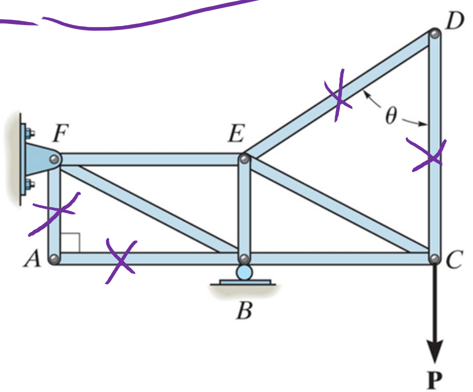
Therefore for the structure to the above-left, there are no external or support reaction forces on joint L; thus LE is a ZFM. We see the same for joints C & D in bottom-middle structure (links DA & CA are ZFM), and joint E in bottom-right structure (link BE is ZFM).

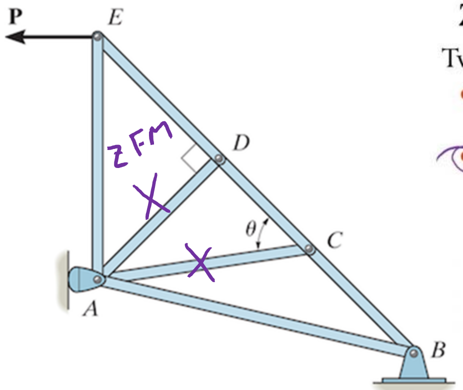
Zero-force members

- Particular members in a structure may experience no force for certain loads.
- Zero-force members are used to increase stability
- Identifying members with zero-force can expedite analysis.

Two situations:

- Joint with two non-collinear members, no external or support reaction applied to the joint → **Both members are zero-force members**.
- Joint with two collinear member, plus third non-collinear, no external or support reaction applied to ~~non-collinear member~~ the joint → **Non-collinear member is a zero-force member**.

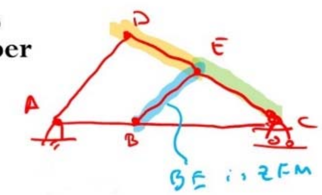




Zero-force members

Two situations:

- Two non-collinear members , no external or support at jt → **Both members are ZFM**
- Two collinear member, plus third non-collinear, no loads on third member → **Non-collinear member is ZFM**.



Frames and machines

Frames and machines are two common types of structures that have at least **one multi-force member.** (Recall that trusses have **only** two-force members.) Therefore, it is not appropriate to use Method of Joints or Method of Sections for frames and machines.

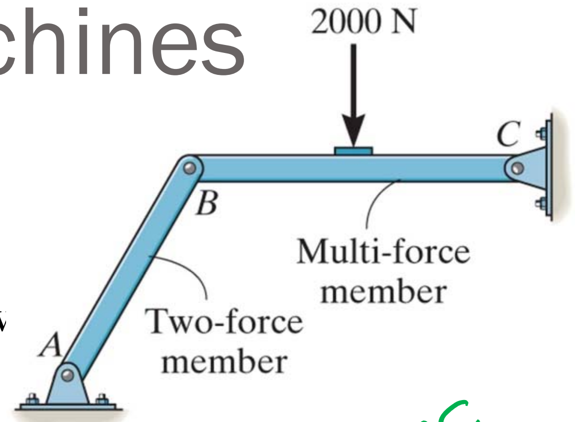
Frames are generally **stationary** and used to support various external loads.

Frames and machines

Frames and machines are two common types of structures that have at least **one multi-force member.** (Recall that trusses have **only** two-force members.) Therefore, it is not appropriate to use Method of Joints or Method of Sections for frames and machines.

Machines contain **moving parts** and are designed to alter the effect of forces.

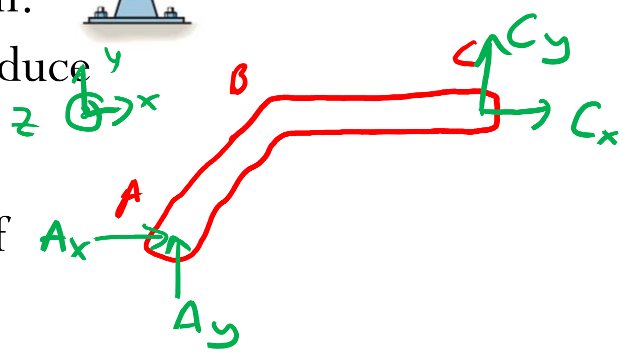
Forces/Moment in frames and machines



LINK 3 or more force

The members can be truss elements, beams, pulleys, cables, and other components. The general solution method is the same:

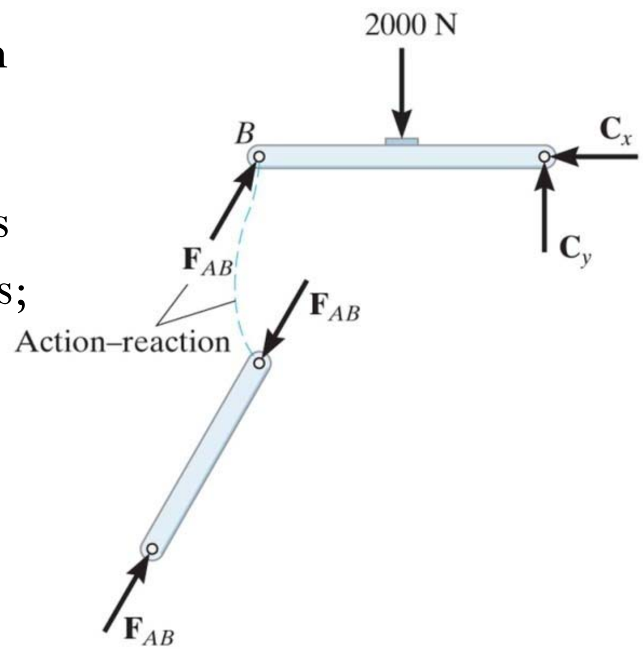
1. Identify external support reactions on entire frame or machine. (Draw FDB of entire structure. Set the structure into external equilibrium: $\sum F_x = 0, \sum F_y = 0, \sum M_{most\ efficient\ pt} = 0$. This step will generally produce more unknowns than there are relevant equations of equilibrium.)



2. Identify zero-force & two-force member(s) to simplify direction of unknown force(s).

3. Draw FDBs of individual subsystems (members). (Isolate part(s) of the structure, setting each part into equilibrium $\sum F_x = 0, \sum F_y = 0$, $\sum M_{most\ efficient\ pt} = 0$. The unknown forces or couples must appear in one or more free-body diagrams.)

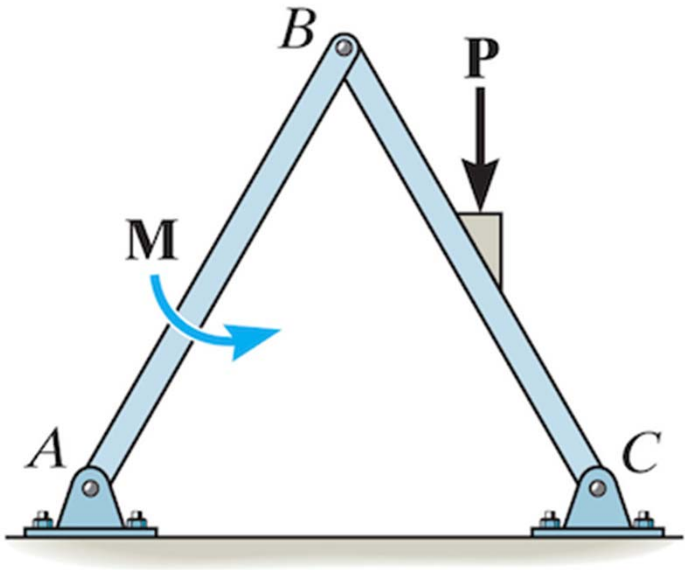
4. Solve for the requested unknown forces or moments. (Look for ways to solve efficiently and quickly: single equations and single unknowns; equations with least # unknowns.)

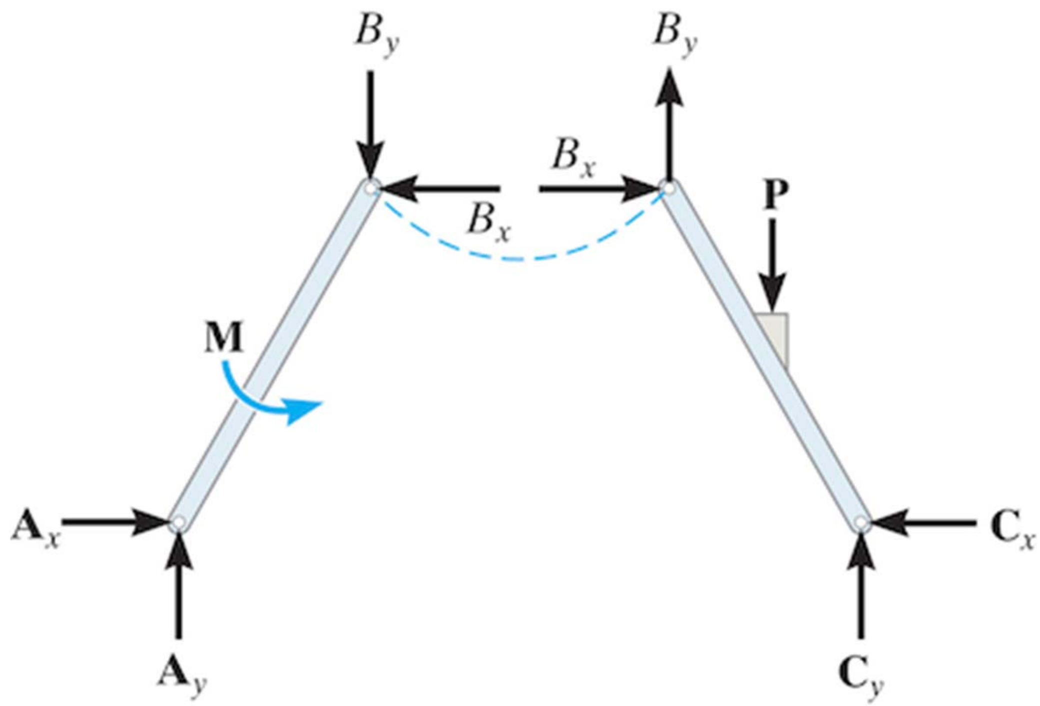


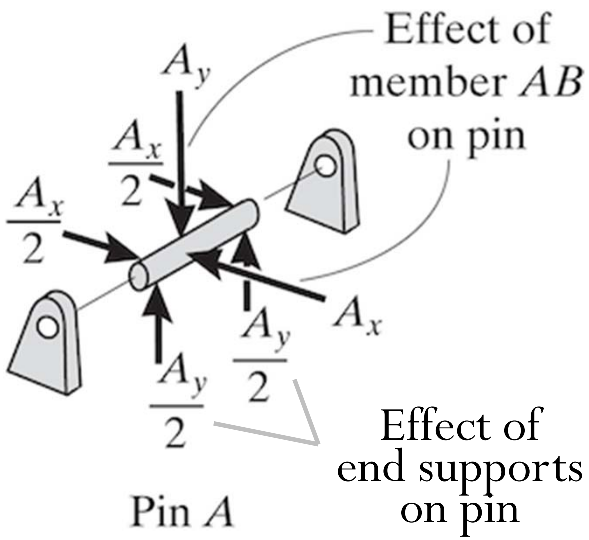
Problems are going to be **challenging** since there are usually several unknowns (and several solution steps). A lot of practice is needed to develop good strategies and ease of solving these problems.

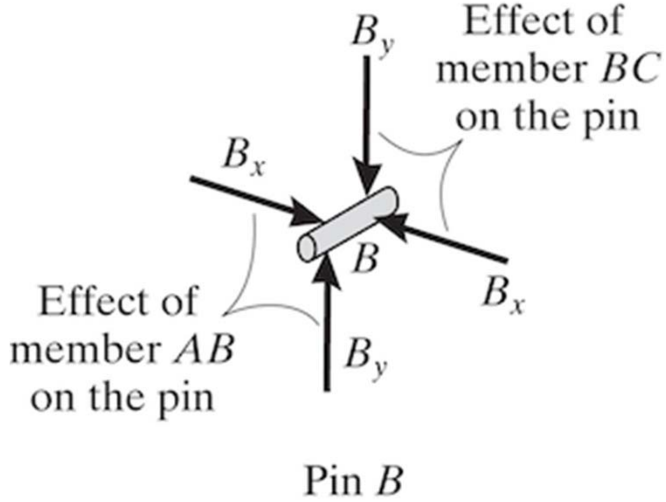
A note about why do we not draw FBD of the pin joint between members:

For the frames, we are interested in forces and/or moments on the rigid body members. Because this method examines individual members, we can ignore the pin that connects the members and directly consider that adjacent members experience equal and opposite forces at the joints.

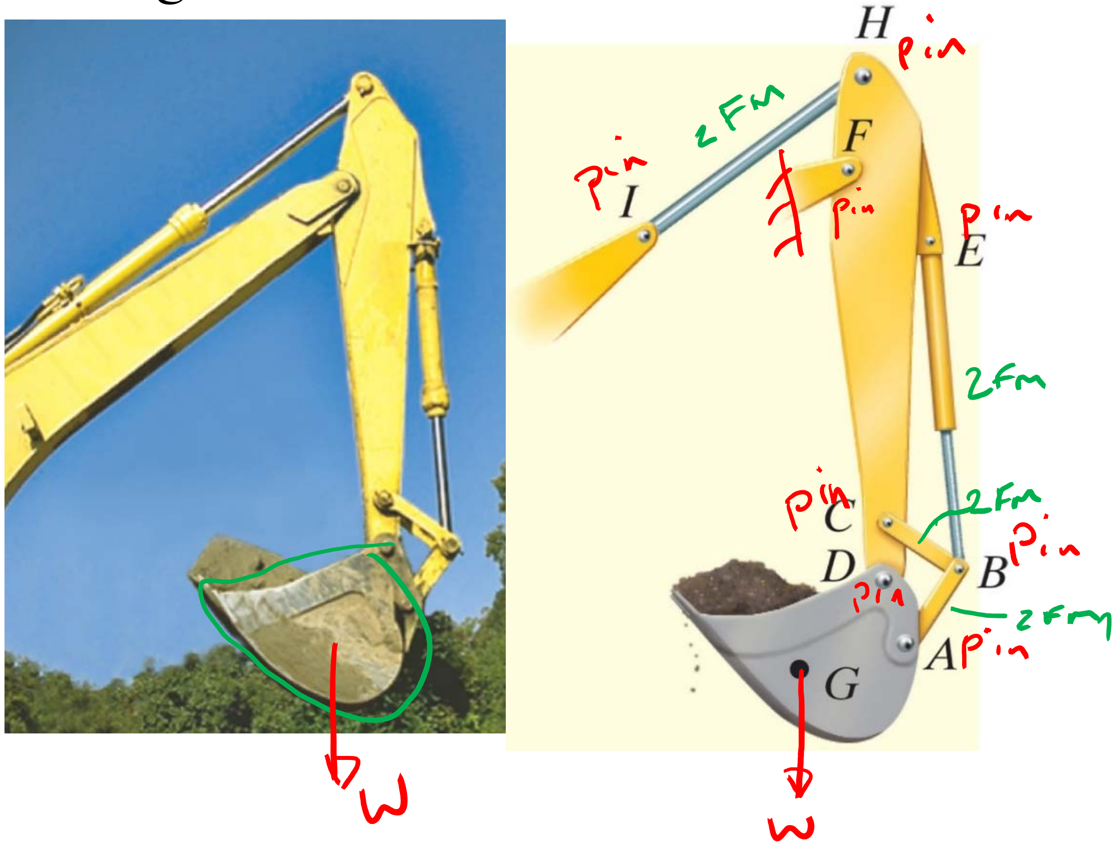








Draw the FBD of the members of the backhoe. The bucket and its contents have a weight W.



1) ID: support rxn forces

2) ID: 2FM or 2FM

if 2FM, draw FBD with forces along the line of action

3) Other joints, not 2FM, draw these with 2 components (x, y) for unknown forces

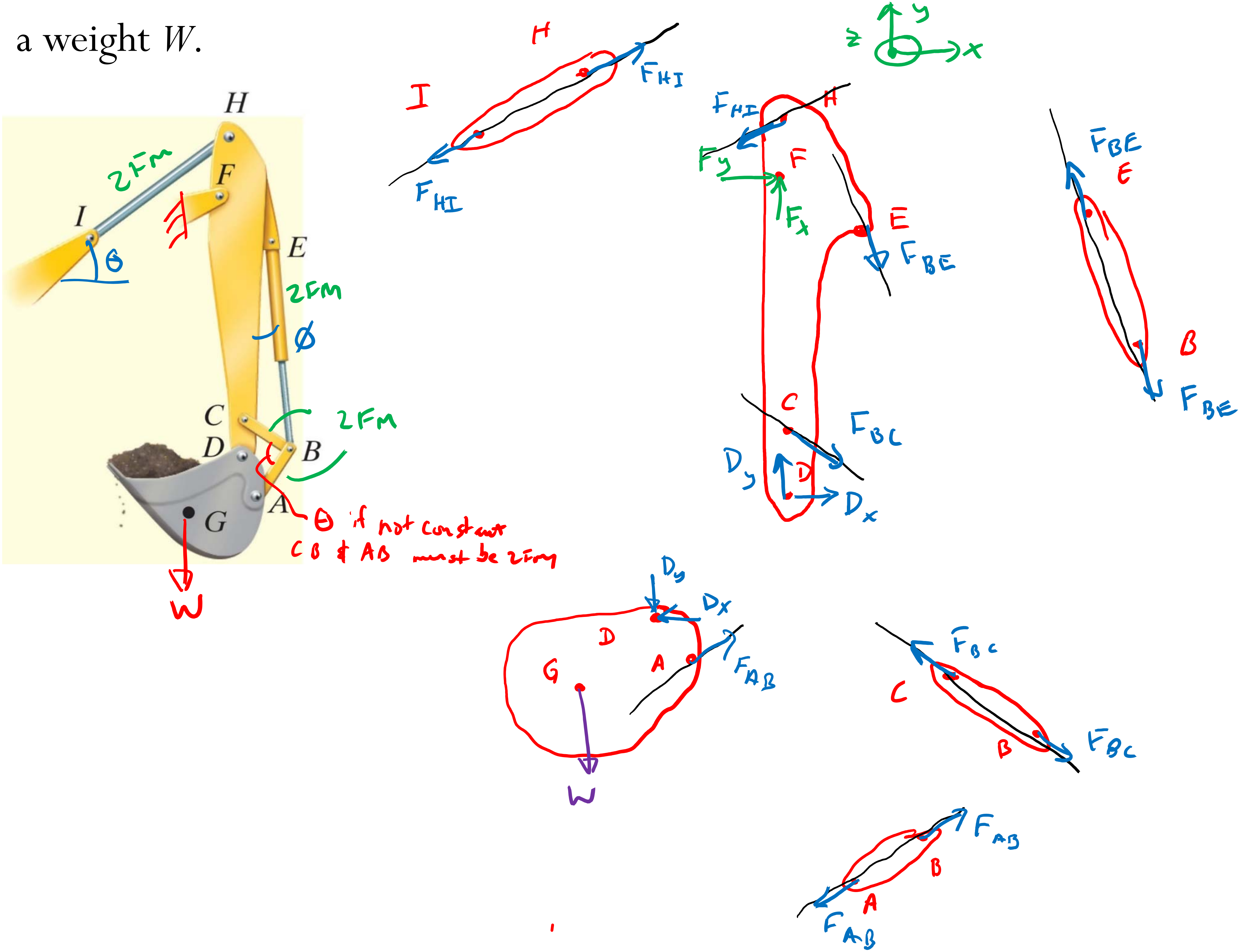
Hydraulic Cylinders

IH, EB

For Static Problem, assume length of H.C. is constant.

$\therefore$ Assume a HC $\equiv$ 2FM

Draw the FBD of the members of the backhoe. The bucket and its contents have a weight W.



A 500 kg elevator car is being hoisted by a motor using a pulley system. If the car travels at a constant speed, determine the force developed in the cables. Neglect the cable and pulley masses.

We'll label the tension in the rightmost cable T_1, and tension in the leftmost cable T_2. Which is an equation for equilibrium of pulley C?

A. $T_1 + 2T_2 = 0$

B. $2T_1 + T_2 = 0$

C. $T_1 - T_2 = 0$

D. $2T_1 - T_2 = 0$

E. $T_1 - 2T_2 = 0$

y x

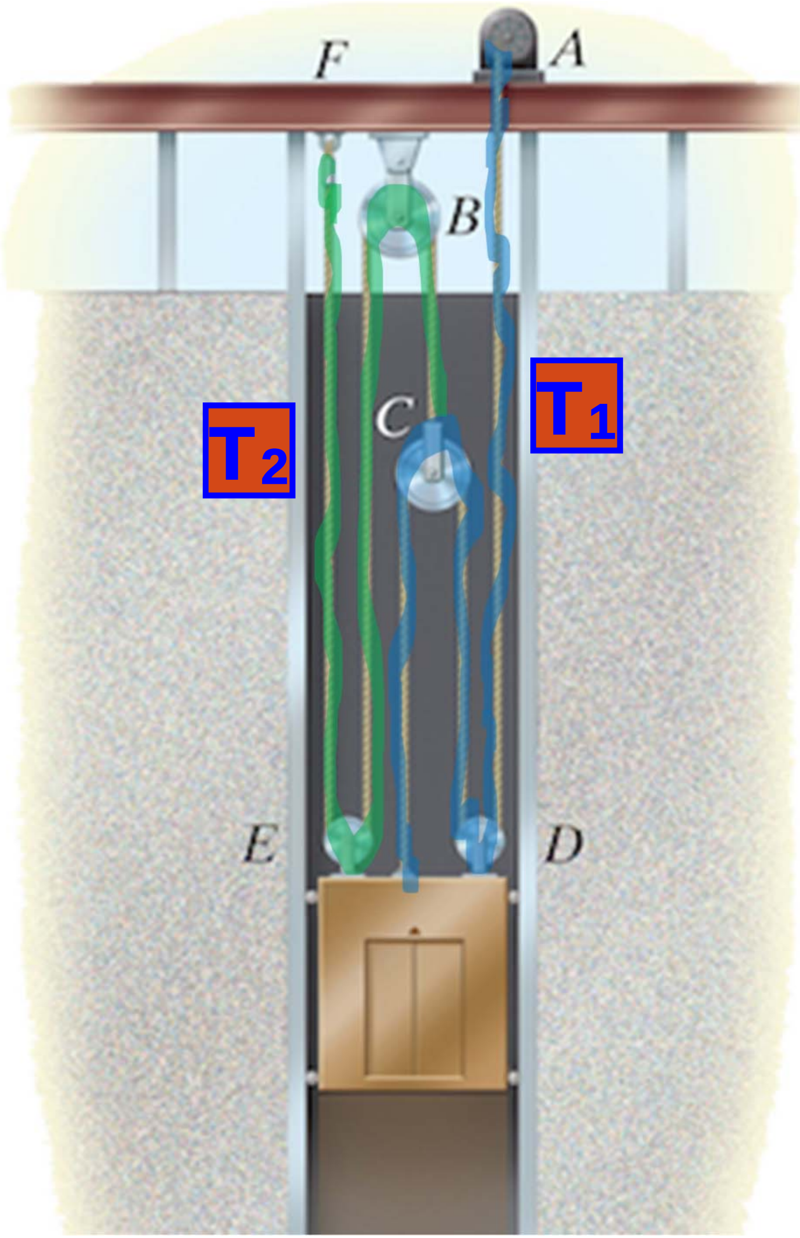
T_2

C

T_1 T_1

$\Sigma F_y = 0$

$T_2 - 2T_1 = 0$



A 500 kg elevator car is being hoisted by a motor using a pulley system. If the car travels at a constant speed, determine the force developed in the cables. Neglect the cable and pulley masses.

We'll label the tension in the rightmost cable T_1, and tension in the leftmost cable T_2. Which is an equation for equilibrium of the car?

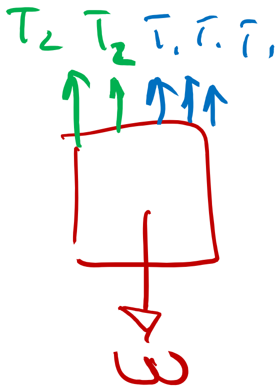
A. $3T_1 + 2T_2 + 500(9.81)\text{ N} = 0$

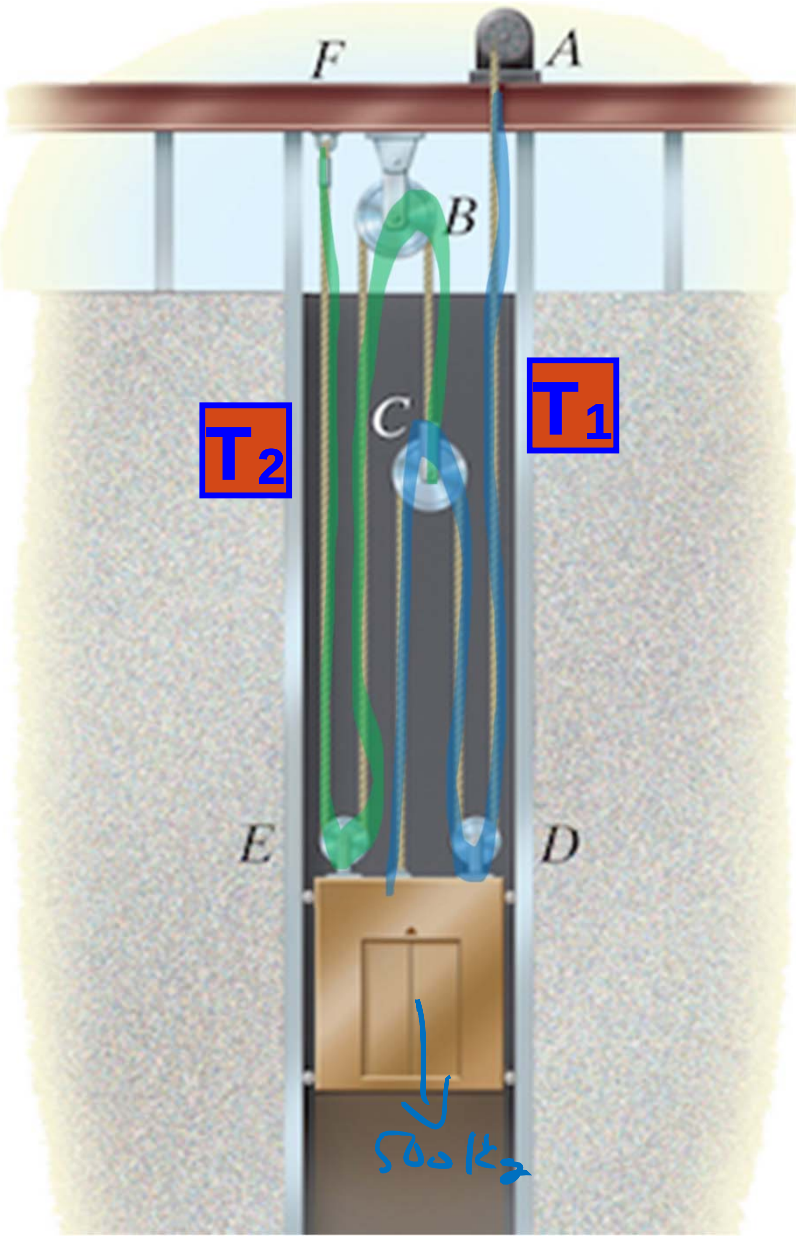
B. $3T_1 - 4T_2 + 500(9.81)\text{ N} = 0$

C. $3T_1 + 2T_2 - 500(9.81)\text{ N} = 0$

D. $3T_1 - 2T_2 - 500(9.81)\text{ N} = 0$

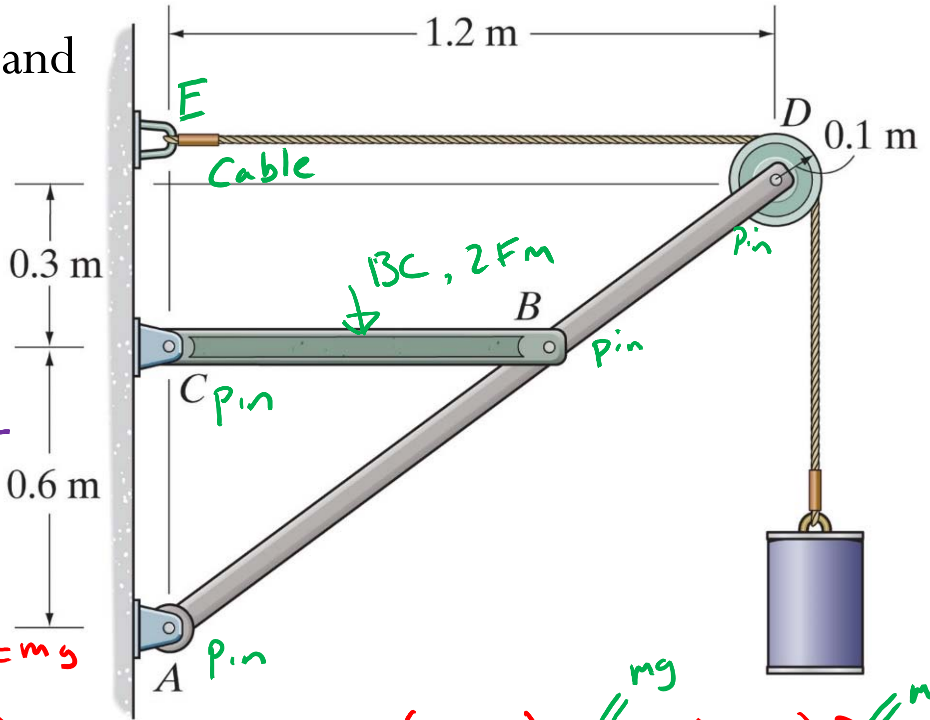
E. None of the above





Here are some sample problems that have already been solved

The frame supports a 50kg cylinder. Determine the horizontal and vertical components of reaction at A and the force at C



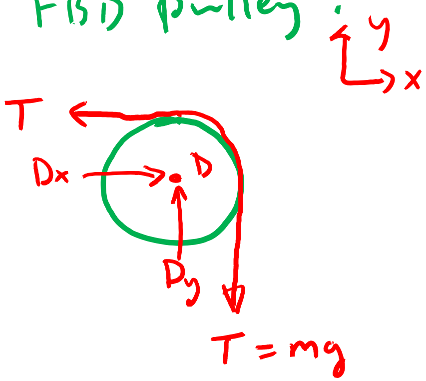
Find: A_x, A_y, F_{BC}

ID: 2FM ?

ID: supports

FBD: BC (2FM)

FBD pulley:



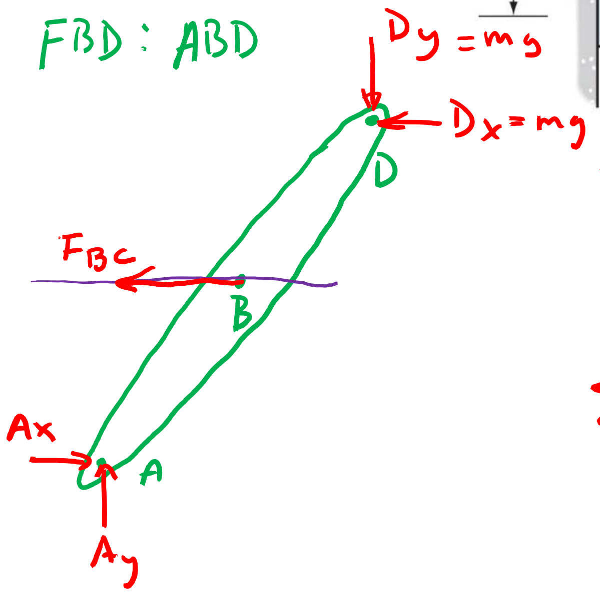
$\Sigma F_x: D_x - T = 0$

$\boxed{D_x = T = mg}$

$\Sigma F_y: D_y - T = 0$

$\boxed{D_y = mg}$

FBD: ABD



$+\circlearrowleft \Sigma M_A: (0.9\,m)\underbrace{D_x}_{mg} - (1.2\,m)\underbrace{D_y}_{mg} + (0.6\,m) F_{BC} = 0$

$\boxed{F_{BC} = 245\,N}$ $m = 50\,kg$

$\Sigma F_x: A_x - F_{BC} - D_x = 0$

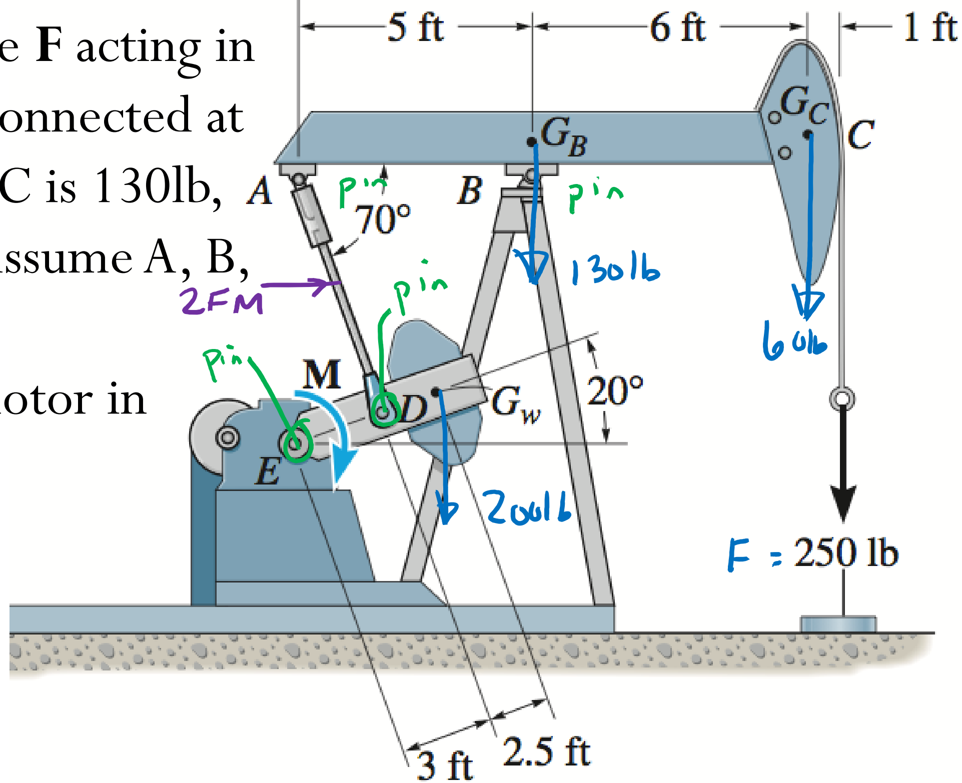
$\boxed{A_x = 736\,N}$

$\Sigma F_y: A_y - D_y = 0$

$\boxed{A_y = 490\,N}$

Given: The pumping unit used to recover oil has force **F** acting in the wireline at the well head. The pitman, *AD*, is pin connected at its ends and has negligible weight. Weight of beam ABC is 130lb, horsehead at C is 60lb, counterweight at D is 200lb. Assume A, B, C, G_B and G_C are collinear.

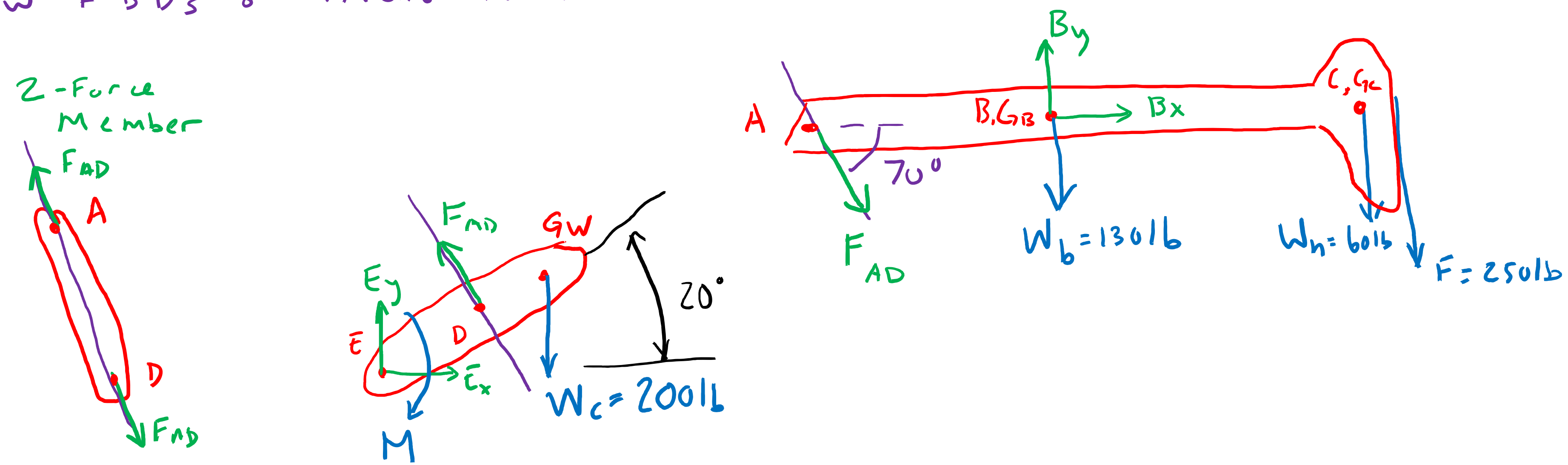
Find: The torque **M** which must be exerted by the motor in order to overcome this load.

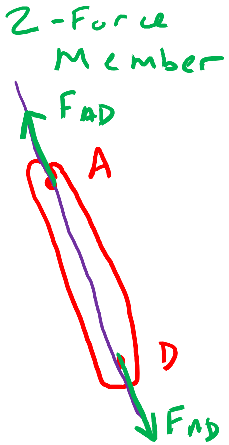


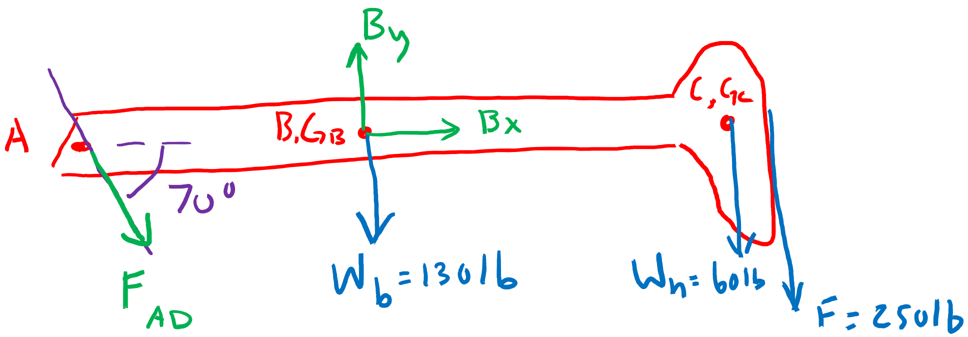
1) Identify any two-force members ⇒ AD
Any zero force members? No

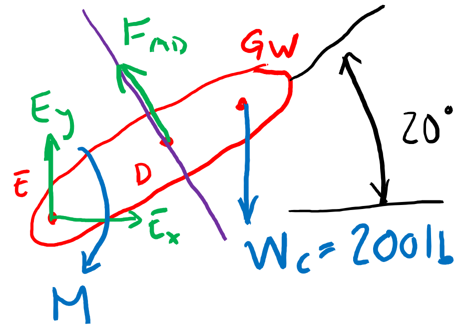
2) Identify support reaction types.
Recall supports are ways to secure the structure to ground.
⇒ pin supports at B & E

3) Draw FBDs of individual members.









4) Solve for unknowns ⇒ Find M

$$+\circlearrowleft \sum M_E : \quad -M + (3ft)F_{AD} - (5.5ft)W_c \cos 20° = 0$$

$$M = (3ft)F_{AD} - (5.5ft)(200 lb)\cos 20°$$

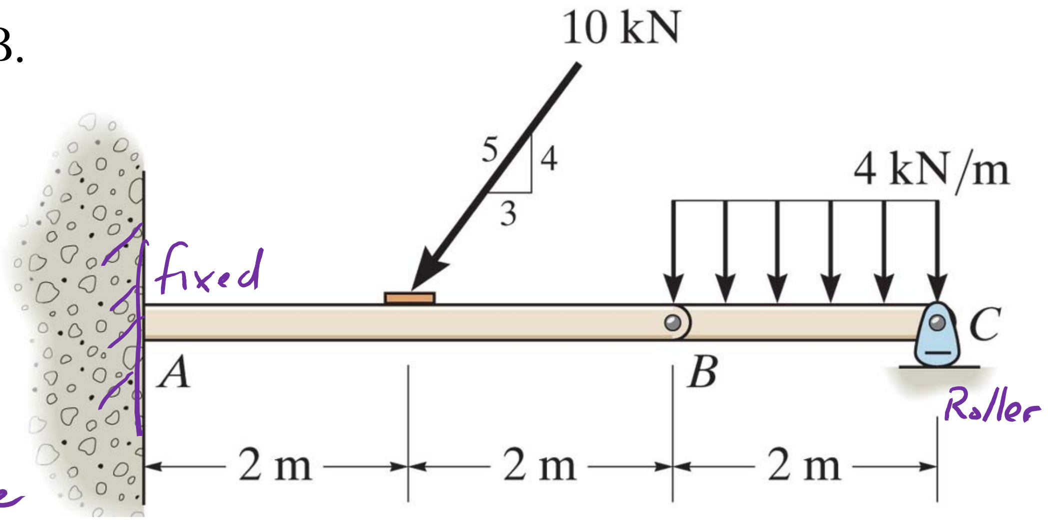
$$+\circlearrowleft \sum M_B : \quad (5ft)F_{AD}\sin(70°) - (6ft)W_h - (7ft)F = 0$$

$$F_{AD} = \frac{(6ft)(60lb) + (7ft)(250lb)}{(5ft)\sin 70°} \Rightarrow \boxed{F_{AD} = 449 lb}$$

$$\therefore M = (3ft)(449 lb) - (5.5ft)\,200 lb \cos 20°$$

$$\Rightarrow \boxed{M = 314 lb \cdot ft}$$

The compound beam shown is pin-connected at B. Determine the components of reaction at its supports. Neglect its weight and thickness.

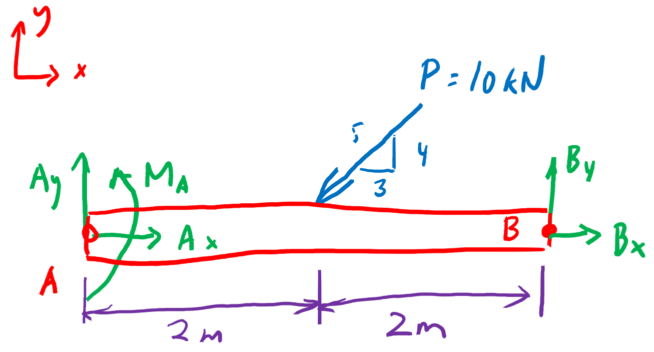


Find: A_x, A_y, M_A, C_y

1) Identify 2 force or zero force members → None

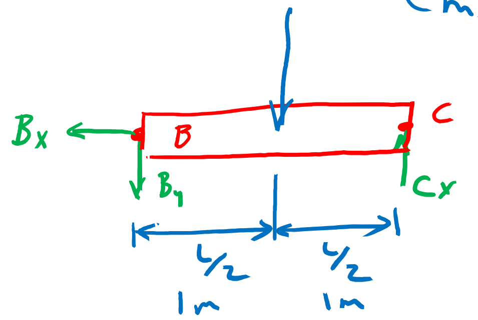
2) Identify support reaction types ⇒ fixed@A, roller@C

3) Draw FBDs of individual members



5 unknowns

$wL = \left(4 \frac{kN}{m}\right) 2m = 8kN$



3 unknowns

4) Solve for unknowns

Start with solving for unknowns on BC:

$\Sigma F_x: \ -B_x = 0 \ \Rightarrow \ \boxed{B_x = 0}$

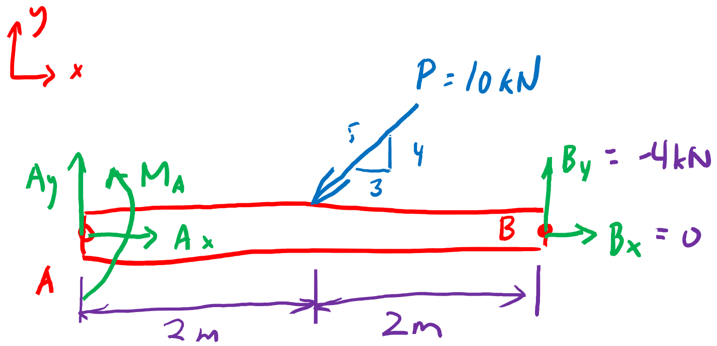
$\Sigma F_y: \ -B_y - wL + C_y = 0$

$B_y = C_y - 8kN$

$+\circlearrowleft \Sigma M_B: \ -\left(\frac{L}{2}\right) wL + L C_y = 0$

$C_y = \frac{(1m)(8kN)}{2m} \rightarrow \boxed{C_y = 4kN} \uparrow$ upward

$\therefore B_y = 4kN - 8kN = -4kN \rightarrow \boxed{B_y = -4kN}$



Now solve for remaining unknowns on left side:

$$\Sigma F_x : A_x - P\left(\tfrac{3}{5}\right) + \underbrace{B_x}_{=0} = 0$$

$$A_x = P\tfrac{3}{5}$$

$$\boxed{A_x = 6kN} \text{ toward right}$$

$$\Sigma F_y : A_y - P\left(\tfrac{4}{5}\right) + B_y = 0$$

$$A_y = P\left(\tfrac{4}{5}\right) - (-4kN)$$

$$\boxed{A_y = 12kN} \text{ upward}$$

$$\circlearrowleft + \; \Sigma M_A : M_A - (2m)\,P\left(\tfrac{4}{5}\right) + (4m)\,B_y = 0$$

$$M_A = \left(\tfrac{8}{5}m\right)(10kN) - (4m)(-4kN)$$

$$\boxed{M_A = 32\,kN \cdot m} \circlearrowleft \text{ ccw}$$