Statics - TAM 211

Lecture 28

November 30, 2018

Chap 9.5

Announcements

- Upcoming deadlines:
- Friday (11/30)
  - Written Assignment 10
- **Friday (11/30) all in Teaching Building A418-420**
  - 8:00 am: Quiz 5, On paper. Chapter 7+8 (Internal forces, Friction)
  - 9:00 am: Lecture 28 (Fluid Pressure)
  - 10:00 am: Discussion section for ALL students
- Tuesday (12/4)
  - Prairie Learn HW 11
- **Reminder: Discussion Section**
  - **12% of final grade**
  - **Attendance + Participation**
  - **No grade given for discussion section if > 5 minutes late**

The Submarine Sports Car
Price $2,000,000

- **NO CLASS ON MONDAY DEC 3**

https://www.hammacher.com/Product/12531

Recap: Calculation of composite area, we divide the area A into parts A_1, A_2, A_3

Centroid

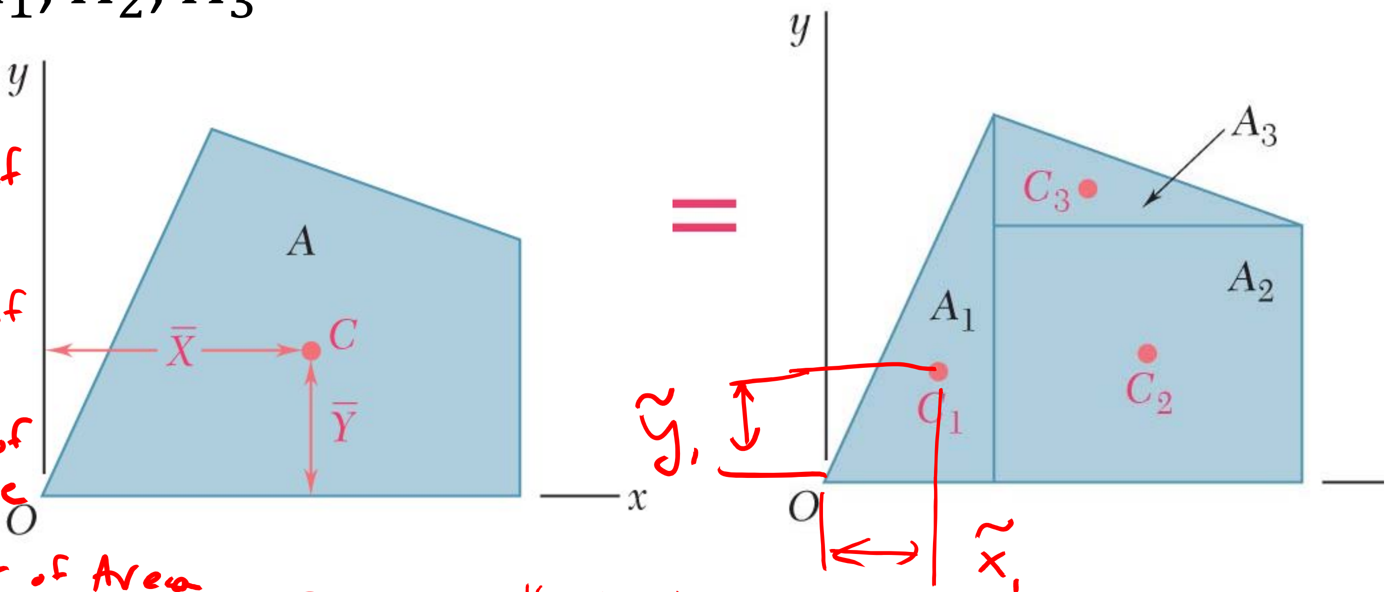
Center of Gravity

Center of Mass

3D Center of Volume

2D Center of Area

1D Center of Line



$\tilde{x}$ "tilda"

$\bar{x}$ "bar"

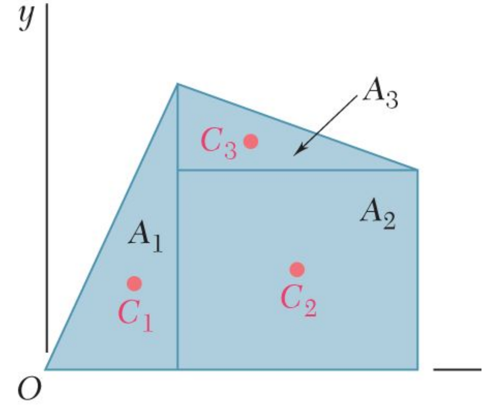
$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} \qquad \bar{x} = \frac{\sum \tilde{x}W}{\sum W}$$

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} \qquad \bar{y} = \frac{\sum \tilde{y}W}{\sum W}$$

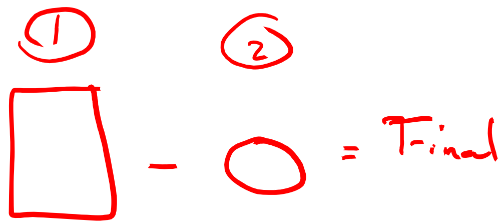
$$\bar{z} = \frac{\sum \tilde{z}A}{\sum A} \qquad \bar{z} = \frac{\sum \tilde{z}W}{\sum W}$$

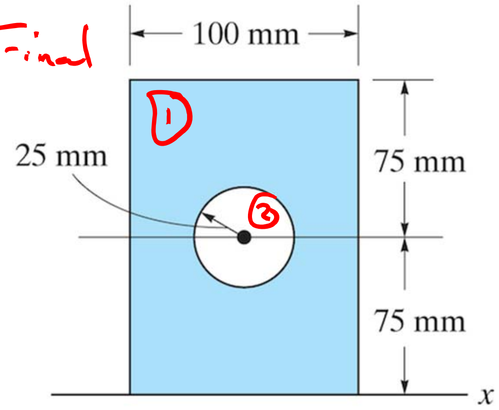
Similarly for mass (m), volume (V), or line (L)

Recap: Composite bodies – Analysis Procedure



1. Identify possible axis (axes) of symmetry
2. Divide the body into finite number of simple shapes
   - Select the least number of shapes *
3. Consider "holes" as "negative" parts
4. Establish coordinate axes
5. Make a table to help with bookkeeping
6. Determine total centroid location by applying equations





$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} \qquad \bar{y} = \frac{\sum \tilde{y}A}{\sum A}$$

Segment #	W, m, A, V, or L (units)	Moment arm [Coord of part] (units)			Summations (units)		
		$\tilde{x}$	$\tilde{y}$	$\tilde{z}$	$\tilde{x}_i A_i$	$\tilde{y}_i A_i$	$\tilde{z}_i A_i$
1							
2							
3	$\Sigma A=$				$\Sigma\tilde{x}A=$	$\Sigma\tilde{y}A=$	$\Sigma\tilde{z}A=$

Chapter 9 Part II – Fluid Pressure

Chap 9.5

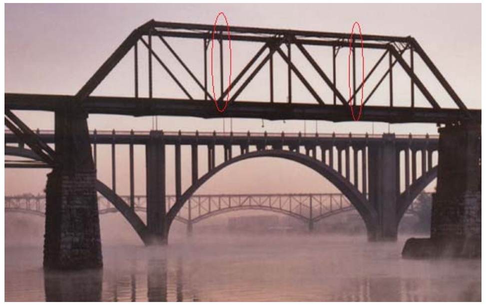
Goal and objective

- Present a method for finding the resultant force of a pressure loading caused by a fluid

Mechanics is a branch of the physical sciences that is concerned with the **state of rest or motion of bodies that are subjected to the action of forces**

SOLIDS

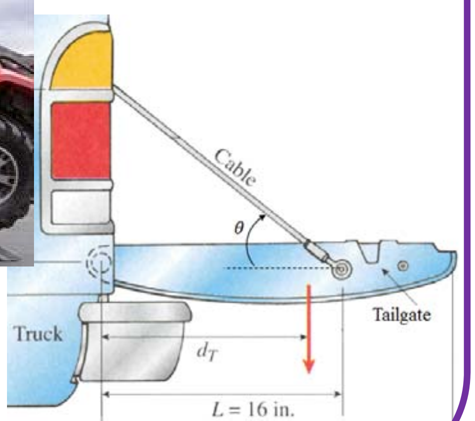
Rigid Bodies



TAM 210/211: Statics

TAM212: Dynamics

Deformable Bodies



TAM 251: Solid Mechanics

FLUIDS

What Makes a Fluid or Solid?

Honey

Rock

What is Sand?

Pressure Force

$$P = \frac{F}{A}$$

Ocean / Top

Air mountain

$P_{Top\,m}$

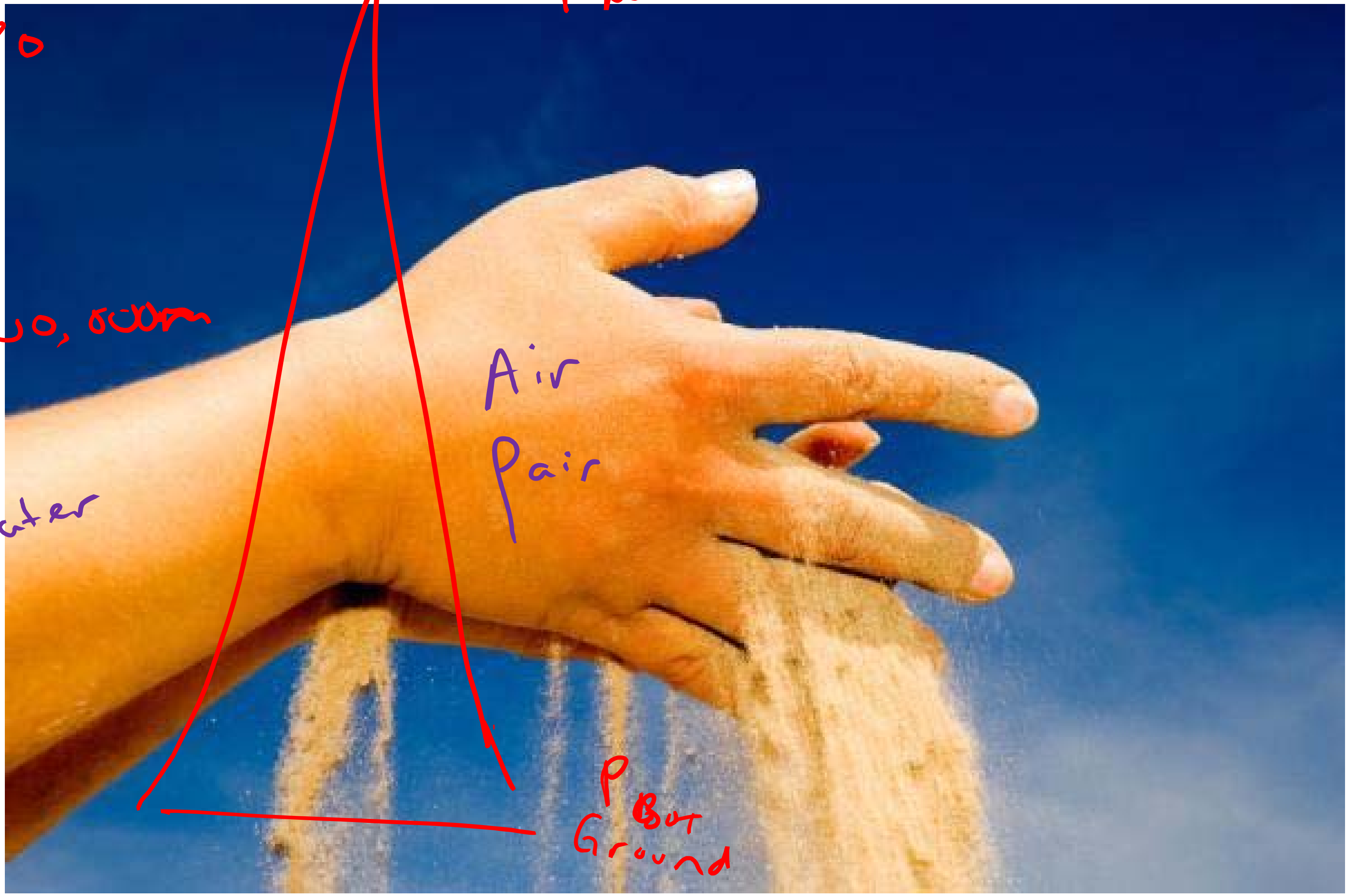
$P_{Top\,o}$

100,000,000m

Water

ρ_{water}

P_{Bot}



h - altitude
height
depth

ρ = density

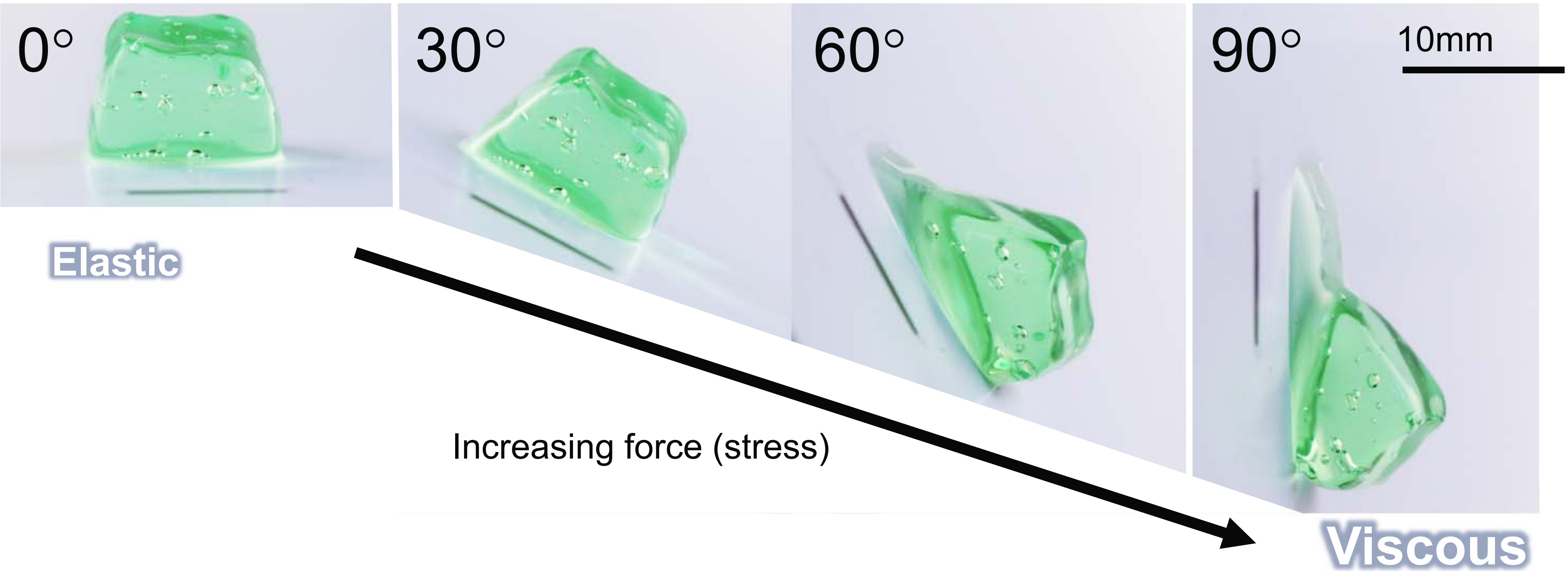
$\rho_{water} > \rho_{air}$

$$P_{Top\,m} < P_{Bot\,m} = P_{Top\,ocean} < P_{Bot\,ocean}$$

Particles swollen with water – ‘Squishy Baff’

disposable diaper

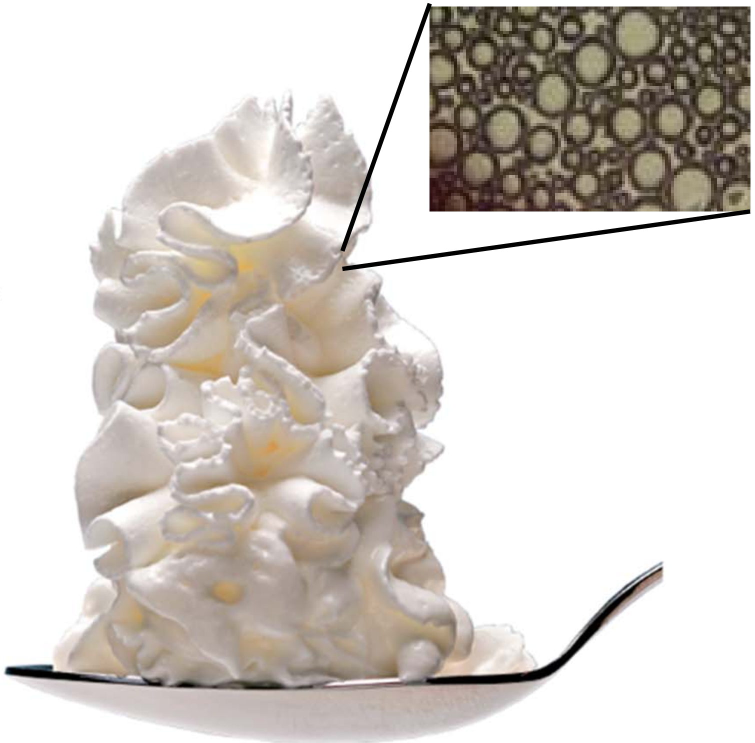
Aloe Gel



They act like a solid…

But they flow like a fluid once enough stress is applied.

Whipping cream (liquid) + air (gas) = Foam (solid)



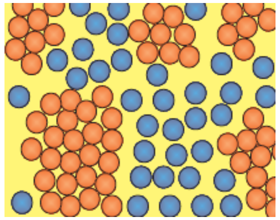
with compressed air

mechanical beating

They look like a fluid...

Video

cornstarch (small, hard particles) + water =



But they may bear static loads like solids

Summary

Water takes shape of its container. Rock does not.

Water and rock fit classical definitions of fluid and solid, respectively

Sand and Squishy Baff take the shape of containers, but are composed of solid particles

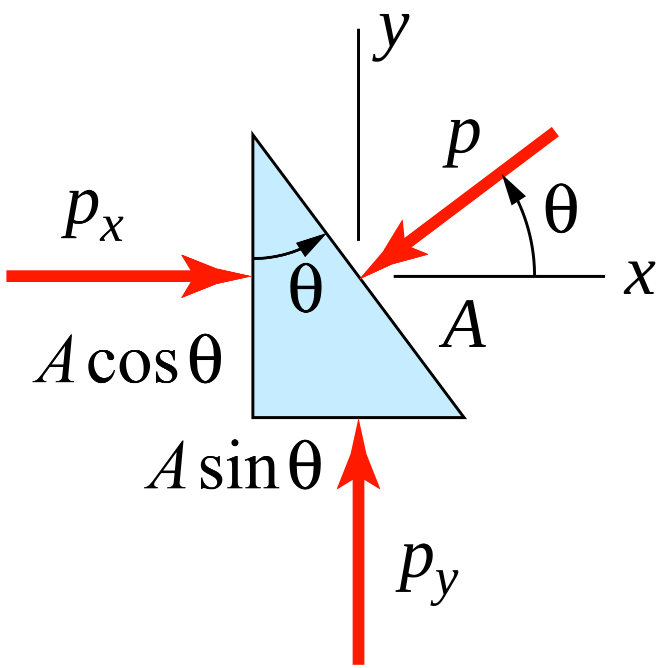
Sand and Squishy Baff are granular materials, which have properties of both fluids and solid

The aloe gel holds its shape and can trap air bubbles, until a certain amount of stress is applied.

Aloe gel is a suspension of particles, which is able to bear static load like a solid but behaves like a fluid when "enough" stress is applied.

Fluids

Pascal's law: A fluid at rest creates a pressure p at a point that is the ***same*** in ***all*** directions. Recall: $p = F/A$, or $F = pA$



For equilibrium of **an infinitesimal element**,

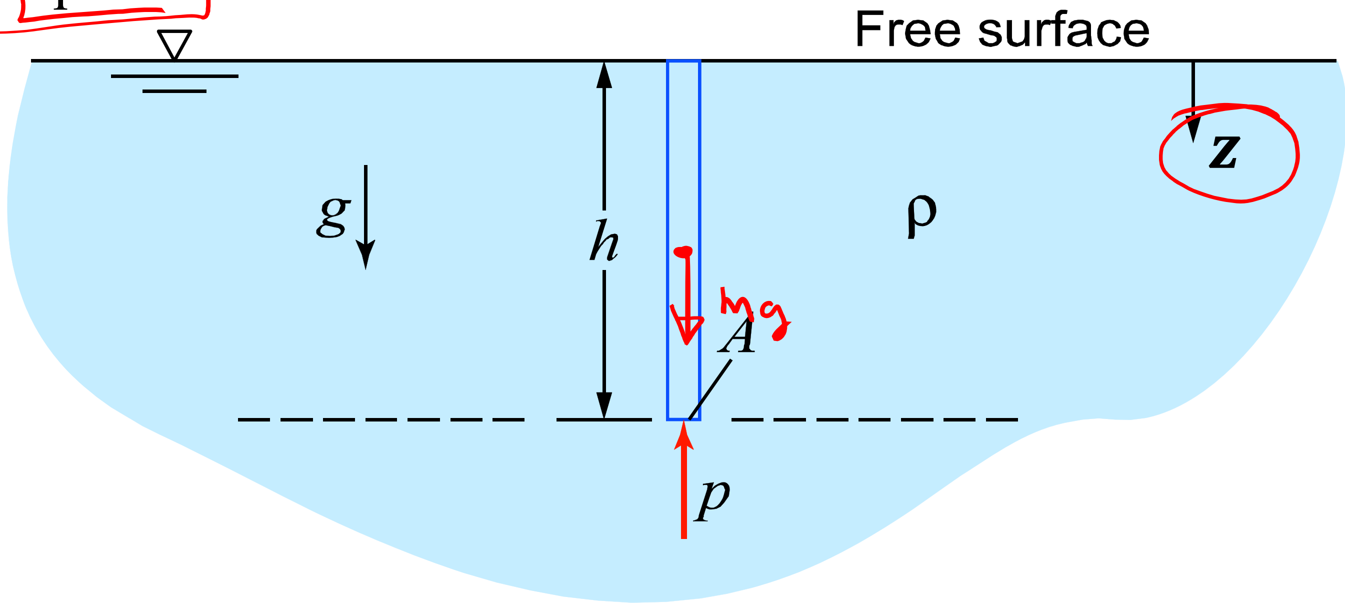
$$\Sigma F_x = 0: \quad p_x(A\cos\theta) - pA\cos\theta = 0 \quad \Rightarrow \quad p_x = p,$$
$$\Sigma F_y = 0: \quad p_y(A\sin\theta) - pA\sin\theta = 0 \quad \Rightarrow \quad p_y = p.$$

Thus, $p_x = p_y = p$ for any angle θ. The Pascal's law holds for fluids, but not solids.

Incompressible: An incompressible fluid is one for which the mass density ρ is independent of the pressure p. Liquids are generally considered incompressible. Gases are compressible, but may be approximated as incompressible if the pressure variations are relatively small.

Fluid Pressure

For an incompressible fluid at rest with mass density ρ, the pressure varies linearly with depth z



Summing forces in the vertical direction gives

density * volume = mass

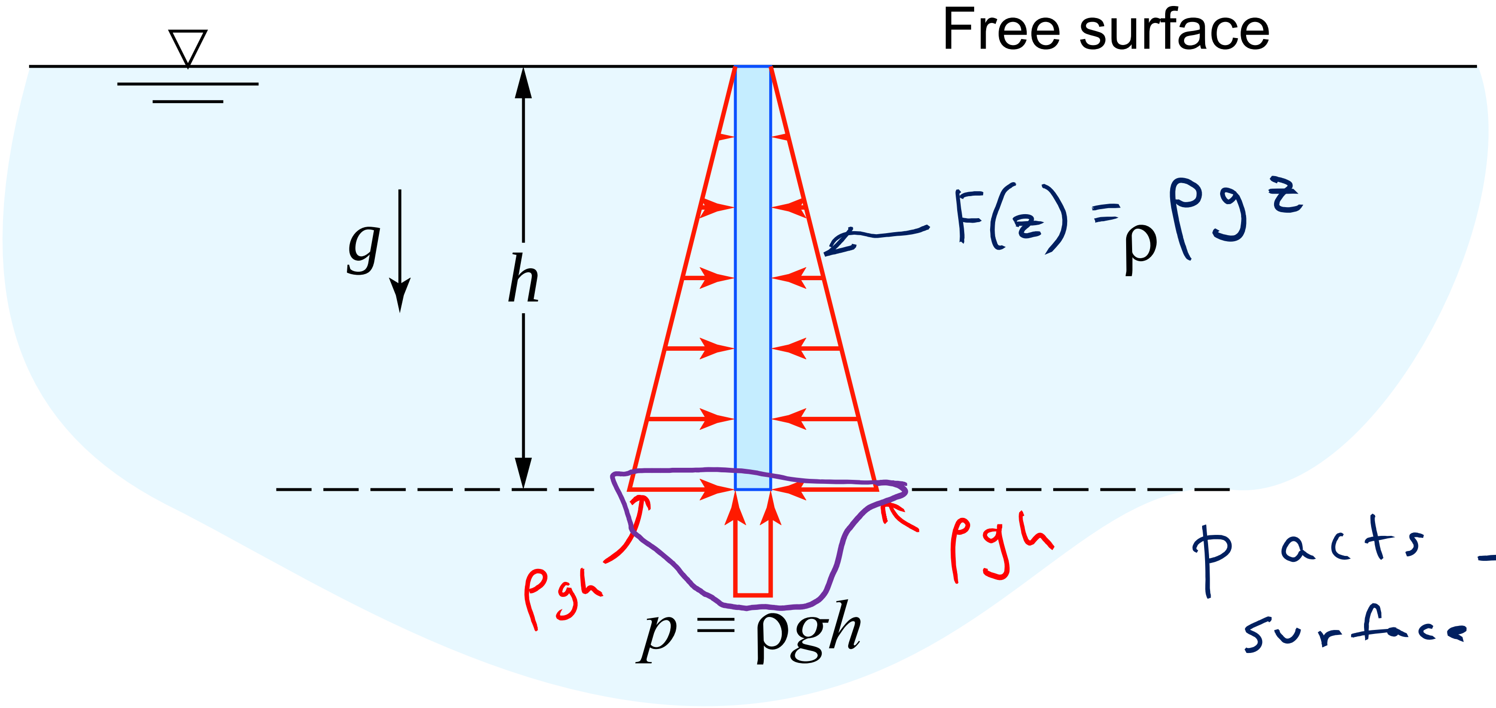
$$\Sigma F_x = 0: \quad mg - pA = 0 \quad \Rightarrow \quad (\rho(Ah))g - pA = 0 \quad \text{or} \quad p = \rho g h\,.$$

In general, this result is written as $p = \rho\, g\, z = \gamma\, z$

where $\gamma = \rho\, g$ is called the specific weight (weight per unit volume).

For fresh water: γ =62.4 lb/ft^3 (9810 N/m^3)

Observe that the pressure varies *linearly* from the free surface, and is *constant* along any horizontal plane (since h is constant):



p acts ⊥ to surface of object

or

$F_{pressure}$

The tank is filled with water to a depth of $d = 4$ m. Determine the resultant force the water exerts on side A of the tank. ($\rho = 1000\ \text{kg/m}^3$)

FBD of face A

$w(0) = 0$

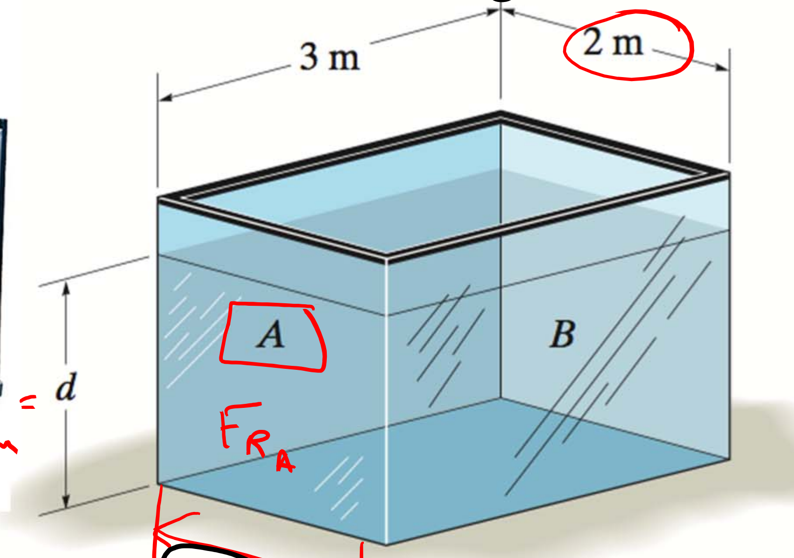
$p(z) = \rho g z = \frac{\text{Force}}{\text{Area}}$

d_R, F_R, z

$w(z) = \rho g z \cdot a$

$w(d) = ? = (\rho g d)\, a$

Force $= pA$



- Distributed load due to fluid pressure = (pressure) * (surface width)

$F_R = \frac{1}{2}(d) \cdot w(d) = \frac{1}{2} d\, (\rho g d a)$

$\Rightarrow$ $F_R = 157$ kN

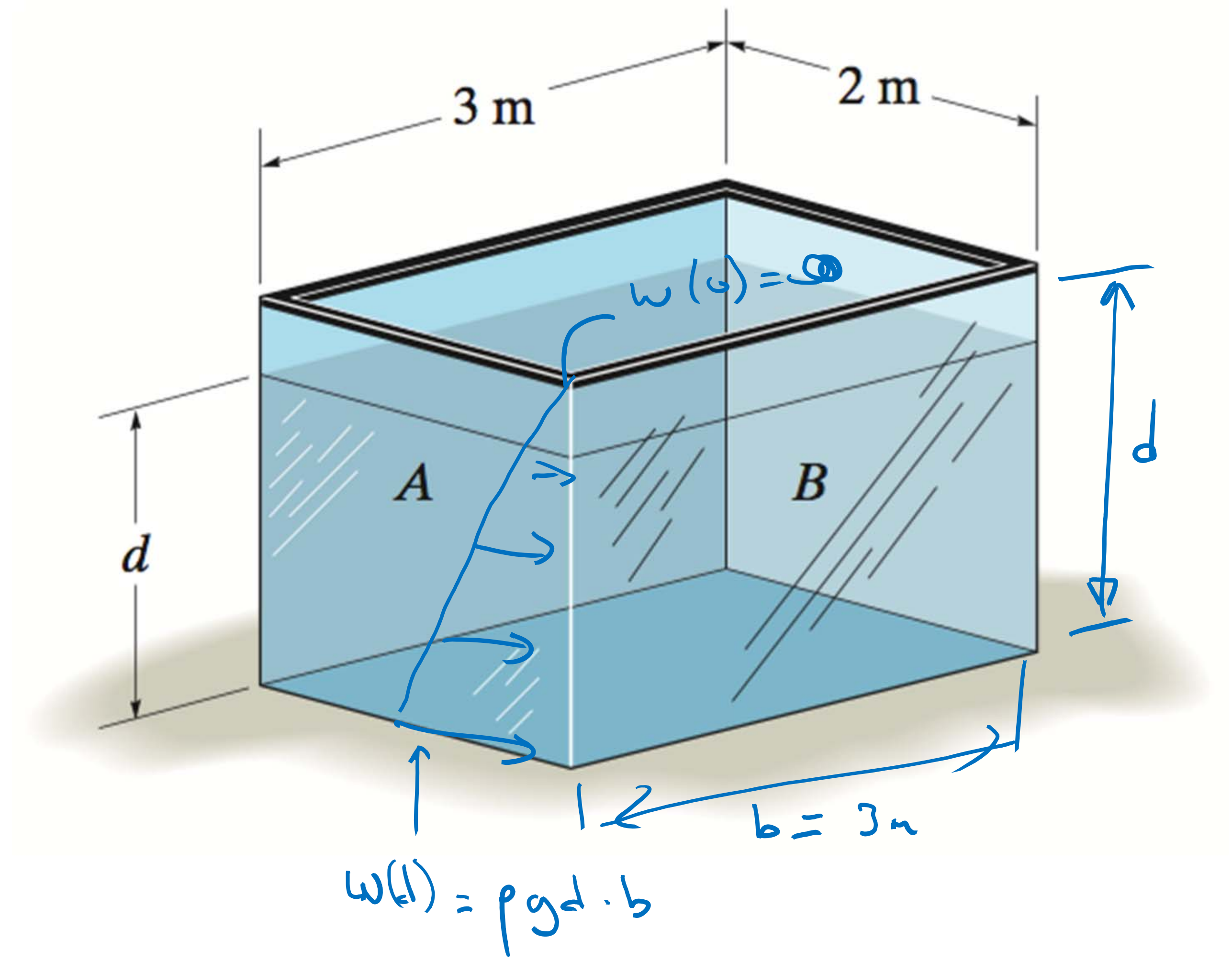
$d_R = ? = \frac{2}{3} d$

The tank is filled with water to a depth of $d = 4$ m. Determine the resultant force the water exerts on side B of the tank. ($\rho_{water} = 1000\ \mathrm{kg/m^3}$)

$F_R = \frac{1}{2} \cdot d \cdot w(d)$

$= \frac{1}{2} d^2 \rho g b$

$F_{R_B} = 235\ \mathrm{kN}$



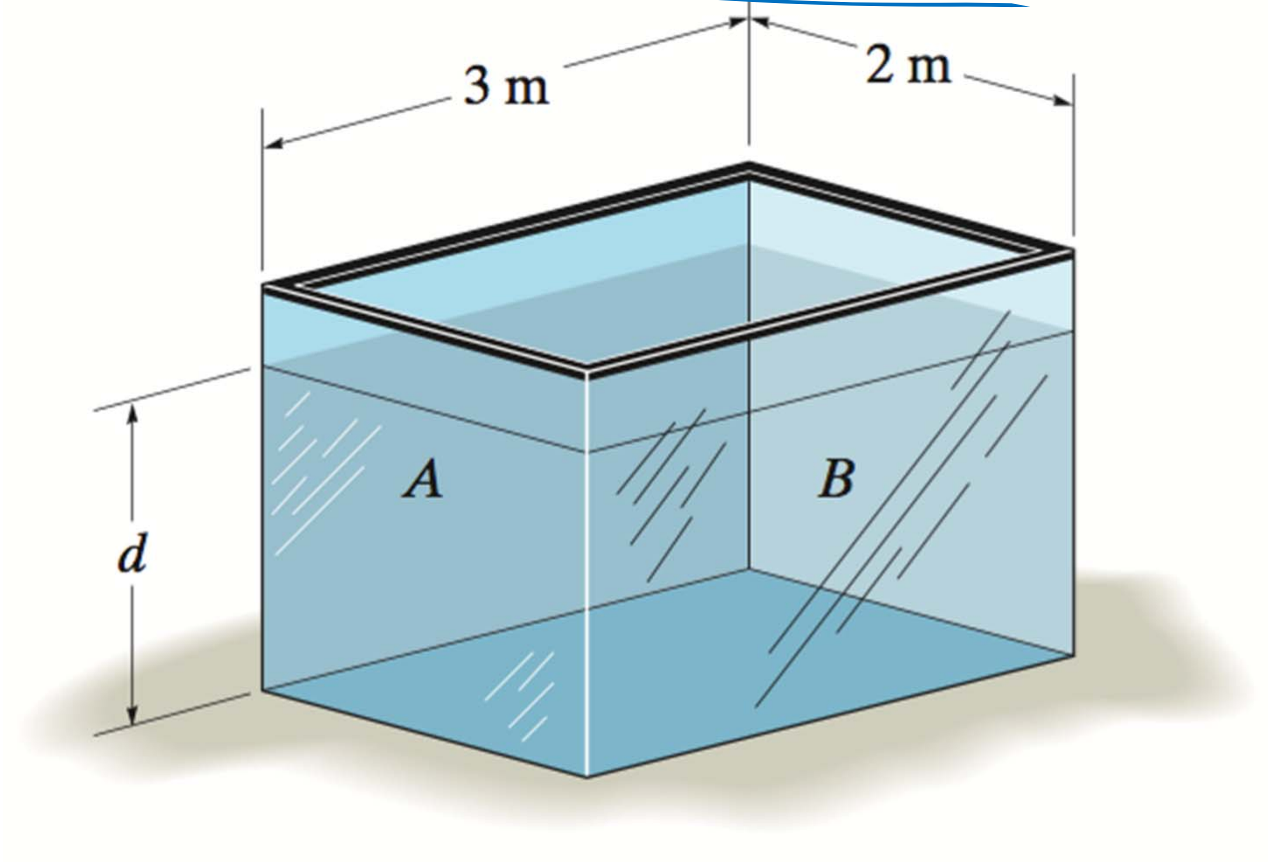
If the tank is filled **with oil instead**, what depth d should it reach so that it creates the same resultant forces on side A. ($\rho_{oil} = 900\ \text{kg/m}^3$)

F_{R_A}

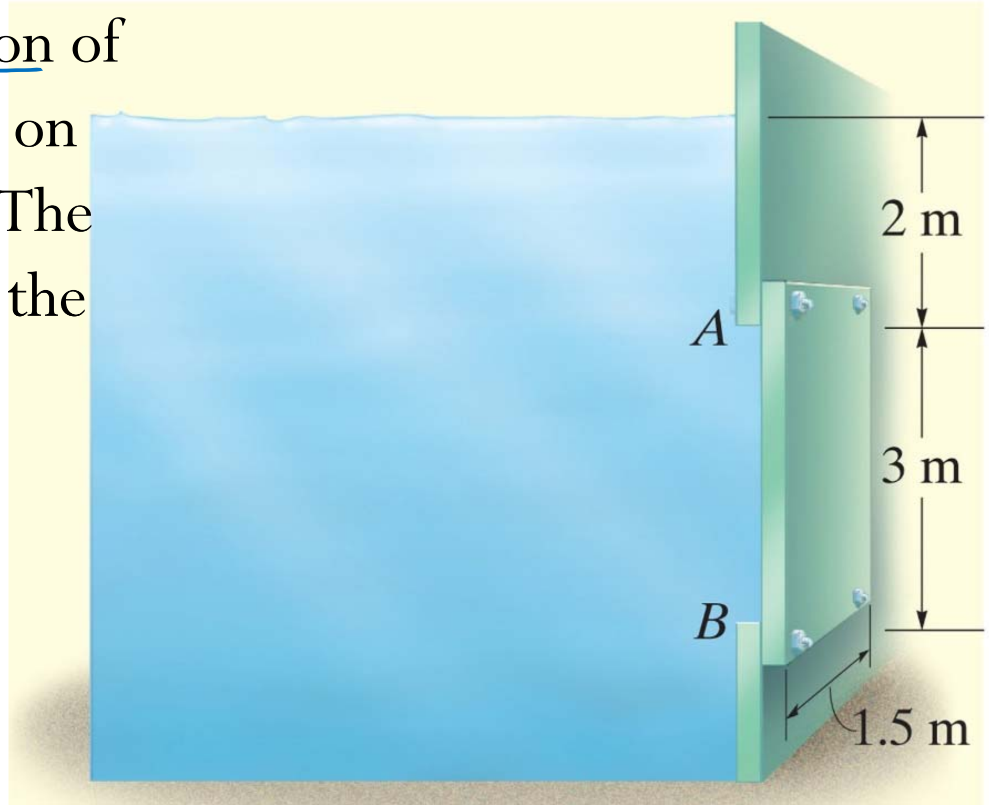
$$\frac{1}{2} d_{R_w}^2 \rho_w g \cancel{a} = \frac{1}{2} d_{R_o}^2 \rho_o g \cancel{a}$$

$$d_{R_o} = \sqrt{\frac{\rho_w}{\rho_o}}\, d_w$$

$$= 1.05\, d_w$$



Determine the magnitude and location of the resultant hydrostatic force acting on the submerged rectangular plate AB. The plate has width 1.5m. The density of the water is 1000 kg/m^3



hydro
water

static