Statics - TAM 211

Lecture 21

November 12, 2018

Chap 7.2

Announcements

- Upcoming deadlines:
- Tuesday (11/12)
  - Prairie Learn HW 8
- Friday (11/16)
  - Written Assignment 8
- Quiz 4
  - Wednesday Nov 14
  - 9:00am-9:50am (class time)
  - Try to arrive early to log in to computer
  - Instructional Lab Building: D211 (ME), D331 (CEE)
  - 3D rigid body (Chap 4)
  - Structural Analysis (Chap 5)

Chapter 7: Internal Forces

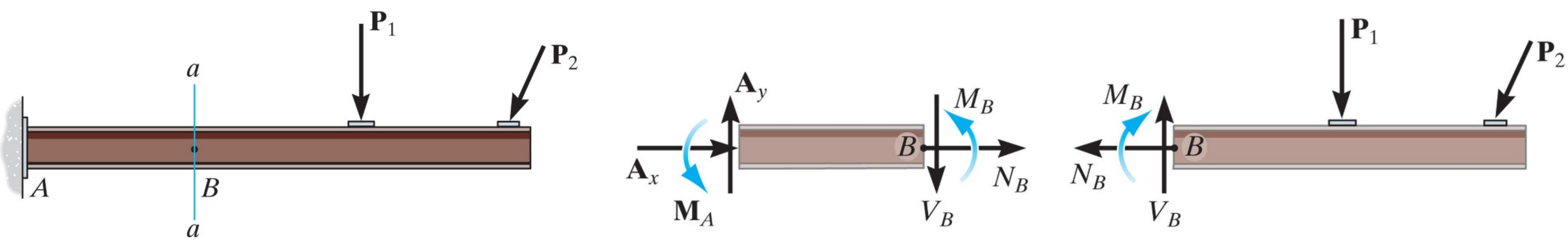
Goals and Objectives

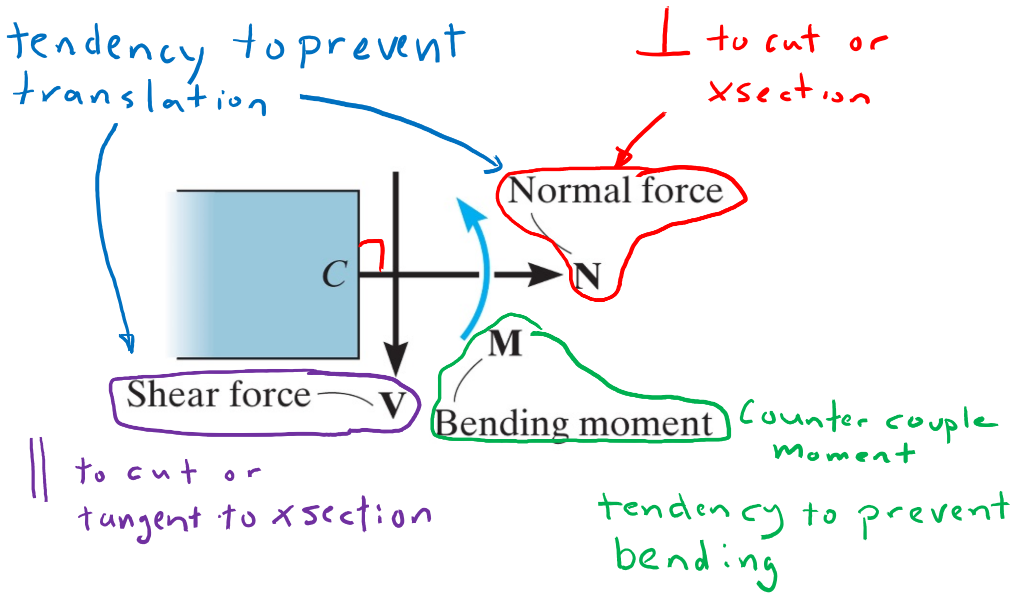
- Determine the internal loadings in members using the method of sections
- Generalize this procedure and formulate equations that describe the internal shear and bending moment throughout a member
- Be able to construct or identify shear and bending moment diagrams for beams when distributed loads, concentrated forces, and/or concentrated couple moments are applied

Recap: Internal loadings in structural members

Structural Design: need to know the loading acting within the member in order to be sure the material can resist this loading

Cutting members at internal points reveal **internal forces and moments.** ⟹ use Method of Sections

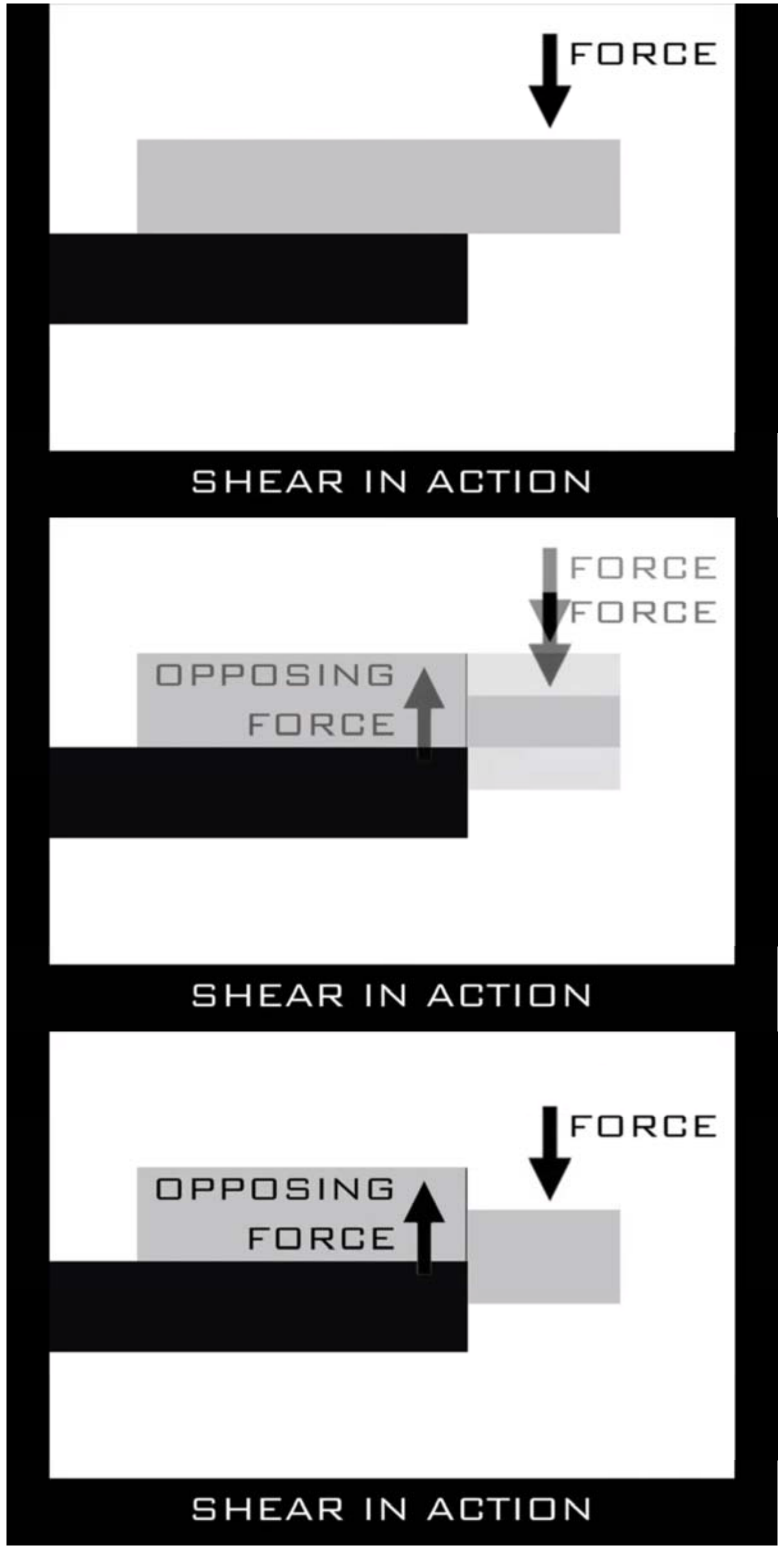




2D

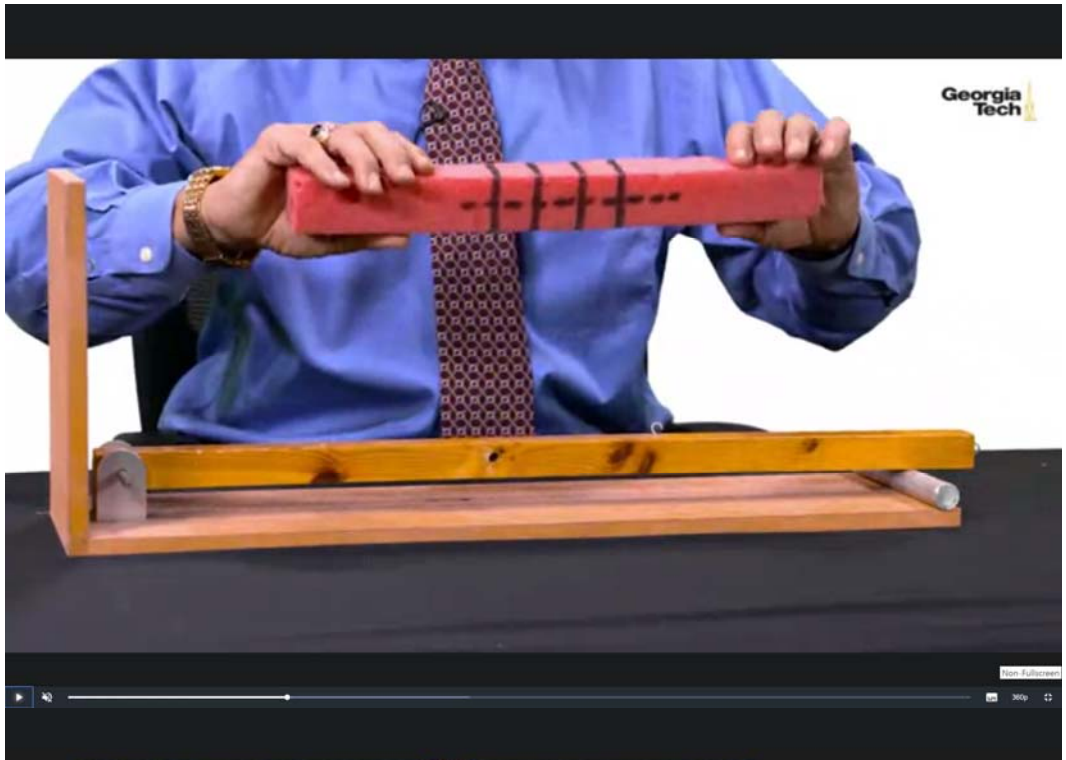
N, V, M ← key labels to learn

Positive Shear Force



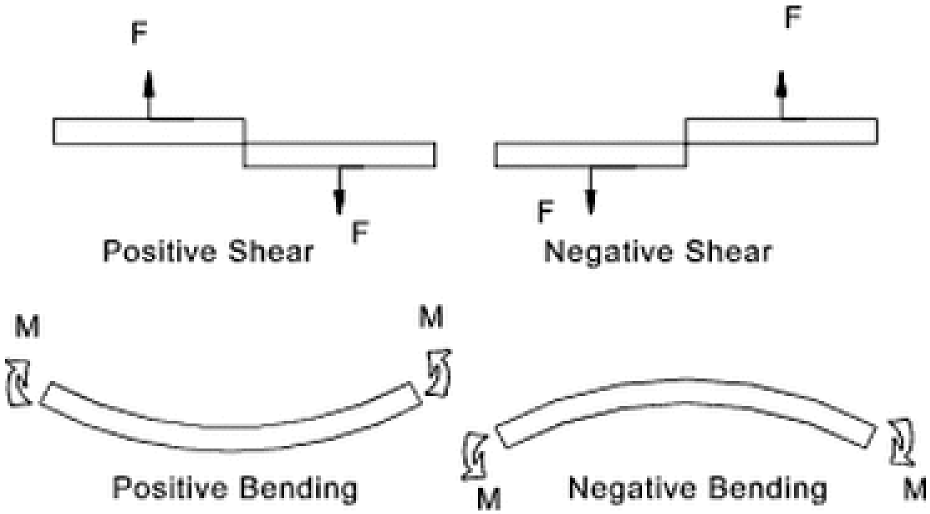
https://youtu.be/-Jbdofrbt6Y

Positive Bending Moment

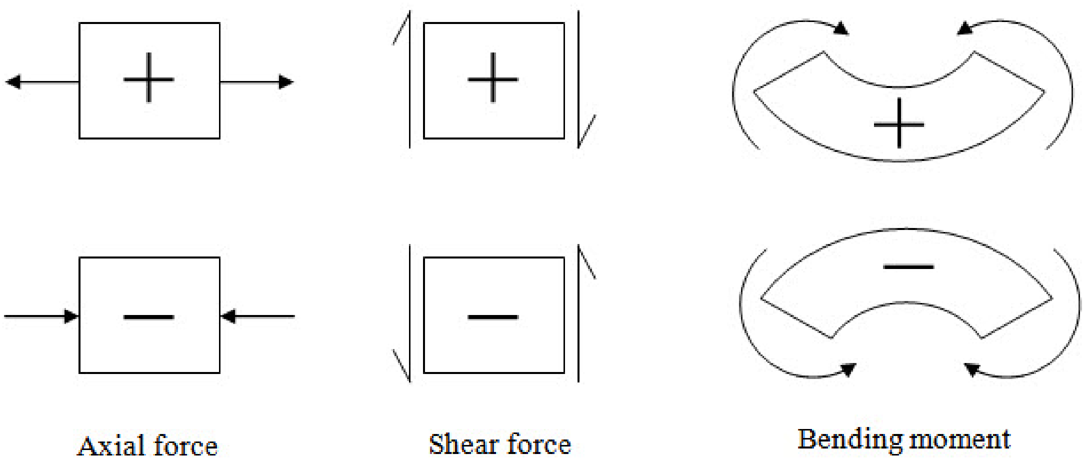


https://fr.coursera.org/lecture/beam-bending/module-2-real-world-beam-bending-examples-c9r68

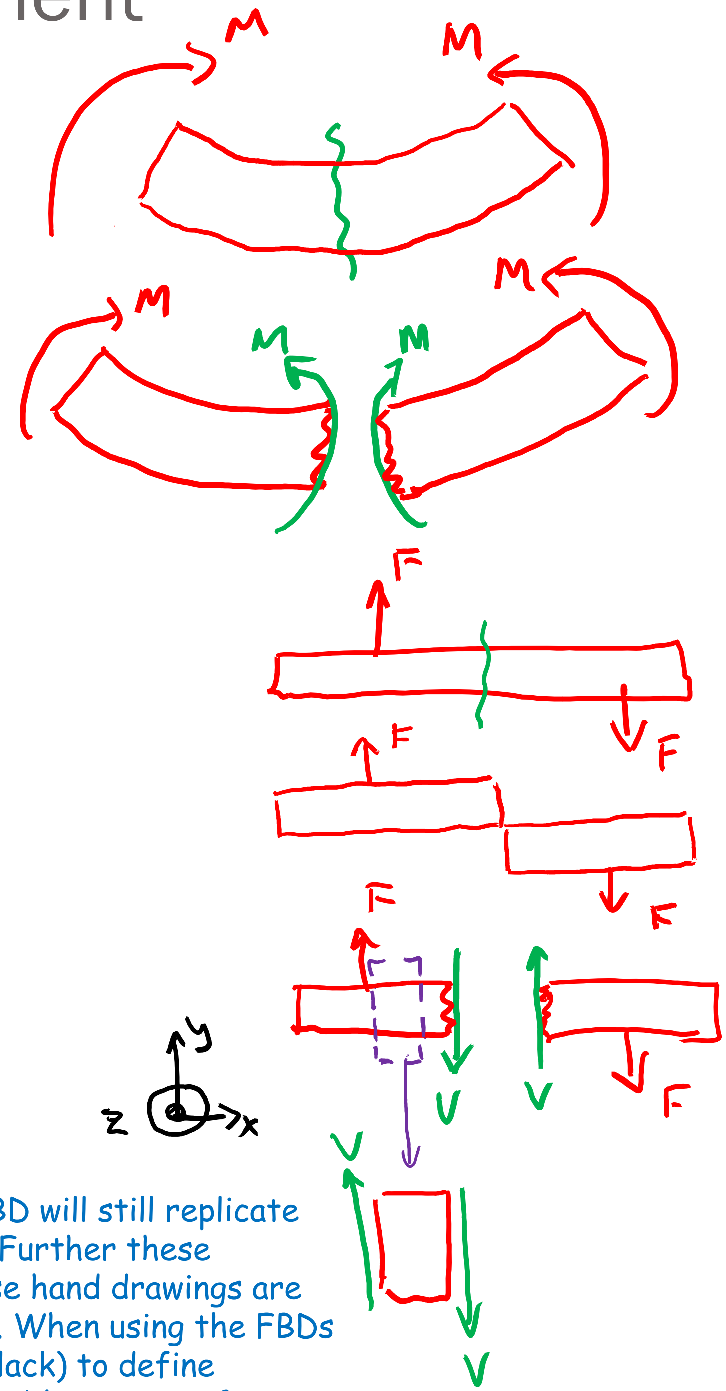
Recap: Shear Force and Bending Moment



http://structureanalysis.weebly.com/bending-moment--shear-force.html

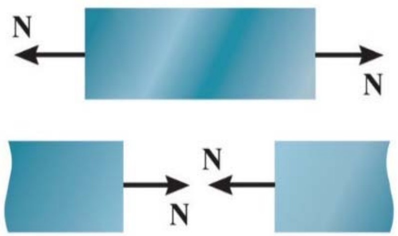


https://ecoursesonline.icar.gov.in/mod/page/view.php?id=125191



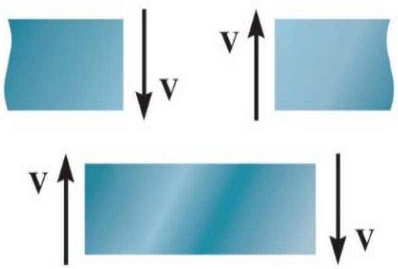
Notes about hand-drawn material: even when cutting a beam (green), the new smaller FBD will still replicate the bending moments or shear forces on the cut surfaces on the left or right segment. Further these replicated moments/forces should be drawn to be equal and in opposite directions. These hand drawings are for when the bending moments and shear forces are drawn to be in the “positive” sense. When using the FBDs to write out the eqns of equilibrium, use the axes of your coordinate system diagram (black) to define whether the vectors are pointing in a positive or negative direction – see any example problem to see if a particular force or moment is + or – in the eqn.

Recap: Sign conventions:



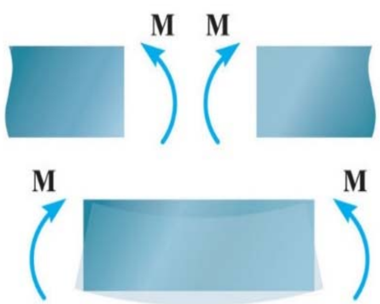
Positive normal force

Tension



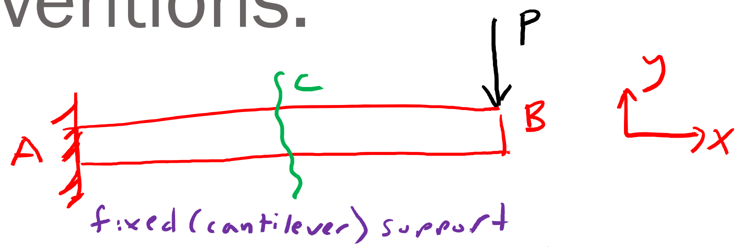
Positive shear

Clockwise rotation



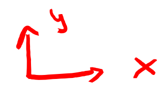
Positive moment

Concave up

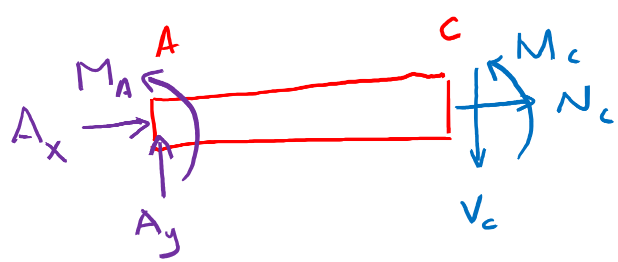


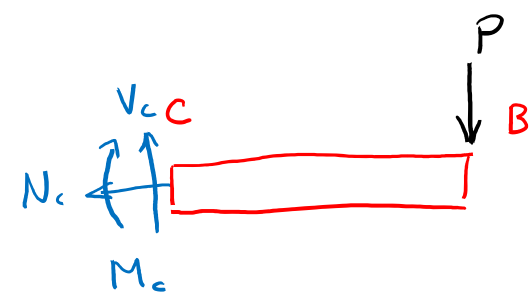
If beam AB is cut at C, draw FBDs of sections AC, CB illustrating assumptions of N, V, M drawn in positive directions.

FBD AC:

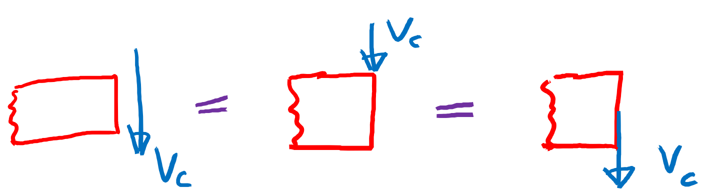


FBD CB:



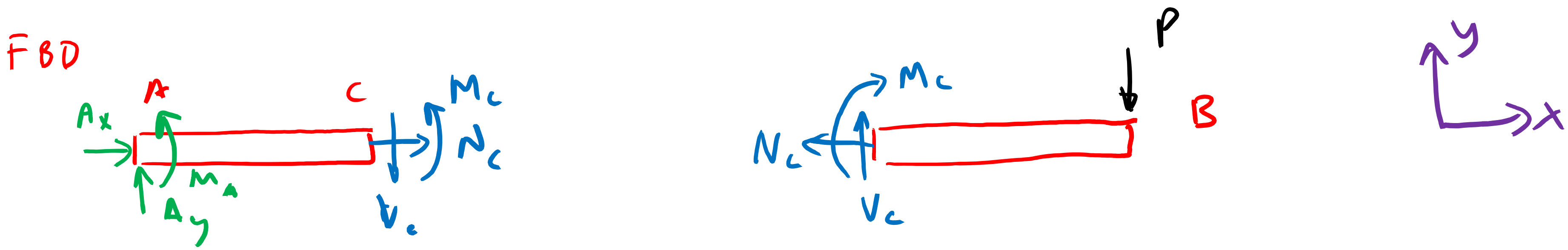


Note: although draw V off the side of the cut section, V is actually applied at the cut.



Recap: What sign to give N, V and M terms in equations of equilbrium?

Follow the positive orientations of the coordinate system.



$+\rightarrow \Sigma F_x: \; A_x + N_c = 0$

$+\uparrow \Sigma F_y: \; A_y - V_c = 0$

$+\circlearrowleft \Sigma M_A: \; M_A + M_c - r_{AC} V_c = 0$

$+\rightarrow \Sigma F_x: \; -N_c = 0$

$+\uparrow \Sigma F_y: \; V_c - P = 0$

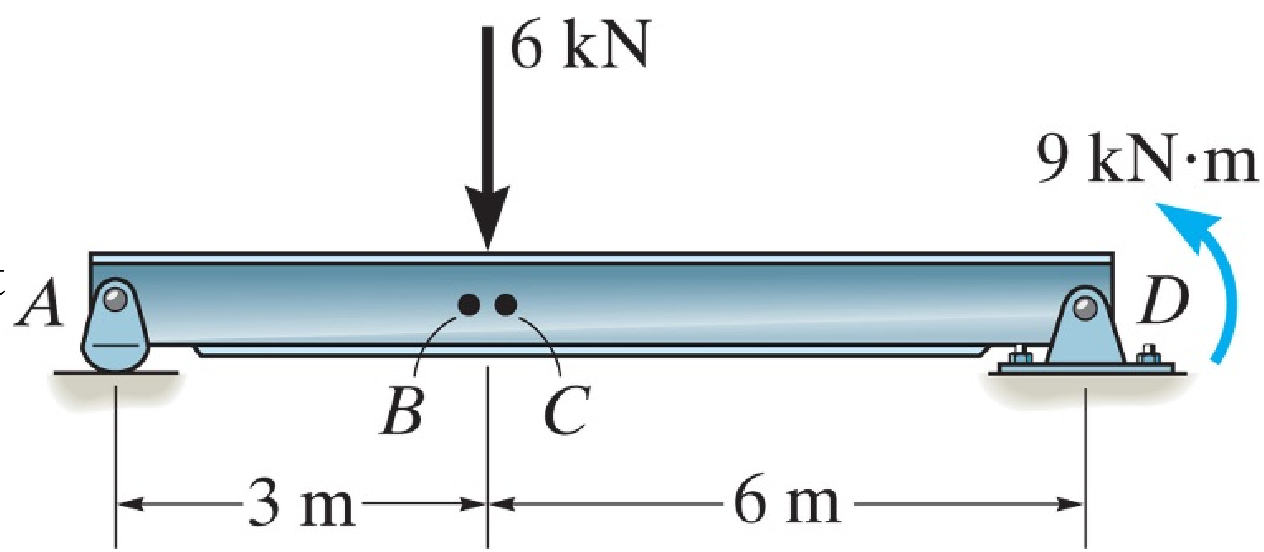
$+\circlearrowleft \Sigma M_B: \; -M_c - r_{CB} V_c = 0$

couple moment
$\Rightarrow$ free vectors

Procedure for analysis:

1. Find support reactions (free-body diagram of entire structure)
2. Pass an imaginary section through the member
3. Draw a free-body diagram of the segment that has the least number of loads on it
4. Apply the equations of equilibrium

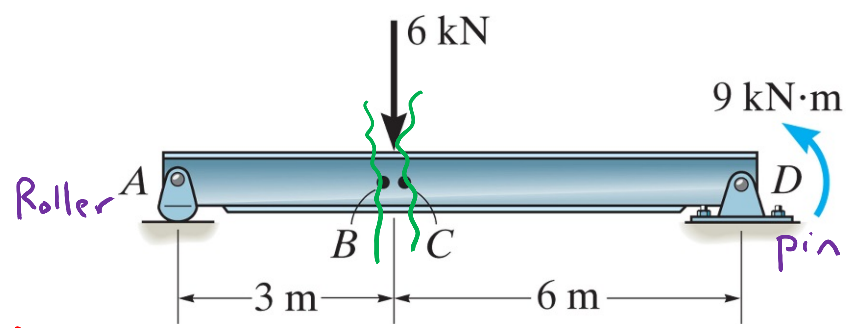
Find the internal forces and moments at B (just to the left of load P) and at C (just to the right of load P)



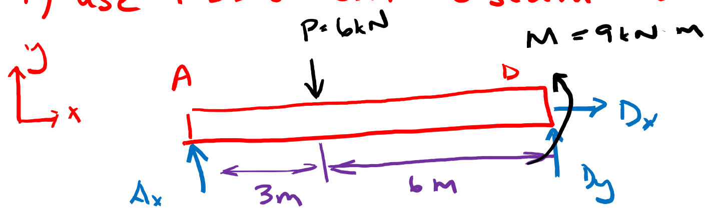
Procedure for analysis:

1. Find support reactions (free-body diagram of entire structure)
2. Pass an imaginary section through the member
3. Draw a free-body diagram of the segment that has the least number of loads on it
4. Apply the equations of equilibrium

Find the internal forces and moments at B (just to the left of P) and at C (just to the right of P)



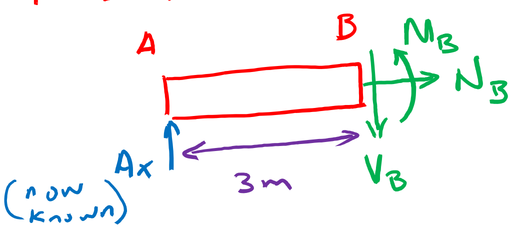
1) use FBD of entire beam AD to find unknown rxn forces



3 unknowns (Ax, Dx, Dy) → 3 eqns ΣFx, ΣFy, ΣM
can solve for unkn rxns

Steps 2, 3, 4:

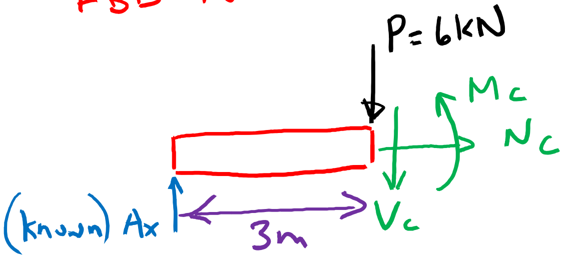
FBD for AB



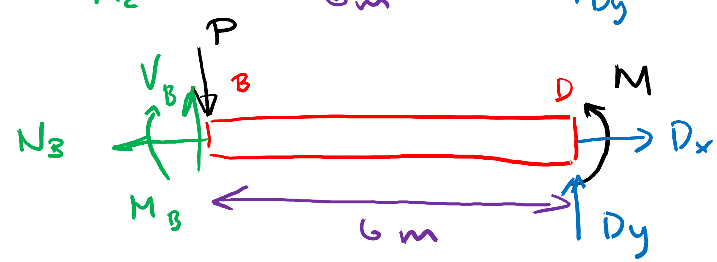
3 unknowns (NB, VB, MB) → 3 eqns: ΣFx, ΣFy, ΣMB
can solve for unknown internal forces

Note: Could have also used FBDs of CD or BD

FBD for AC



3 unknowns (Nc, Vc, Mc)
3 eqns: ΣFx, ΣFy, ΣMc
solve for unknowns



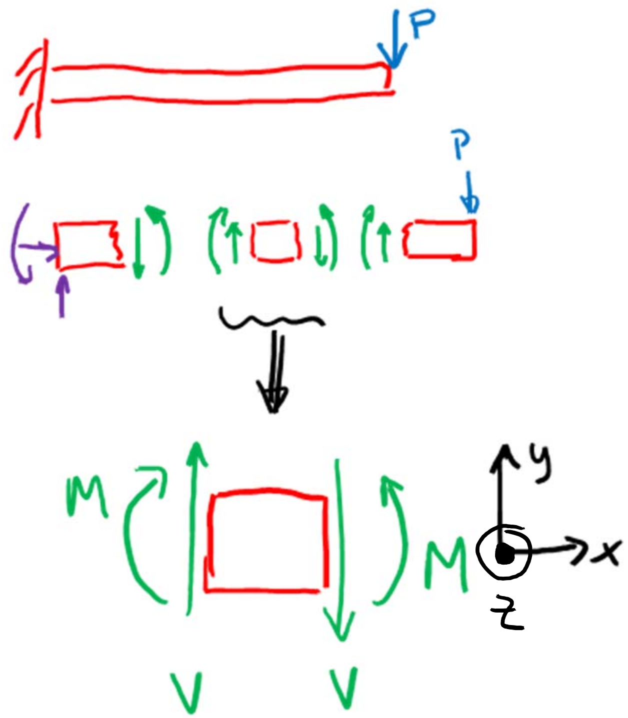
How to orient positive V and M on a FBD?

"Positive" sign convention:

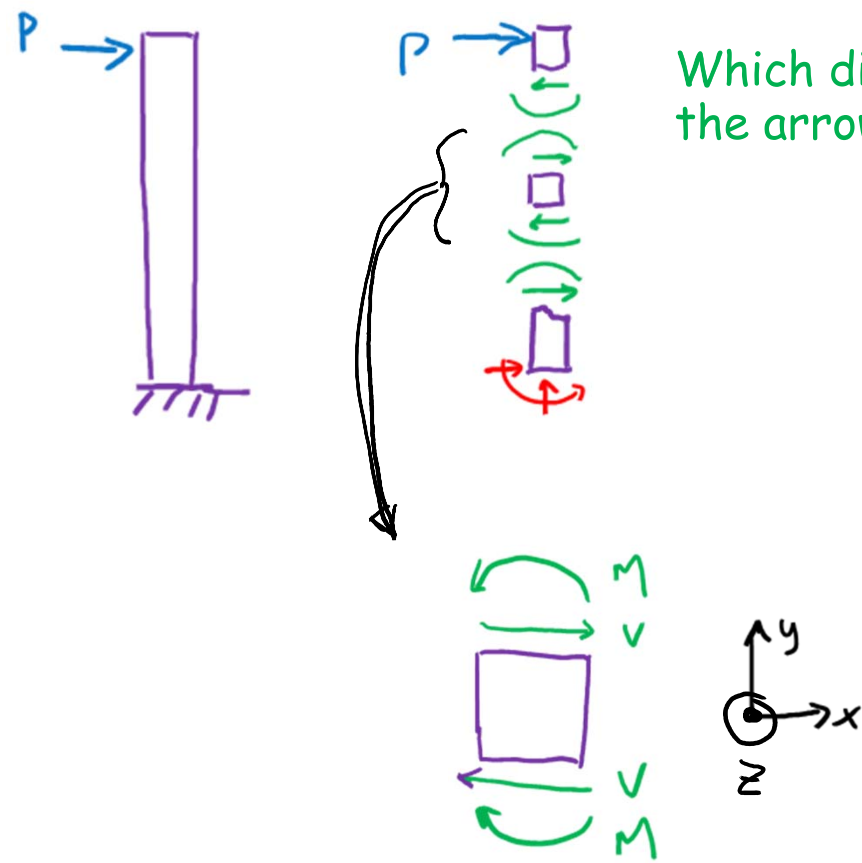
"Positive shear will create a clockwise rotation" ⟹ Draw V arrows to create CW rotation

"Positive bending moment will create bend that is concave upward"

Horizontal Beam

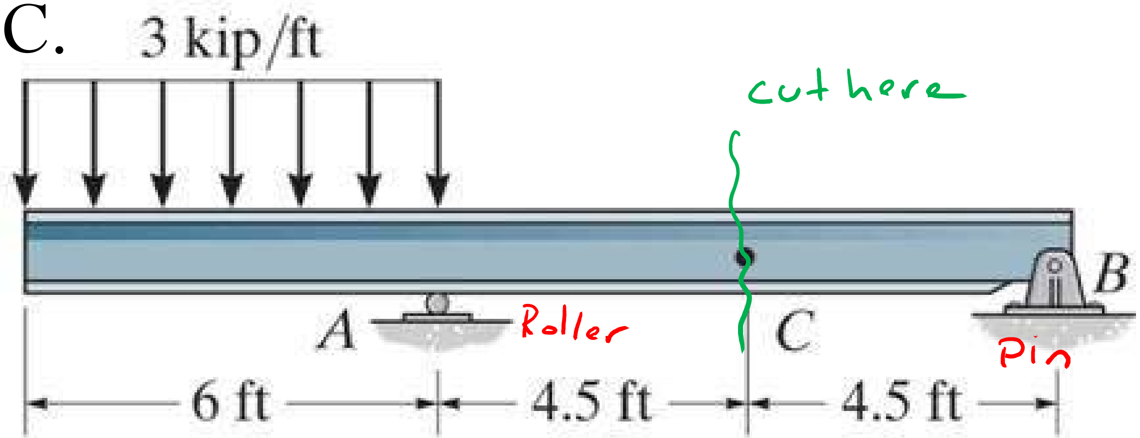


Vertical Beam

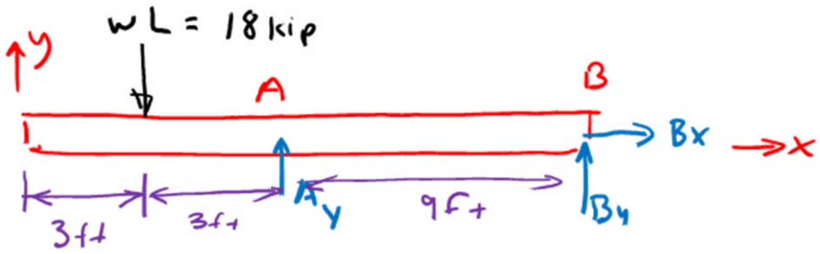


Which direction of the arrow for M?

Find the internal forces at point C.



FBD of entire beam



3 unknowns (A_y, B_x, B_y)

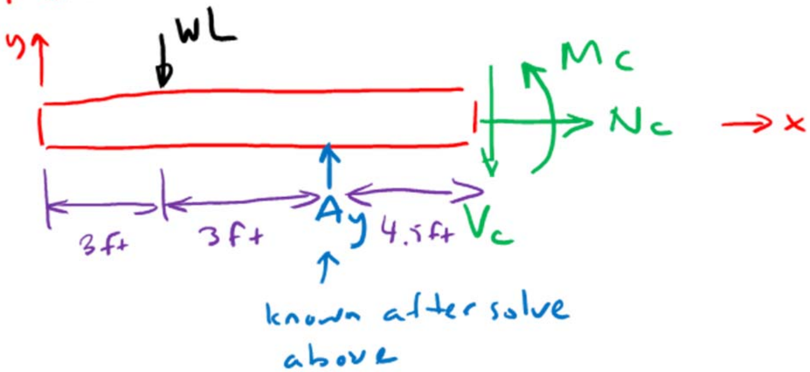
use 3 EoE to solve for A_y, B_x, B_y.

ΣF_x : $\boxed{B_x = 0}$, ΣF_y : $A_y + B_y - wL = 0$ → $\boxed{B_y = -6 \text{ kip}}$

+↺ ΣM_B : $(12 ft)\, wL - (9 ft)\, A_y = 0$ → $\boxed{A_y = 24 \text{ kip}}$

B ↓ B_y

FBD of left section :



3 unknowns (N_c, V_c, M_c); assuming know A_y

use EoE :

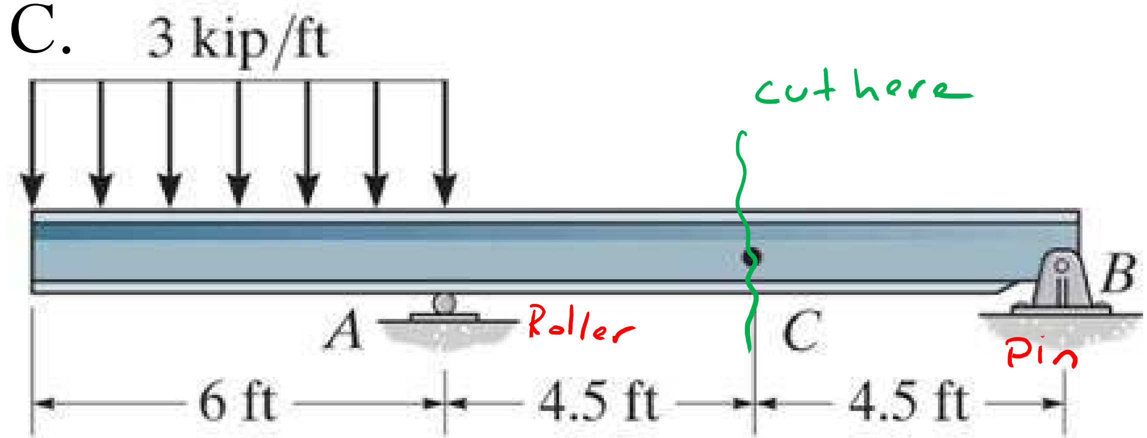
ΣF_x : $\boxed{N_c = 0}$

ΣF_y : $A_y - wL - V_c = 0$ ⇒ $\boxed{V_c = 6 \text{ kip}}$

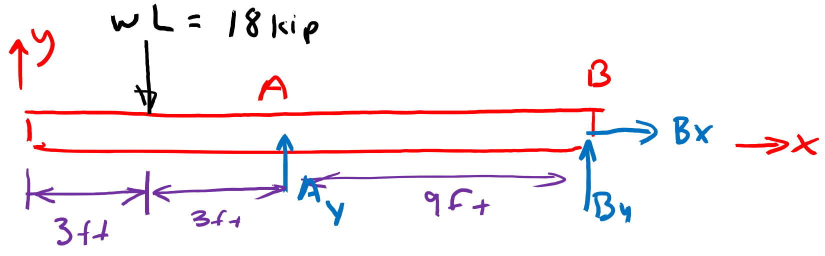
+↺ ΣM_c : $M_c - (4.5 ft)\, A_y + (7.5 ft)\, wL = 0$

⇒ $\boxed{M_c = -27 \text{ kip ft}}$

Find the internal forces at point C.



FBD of entire beam

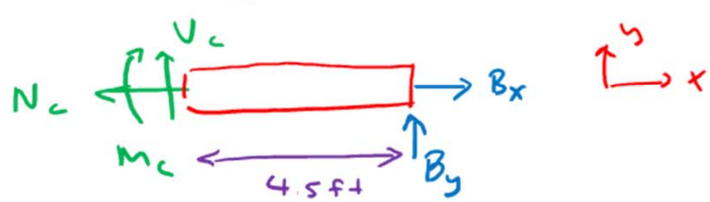


3 unknowns (A_y, B_x, B_y)

use 3 EoE to solve for A_y, B_x, B_y. $\boxed{A_y = 24\text{ kip}}$ $\boxed{B_x = 0}$ $\boxed{B_y = -6\text{ kip}}$

Alternatively, could examine right section:

FBD of right section



3 unknowns (N_c, V_c, M_c) assuming know B_x, B_y

use EoE:

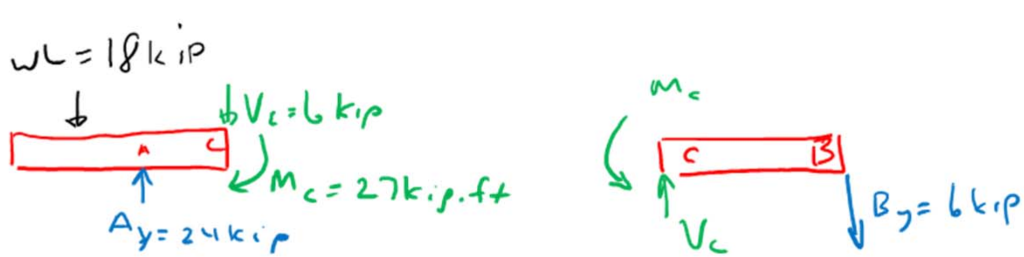
ΣF_x : $B_x - N_c = 0 \Rightarrow \boxed{N_c = 0}$

ΣF_y : $B_y + V_c = 0 \rightarrow \boxed{V_c = 6\text{ kip}}$

$+\circlearrowleft \Sigma M_c$: $-M_c + (4.5ft)B_y = 0$

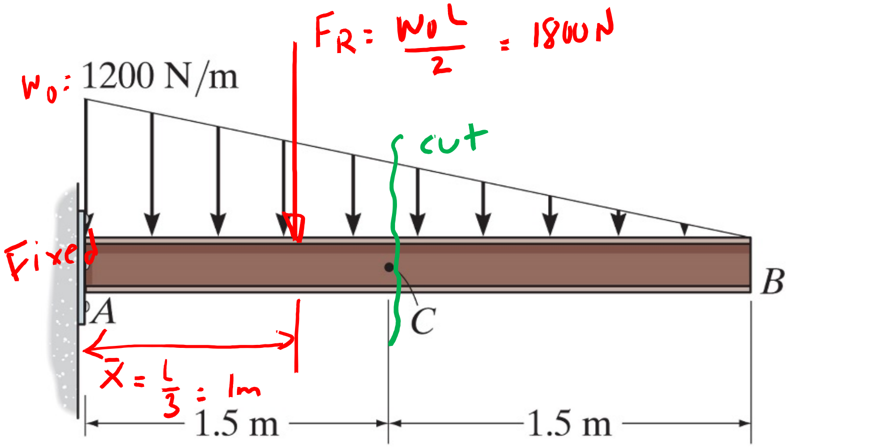
$\rightarrow \boxed{M_c = -27\text{ kip.ft}}$

∴ Actual Forces & Moments:

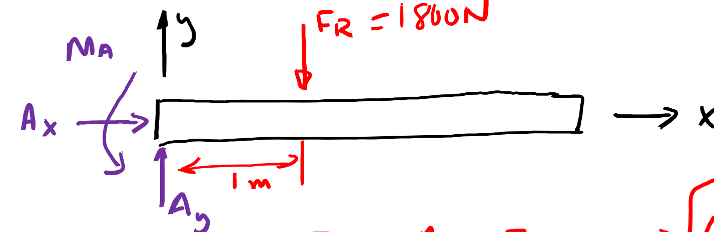


Note changes in directions of arrows for B_y & M_c from original FBDs due to negative values in solutions.

Find the internal forces and moments at C



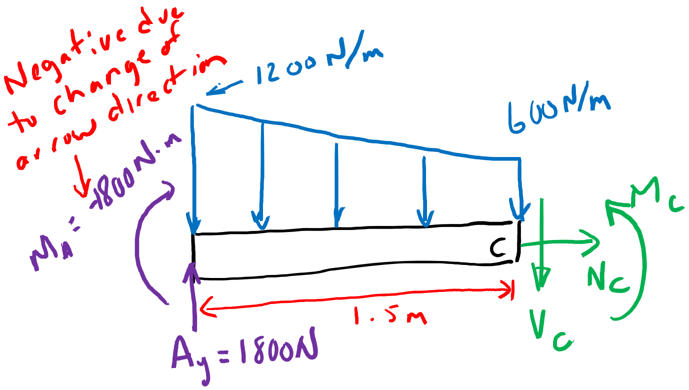
FBD of entire beam:

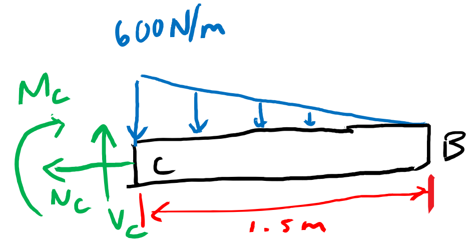


ΣF_x: $\boxed{A_x = 0}$, ΣF_y: $A_y - F_R = 0 \Rightarrow \boxed{A_y = F_R = 1800N}$ $A_y \uparrow$

$\circlearrowleft \Sigma M_A$: $M_A - (1m)(1800N) = 0 \Rightarrow \boxed{M_A = 1800N}$ $M_A \circlearrowleft$

Let's look at FBDs of Left & Right sides when cut at C:





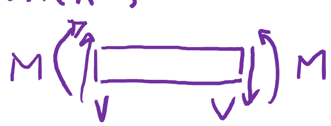
3 unknowns

N_C, V_C, M_C

3 unknowns

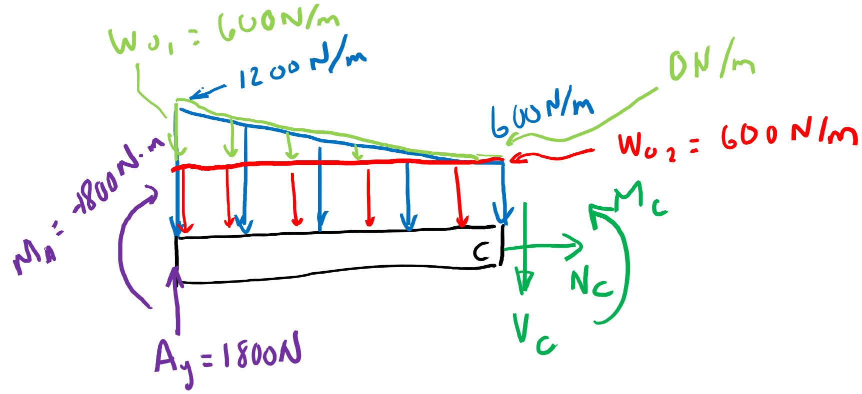
N_C, V_C, M_C

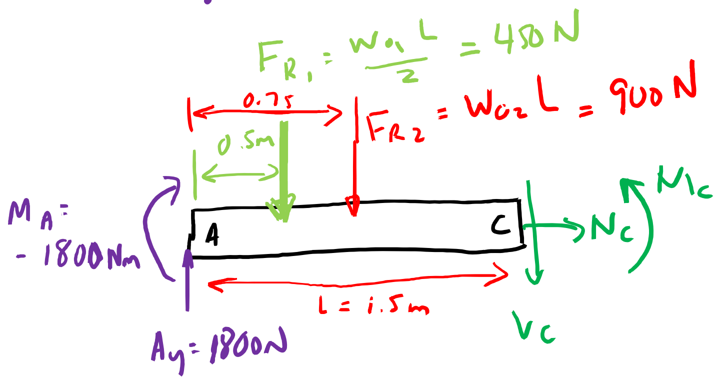
Let's draw rxn force & moment arrows following positive conventions for shear & bending moments



We can solve for unknown internal forces with either left or right side:

Left side:
Divide distributed load into F_{R_1} for triangle and F_{R_2} for rectangle





ΣF_x: $\boxed{N_c = 0}$

ΣF_y: $A_y - F_{R_1} - F_{R_2} - V_c = 0$

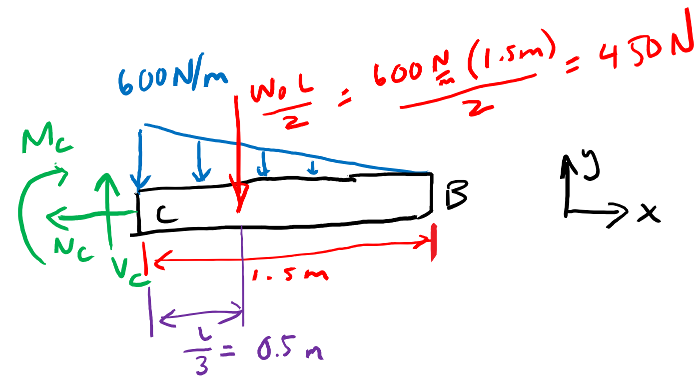
$V_c = 1800N - 450N - 900N$

$\boxed{V_c = 450N}$ $V_c \downarrow$

$+\circlearrowleft \Sigma M_A$: $-M_A - (0.5m) F_{R_1} - (0.75m) F_{R_2} - (1.5m) V_c + M_c = 0$

$\boxed{M_c = -225 \text{ Nm}}$ Since negative ⇒ assumed arrow direction on FBD is incorrect; should be $\circlearrowright M_c$

Right side:
simply find F_R for distributed load



ΣF_x: $-N_c = 0 \rightarrow \boxed{N_c = 0}$

ΣF_y: $V_c - 450N \rightarrow \boxed{V_c = 450N}$ $V_c \uparrow$

$+\circlearrowleft \Sigma M_c$: $-M_c - (0.5m)(450N) = 0$

$\boxed{M_c = -225 \text{ N m}}$ $M_c \circlearrowright$

Note that choosing left FBD takes more steps, but get the same result.

What are internal forces along the length of the beam?

Shear Force and Bending Moment Diagrams

Goal: provide detailed knowledge of the variations of internal shear force and bending moments (V and M) throughout a beam when perpendicular distributed loads, concentrated forces, and/or concentrated couple moments are applied.

Normal forces (N) in such beams are zero, so we will not consider normal force diagrams.

Procedure

1. Find support reactions (free-body diagram of entire structure)
2. Specify coordinate *x* (start from left)
3. Divide the beam into sections according to loadings
4. Draw FBD of a section
5. Apply equations of equilibrium to derive V and M as functions of *x*: V(*x*), M(*x*)