

Statics - TAM 211

Lecture 24

November 21, 2018

Chap 8.1

Announcements

□ Upcoming deadlines:

- Friday (11/23)
 - Written Assignment 9
- Tuesday (11/27)
 - Prairie Learn HW 10
- Quiz 5 → Paper

● Week of Nov 26

Nov 30 Friday 8am Quiz
Lecture 9, Discussion 10

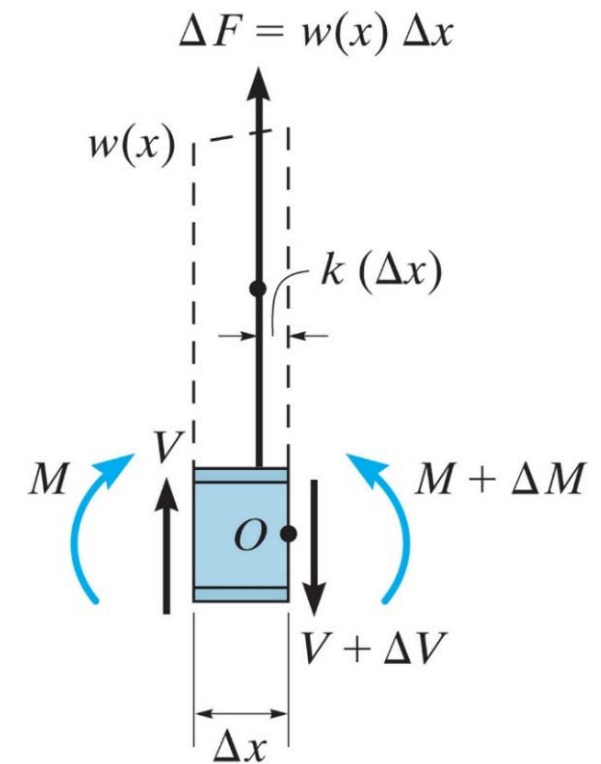
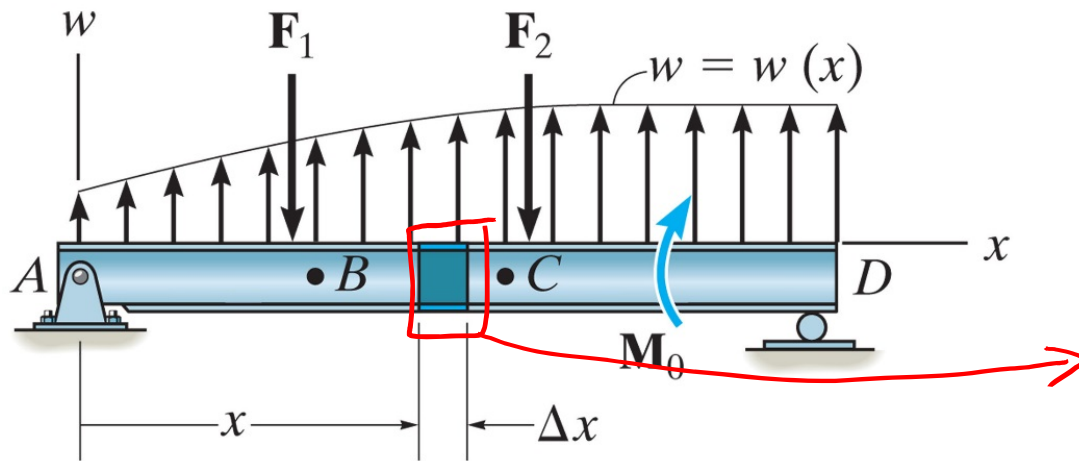
● Discussion Section 8%
attendance + class participation



tuchong.com

- **Prof. H-W office hours**
 - **Monday 3-5pm (Room C315 ZJUI Building)**
 - **Wednesday 7-8pm (Residential College Lobby)**

Recap: Relations Among Distributed Load (w), Shear Force (F) and Bending Moments (M)



Relationship between distributed load and shear:

$$\frac{dV}{dx} = w$$

Slope of shear force = distributed load intensity

$$\Delta V = V_2 - V_1 = \int w dx$$

Change in shear force = area under loading curve

Relationship between shear and bending moment:

$$\frac{dM}{dx} = V$$

Slope of bending moment = shear force

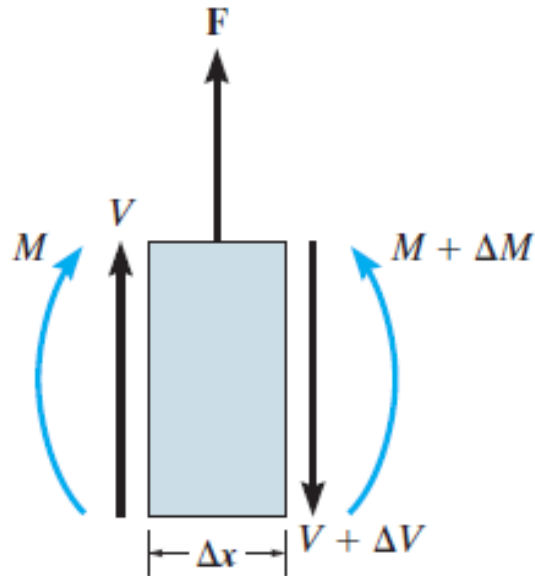
$$\Delta M = M_2 - M_1 = \int V dx$$

Change in moment = area under shear curve

Note that for a concentrated force or moment, $w = 0$. Therefore, $\frac{dV}{dx} = w = 0$, so $V(x)$ must be constant.

Recap: Relationships Among Concentrated Force (F) or Moment (M_o), Shear Force (V) and Bending Moments (M)

Wherever there is an external concentrated force or a concentrated moment, there will be a change (jump) in shear or moment, respectively.



$$\Sigma F_y:$$

$$V + F - (V + \Delta V) = 0$$

$$\Delta V = F$$

Jump in shear force due to concentrated point force F

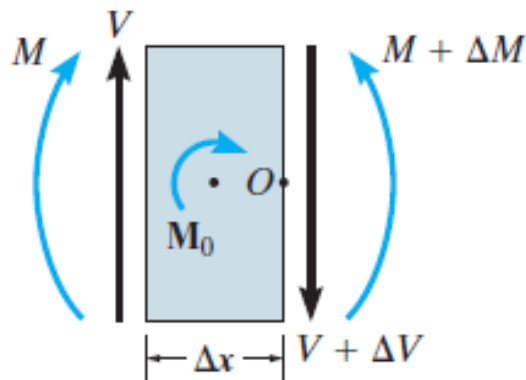
$$\Sigma M_o:$$

$$(M + \Delta M) - M - M_o - V(\Delta x) = 0$$

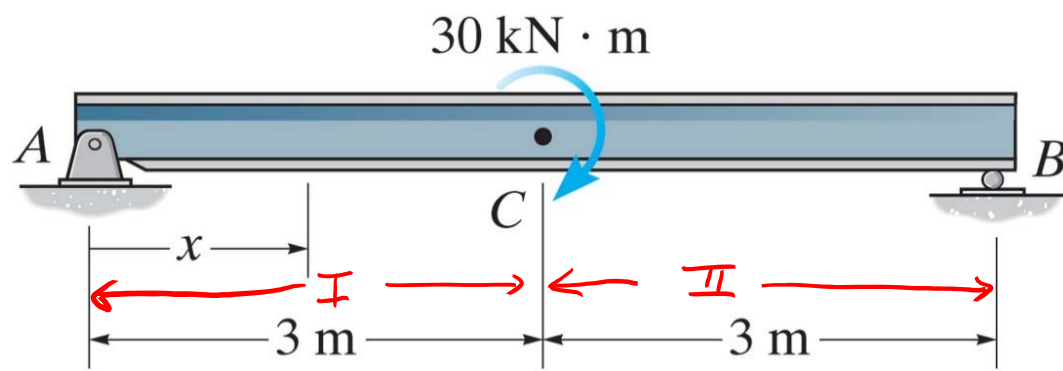
$$\Delta M = M_o + V(\Delta x)$$

$$\Delta M = M_o, \text{ when } \Delta x \rightarrow 0$$

Jump in bending moment due to concentrated couple moment M_o



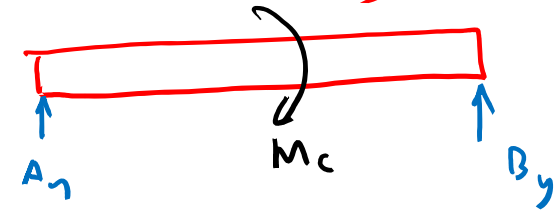
Note: the text, these notes, and convention assume that an applied concentrated moment M_o in clockwise direction results in a positive change in $M(x)$



Draw the shear force and moment diagrams for the beam.

Exercise: derive $V(x)$ and $M(x)$ as shown

① FBD entire body



$$\sum F_y:$$

$$\sum M_c:$$

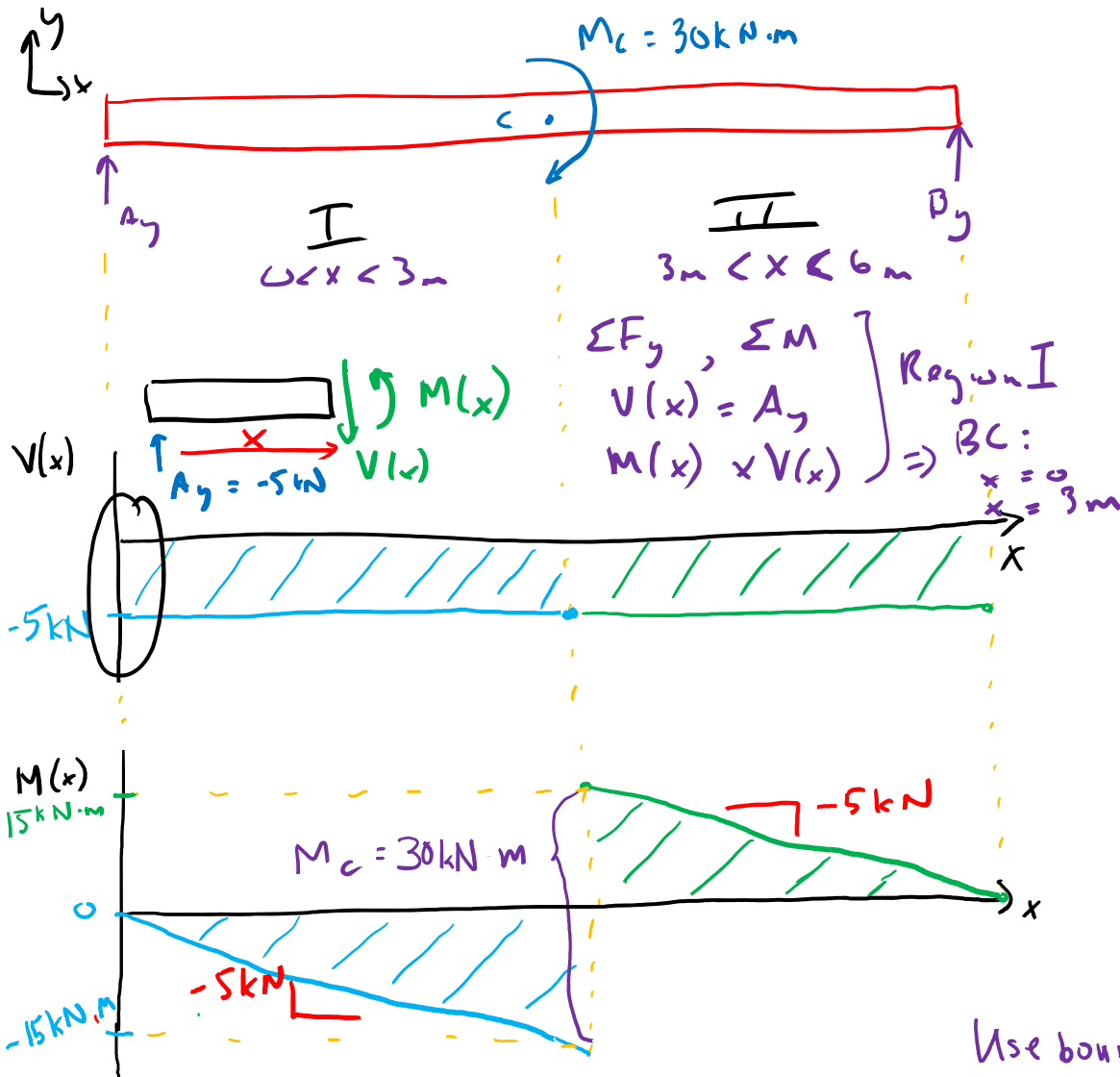
$$\Rightarrow A_y = -5 \text{ kN}$$

$$B_y = 5 \text{ kN}$$

For concentrated M_c :

$$W = 0 \rightarrow \frac{dU}{dx} = W = 0$$

$$\Rightarrow v(x) = \text{const} = -5 \text{ kN}$$



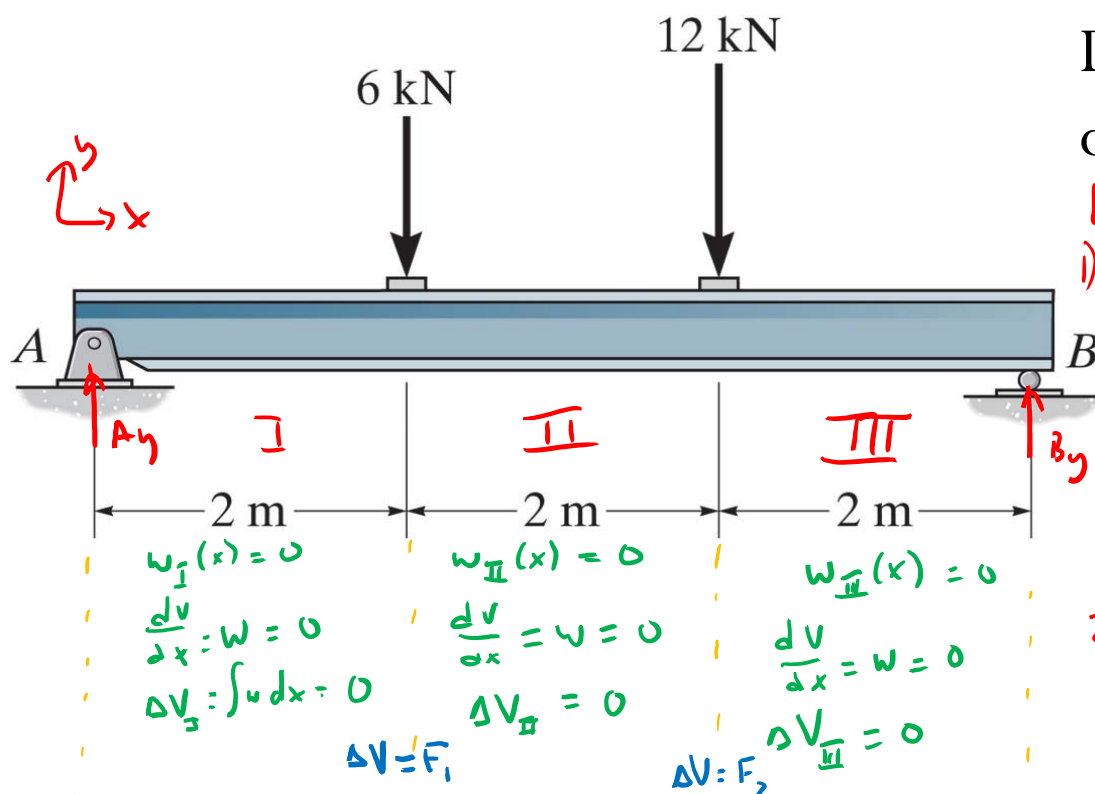
$$\Delta M = M_c$$

$$M_c = +30 \text{ kN}\cdot\text{m}$$

\therefore \uparrow jump in $M(x)$ plot

$$\frac{dM}{dx} = V = -5 \text{ kN}$$

Use boundary conditions to determine where to start $M(0)$



Draw the shear force and moment diagrams for the beam.

[solution written out side of class]

1) Find support reactions

$$\uparrow \sum F_y: A_y + B_y - 6 \text{ kN} - 12 \text{ kN} = 0$$

$$\uparrow \sum M_A: (2\text{m})6\text{kN} - (4\text{m})(12\text{kN}) + (6\text{m})B_y = 0$$

$$B_y = 10 \text{ kN} \Rightarrow A_y = 8 \text{ kN}$$

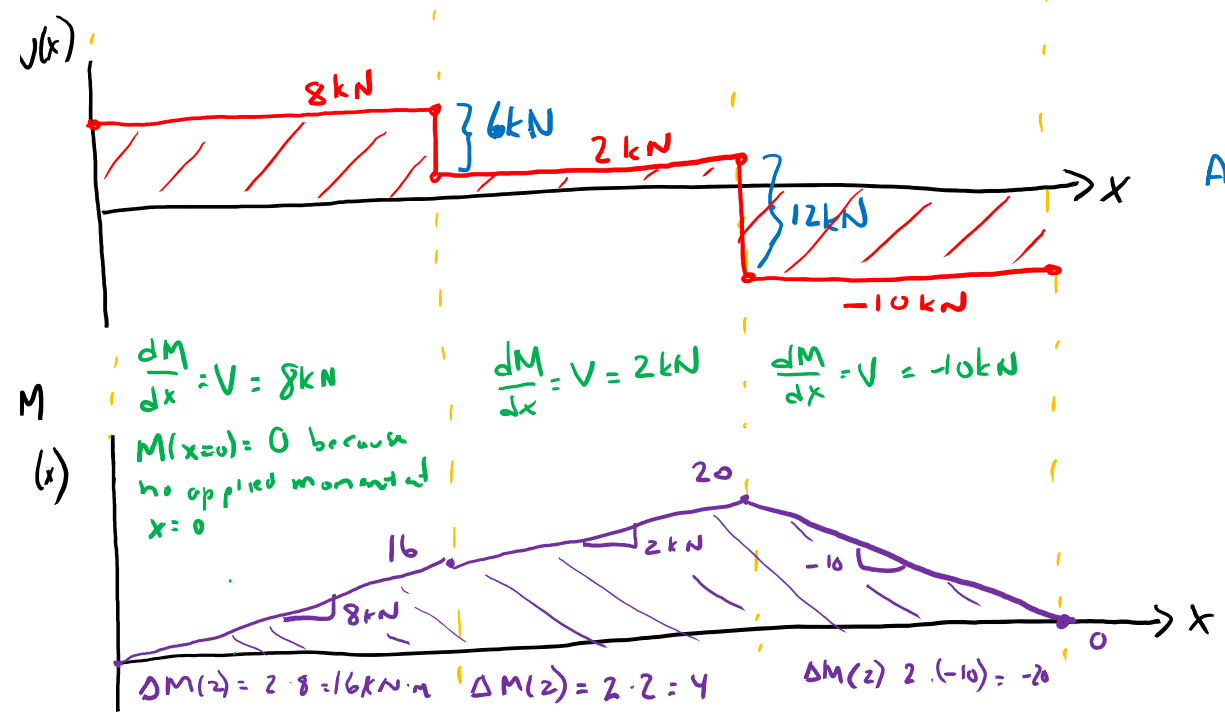
2) Quickly draw $V(x)$ & $M(x)$

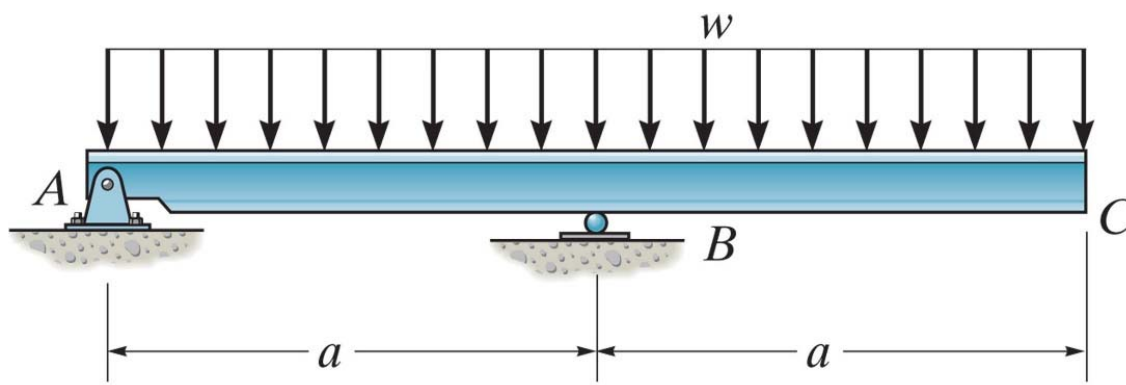
within a region use:

$$\frac{dV}{dx} = w, \quad \Delta V = V_2 - V_1 = \int w dx$$

$$\frac{dM}{dx} = V, \quad \Delta M = M_2 - M_1 = \int V dx$$

At locations of applied loads, use $\Delta V = F$





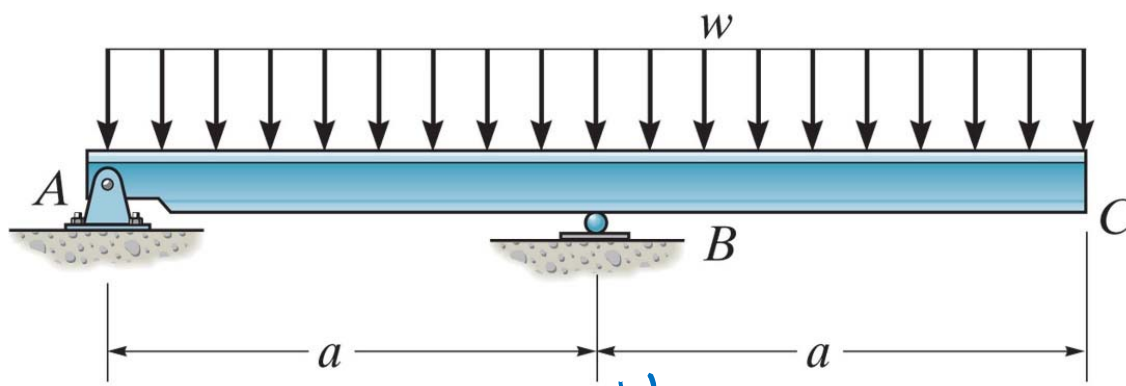
Draw the shear force and moment diagrams for the beam.

What's different about this beam?

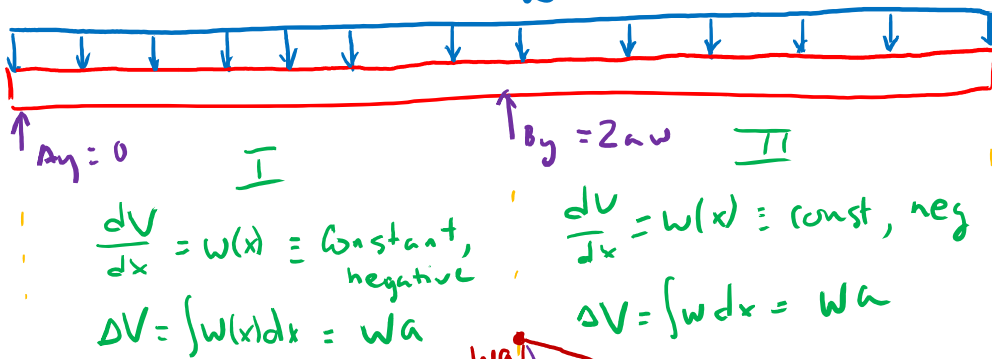
Ans: Distributed load w is also over support B !

→ Can't automatically draw V & M diagram for a rectangular distributed load

Draw the shear force and moment diagrams for the beam.

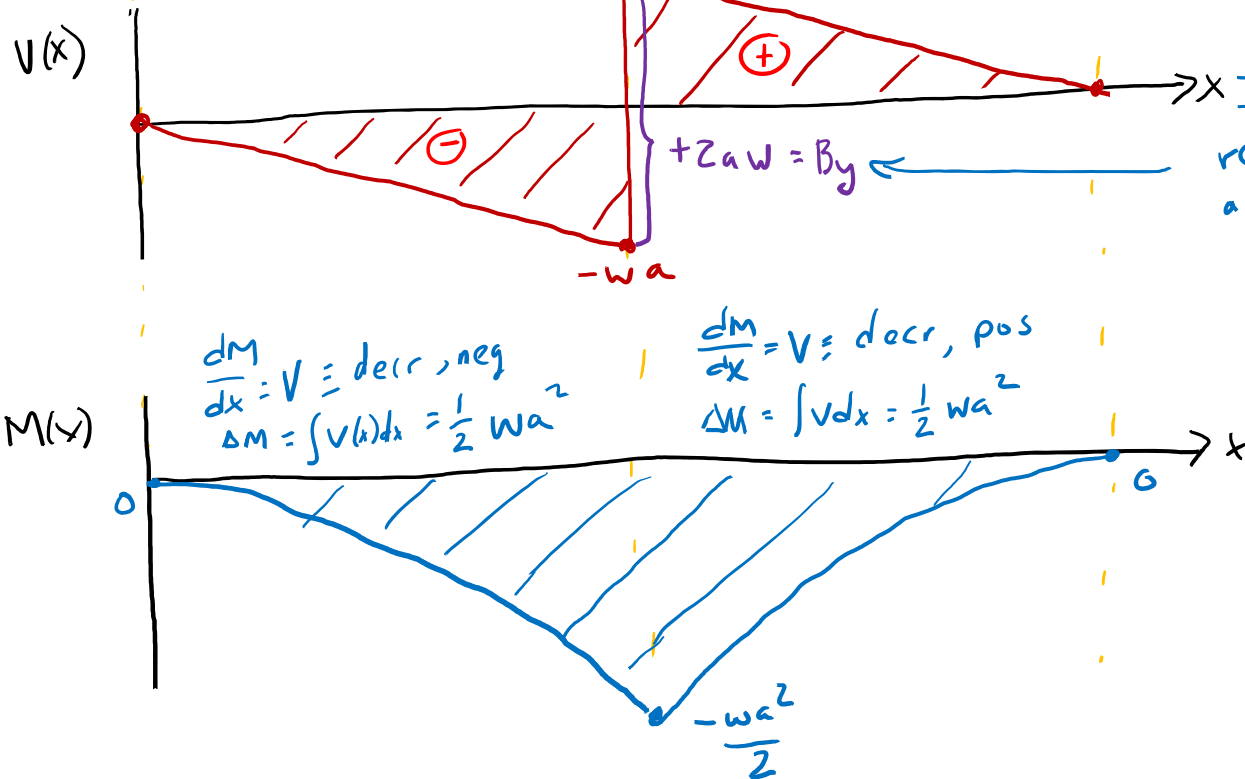


Prove to yourself that
 $A_y = 0$, $B_y = 2aw$



$\frac{dV}{dx} = w(x) \equiv \text{constant, negative}$
 $\Delta V = \int w(x) dx = wa$

$\frac{dV}{dx} = w(x) \equiv \text{const, neg}$
 $\Delta V = \int w dx = wa$



$\frac{dM}{dx} = V \equiv \text{decr, neg}$
 $\Delta M = \int V(x) dx = \frac{1}{2} wa^2$

$\frac{dM}{dx} = V \equiv \text{decr, pos}$
 $\Delta M = \int V dx = \frac{1}{2} wa^2$

In this scenario, reaction support force B_y acts like an upward concentrated load!

Chapter 8: Friction

Goals and Objectives

- Sections 8.1-8.2
- Introduce the concept of dry friction
- Analyze the equilibrium of rigid bodies subjected to this force

Friction

Friction is a force that resists the movement of two contacting surfaces that slide relative to one another. This force acts tangent to the surface at the points of contact and is directed so as to oppose the possible or existing motion between the surfaces.

Dry Friction (or Coulomb friction) occurs between the contacting surfaces of bodies when there is no lubricating fluid.



Figure: 08_COC

The effective design of each brake on this railroad wheel requires that it resist the frictional forces developed between it and the wheel. In this chapter we will study dry friction, and show how to analyze friction forces for various engineering applications.

Friction

In designing a brake system for a bicycle, car, or any other vehicle, it is important to understand the frictional forces involved.

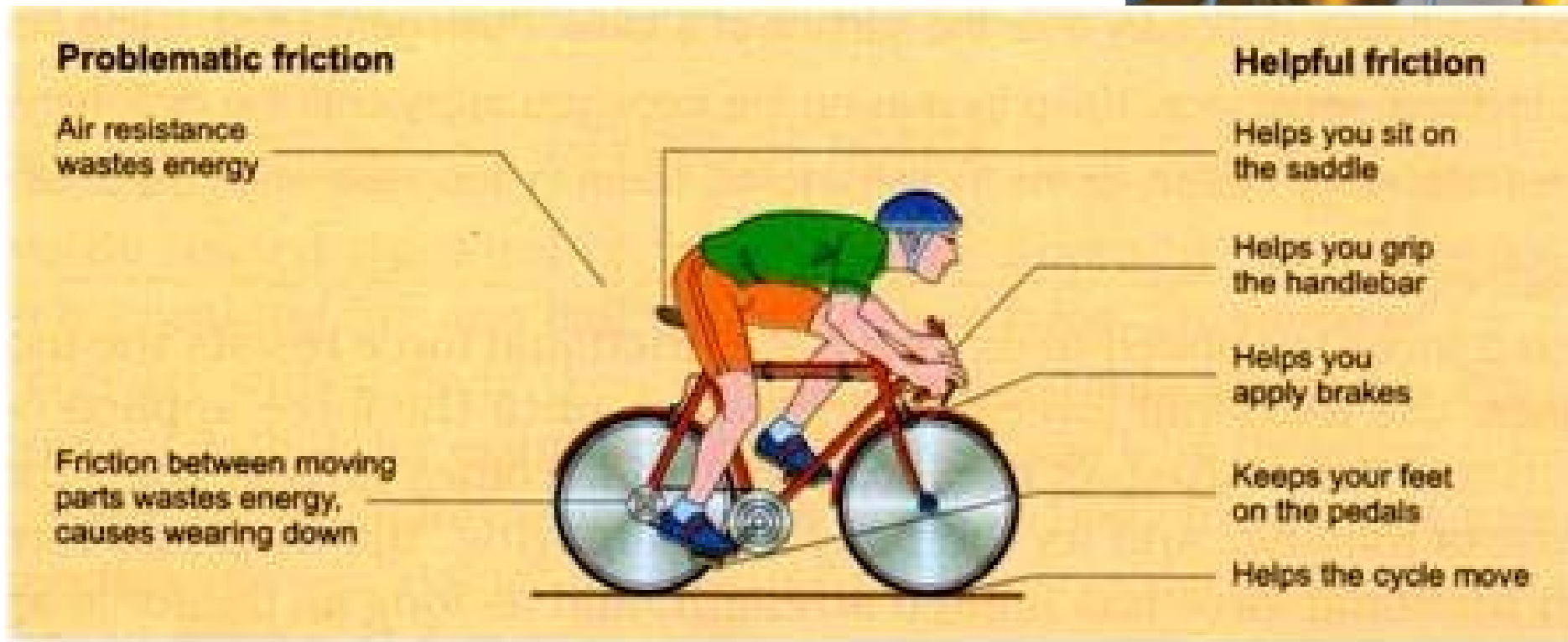


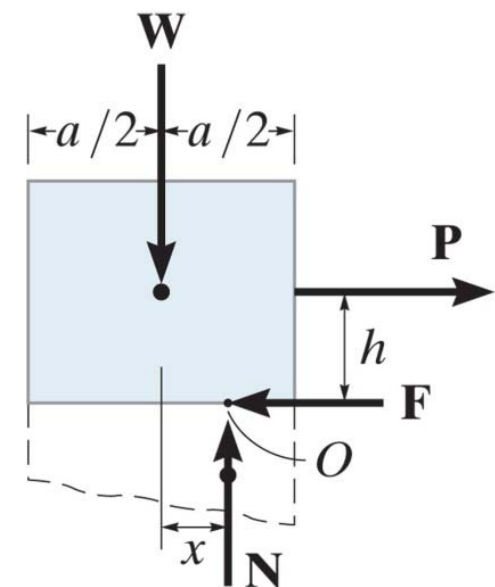
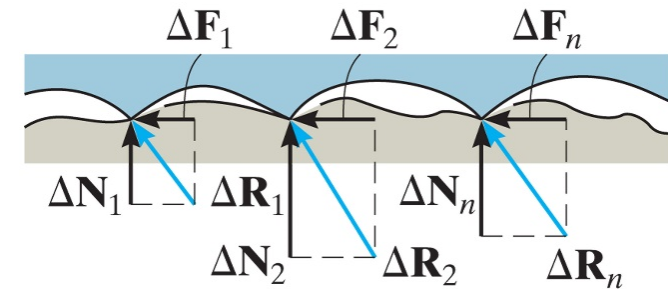
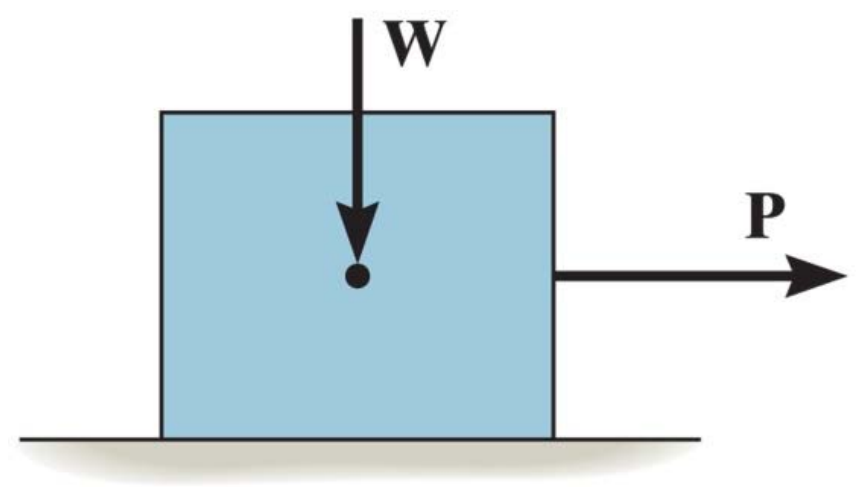
Fig. 8.6 How friction helps and creates problems when you cycle

This is a good link to explain different types of forces:

<http://www.yourarticlelibrary.com/science/4-important-types-of-force-explained-with-diagram/31675>

Dry friction

- Consider the effects of pulling horizontally a block of weight \mathbf{W} which is resting on a **rough** surface.
- The floor exerts an uneven distribution of normal forces $\Delta\mathbf{N}_n$ and frictional forces $\Delta\mathbf{F}_n$ along the contacting surface.
- These distributed loads can be represented by their equivalent resultant normal forces \mathbf{N} and frictional forces \mathbf{F}



Dry friction

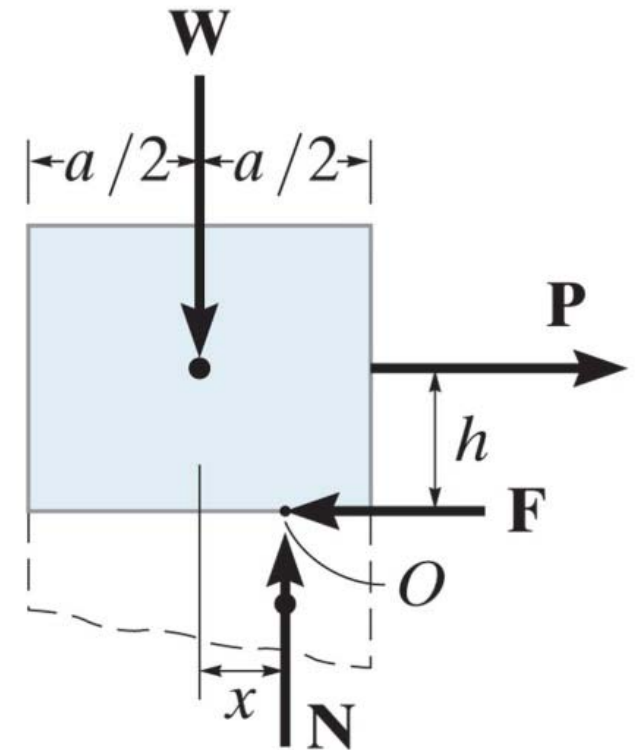
- **Equilibrium:** to avoid tipping of the block, the following equilibrium should be satisfied:

$$\sum M_O = -Ph + Wx = 0 \rightarrow x = \frac{Ph}{W}$$

- **Impending motion:** the maximum force F_S before slipping begins is given by

$$F_S = \mu_s N$$

where μ_s is called the coefficient of static friction



1. $P = 0 \rightarrow$ no motion; no friction
2. $P < F_s \Rightarrow P < \mu_s W \rightarrow$ no motion; friction force $|F| = |P|$
3. $P = F_s = \mu_s W \rightarrow$ no motion, but on the point of sliding
4. $P > F_s \rightarrow$ box begins to slide, since $\sum F_x > 0$

When $P > F_s$, the frictional force is no longer a function of the coefficient of static friction, but instead it will drop to a smaller value F_k , i.e.,

$$F_k = \mu_k N$$

where μ_k is called the coefficient of kinetic friction, or dynamic friction. Typical values for μ_k are approximately 25% smaller than the ones for μ_s .

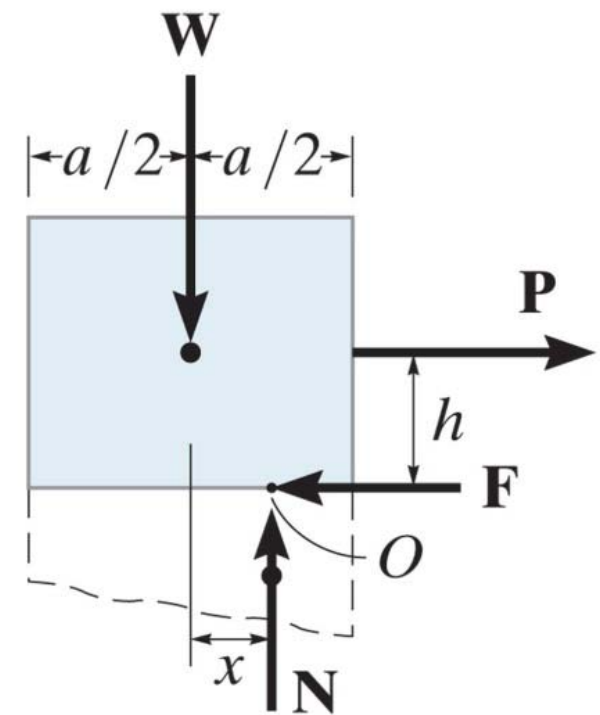
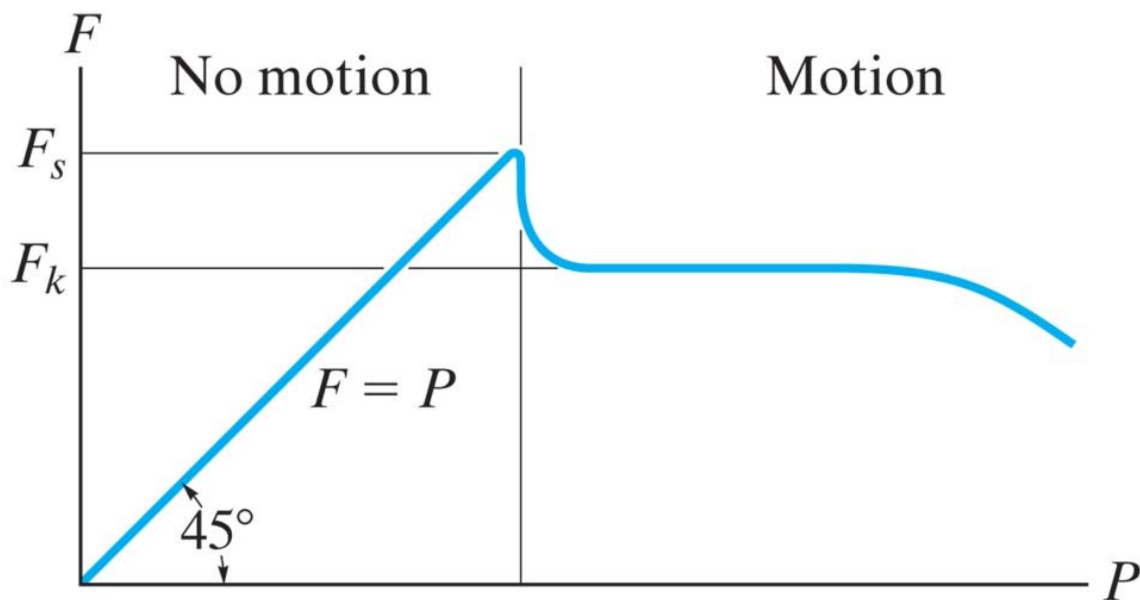
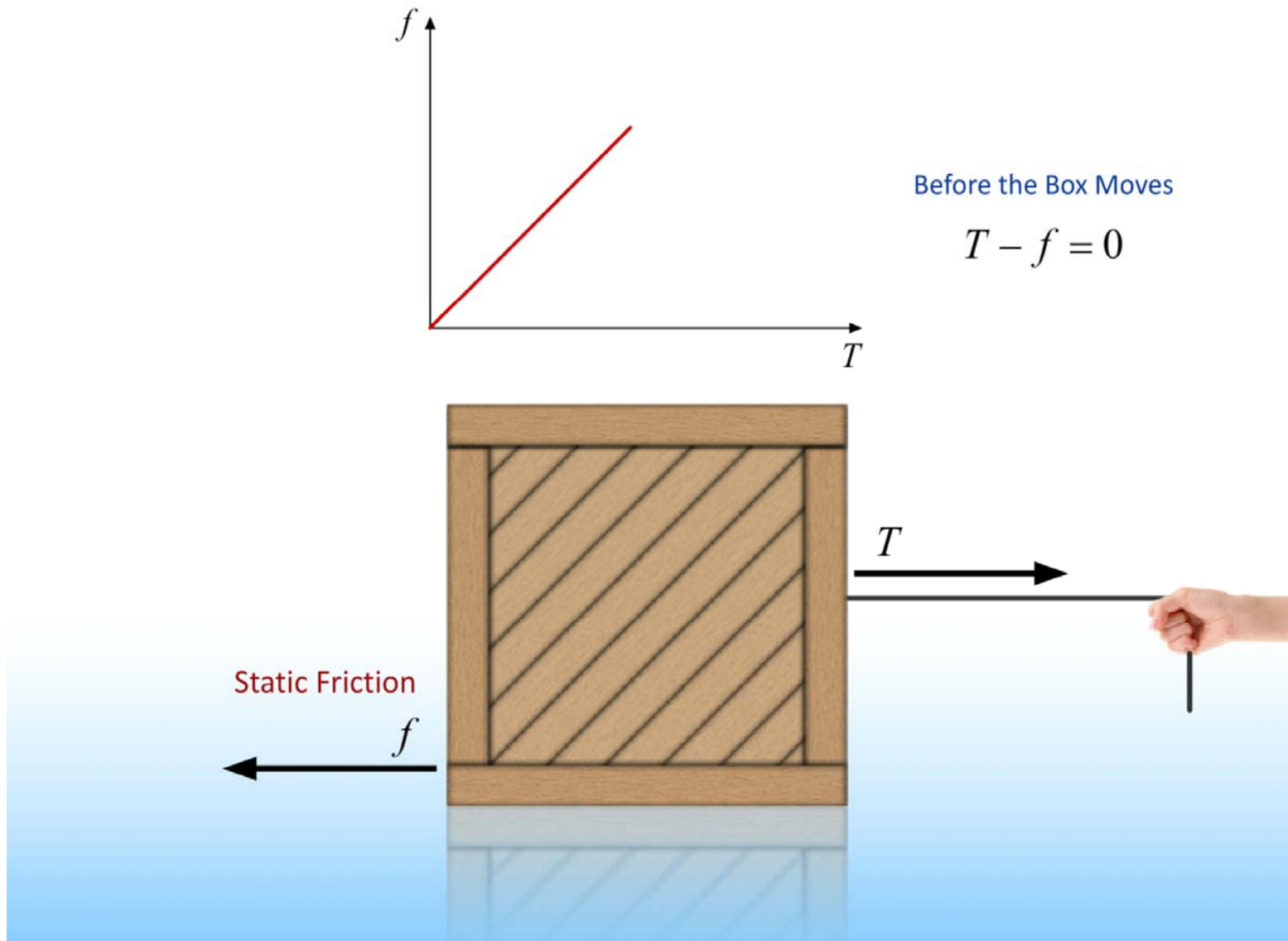
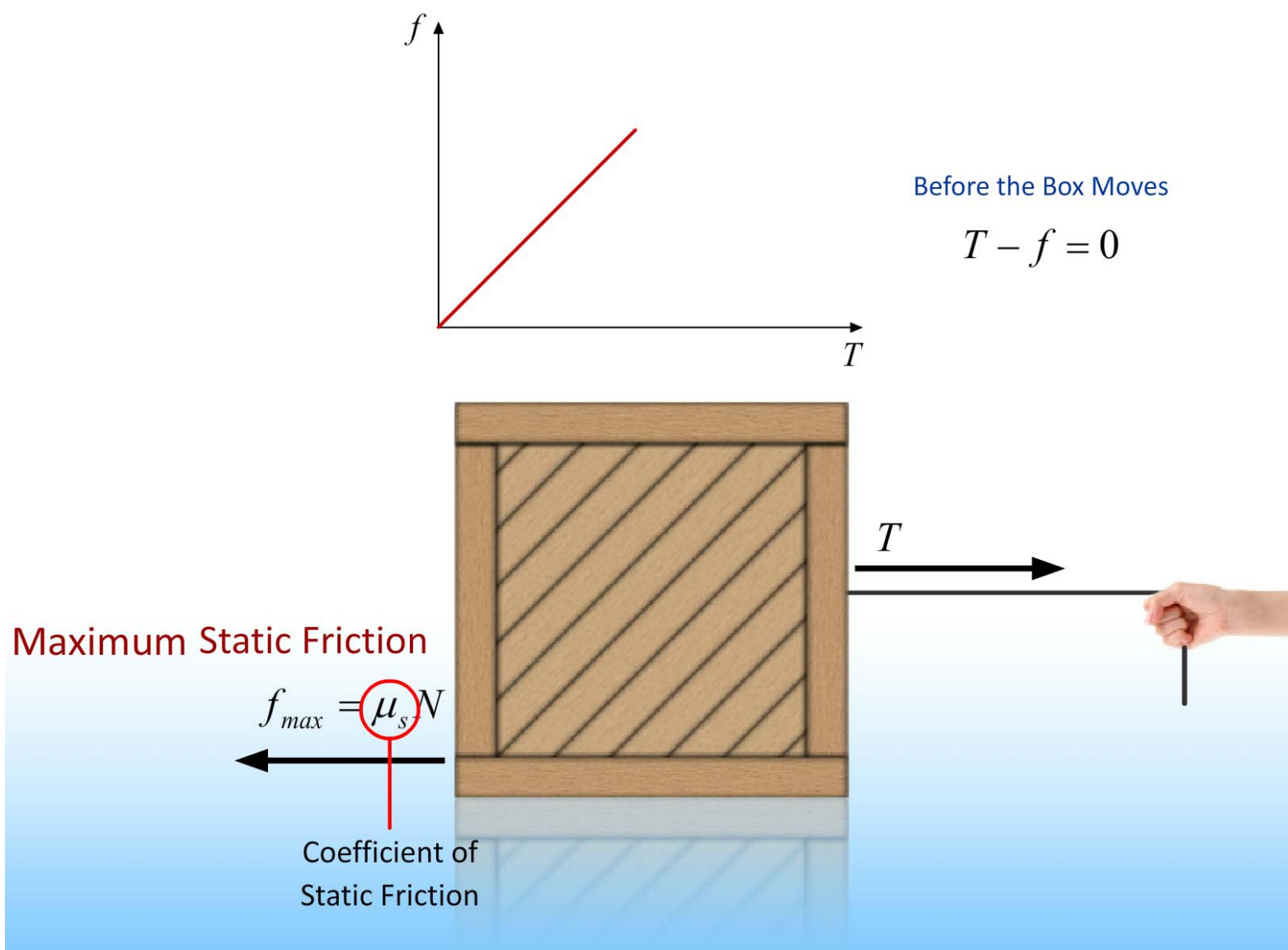
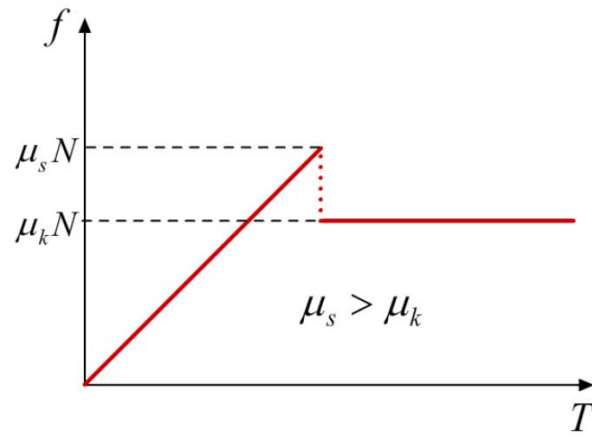


Table 8–1 Typical Values for μ_s

Contact Materials	Coefficient of Static Friction (μ_s)
Metal on ice	0.03–0.05
Wood on wood	0.30–0.70
Leather on wood	0.20–0.50
Leather on metal	0.30–0.60
Aluminum on aluminum	1.10–1.70

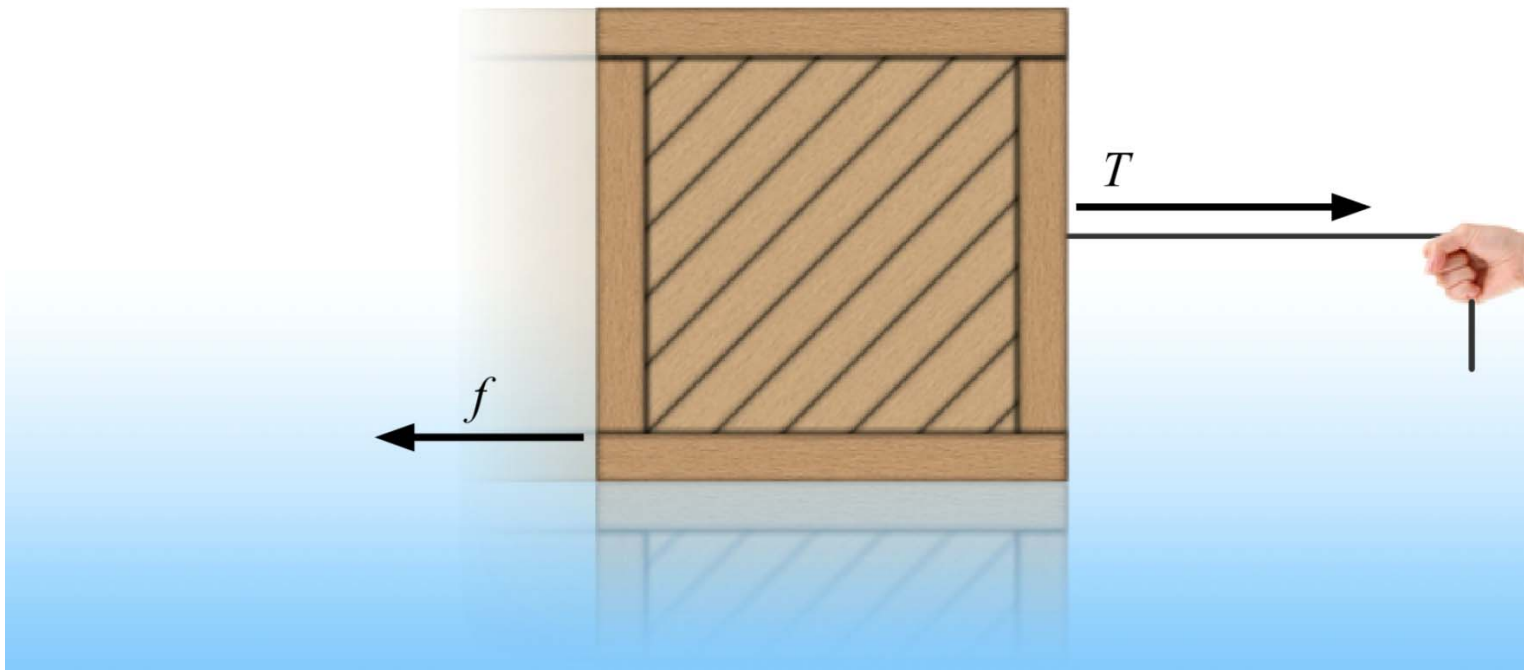






Before the Box Moves

$$T - f = 0$$



Summary: Dry friction

- Friction acts tangent to contacting surfaces and in a direction opposed to motion of one surface relative to another
- Friction force F is related to the coefficient of friction and normal force N
 - Static friction: $F_s \leq \mu_s N$
 - Kinetic friction: $F_k = \mu_k N$
- Magnitude of coefficient of friction depends on the two contacting materials
- Maximum static frictional force occurs when motion is impending
- Kinetic friction is the tangent force between two bodies after motion begins. Less than static friction by $\sim 25\%$.

Components of a Contact Force

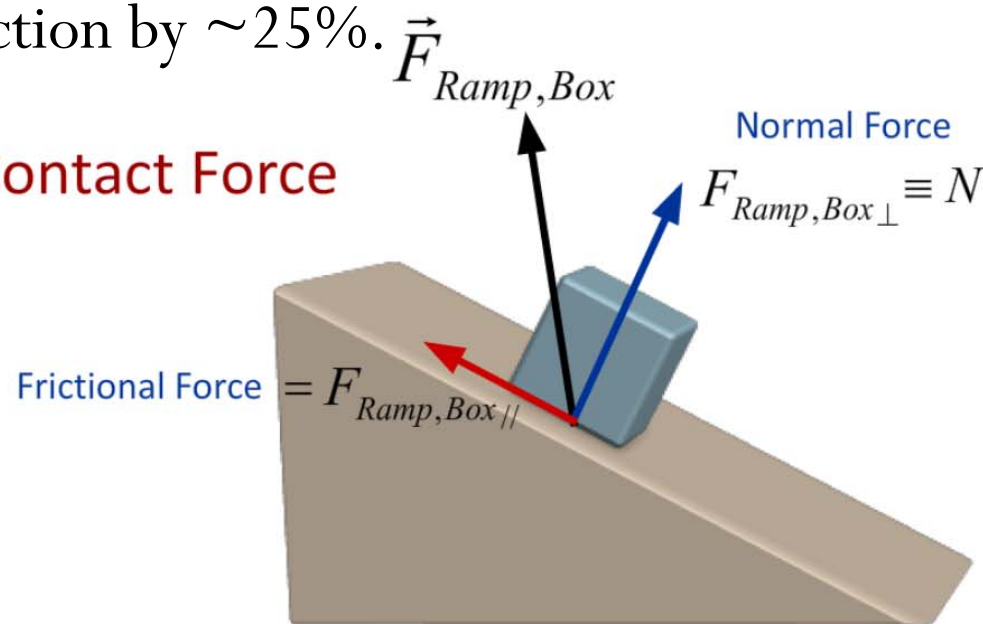
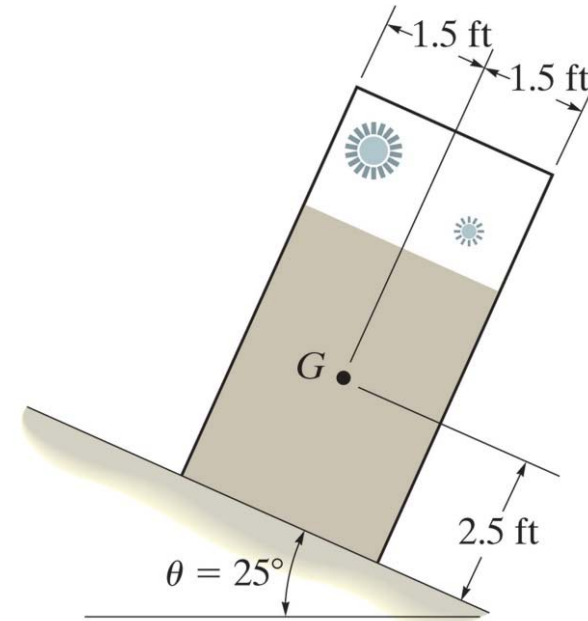


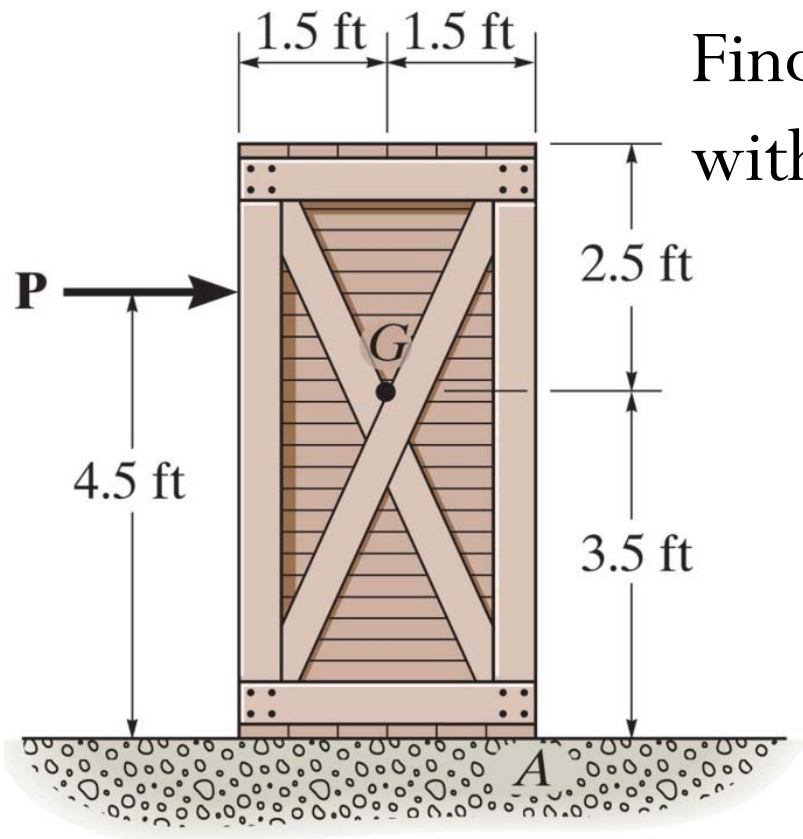
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It is observed that when the bed of the dump truck is raised to an angle of $\theta = 25^\circ$ the vending machines will begin to slide off the bed. Determine the static coefficient of friction between a vending machine and the surface of the truck bed.



Find unknown μ_s .
Compare 2 possible cases of "impending motion" (slip or Tip)



Find the maximum force P that can be applied without causing movement of the crate.

Find P

2 cases of impending motion (Slip or Tip)