Statics - TAM 211

Lecture 27

November 28, 2018

Chap 9.2

Announcements

- Upcoming deadlines:
- Friday (11/30)
  - Written Assignment 10
- **Friday (11/30) all in Teaching Building A418-420**
  - 8:00 am: Quiz 5, On paper. Chapter 7+8 (Internal forces, Friction)
  - 9:00 am: Lecture 28 (~~Center of Gravity/Composite Areas~~) *Moments of Inertia*
  - 10:00 am: Discussion section for ALL students
- Tuesday (12/4)
  - Prairie Learn HW 11

- **Reminder: Discussion Section**
  - **12% of final grade**
  - **Attendance + Participation**
  - **No grade given for discussion section if > 5 minutes late**

Chapter 9: Center of Gravity and Centroid

Goals and Objectives

- Understand the concepts of center of gravity, center of mass, and centroid.

- Determine the location of the center of gravity and centroid for a system of discrete particles and a body of arbitrary shape.

Recap:

Center of Gravity	Center of Mass	Center of Volume	Center of Area	Center of Line

Given: $dW = g\,dm$
Provided $g =$ constant

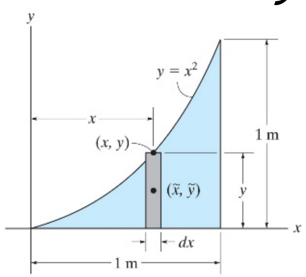
For homogeneous material, $\rho =$ constant.
Therefore, $dm = \rho\,dV$

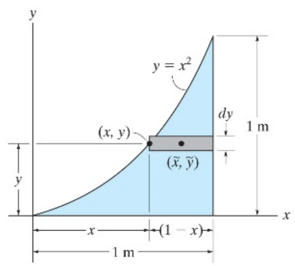
$$\bar{x} = \frac{\int \tilde{x}\,dW}{\int dW} \qquad \bar{x} = \frac{\int \tilde{x}\,dm}{\int dm} \qquad \bar{x} = \frac{\int \tilde{x}\,dV}{\int dV} \qquad \bar{x} = \frac{\int \tilde{x}\,dA}{\int dA} \qquad \bar{x} = \frac{\int \tilde{x}\,dL}{\int dL}$$

$$\bar{y} = \frac{\int \tilde{y}\,dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\,dm}{\int dm} \qquad \bar{y} = \frac{\int \tilde{y}\,dV}{\int dV} \qquad \bar{y} = \frac{\int \tilde{y}\,dA}{\int dA} \qquad \bar{y} = \frac{\int \tilde{y}\,dL}{\int dL}$$

$$\bar{z} = \frac{\int \tilde{z}\,dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\,dm}{\int dm} \qquad \bar{z} = \frac{\int \tilde{z}\,dV}{\int dV} \qquad \bar{z} = \frac{\int \tilde{z}\,dA}{\int dA} \qquad \bar{z} = \frac{\int \tilde{z}\,dL}{\int dL}$$

If use rectangular strip,
simplify to $dA = y\,dx$ and $\tilde{x} = x,\ \tilde{y} = y/2.$
and $dA = x\,dy$ and $\tilde{x} = x/2,\ \tilde{y} = y.$





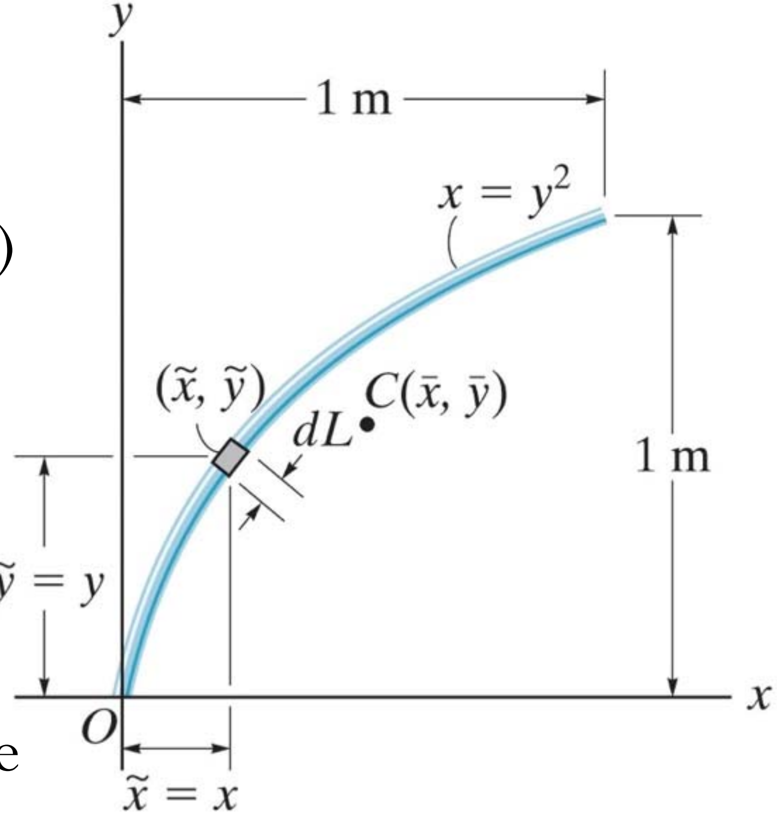
$$dL = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right) dx$$

Or

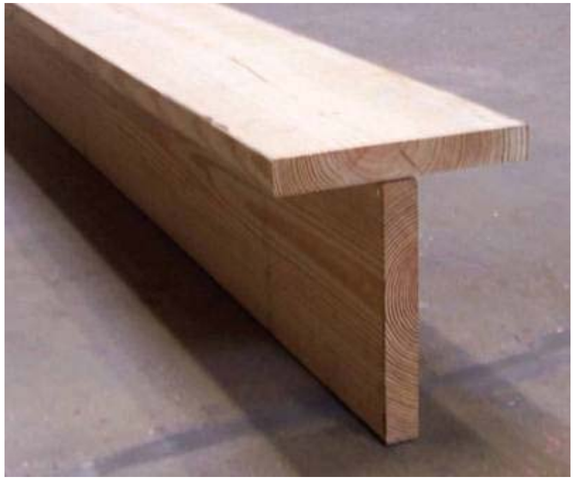
$$dL = \left(\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\right) dy$$

Recap: Centroid – Analysis Procedure

1. Select an appropriate coordinate system
2. Define the appropriate element (dL, dA, or dV)
3. Express (2) in terms of the coordinate system
4. Identify any symmetry
5. Express the moment arms (centroid) of (2)
6. Substitute (3) and (4) into the integral and solve



Composite bodies



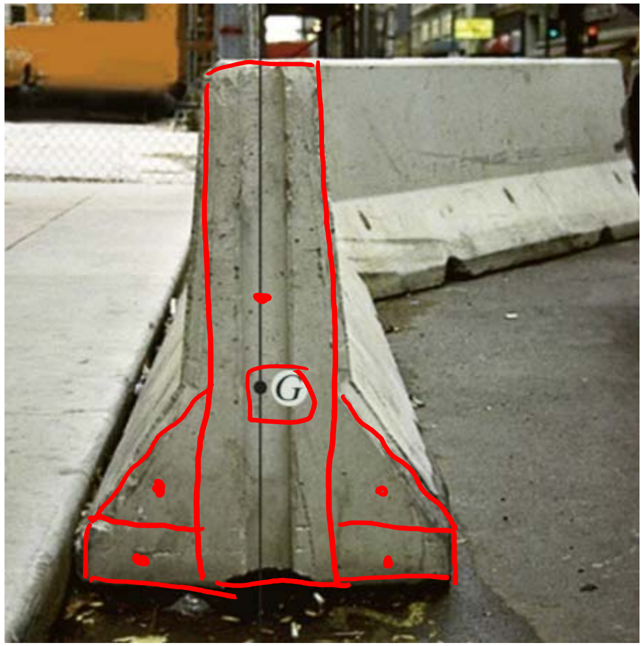
The I-beam (top) or T-beam (bottom) shown are commonly used in building various types of structures.

How can we easily determine the location of the centroid for different beam shapes?

Composite bodies

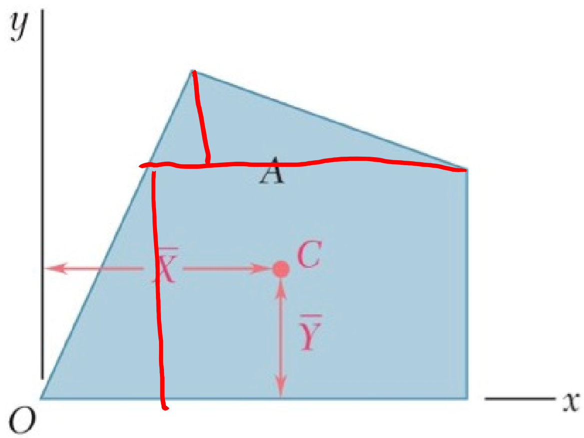
A composite body consists of a series of connected simpler shaped bodies.

Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.

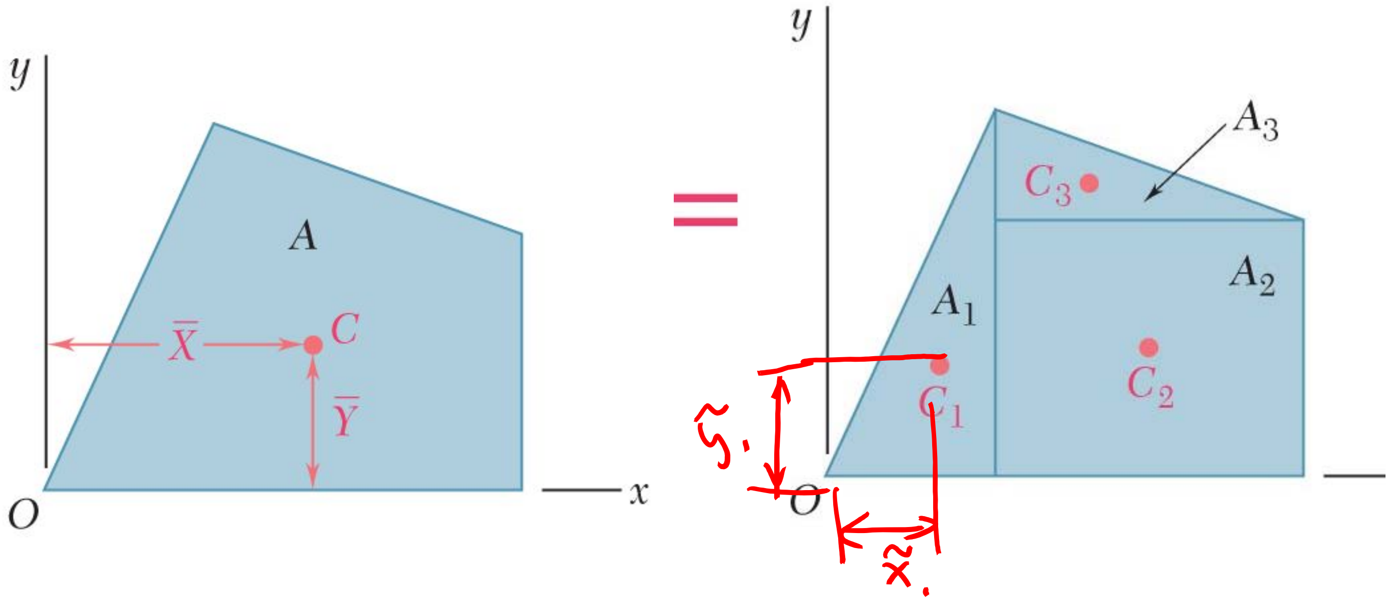


2D
Triangle
+
square
+
Semicircle

3D
Cone
+
cylinder
+
$\frac{1}{2}$ sphere



For example, the centroid of the area A is located at point C of coordinates $\bar{x}$ and $\bar{y}$. In the case of a composite area, we divide the area A into parts A_1, A_2, A_3



$$\bar{x}\ A_{total} = \sum_{i=1}^{n=3} \widetilde{x_i}\ A_i$$

$$\bar{y}\ A_{total} = \sum_{i=1}^{n} \widetilde{y_i}\ A_i$$

Where: $A_{total} = \sum_{i=1}^{n} A_i$

Therefore:

$$\bar{x} = \frac{\sum_{i=1}^{n} \widetilde{x_i}\ A_i}{\sum_{i=1}^{n} A_i}, \text{ shorthand: } \bar{x} = \frac{\sum \tilde{x} A}{\sum A}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} \widetilde{y_i}\ A_i}{\sum_{i=1}^{n} A_i} \text{ or } \bar{y} = \frac{\sum \tilde{y} A}{\sum A}$$

Composite bodies

$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W} \qquad \bar{x} = \frac{\sum \tilde{x}A}{\sum A}$$

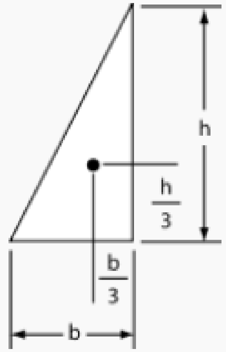
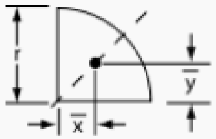
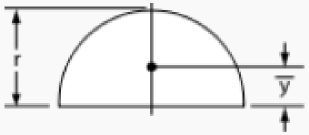
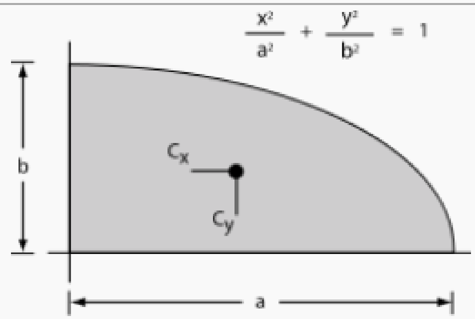
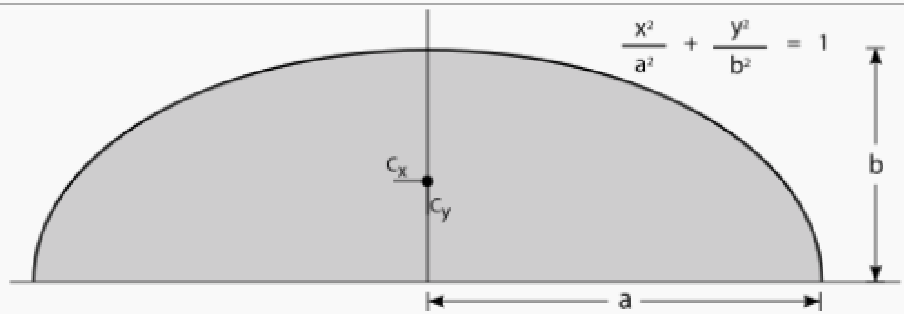
$$\bar{y} = \frac{\sum \tilde{y}W}{\sum W} \qquad \bar{y} = \frac{\sum \tilde{y}A}{\sum A}$$

$$\bar{z} = \frac{\sum \tilde{z}W}{\sum W} \qquad \bar{z} = \frac{\sum \tilde{z}A}{\sum A}$$

Similarly for mass (m), volume (V), or line (L)

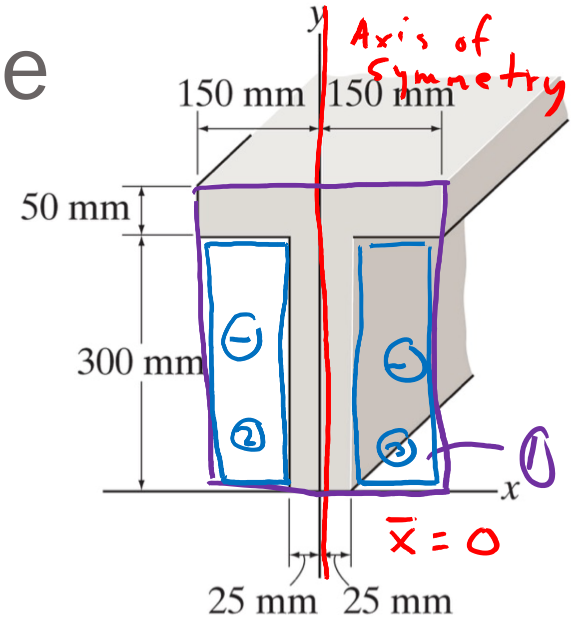
$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW}$$

Centroid of typical 2D shapes

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Right-triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

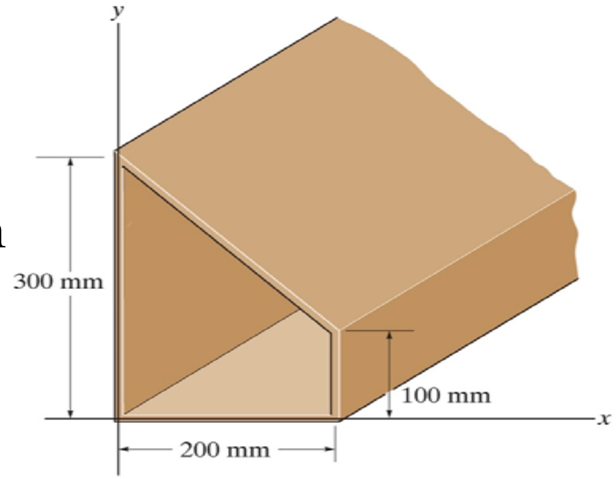
http://en.wikipedia.org/wiki/List_of_centroids

Composite bodies – Analysis Procedure



1. Identify possible axis (axes) of symmetry
2. Divide the body into finite number of simple shapes
   - Select the least number of shapes
3. Consider "holes" as "negative" parts
4. Establish coordinate axes
5. Make a table to help with bookkeeping
6. Determine total centroid location by applying equation

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} \qquad \bar{y} = \frac{\sum \tilde{y}A}{\sum A}$$

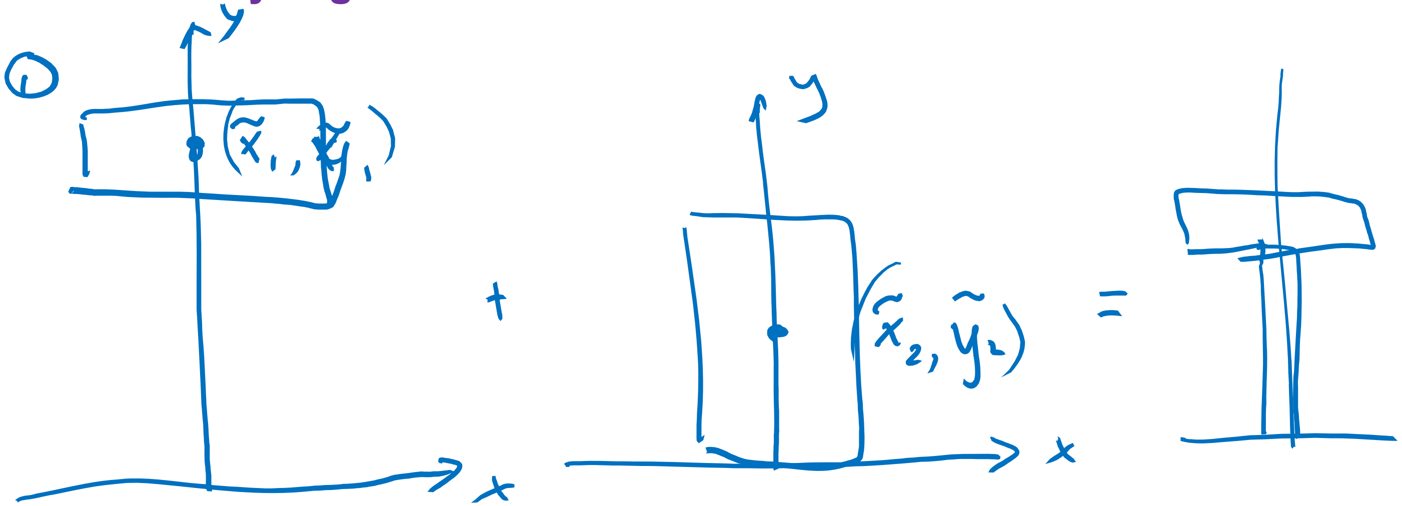


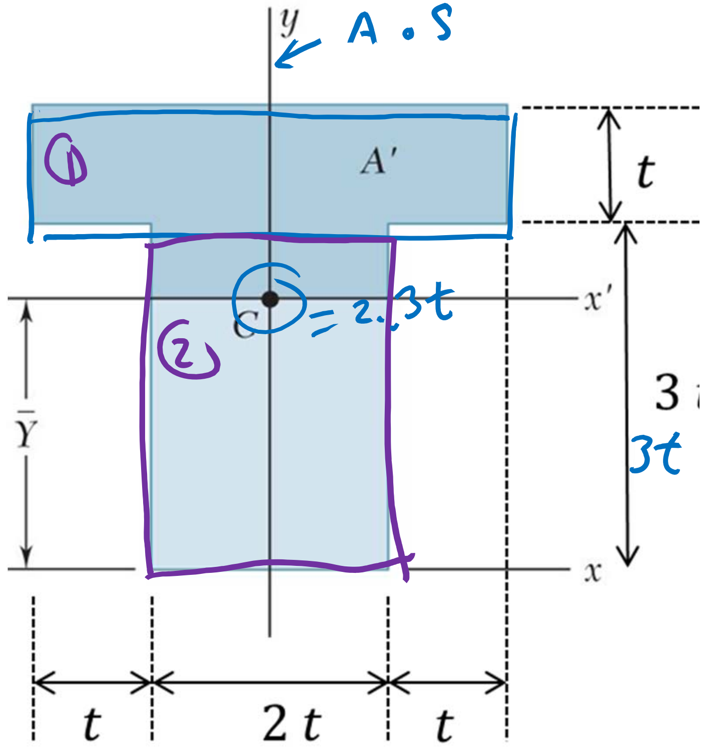
Segment #	W, m, A, V, or L (units)	Moment arm [Coord of part] (units)			Summations (units)		
		$\tilde{x}$	$\tilde{y}$	$\tilde{z}$	$\tilde{x}_i\, A_i$	$\tilde{y}_i\, A_i$	$\tilde{z}_i\, A_i$
	$\Sigma A=$				$\Sigma\tilde{x}A=$	$\Sigma\tilde{y}A=$	$\Sigma\tilde{z}A=$

Find the centroid of the area.

Any axis of symmetry? Yes, y-axis $\Rightarrow \boxed{\bar{x} = 0}$

How many segments? 2





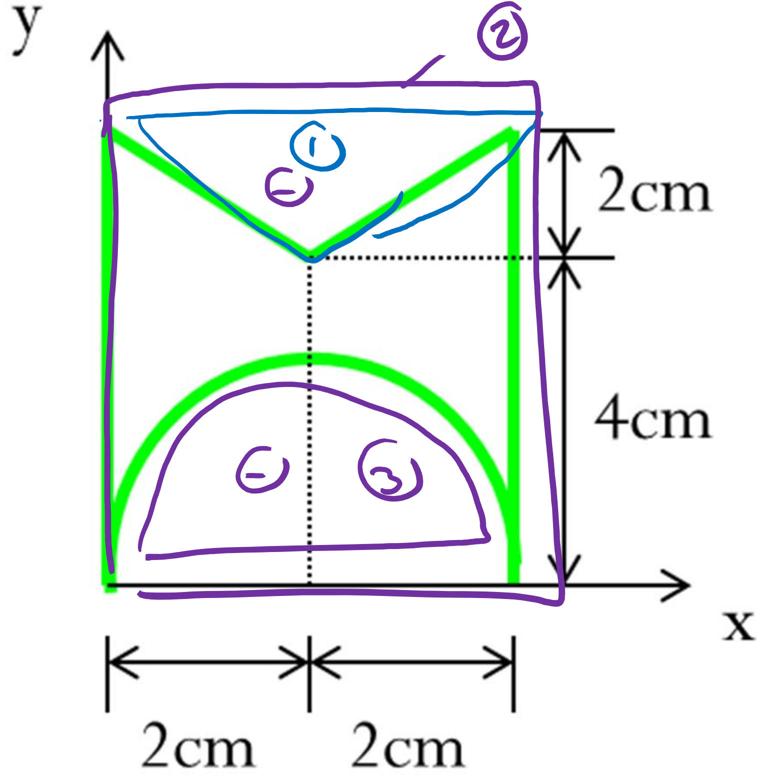
Seg #	Area	$\tilde{x}$	$\tilde{y}$	$\tilde{x}_i A_i$	$\tilde{y}_i A_i$
1	$4t^2$	0	$3.5t$	0	$14t^3$
2	$6t^2$	0	$1.5t$	0	$9t^3$
	$\Sigma A = 10t^2$			$\Sigma\tilde{x}A = 0$	$\Sigma\tilde{y}A = 23t^3$

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} = \frac{0}{10t^2} = 0 \checkmark$$

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} = \frac{23t^3}{10t^2} = 2.3t$$

A rectangular area has semicircular and triangular cuts as shown. What is the centroid of the resultant area?

Any axis of symmetry?
How many segments?



Seg #	Area (cm^2)	$\tilde{x}$	$\tilde{y}$	$\tilde{x}_i A_i$	$\tilde{y}_i A_i$
1	-4	2	$16/3$		$-64/3$
2	24	2	3		72
3	-2π	2	$\frac{8}{3\pi}$		$-16/3$
	$\Sigma A=$ 13.7	2		$\Sigma\tilde{x}A=$	$\Sigma\tilde{y}A=$ 45.3

	A	$\tilde{y}$	$\tilde{y}A$
3a	16	1	
3b	$-\frac{\pi a b}{2}$	$\frac{8}{3\pi}$	

$-\frac{\pi \cdot 2 \cdot 2}{2} = -2\pi$

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A}$$

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} = \frac{45.3}{13.7} = \boxed{3.3\text{ cm} = \bar{y}}$$

Locate the centroid of the **cross section area**.

Any axis of symmetry? No

How many segments? 4

Use Lines

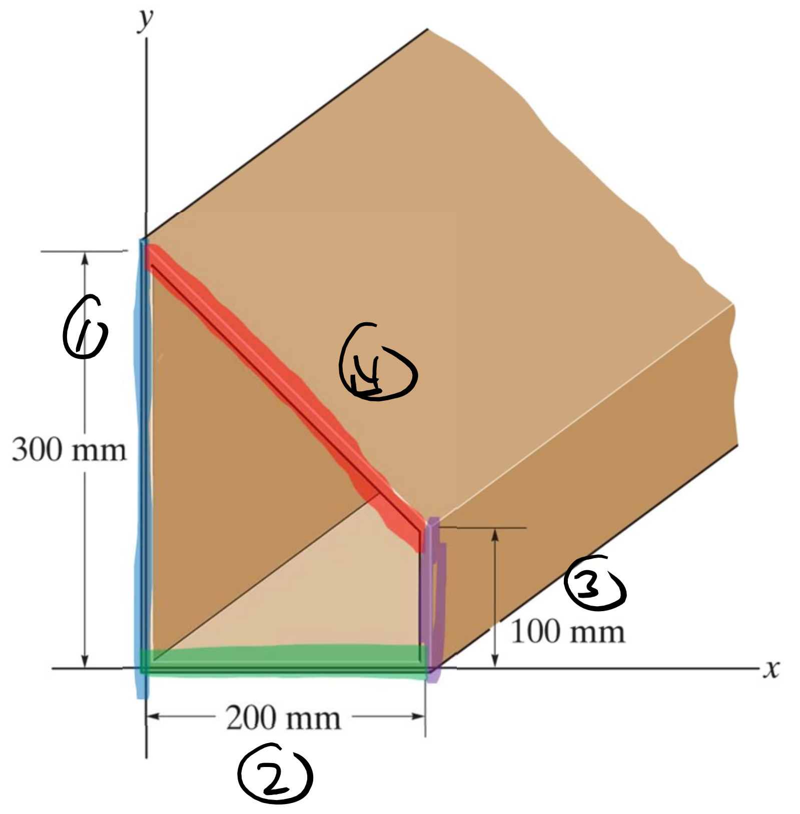
	L (mm)	$\tilde{x}$	$\tilde{y}$	$\tilde{x}L$	$\tilde{y}L$
1	300	0	150	0	45k
2	200	100	0	20k	0
3	100	200	50	20k	5k
4	$200\sqrt{2}$	100	200	$20\sqrt{2}$k	$40\sqrt{2}$k

$\Sigma L = 883$

68

107

$\bar{x} = 77$ $\bar{y} = 121$

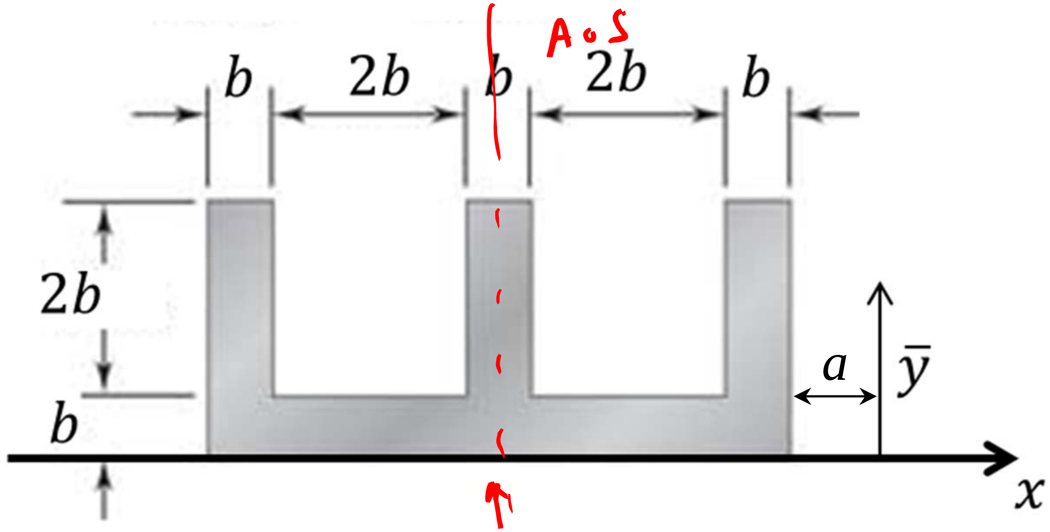


Find the centroid of the area.

Any axis of symmetry?
How many segments?

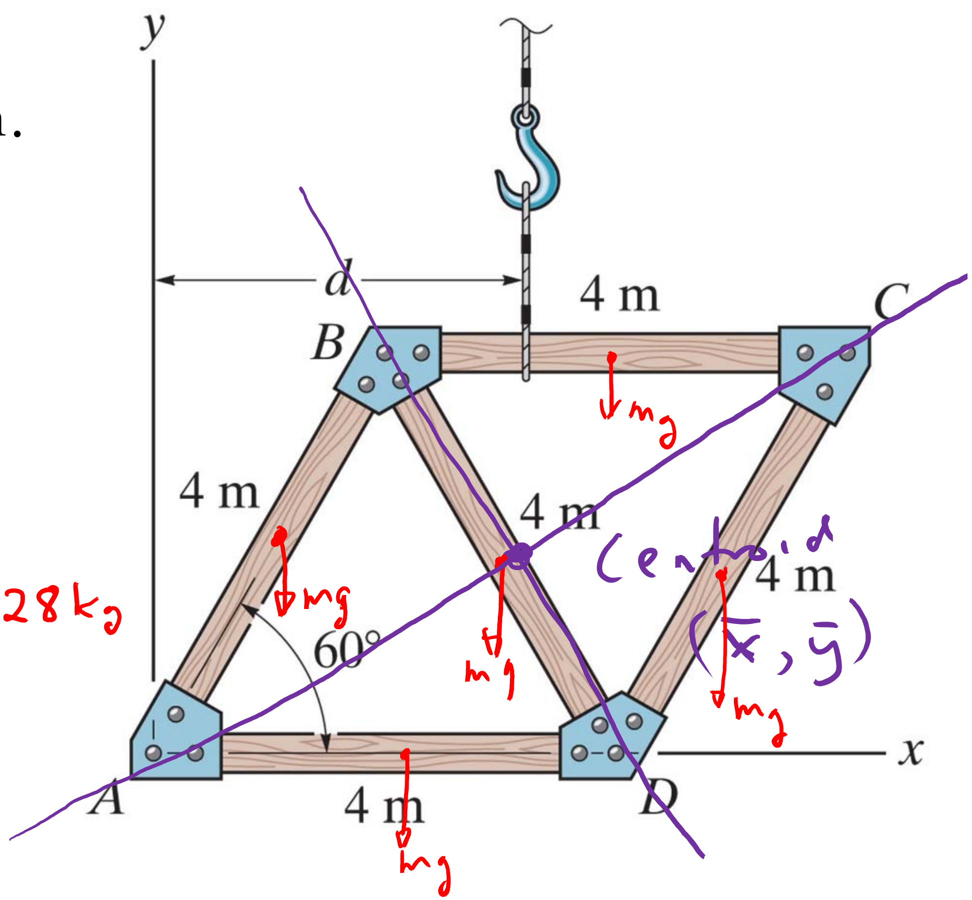
$\bar{x} = \frac{7}{2} b$

$\bar{y} = 1.2b$



$\bar{x} = -\left(a + b + 2b + \frac{b}{2}\right)$

A truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. Determine the distance d to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.



Any axis of symmetry?
How many segments? 5 lines

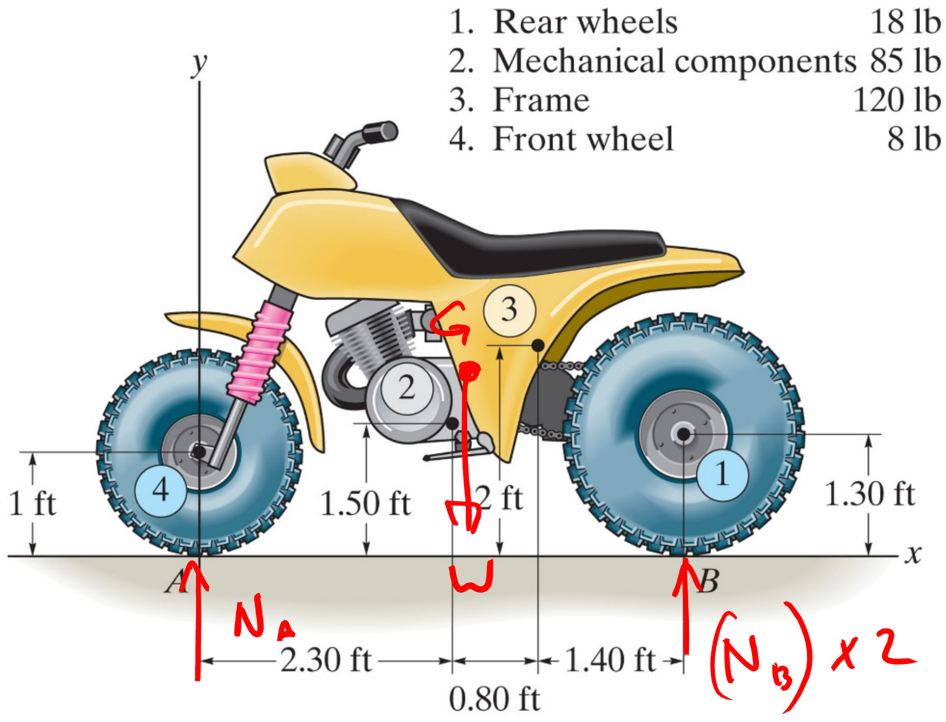
Each segment has mass $m = (7\,\frac{kg}{m})(4\,m) = 28\,kg$

So use $\bar{x} = \dfrac{\sum \tilde{x}_i m_i}{\sum m_i}$

For this problem, since distance "d" is the only needed value, then only $\bar{x}$ is needed.

As an aside that I mentioned in class, this structure has two axes of symmetry: one between AC, another between BD; like the supports for a diamond-shaped kite. Therefore, if one can quickly identify two axes of symmetry, the intersection is the centroid.
http://mathdemos.org/mathdemos/centroids/centroid.html

Determine the location of the center of gravity of the three-wheeler. If the three-wheeler is symmetrical with respect to the x-y plane, determine the normal reaction each of its wheels exerts on the ground.



Any axis of symmetry?
How many segments?

FIND $\bar{x}$, $\bar{y}$ ⇒ G

FBD: to solve for N_A, N_B

To solve this problem,

1. Determine the location of the center of gravity (CoG) of the vehicle.
2. Draw the FBD of the vehicle in the x-y plane.
3. To solve for the unknown normal reaction forces at the points of contact of the wheels, use the equations of equilibrium. Note that the total weight (W) would be located at the CoG found in step 1, and that there are two normal forces at B due to two back wheels of the "three-wheeler".