Statics - TAM 211

Lecture 26

November 26, 2018

Chap 9.1

Announcements

- Upcoming deadlines:
- Tuesday (11/27)
  - Prairie Learn HW 10
- Friday (11/30)
  - Written Assignment 10
- **Friday (11/30) all in Teaching Building A418-420**
  - 8:00 am: Quiz 5, On paper. Chapter 7+8 (Internal forces, Friction)
  - 9:00 am: Lecture 28 (Center of Gravity/Composite Areas)
  - 10:00 am: Discussion section for ALL students

- **Reminder: Discussion Section**
  - **12% of final grade**
  - **Attendance + Participation**
  - **No grade given for discussion section if > 5 minutes late**

Recap: Dry Friction Problem Procedure

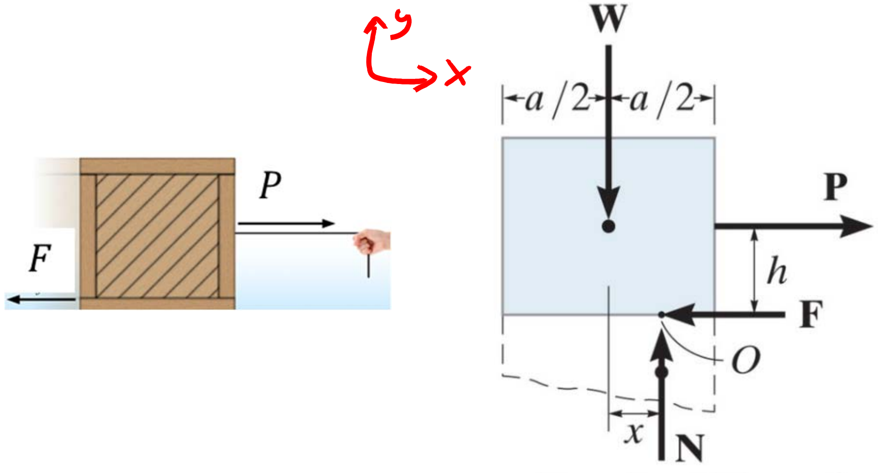
A. Draw FBD for each body
   - Friction force vector points in opposite direction of impending motion

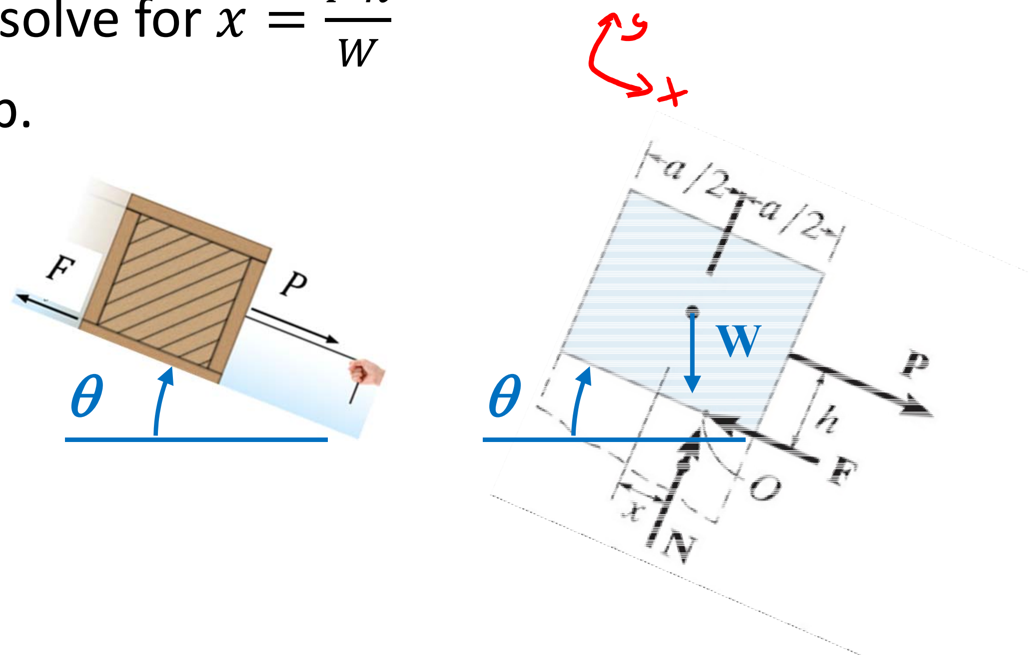
B. Determine # unknowns

C. Apply equations of equilibrium

   i. If checking for slipping:
      - Examine $\sum F_x = 0, \sum F_y = 0,$ and case when slipping starts $F_s = \mu_s N$

   ii. If checking for tipping:
      - Examine $\sum M_O = 0 = -Ph + Wx,$ solve for $x = \frac{P\,h}{W}$
      - If $x>a/2$, then tip. If $x<a/2$, then slip.





Chapter 9: Center of Gravity and Centroid

Goals and Objectives

- Understand the concepts of center of gravity, center of mass, and centroid.

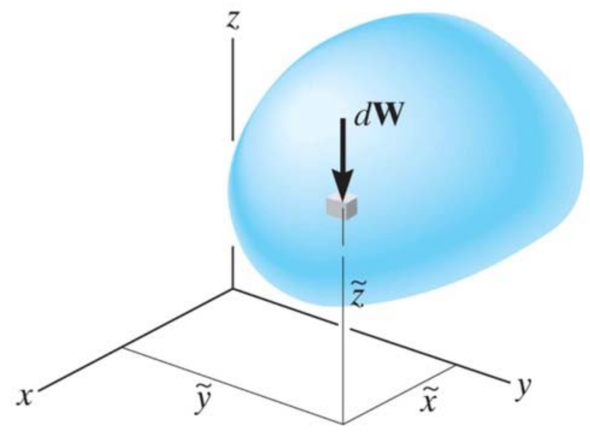
- Determine the location of the center of gravity and centroid for a system of discrete particles and a body of arbitrary shape.

Center of gravity

To design the structure for supporting a water tank, we will need to know the weight of the tank and water as well as the locations where the resultant forces representing these distributed loads act.

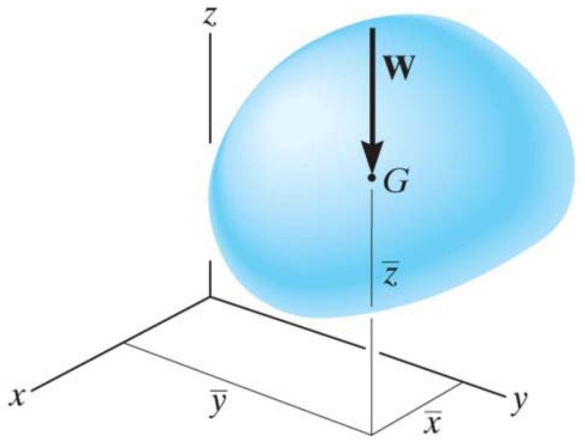
How can we determine these resultant weights and their lines of action?

Center of gravity



A body is composed of an infinite number of particles, and so if the body is located within a gravitational field, then each of these particles will have a weight *dW*.

The **center of gravity (CG)** is a point, often shown as G, which locates the resultant weight of a system of particles or a solid body.



From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G.

If *dW* is located at point $(\tilde{x}, \tilde{y}, \tilde{z})$ then

$$\bar{x}\, W = \int \tilde{x}\, dW$$

$$\bar{y}\, W = \int \tilde{y}\, dW$$

$$\bar{z}\, W = \int \tilde{z}\, dW$$

$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW}$$

$$\bar{y} = \frac{\int \tilde{y}\, dW}{\int dW}$$

$$\bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

Center of Mass

Given: $dW = g\,dm$
Provided that $g =$ constant:

$$\bar{x} = \frac{\int \tilde{x}\,dm}{\int dm}$$

$$\bar{y} = \frac{\int \tilde{y}\,dm}{\int dm}$$

$$\bar{z} = \frac{\int \tilde{z}\,dm}{\int dm}$$

Center of Volume

For homogeneous material,
$\rho =$ constant.
Therefore, $dm = \rho\,dV$

$$\bar{x} = \frac{\int \tilde{x}\,dV}{\int dV}$$

$$\bar{y} = \frac{\int \tilde{y}\,dV}{\int dV}$$

$$\bar{z} = \frac{\int \tilde{z}\,dV}{\int dV}$$

Center of Area

$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA}$$

$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA}$$

$$\bar{z} = \frac{\int \tilde{z}\,dA}{\int dA}$$

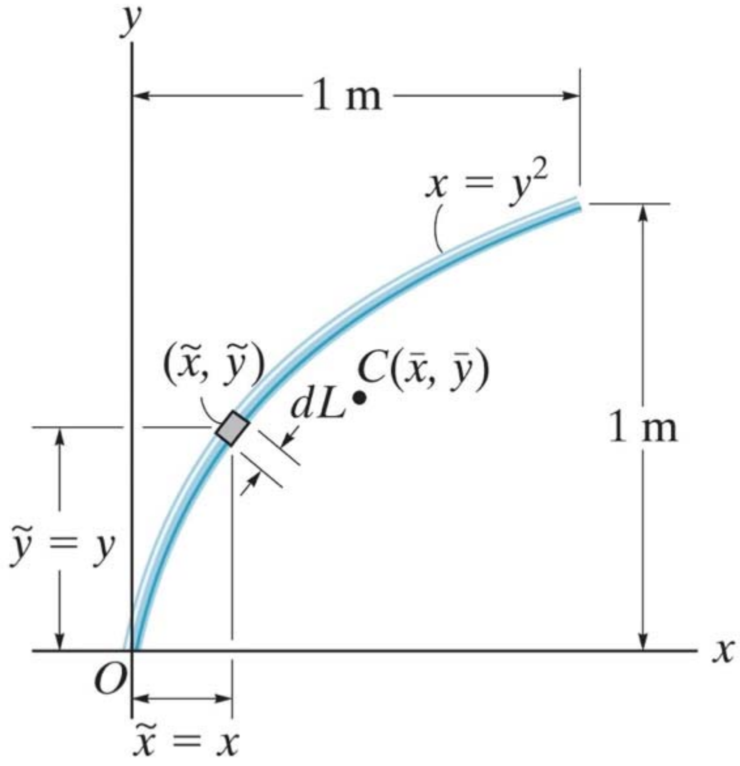
If use rectangular strip,
simplify to $dA = y\,dx$ and $\tilde{x} = x,\ \tilde{y} = y/2$.
and $dA = x\,dy$ and $\tilde{x} = x/2,\ \tilde{y} = y$.

Center of Line

$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL}$$

$$\bar{y} = \frac{\int \tilde{y}\, dL}{\int dL}$$

$$\bar{z} = \frac{\int \tilde{z}\, dL}{\int dL}$$



$y = f(x)$ or $x = f(y)$

Use Pythagorean Theorem:

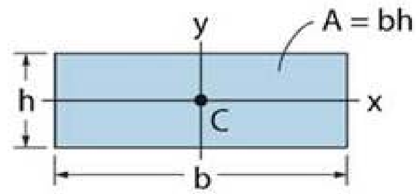
$$dL = \sqrt{(dx)^2 + (dy)^2}$$

$$dL = \sqrt{\left(\frac{dx}{dx}\right)^2 (dx)^2 + \left(\frac{dy}{dx}\right)^2 (dx)^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right) dx$$

Or

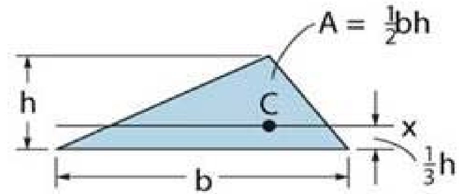
$$dL = \sqrt{\left(\frac{dx}{dy}\right)^2 (dy)^2 + \left(\frac{dy}{dy}\right)^2 (dy)^2} = \left(\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\right) dy$$

Centroid



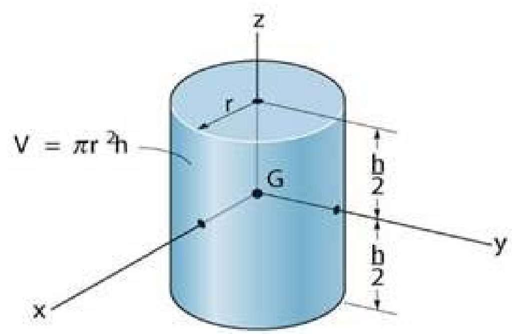
Rectangular area

The centroid, C, is a point defining the geometric center of an object.



Triangular area

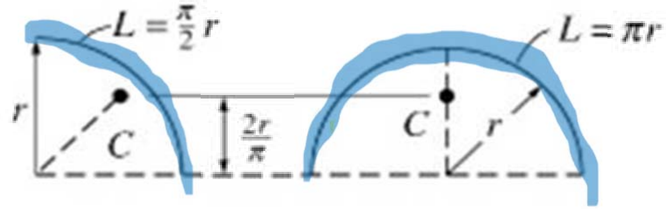
The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogeneous (density or specific weight is constant throughout the body).



Cylinder

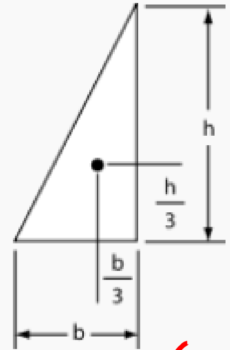
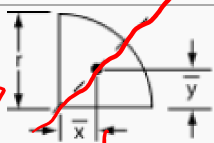
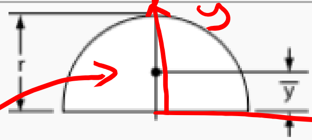
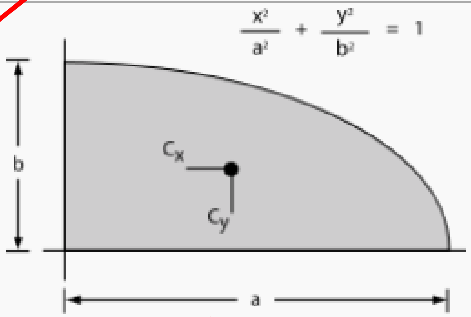
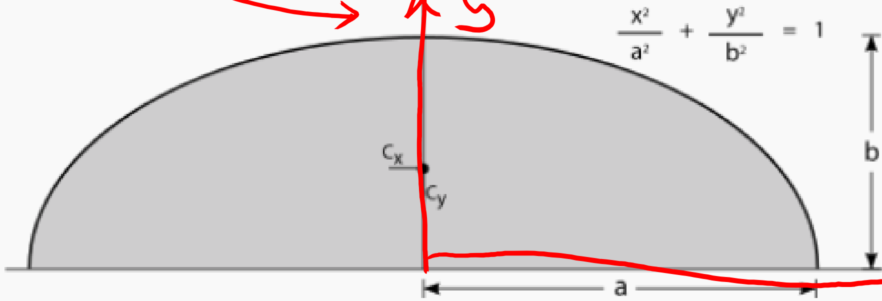
If an object has an axis of symmetry, then the centroid of object lies on that axis.

In some cases, the centroid may not be located on the object.



Quarter and semicircle arcs

Centroid of typical 2D shapes

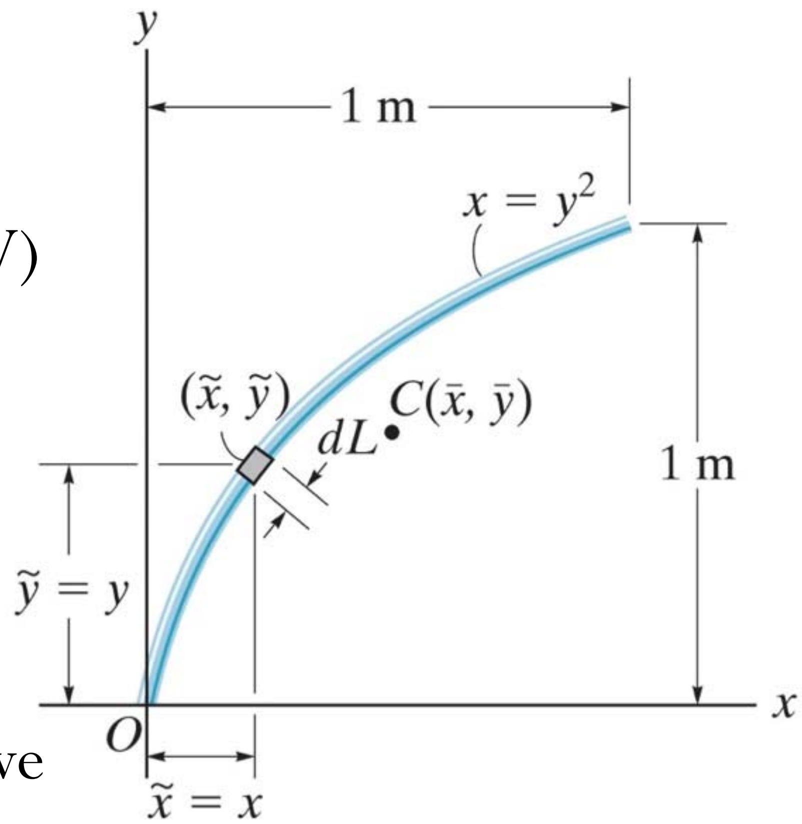
Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Right-triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

Axis of symmetry

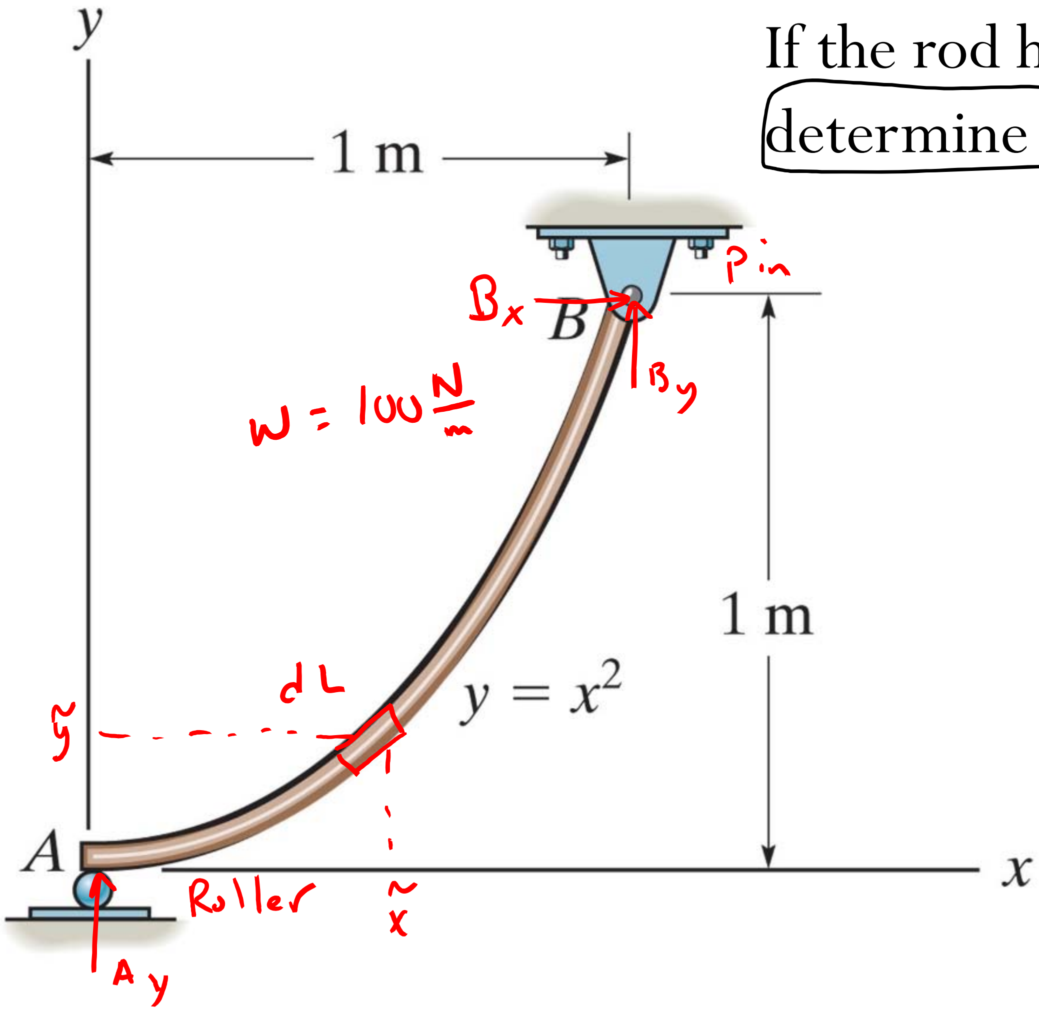
http://en.wikipedia.org/wiki/List_of_centroids

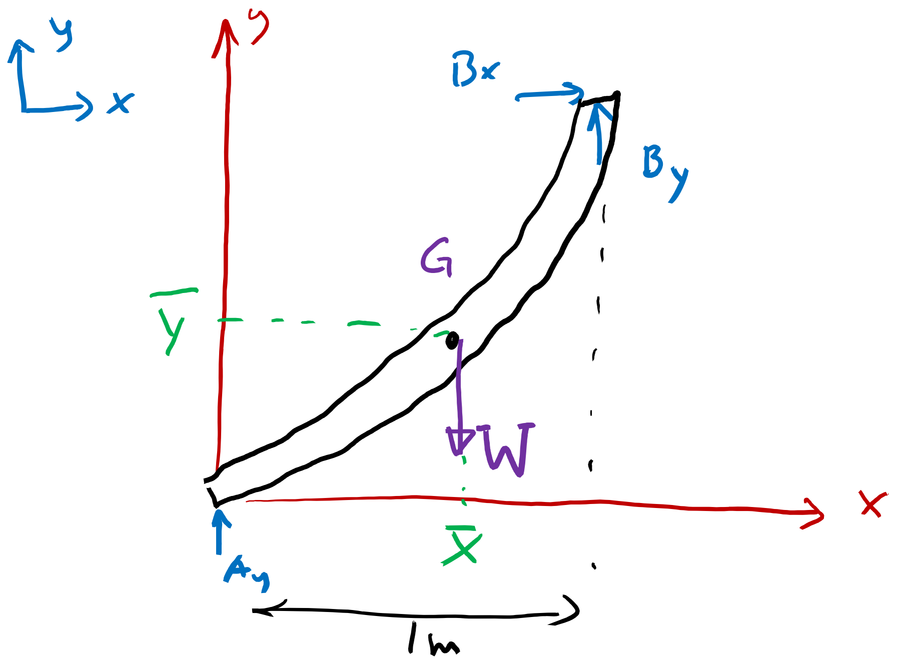
Centroid – Analysis Procedure

1. Select an appropriate coordinate system
2. Define the appropriate element (dL, dA, or dV)
3. Express (2) in terms of the coordinate system
4. Identify any symmetry
5. Express the moment arms (centroid) of (2)
6. Substitute (3) and (4) into the integral and solve



If the rod has a weight per unit length of 100 N/m, determine the reaction supports at A and B.





EOE:

$\xrightarrow{+} \sum F_x : \boxed{B_x = 0}$

$+\uparrow \sum F_y : \; A_y + B_y - \bar{W} = 0$

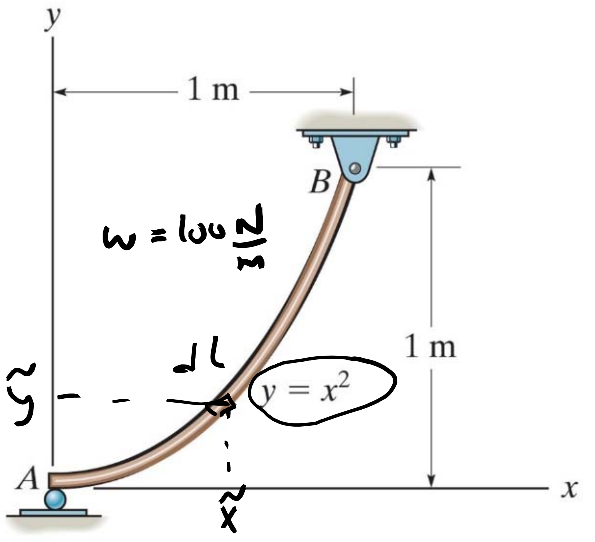
$+\circlearrowleft \sum M_B : \; -(1\,\text{m})A_y + (1 - \bar{x})\bar{W} = 0$

UNKNOWNS: $\bar{x}, A_y, B_y, \bar{W}$

After solving for $\bar{x}$ & $\bar{W}$ (on next slide)

$\therefore \boxed{A_y = 63.1\text{ N}}$

$\boxed{B_y = 84.8\text{ N}}$



solve $\bar{x}$ & W

$$y = x^2 \Rightarrow dy = 2x\,dx$$

$$\tilde{x} = x$$

$$\tilde{y} = y$$

$$dL = \sqrt{(dx)^2 + (dy)^2}$$

use $$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$= \sqrt{1 + (2x)^2}\,dx$$

$$= \sqrt{4x^2 + 1}\,dx$$

$$\bar{x} = \frac{\int \tilde{x}\,dL}{\int dL} = \frac{\int_0^1 x\sqrt{4x^2+1}\,dx}{\int_0^1 \sqrt{4x^2+1}\,dx}$$

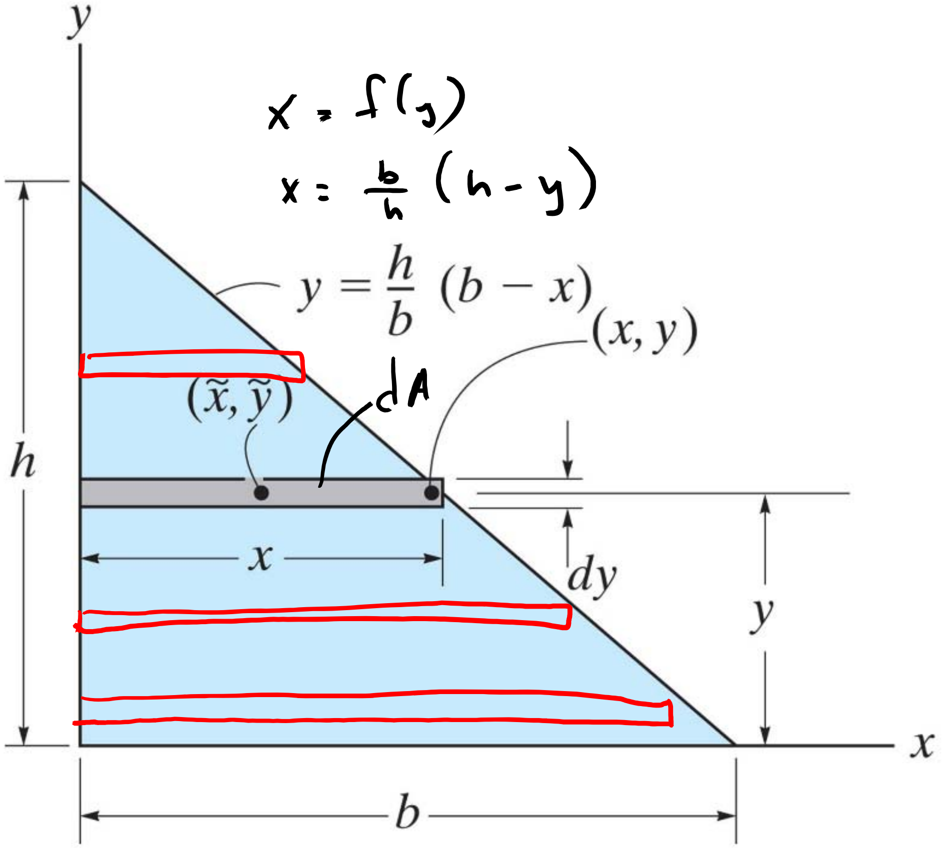
$$= \frac{2\int_0^1 x\sqrt{x^2 + \frac{1}{4}}\,dx}{2\int_0^1 \sqrt{x^2+\frac{1}{4}}\,dx} = \frac{0.848\ m^2}{1.478\ m} = \boxed{0.574m = \bar{x}}$$

$$W = w\int_0^1 dL$$

$$= 100\frac{N}{m}\int_0^1 \sqrt{4x^2+1}\,dx = 100\frac{N}{m}(1.478m)$$

$$\Rightarrow \boxed{W = 147.8N}$$

Determine the distance y measured from the x axis to the centroid of the area of the triangle.



$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA} \qquad \bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$$\tilde{x} = \frac{x}{2}, \quad \tilde{y} = y$$

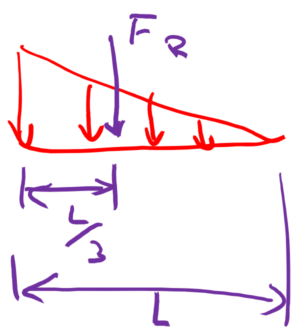
$$\bar{y} = \frac{\int y\, dA}{\int dA} \qquad dA = x\, dy$$

$$= \frac{\int_0^h y\, x\, dy}{\int_0^h x\, dy} = \frac{\int_0^h y\left(\frac{b}{h}(h-y)\right) dy}{\int_0^h \frac{b}{h}(h-y)\, dy}$$

$$= \frac{\frac{b}{h}\int_0^h (yh - y^2)\, dy}{\frac{b}{h}\int_0^h (h-y)\, dy}$$

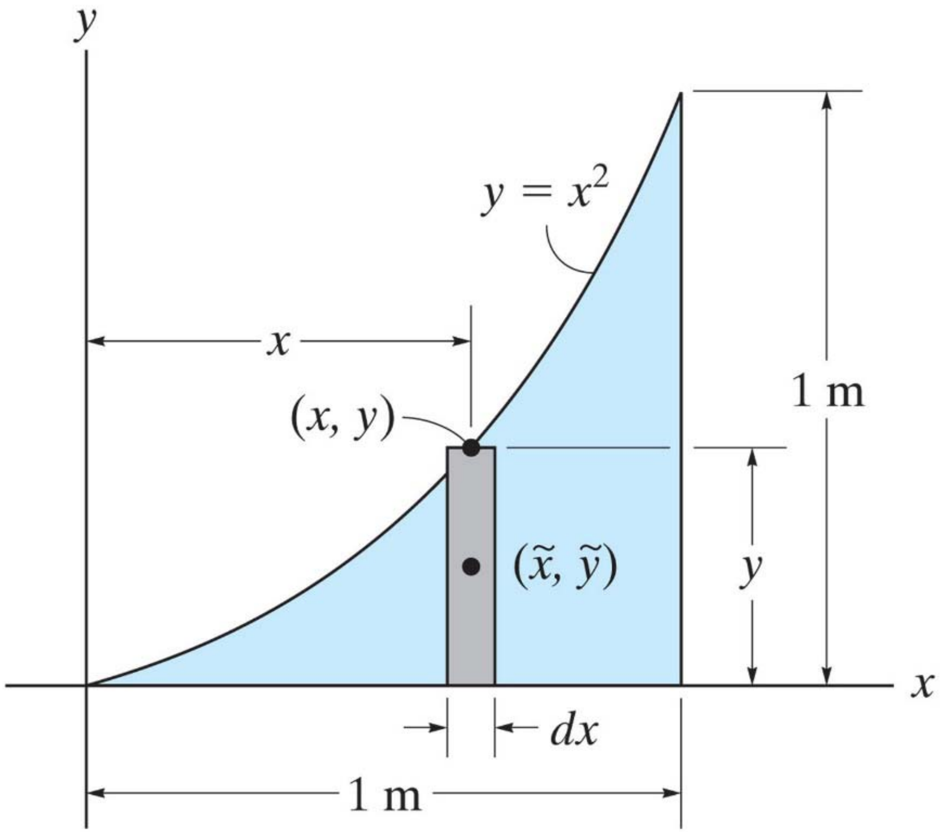
$$= \frac{\left(h\frac{y^2}{2} - \frac{y^3}{3}\right)\Big|_0^h}{\left(hy - \frac{y^2}{2}\right)\Big|_0^h} = \frac{h}{3}$$

$$\therefore \bar{y} = \frac{h}{3}$$



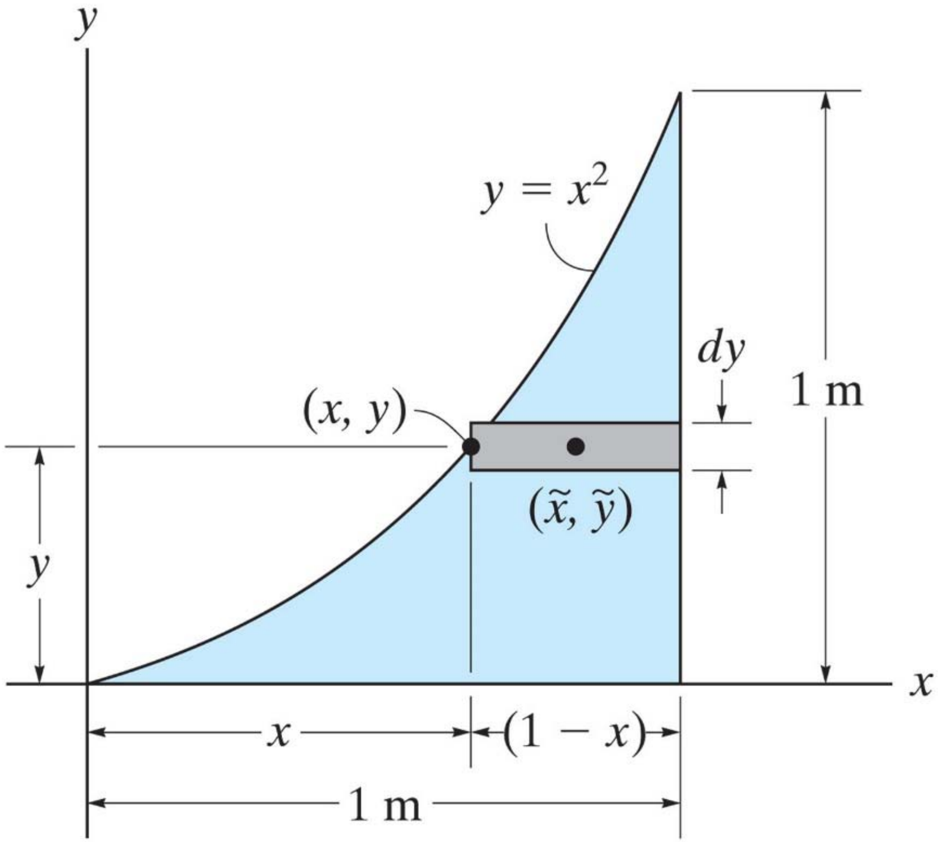
cf. Chap 4 distributed load of triangle

See Text Example: 9.3



Locate the centroid of the area.

See Text Example: 9.5



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