Statics - TAM 211

Lecture 23

November 19, 2018

Chap 7.3

Announcements

- Upcoming deadlines:
- Tuesday (11/20)
  - Prairie Learn HW 9
- Friday (11/23)
  - Written Assignment 9
- **Prof. H-W office hours**
  - **Monday 3-5pm (Room C315 ZJUI Building)**
  - **Wednesday 7-8pm (Residential College Lobby)**

Chapter 7: Internal Forces

Goals and Objectives

- Determine the internal loadings in members using the method of sections
- Generalize this procedure and formulate equations that describe the internal shear force and bending moment throughout a member
- Be able to construct or identify shear a force nd bending moment diagrams for beams when distributed loads, concentrated forces, and/or concentrated couple moments are applied

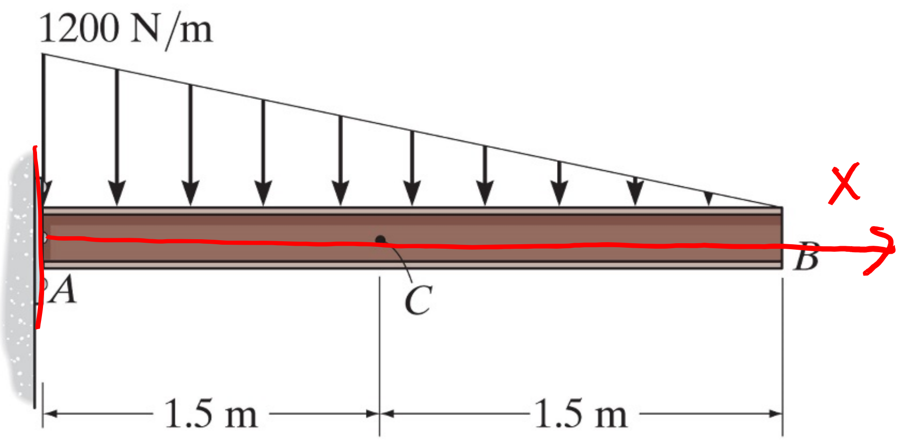
Recap: Shear Force and Bending Moment Diagrams

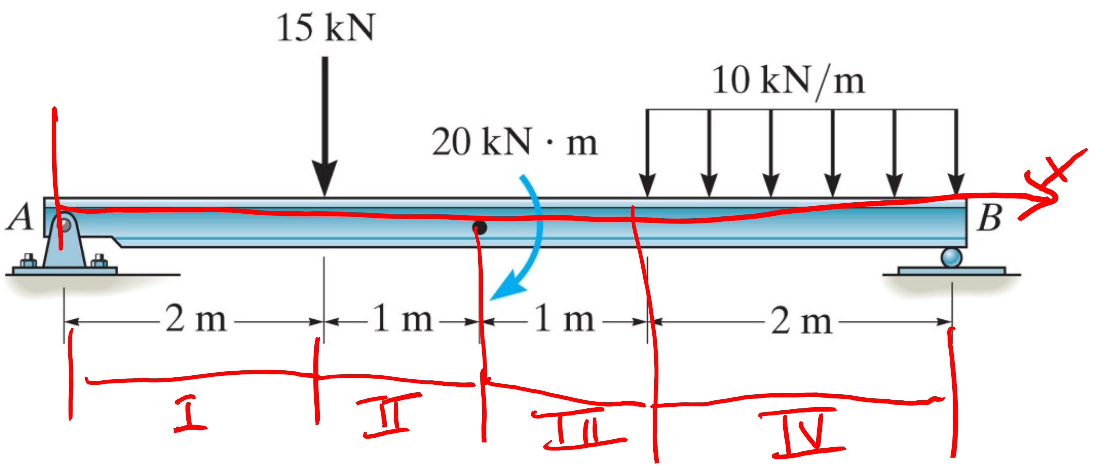
Goal: provide detailed knowledge of the variations of internal shear force and bending moments (V and M) throughout a beam when perpendicular distributed loads, concentrated forces, and/or concentrated couple moments are applied.

Normal forces (N) in such beams are zero, so we will not consider normal force diagrams

Procedure

1. Find support reactions (free-body diagram of entire structure)
2. Specify coordinate x (start from left)
3. Divide the beam into sections according to loadings
4. Draw FBD of a section
5. Apply equations of equilibrium to derive V and M as functions of x: $V(x)$, $M(x)$

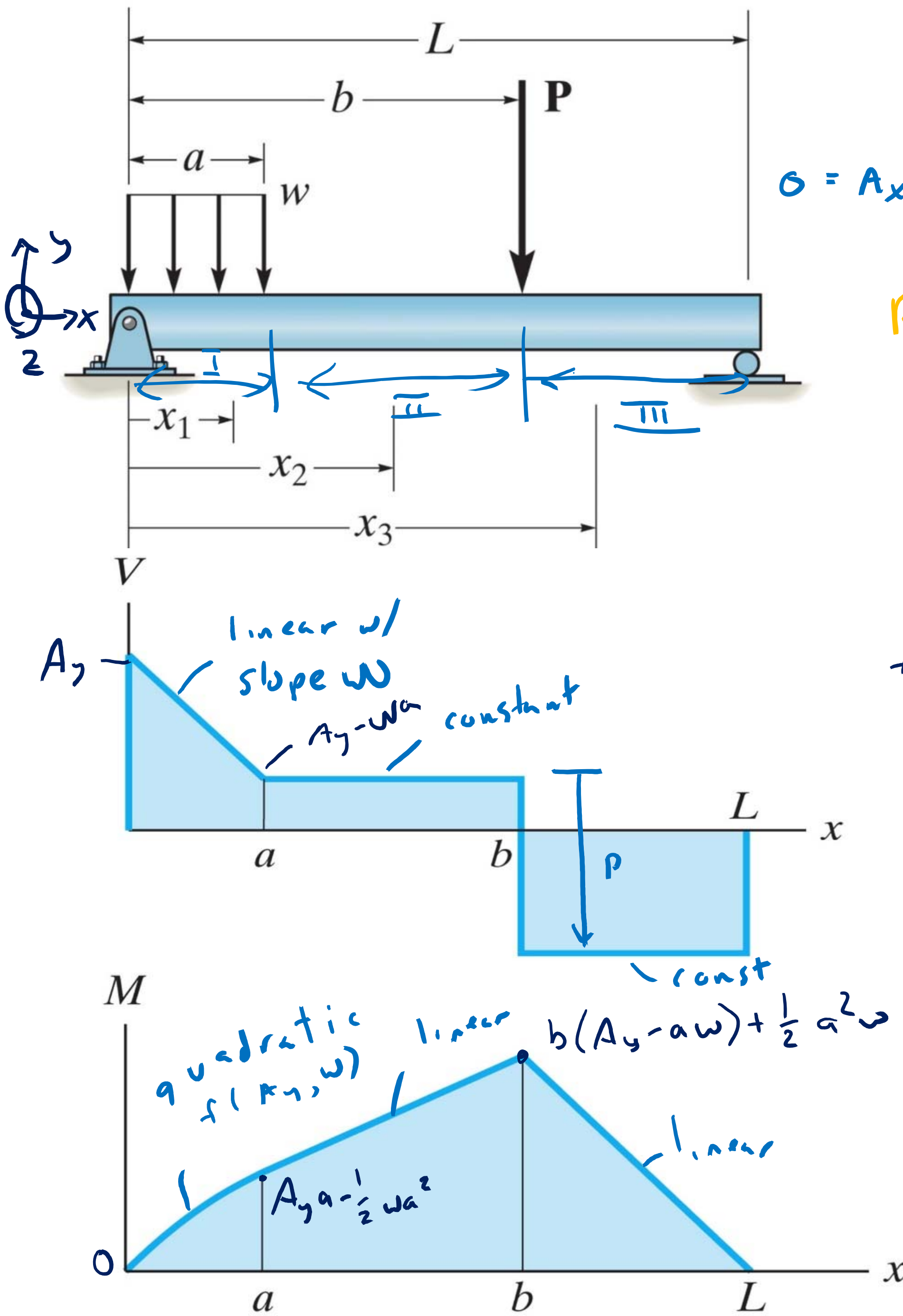




- For the following examples, practice deriving the final expressions (i.e., equations) for V(x) and M(x) by creating the appropriate FBD for a specific region.
- Then determine the values for V and M at the start and end points for the region, i.e., find their values at the Boundary Conditions.
- Note that the different load types (distributed loads, concentrated point forces, and concentrated couple moments) have characteristic V(x) and M(x) behaviors that can be easily plotted, especially if one knows the relationships between w, V, and M, and the Boundary Conditions at each region.

Explore and re-create the shear force and bending moment diagrams for the beam.

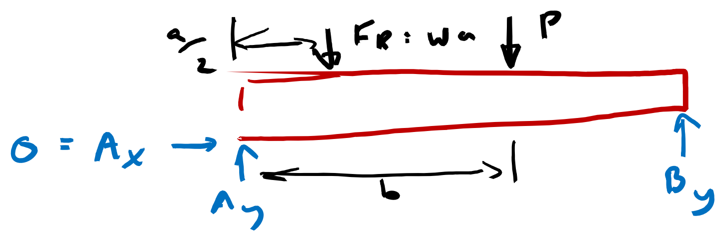
Example: single concentrated load, rectangular distributed load



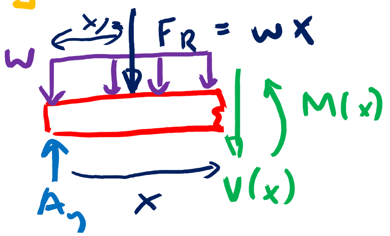
$A_y = aw + P - B_y$ ∴ const

$B_y = \frac{1}{L}\left(\frac{a^2 w}{2} + bP\right)$ = const.

(1) Find Rxn loads



Region I : $0 < x < a$



$+\uparrow \Sigma F_y : -wx - V(x) + A_y = 0$

$$V(x) = A_y - wx$$

linear w/ negative slope (w)

$+\circlearrowleft \Sigma M_A : M(x) - \left(\frac{x}{2}\right)(w \cdot x) - x \cdot V(x)$

$$M(x) = A_y x - \frac{1}{2} w x^2$$

quadratic

BC:

$x = 0$ $\quad V(0) = A_y$, $M(0) = 0$

$x = a$ $\quad V(a) = A_y - wa$, $M(a) = A_y a - \frac{1}{2} w a^2$

Region II $a < x < b$

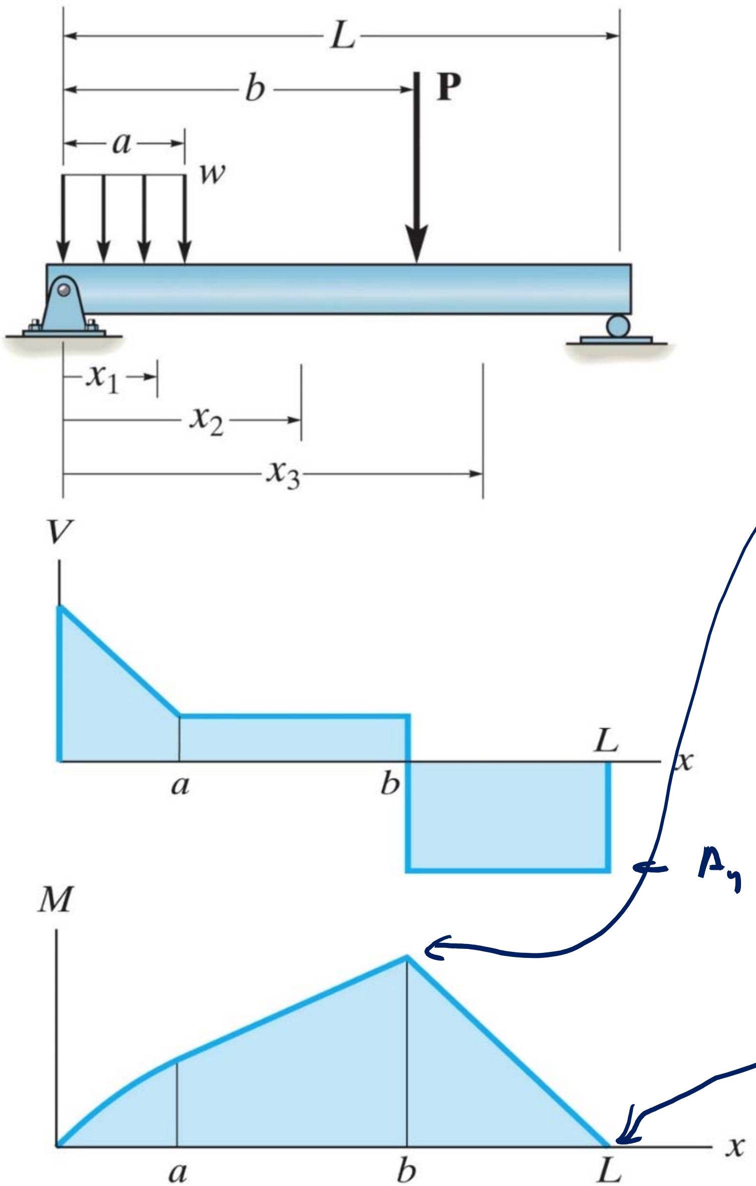
$$V(x) = A_y - wa \quad \text{constant}$$
$$M(x) = (A_y - aw)x + \frac{1}{2}a^2 w$$

linear w/ slope V(x)

BC: $x = b^{(-)}$ ↖ just before load P

Explore and re-create the shear force and bending moment diagrams for the beam.

Example: single concentrated load, rectangular distributed load



Region III $b < x < L$

$V(x) = A_y - aw - P$ const

$M(x) = x \cdot V(x) + bP + \frac{1}{2}a^2 w$ linear w/ slope $V(x)$

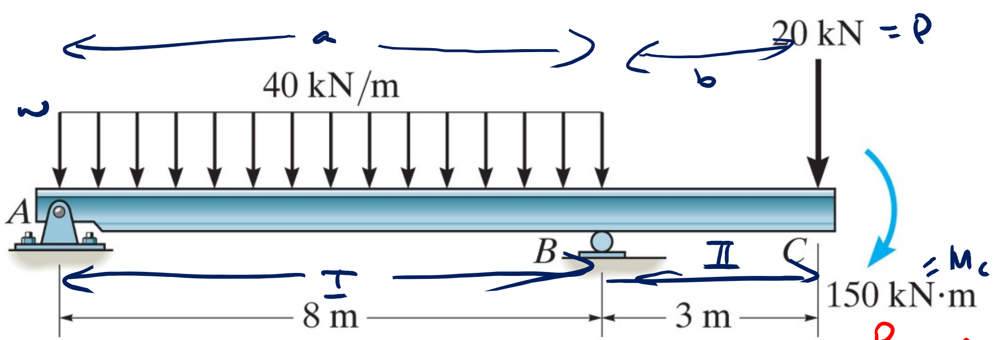
BC: $x = b^{(+)}$ ← position just to right side of P

$M(b^{(+)}) = b(A_y - aw - P) + bP + \frac{1}{2}a^2 w$

$M(L) = 0$

Explore and re-create the shear force and bending moment diagrams for the beam.

Example: concentrated load, rectangular distributed load, concentrated couple moment



Entire body:

$$A_y = wa + P - B_y = 133.75 \text{ kN}$$

$$B_y = \frac{M_c + (a+b)P + \frac{a^2 w}{2}}{a} = 266.25 \text{ kN}$$

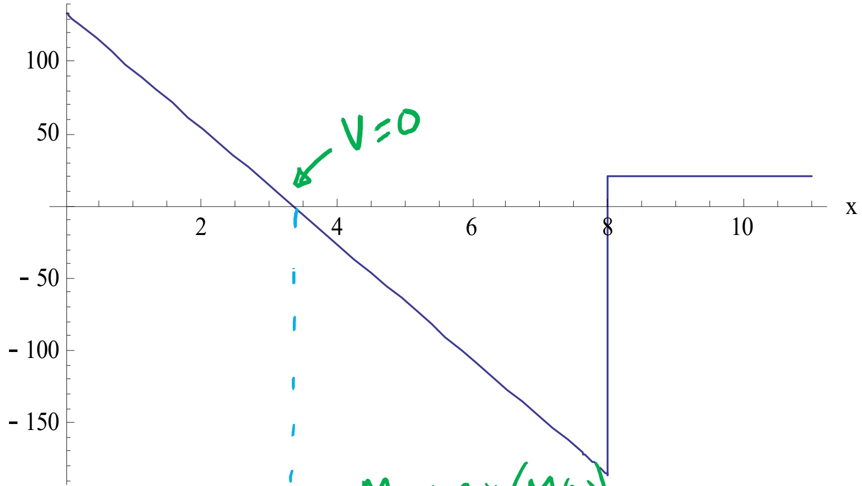
Region I: $0 < x < a$

$V(x) = A_y - wx$ Linear with slope w

$M(x) = (A_y)x - (xw)\frac{x}{2}$ Quadratic

BC: $x = 0$

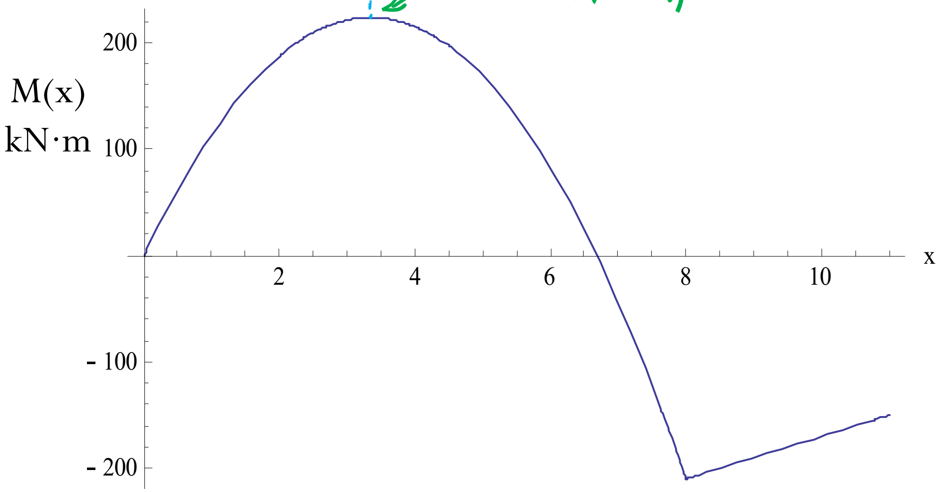
$x = a^{(-)}$



Note for rectangular distributed load:

V(*x*) is linear with slope of *w*. Slope is –*w* since *w* is pointing in –y direction.

M(*x*) is quadratic. When *V(x)* = 0, and *M*(*x*) = max(*M*).



Region II $a < x < b$

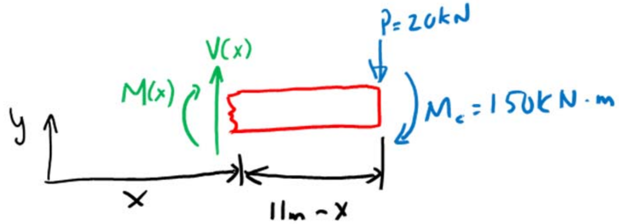
$V(x) = P = 20 \text{ kN}$ constant

$M(x) = (-370 + 20x) \text{ kN}\cdot\text{m}$ Linear with slope V(x)

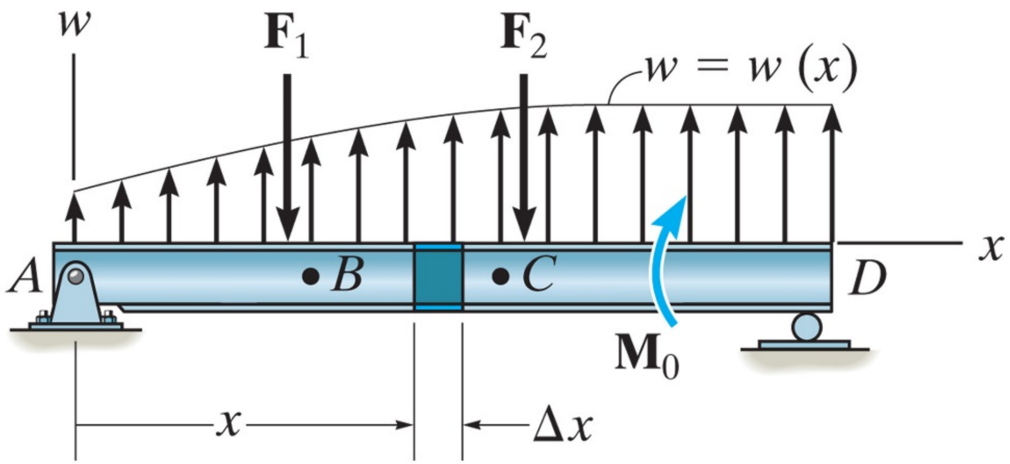
BC: $x = a^{(+)}$

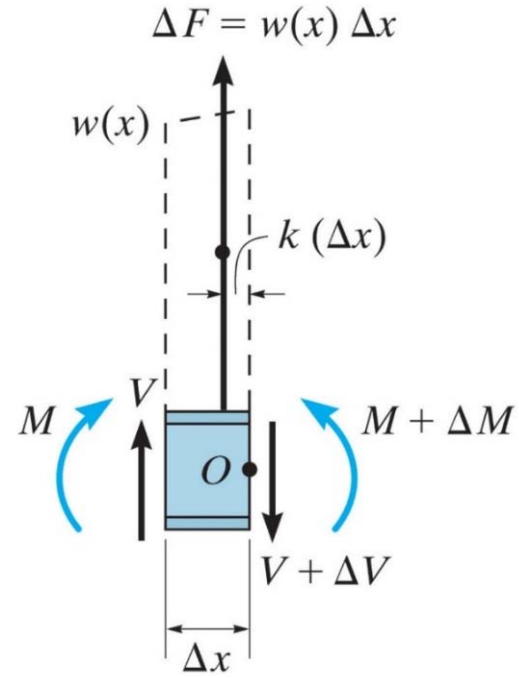
$x = L$

Hint: due to concentrated load P and couple moment M_C at end C, draw FBD for right side segment.



Relationships Among Distributed Load (w), Shear Force (V) and Bending Moments (M)





Relationship between distributed load and shear:

$$\sum F_y = 0: \quad V - (V + \Delta V) + w\Delta x = 0$$

$$\Delta V = w\Delta x$$

Dividing by Δx and letting $\Delta x \to 0$, we get:

$$\frac{dV}{dx} = w$$

Slope of shear force = distributed load intensity

$$\Delta V = V_2 - V_1 = \int w\, dx$$

Change in shear force = area under loading curve

Relationship between shear and bending moment:

$$\sum M_O = 0: \quad (M + \Delta M) - M - V\,\Delta x - w\Delta x\,(k\,\Delta x) = 0$$

$$\Delta M = V\,\Delta x + wk(\Delta x)^2$$

Dividing by Δx and letting $\Delta x \to 0$, we get:

$$\frac{dM}{dx} = V$$

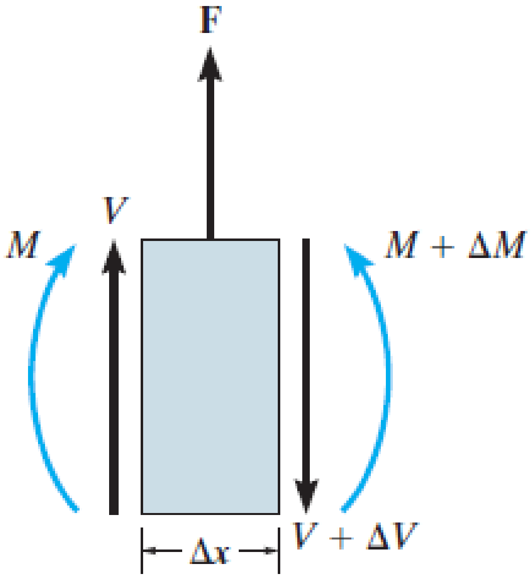
Slope of bending moment = shear force

$$\Delta M = M_2 - M_1 = \int V\, dx$$

Change in moment= area under shear curve

Relationships Among Concentrated Force (F) or Moment (M_O), Shear Force (V) and Bending Moments (M)

Wherever there is an external concentrated force or a concentrated moment, there will be a change (jump) in shear or moment, respectively.

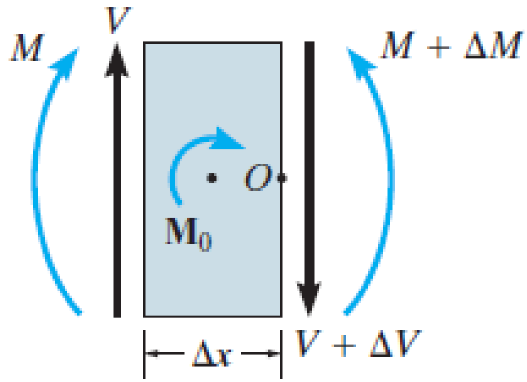


ΣF_y:

$V + F - (V + \Delta V) = 0$

$\Delta V = F$

Jump in shear force due to concentrated point force F



ΣM_O:

$(M + \Delta M) - M - M_O - V(\Delta x) = 0$

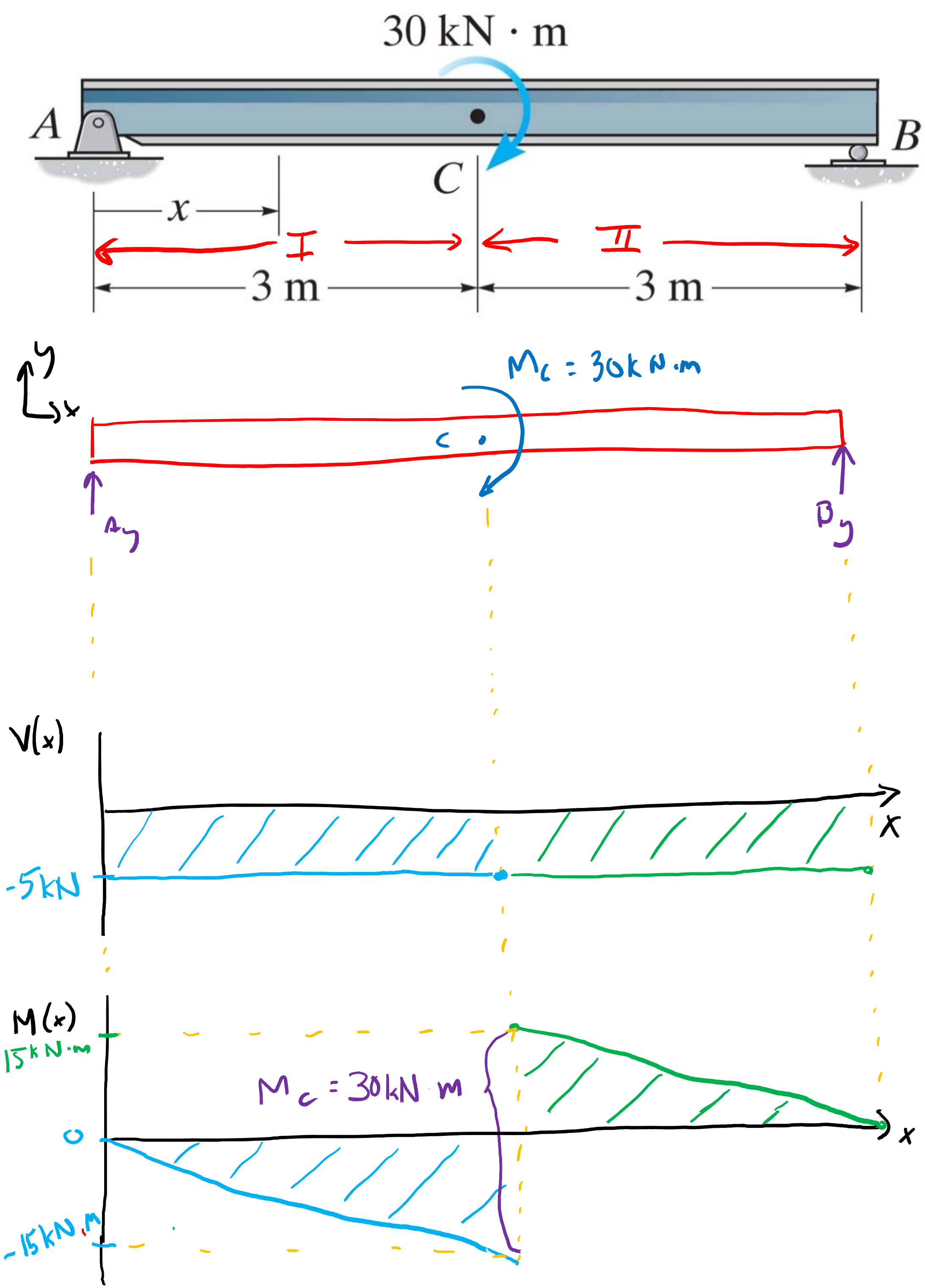
$\Delta M = M_O + V(\Delta x)$

$\Delta M = M_O$, when $\Delta x \rightarrow 0$

Jump in bending moment due to concentrated couple moment M_O

Note: the text, these notes, and convention assume that an applied concentrated moment <u>**M_O in clockwise direction**</u> results in a <u>**positive change in M(x)**</u>

Note that for a concentrated force or moment, $w = 0$.
Therefore, $\frac{dV}{dx} = w = 0$, so <u>V(x) must be constant</u>.



Draw the shear force and moment diagrams for the beam.

Exercise: derive V(x) and M(x) as shown

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