Statics - TAM 211

Lecture 22

November 15, 2018

Chap 7.2

Announcements

- Upcoming deadlines:
- Tuesday (11/20)
  - Prairie Learn HW 9
- Friday (11/23)
  - Written Assignment 9
- **Prof. H-W office hours**
  - **Monday 3-5pm (Room C315 ZJUI Building)**
  - **Wednesday 7-8pm (Residential College Lobby)**

Chapter 7: Internal Forces

Goals and Objectives

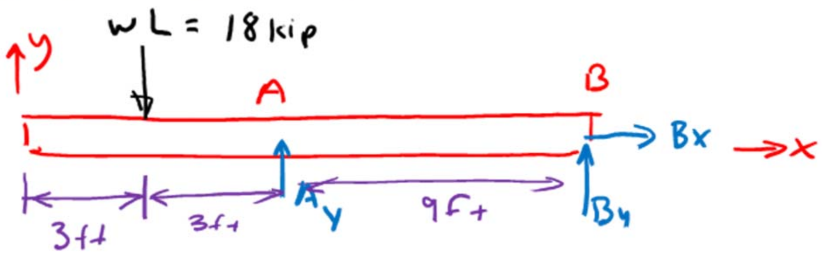
- Determine the internal loadings in members using the method of sections
- Generalize this procedure and formulate equations that describe the internal shear and bending moment throughout a member
- Be able to construct or identify shear and bending moment diagrams for beams when distributed loads, concentrated forces, and/or concentrated couple moments are applied

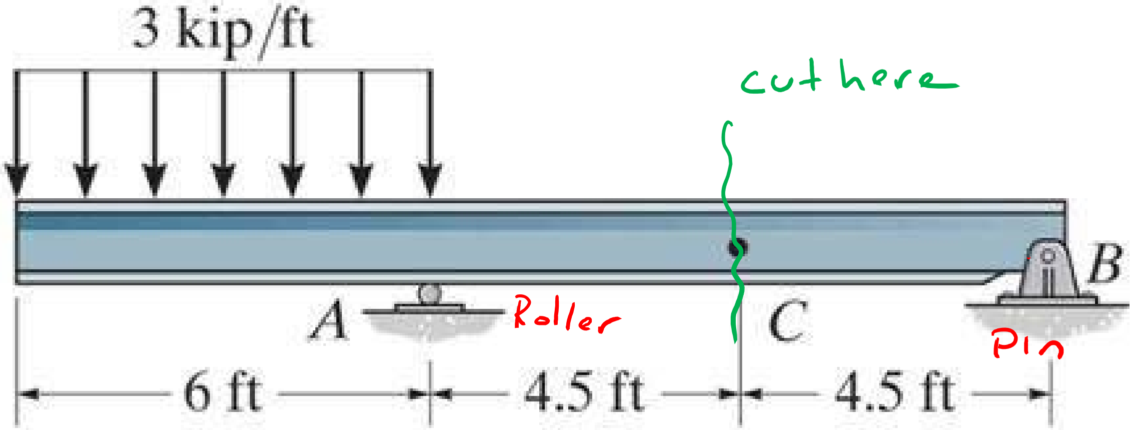
Recap: Procedure for analysis:

1. Find support reactions (free-body diagram of entire structure)
2. Pass an imaginary section through the member
3. Draw a free-body diagram of the segment that has the least number of loads on it
4. Apply the equations of equilibrium

Recap: Find the internal forces at point C.

FBD of entire beam





3 unknowns (A_y, B_x, B_y)

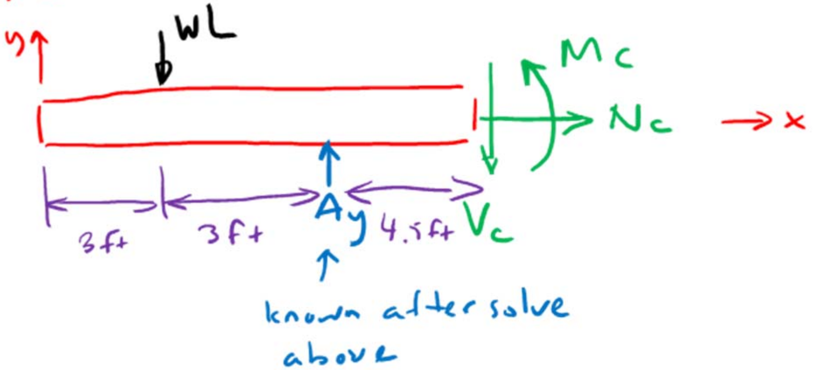
use 3 EoE to solve for A_y, B_x, B_y.

$\Sigma F_x: \boxed{B_x = 0}$, $\quad \Sigma F_y: A_y + B_y - wL = 0 \rightarrow \boxed{B_y = -6 \text{ kip}}$

$+\circlearrowleft \Sigma M_B: (12ft)\, wL - (9ft)\, A_y = 0 \rightarrow \boxed{A_y = 24 \text{ kip}}$

B $\downarrow B_y$

FBD of left section:



3 unknowns (N_c, V_c, M_c), assuming know A_y

use EoE:

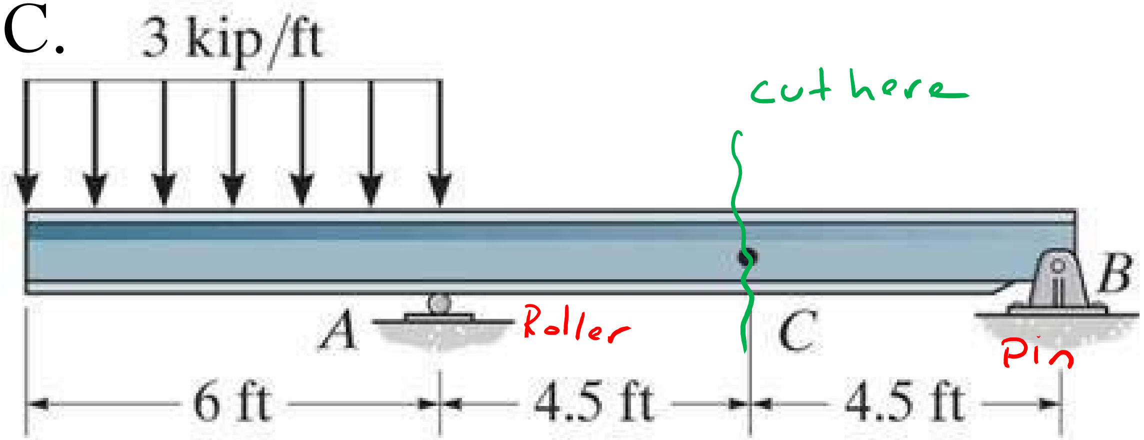
$\Sigma F_x: \boxed{N_c = 0}$

$\Sigma F_y: A_y - wL - V_c = 0 \Rightarrow \boxed{V_c = 6 \text{ kip}}$

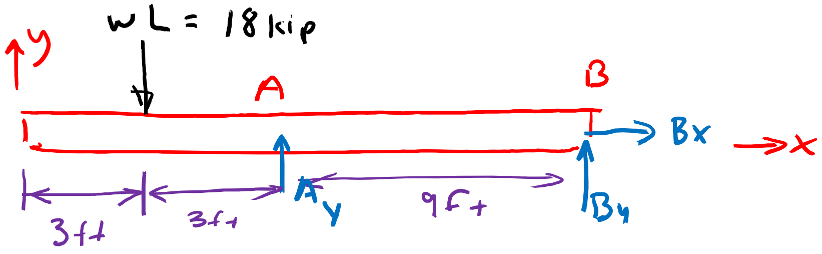
$+\circlearrowleft \Sigma M_c: M_c - (4.5ft)\, A_y + (7.5ft)\, wL = 0$

$\Rightarrow \boxed{M_c = -27 \text{ kip}\cdot\text{ft}}$

Find the internal forces at point C.



FBD of entire beam

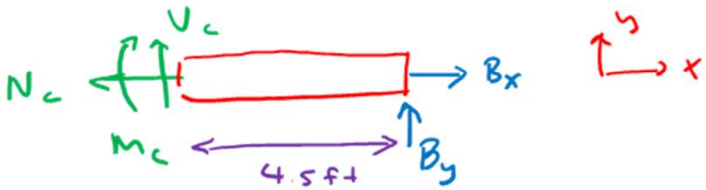


3 unknowns (A_y, B_x, B_y)

use 3 EoE to solve for A_y, B_x, B_y. $\boxed{A_y = 24 \text{ kip}}$ $\boxed{B_x = 0}$ $\boxed{B_y = -6 \text{ kip}}$

Alternatively, could examine right section:

FBD of right section



3 unknowns (N_c, V_c, M_c) assuming know B_x, B_y

use EoE:

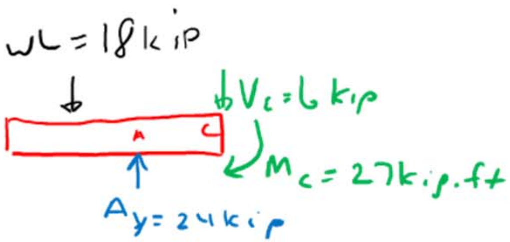
ΣF_x : $B_x - N_c = 0 \Rightarrow \boxed{N_c = 0}$

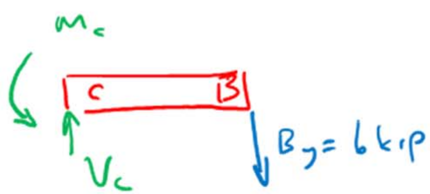
ΣF_y : $B_y + V_c = 0 \rightarrow \boxed{V_c = 6 \text{ kip}}$

$+\circlearrowleft \Sigma M_c$: $-M_c + (4.5ft)B_y = 0$

$\rightarrow \boxed{M_c = -27 \text{ kip.ft}}$

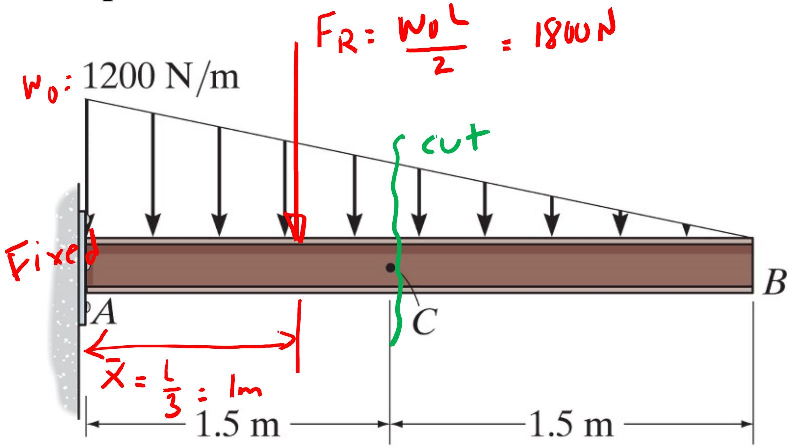
$\therefore$ Actual Forces & Moments:



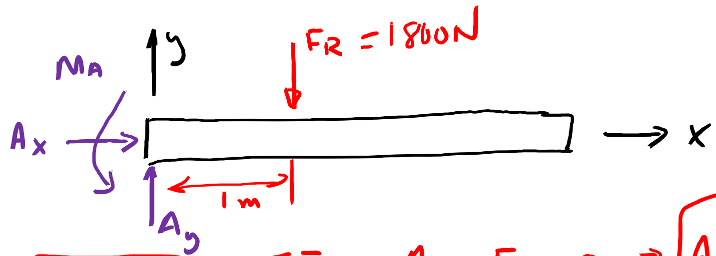


Note changes in directions of arrows for B_y & M_c from original FBDs due to negative values in solutions.

Recap: Find the internal forces and moments at C



FBD of entire beam:



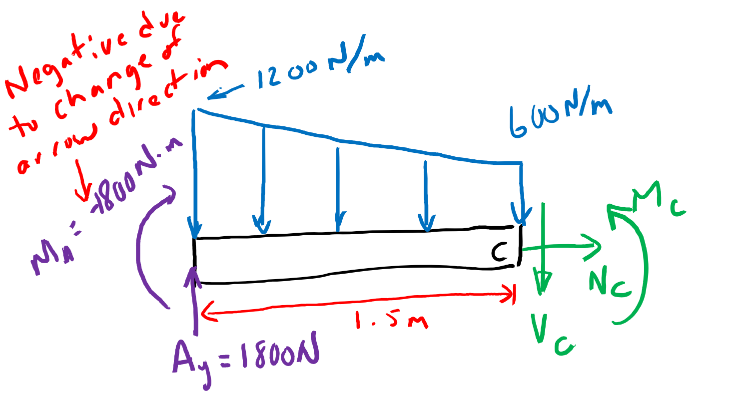
$A_y \uparrow$

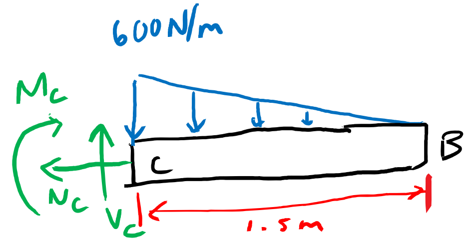
$\Sigma F_x: \boxed{A_x = 0}$, $\Sigma F_y: A_y - F_R = 0 \Rightarrow \boxed{A_y = F_R = 1800\text{N}}$

$\circlearrowleft + \Sigma M_A: M_A - (1\text{m})(1800\text{N}) = 0 \Rightarrow \boxed{M_A = 1800\text{N}}$

$M_A \circlearrowleft$

Let's look at FBDs of Left & Right sides when cut at C:





let's draw rxn force & moment arrows following positive conventions for shear & bending moments

3 unknowns N_c, V_c, M_c

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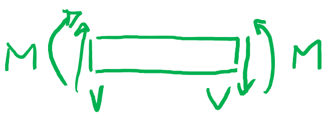
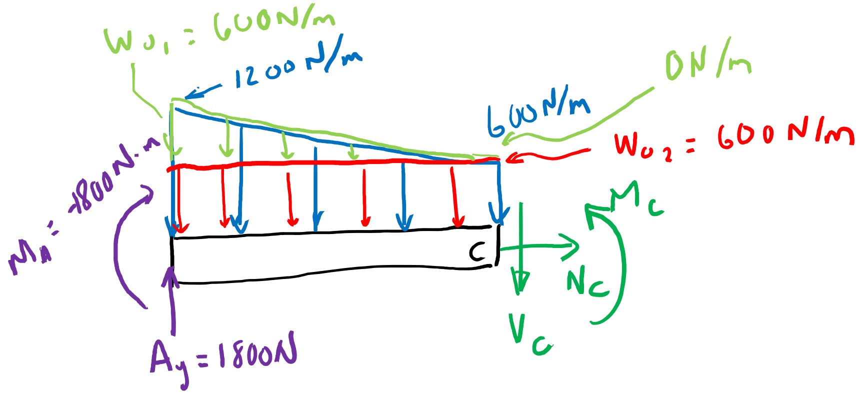
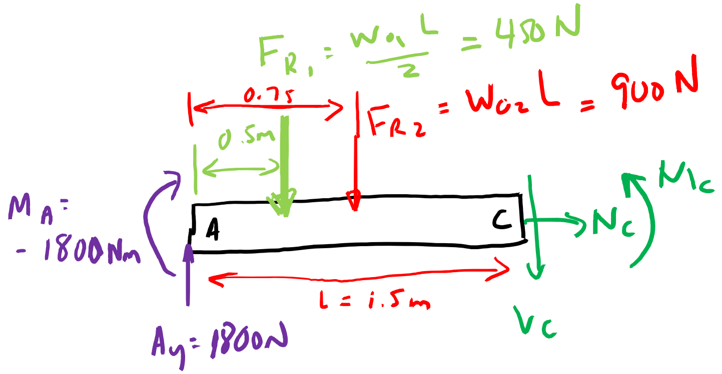


We can solve for unknown internal forces with either left or right side:

Left side:
Divide distributed load into
F_{R_1} for triangle and F_{R_2} for rectangle





ΣF_x: $\boxed{N_c = 0}$

ΣF_y: $A_y - F_{R_1} - F_{R_2} - V_c = 0$

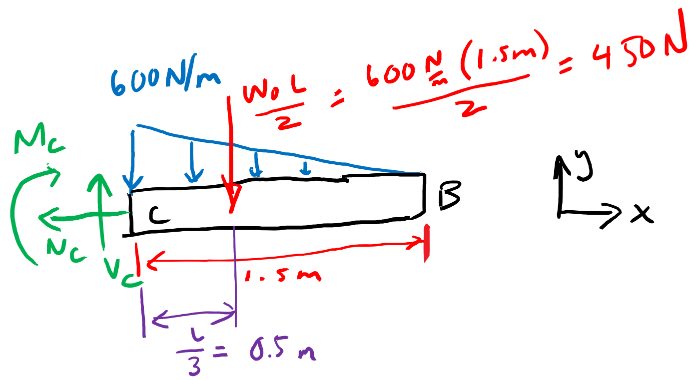
$V_c = 1800N - 450N - 900N$

$\boxed{V_c = 450N}$ $V_c \downarrow$

$+\circlearrowleft \Sigma M_A$: $-M_A - (0.5m)F_{R_1} - (0.75m)F_{R_2} - (1.5m)V_c + M_c = 0$

$\boxed{M_c = -225\,Nm}$ Since negative $\Rightarrow$ assumed arrow direction on FBD is incorrect; should be $\circlearrowright M_c$

Right side:
simply find F_R for distributed load



ΣF_x: $-N_c = 0 \rightarrow \boxed{N_c = 0}$

ΣF_y: $V_c - 450N \rightarrow \boxed{V_c = 450N}$ $V_c \uparrow$

$+\circlearrowleft \Sigma M_c$: $-M_c - (0.5m)(450N) = 0$

$\boxed{M_c = -225\,N{\cdot}m}$ $M_c \circlearrowright$

Note that choosing left FBD takes more steps, but get the same result.

What are internal forces along the length of the beam?

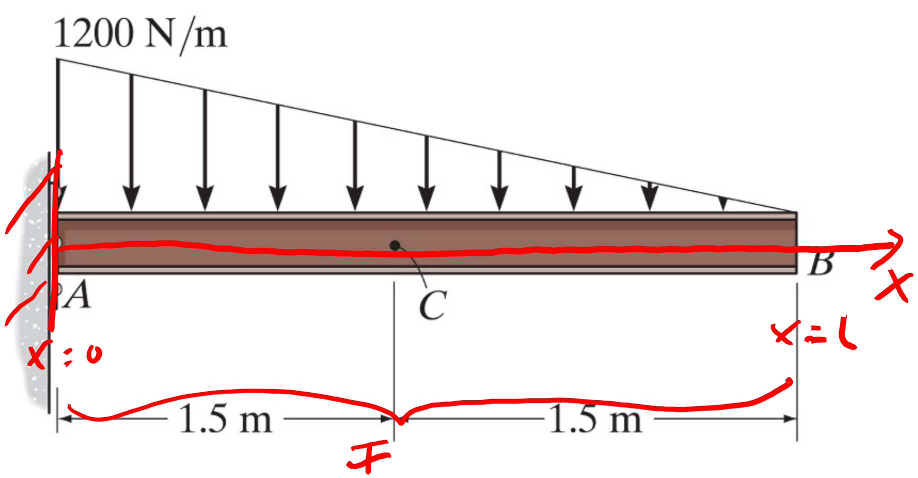
Shear Force and Bending Moment Diagrams

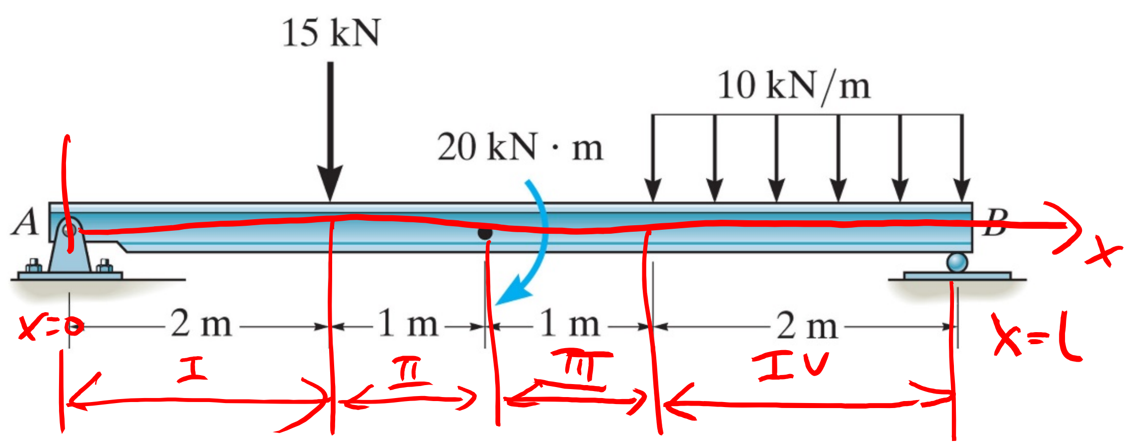
Goal: provide detailed knowledge of the variations of internal shear force and bending moments (V and M) throughout a beam when perpendicular distributed loads, concentrated forces, and/or concentrated couple moments are applied.

Normal forces (N) in such beams are zero, so we will not consider normal force diagrams.

Procedure

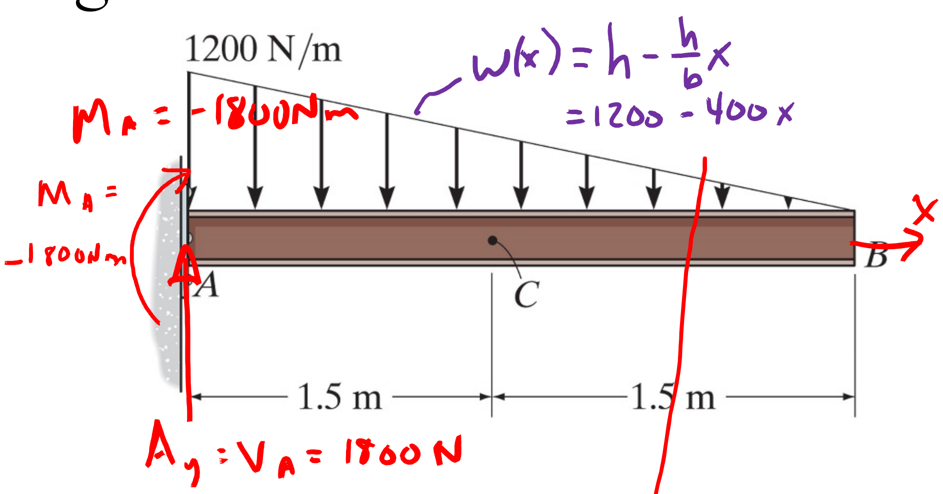
1. Find support reactions (free-body diagram of entire structure)
2. Specify coordinate *x* (start from left)
3. Divide the beam into sections according to loadings
4. Draw FBD of a section
5. Apply equations of equilibrium to derive V and M as functions of *x*: V(*x*), M(*x*)





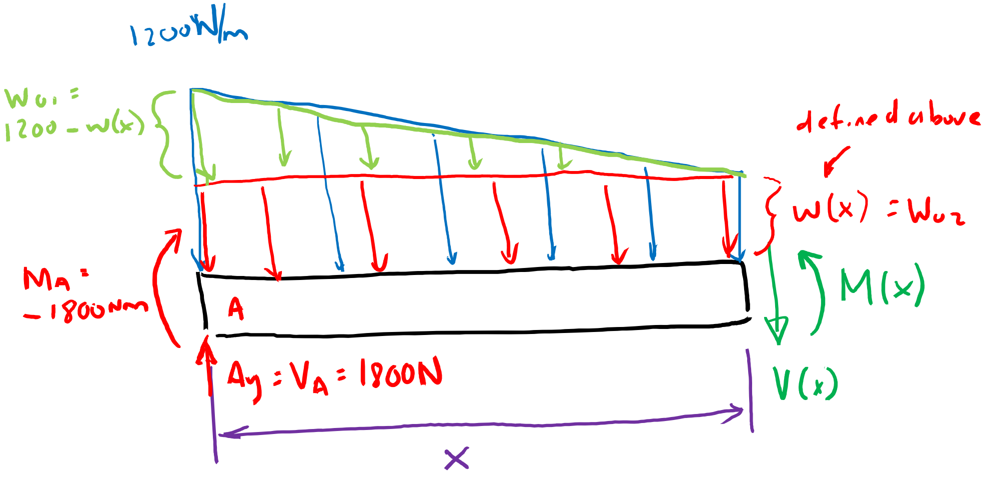
Draw the shear and bending moment diagrams for the beam.

From previous example, we know that the support reactions are: $A_x = 0$, $A_y = 1800N \uparrow$, $M_A = -1800 Nm$



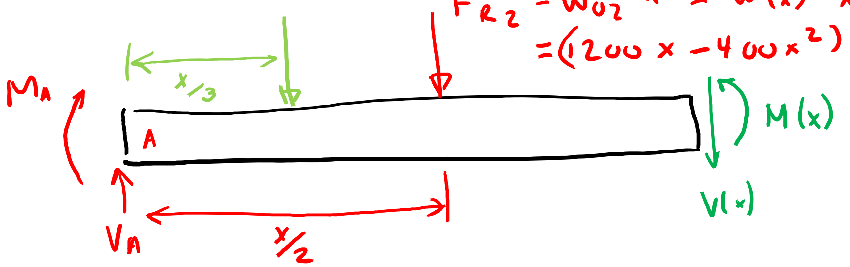
We are interested in finding $V(x)$ & $M(x)$ as these vary along the length of the beam.

So for any length x of the beam, we get the following generic FBD as a function of x.



$F_{R_1} = w_{o_1} \frac{L}{2} = [1200 - w(x)]\frac{x}{2} = (200x^2) N$

$F_{R_2} = w_{o_2} \cdot x = w(x) \; x$
$= (1200x - 400x^2) N$



$$\Sigma F_x: \; A_y - F_{R_1} - F_{R_2} - V(x) = 0$$

$$\boxed{V(x) = (200x^2 - 1200x + 1800)\,N}$$

Quadratic

Boundary conditions:

$V(x=0) = 1800\,N = A_y$

$V(x=L=3m) = 0\,N$

cf. $V(@C=1.5m) = 450\,N$ ✓ w/ previous example

$$+\circlearrowleft \Sigma M_A: \; -M_A - \left(\tfrac{x}{3}\right)F_{R_1} - \left(\tfrac{x}{2}\right)F_{R_2} - x \cdot V(x) + M(x) = 0$$

$$\boxed{M(x) = \left(\tfrac{200}{3}x^3 - 600x^2 + 1800x - 1800\right)\,Nm}$$

3rd Order Polynomial

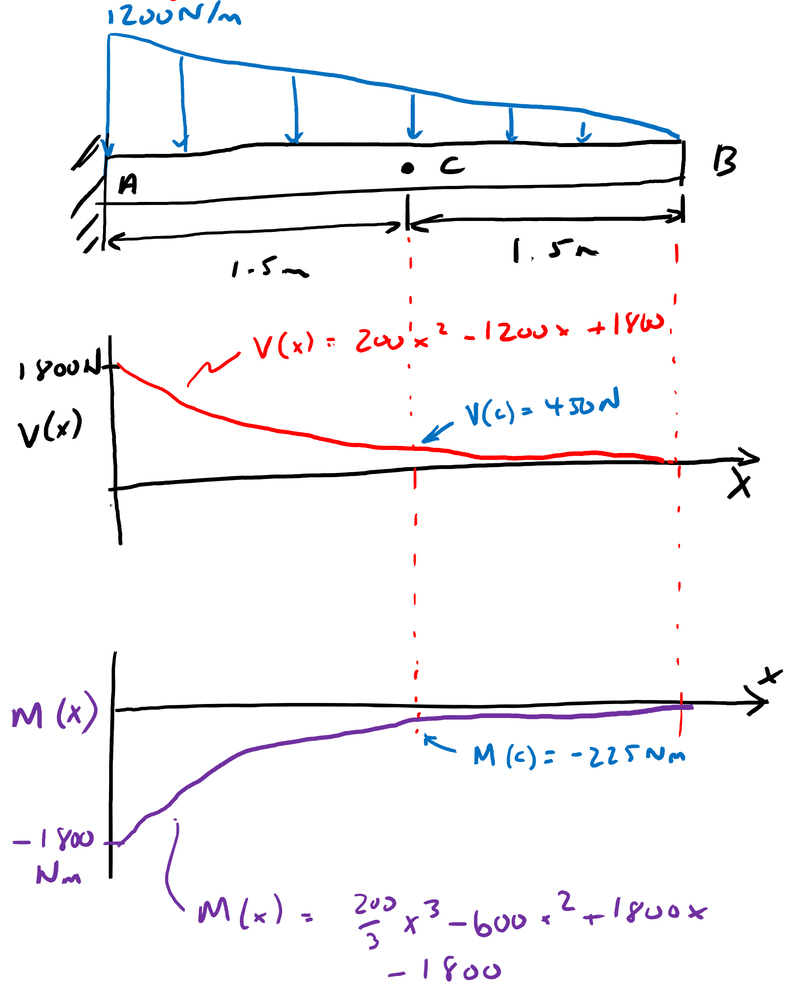
B.C.

$M(0) = -1800\,Nm = M_A$

$M(L) = 0$

cf. $M(@C=1.5m) = -225\,Nm$ ✓ w/ previous

Draw Shear Force $V(x)$ & Bending Moment $M(x)$ diagrams



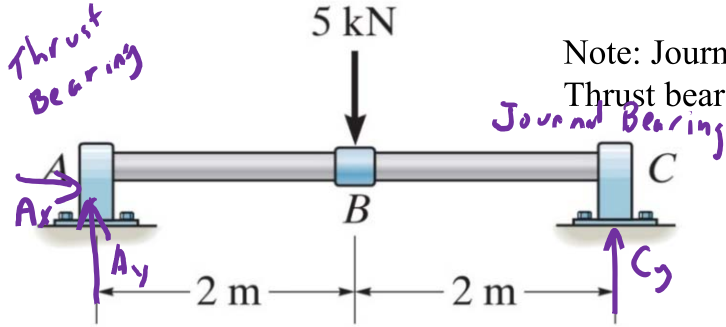
Note that since the applied load is a single distributed load along the entire length of the beam, then V(x) and M(x) are continuous functions. We will see that V(x) and M(x) will be discontinuous functions when multiple loads are applied to a beam, and these discontinuities will happen at the transitions between loading regions.

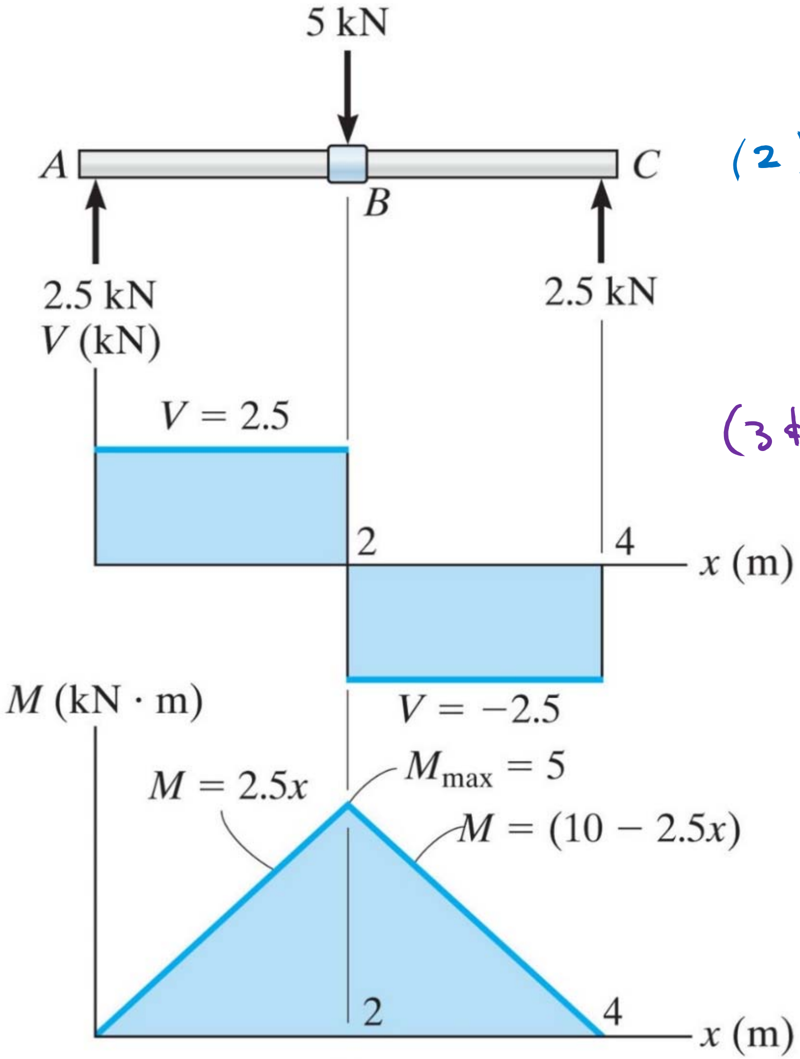
Explore and re-create the shear force and bending moment diagrams for the beam. A is thrust bearing & C is journal bearing.

Example: single concentrated load

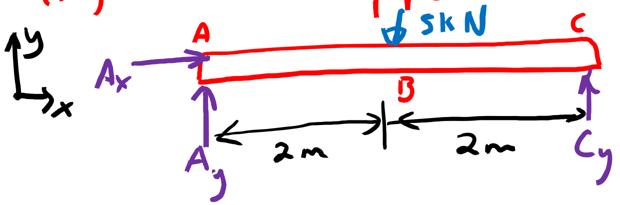
See Example 7.6 in text

Note: Journal bearings only have support reaction forces and moments on axes perpendicular to shaft.
Thrust bearings are similar to journal bearings but with added support reaction force along axis of shaft





(1) Find support reactions:



ΣF_x : $\boxed{A_x = 0}$

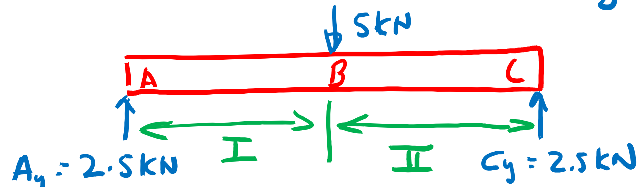
ΣF_y : $A_y + C_y - 5kN = 0$

$+\circlearrowleft\ \Sigma M_A$: $-(2m)5kN + (4m)C_y = 0$

$\rightarrow \boxed{C_y = 2.5kN}$

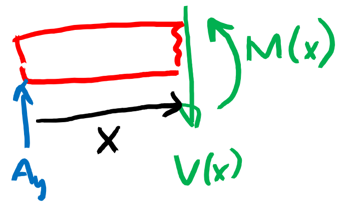
$\therefore \boxed{A_y = 2.5kN}$

(2) Divide beam into regions according to loadings.



(3&4) Draw FBD of a region. Use Eq of Eq to derive V(x) & M(x):

Region I:



ΣF_y : $A_y - V(x) = 0$

$\boxed{V(x) = A_y = 2.5kN}$

constant, positive

$+\circlearrowleft\ \Sigma M_A$: $-x \cdot V(x) + M(x) = 0$ $\therefore$ $M(x) = x \cdot V(x)$

$\boxed{M(x) = x \cdot A_y = 2.5x \text{ kN}\cdot\text{m}}$

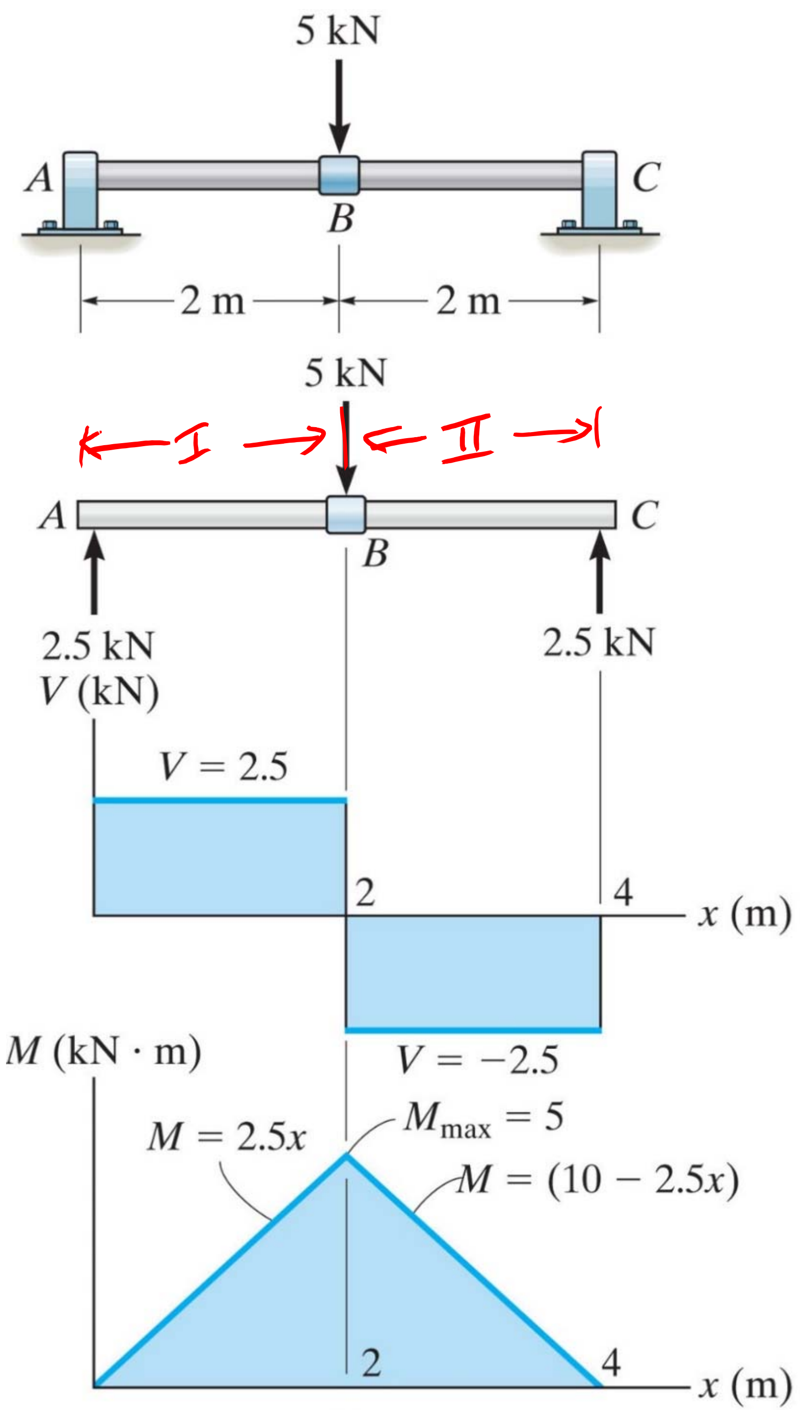
Note: could chose ΣM_x where x is at cut on right side

linear w/ slope A_y

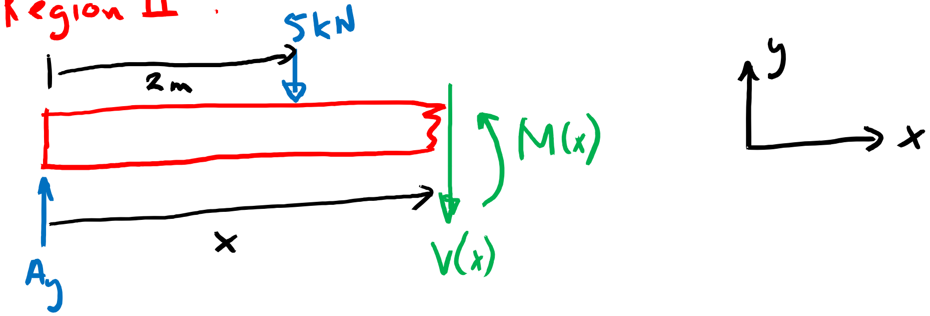
Boundary Conditions: compare results to plots to left

$x = 0$: $V(0) = 2.5kN$, $M(0) = 0$

$x = 2m^{(-)}$ {use (−) as immediately to left of 2m : $V(2m^{(-)}) = 2.5$ kN
$M(2m^{(-)}) = 5$ kN · m



Region II :



ΣF_x : $A_y - V(x) - 5\,kN = 0$

$$\boxed{V(x) = A_y - 5\ kN = -2.5\,kN}$$

constant, negative

$+\circlearrowleft \Sigma M_A$: $-(2m)\,5kN - x \cdot V(x) + M(x) = 0$

$M(x) = 10kN\cdot m + x \cdot V(x)$

$= 10\,kN\cdot m + x\,(A_y - 5\ kN)$

$$\therefore \boxed{M(x) = (10 - 2.5x)\ kN\ m}$$

linear w/ negative slope

cf. BC's :

$x = 2m^{(+)}$: $V(2m^{(+)}) = -2.5kN$, $M(2m^{(+)}) = 5\,kN\cdot m$

$x = 4m$: $V(4) = -2.5\,kN$, $M(4) = 0$

compare results to plots to left

Note for <u>single concentrated load (P)</u>:

- $V(x)$ is <u>constant</u> within a region. $V(x)$ has a step change at location of load that is equivalent to magnitude and direction of applied load (e.g., -P$\hat{j}$ or -5 kN).
- $M(x)$ is <u>linear</u>.

Also note that $V(x) = \frac{d}{dx}M(x)$, or slope of moment diagram