Statics - TAM 211

Lecture 30

December 7, 2018

Chap 9.5

Announcements

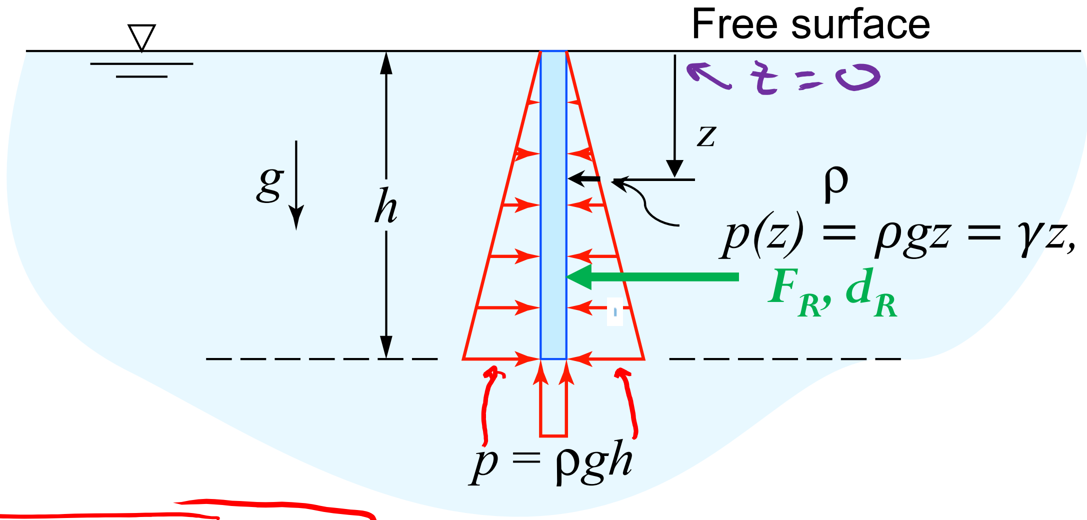
- Check ALL of your grades on Blackboard! Report issues
- Prof. H-W office hours
  - Monday 3-5pm (Room C315 ZJUI Building)
  - Wednesday 7-8pm (Residential College Lobby)
- Upcoming deadlines:
  - Friday (12/7)
    - Written Assignment 11
  - Tuesday (12/11)
    - HW 12
  - Quiz 6
    - Week of Dec 10
    - CoG thru Fluid Pressure: Lectures 26-30 (Chap 9 material)

Chapter 9 Part II – Fluid Pressure

Chap 9.5

Recap: Fluid Pressure

For an incompressible fluid at rest with mass density ρ, the pressure varies linearly with depth z



Object width is b.

$p(z) = \rho g z = \gamma z, \quad w(z) = \rho g z b = \gamma z b$

$$Pressure = \left[\frac{force}{area}\right]$$

- $\boxed{p(z) = \rho\, g\, z = \gamma\, z}$

  where $\gamma = \rho\, g$ is called the specific weight (weight per unit volume).

  For fresh water: γ =62.4 lb/ft^3 (9810 N/m^3), ρ = 1000kg/m^3

  $W_f = \gamma \cdot (\text{Vol})$

- Pressure $p(z)$ or force due to pressure F_R are always perpendicular to the object's surface.

- Distributed load per length due to fluid pressure at depth z is due to pressure and uniform width (b) of object's surface: $\boxed{w(z) = p(z) \cdot b} = \rho g z b = \gamma z b \quad \left[\frac{force}{length}\right]$

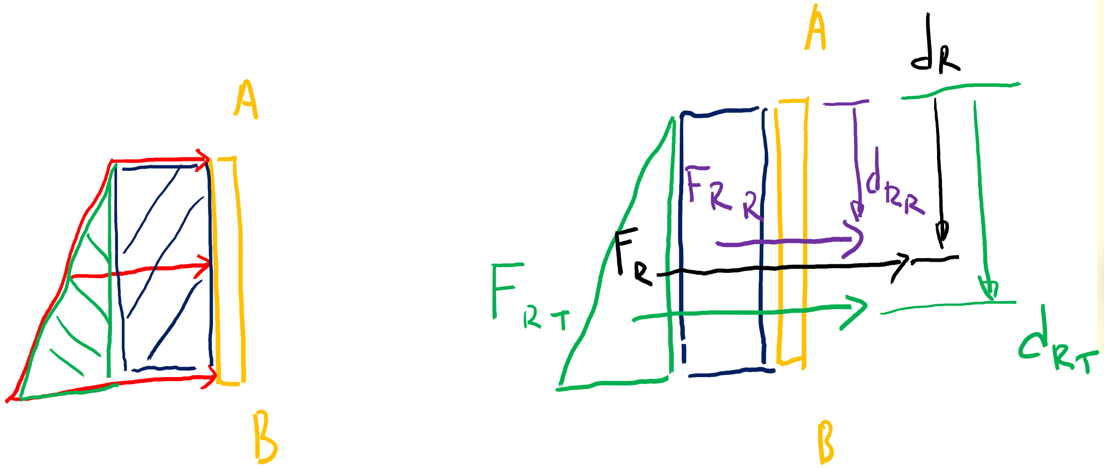
- Determine resultant force (magnitude and location): F_R, d_R
  - If water, this force is called hydrostatic force

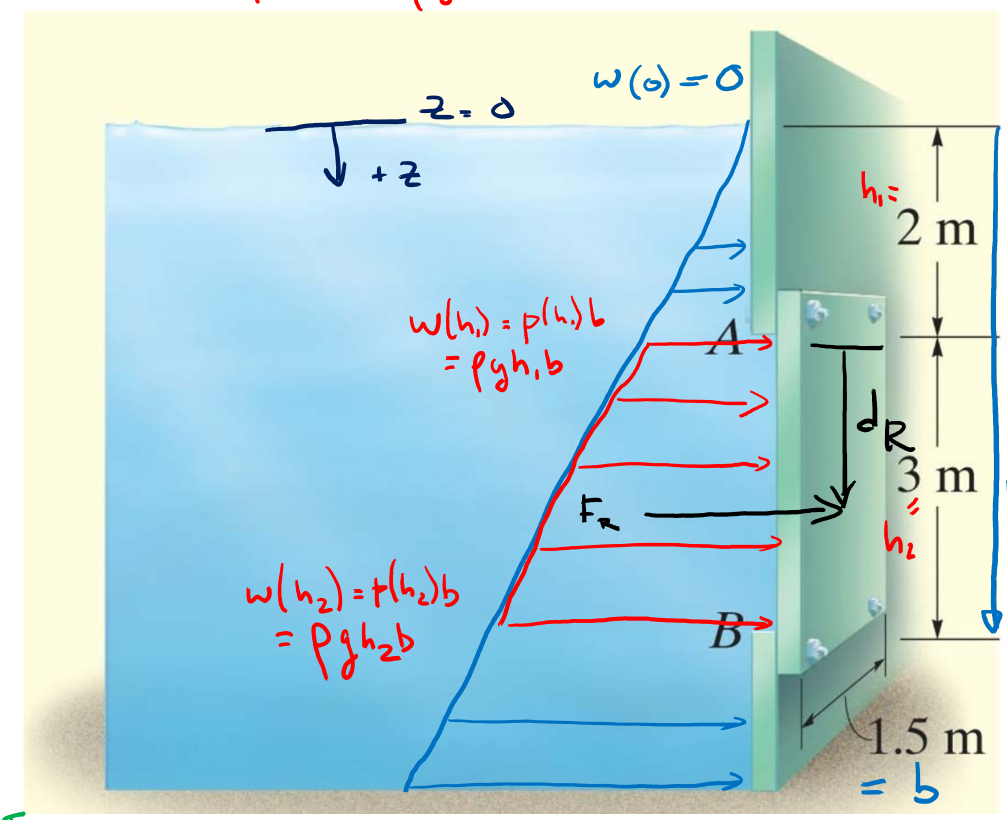
Recap:

Determine the magnitude and location of the resultant hydrostatic force acting on the submerged rectangular plate *AB*. The plate has width 1.5m. The density of the water is 1000 kg/m^3 $= \rho$

$p(z) = \rho g z$

$w(z) = p(z) \cdot b = \rho g z b$

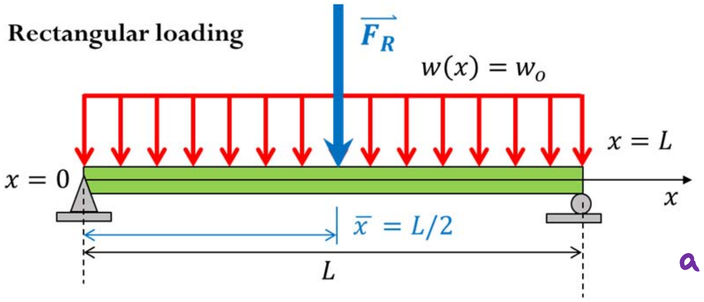




$F_R = F_{RT} + F_{RR} = 154.5\text{N}$

$d_R = \dfrac{d_{RT} F_{RT} + d_{RR} F_{RR}}{F_R} = 1.71 \text{ m below A}$

Simple Shape Centroid Locations

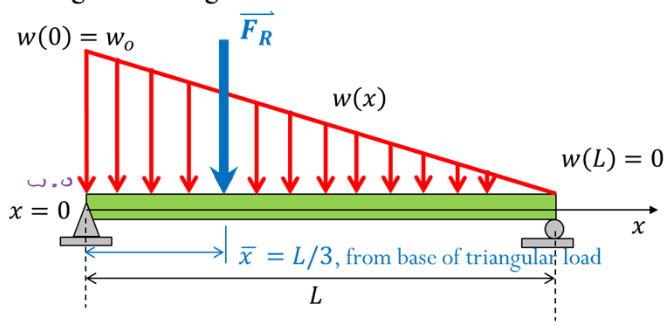


$$w(x) = w_o$$

$$\left|\vec{F_R}\right| = F_R = w_o L$$

$$\bar{x} = \frac{L}{2}$$

a = b

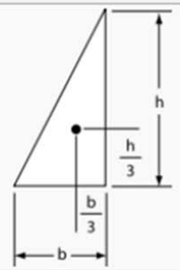
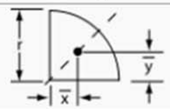
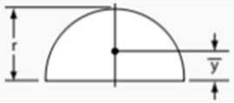
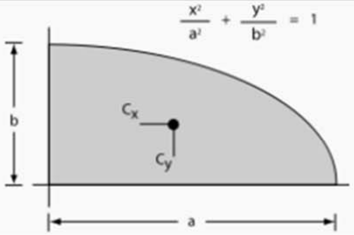
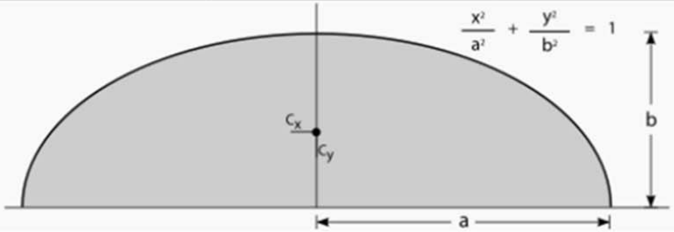


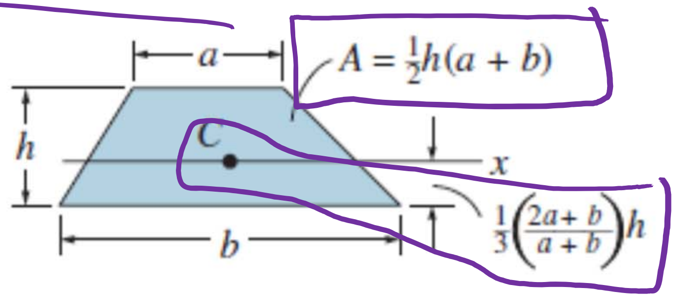
$$w(x) = w_o - \frac{w_o x}{L}$$

$$F_R = w_o \frac{L}{2}$$

$$\bar{x} = \frac{L}{3}$$

a = 0

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Right-triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

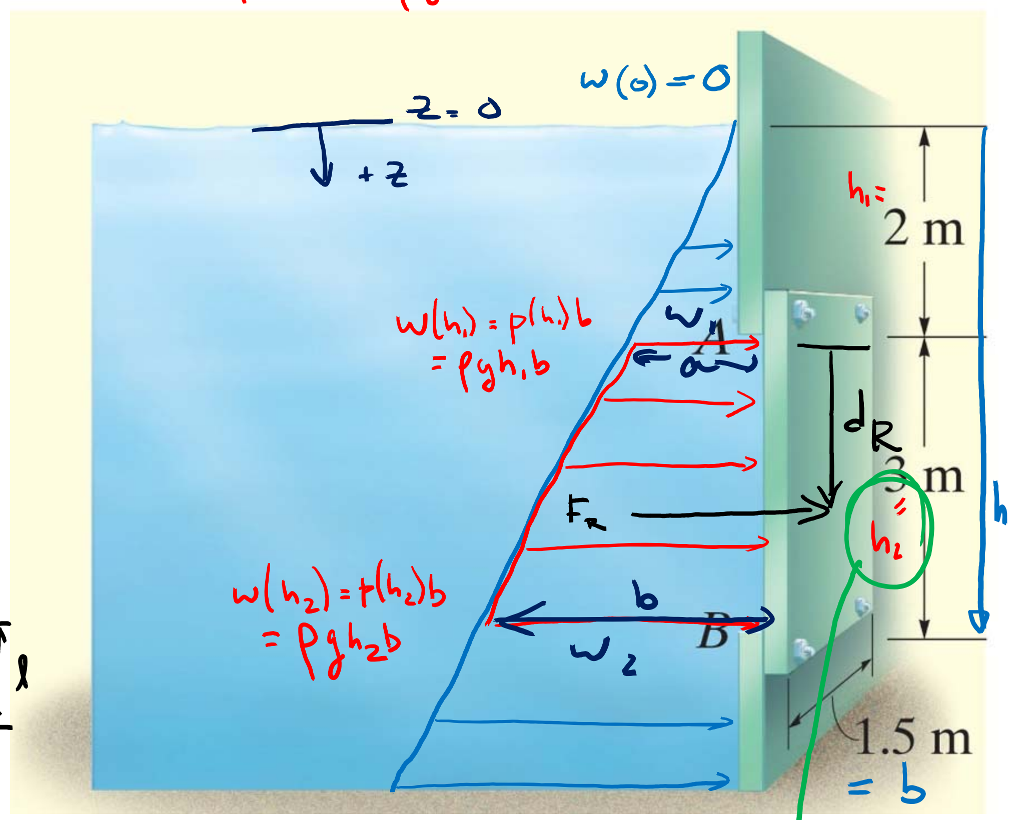


Trapezoidal area

Determine the magnitude and location of the resultant hydrostatic force acting on the submerged rectangular plate *AB*. The plate has width 1.5m. The density of the water is 1000 kg/m^3

$p(z) = \rho g z$

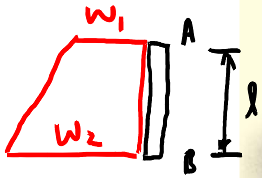
$w(z) = p(z) \cdot b = \rho g z\, b$



Alternative approach to finding centroid of trapezoid:

$$A_{Trap} = \frac{1}{2} l (a + b)$$

$$C_{Trap} : \frac{1}{3} l \left[\frac{2a + b}{a + b} \right]$$



$$\boxed{F_{R_{Trap}} = \frac{1}{2} l (w_1 + w_2)}$$

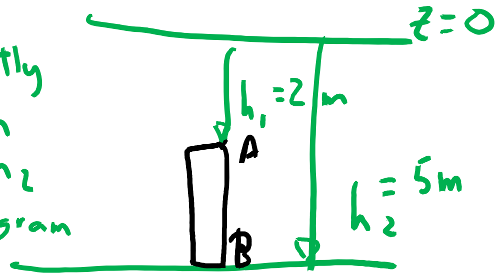
$$= \frac{1}{2} l (\rho g h_1 b + \rho g h_2 b)$$

$$= \frac{1}{2} l \rho g b (h_1 + h_2) = \frac{1}{2} l b (p_1 + p_2)$$

$$\boxed{C_{R_{Trap}} = \frac{1}{3} l \left[\frac{2 w_1 + w_2}{w_1 + w_2} \right]}$$

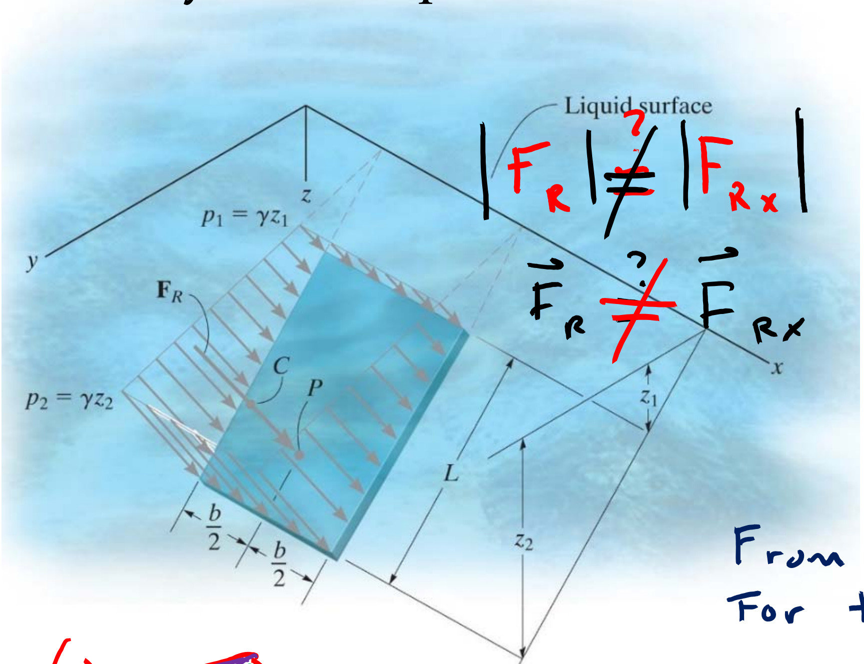
After class correction

The notes on left are correctly written based on definition of h_2 to this new diagram on right side



Fluid Pressure of a flat plate with constant width

For an incompressible fluid at rest with mass density , the pressure varies linearly with depth z

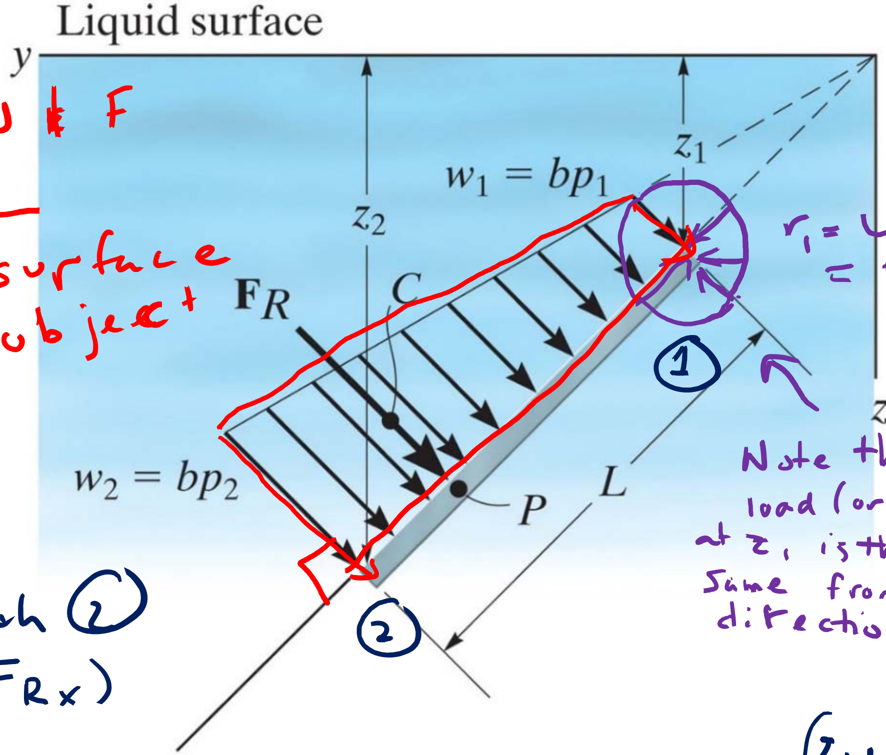


$|F_R| \overset{?}{\neq} |F_{Rx}|$

$\vec{F}_R \overset{?}{\neq} \vec{F}_{Rx}$

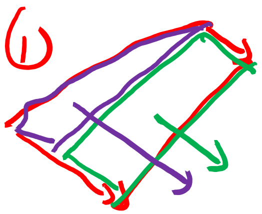
$P \parallel w \parallel F$

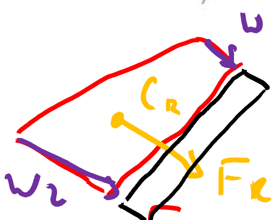
$\perp$ to surface of object



$r_1 = w_1 = p_1 b$

Note that load (or pressure) at z_1 is the same from any direction!





From approach ②

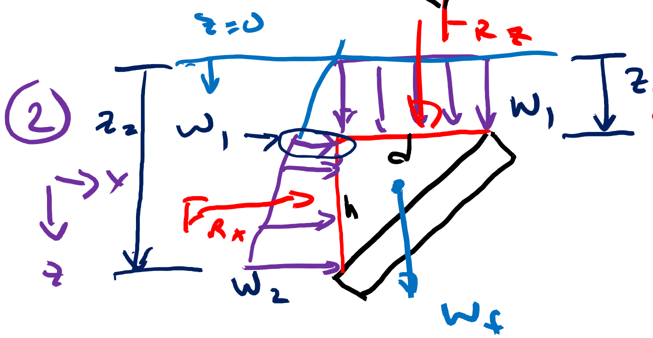
For trap (F_{Rx})

$F_{Rx} = \frac{1}{2} h (w_1 + w_2)$, $C_{Rx} = \frac{1}{3} h \left(\frac{2w_1 + w_2}{w_1 + w_2} \right)$

For approach ①

$F_R = \frac{1}{2} L (w_1 + w_2)$

$C_R = \frac{1}{3} L \left(\frac{2w_1 + w_2}{w_1 + w_2} \right)$



$\Sigma F_x : F_{Rx}$

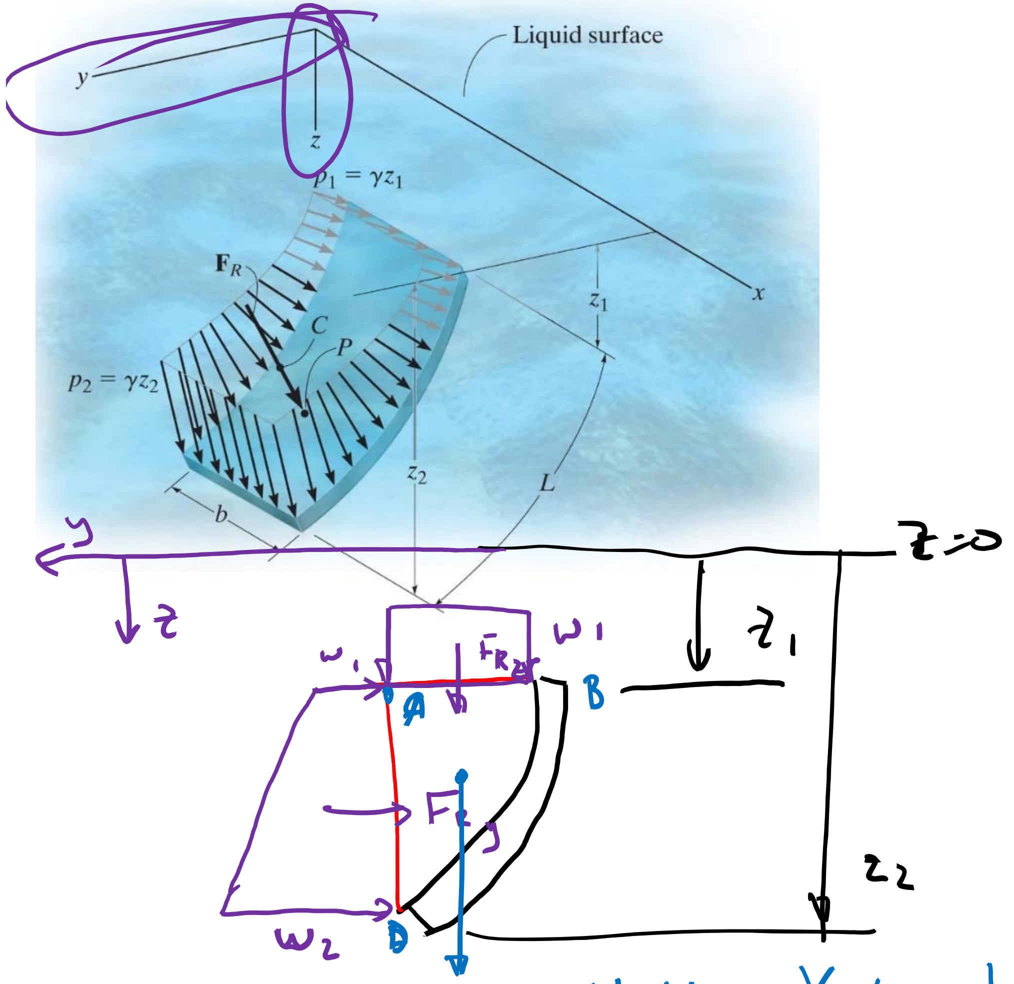
$\Sigma F_z = F_{Rz} + W_f$

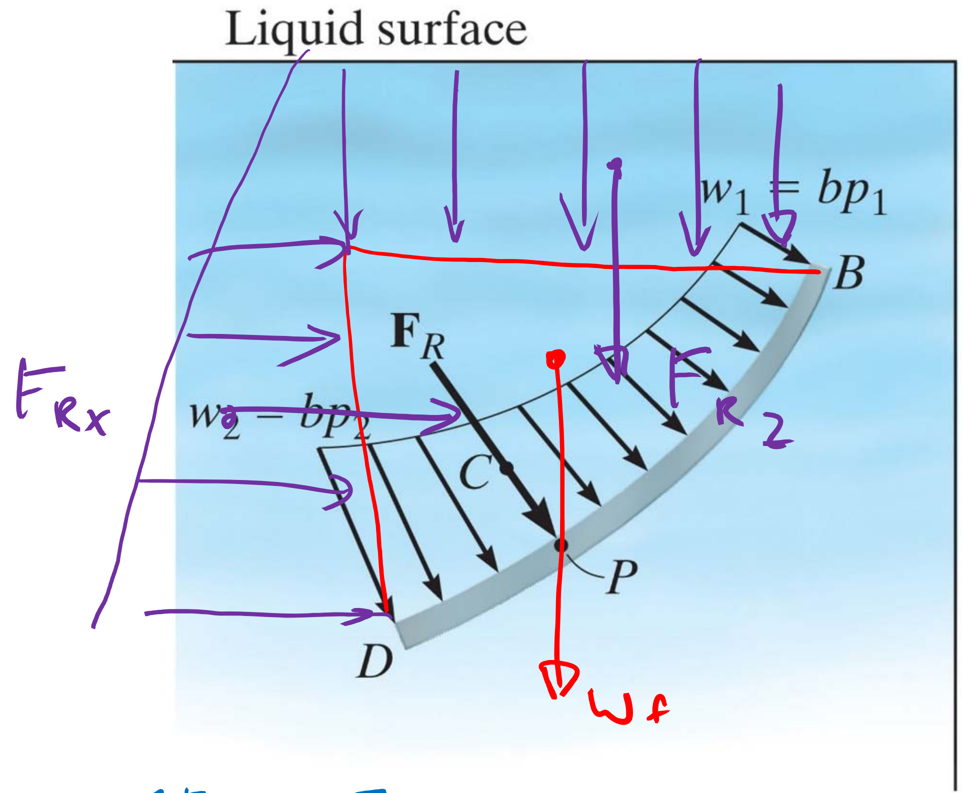
$W_f = \gamma \forall = \gamma \cdot \left(\frac{1}{2} d h \right)$

$F_{Rz} = w_1 d$

Fluid Pressure of a curved plate with constant width

For an incompressible fluid at rest with mass density , the pressure varies linearly with depth z





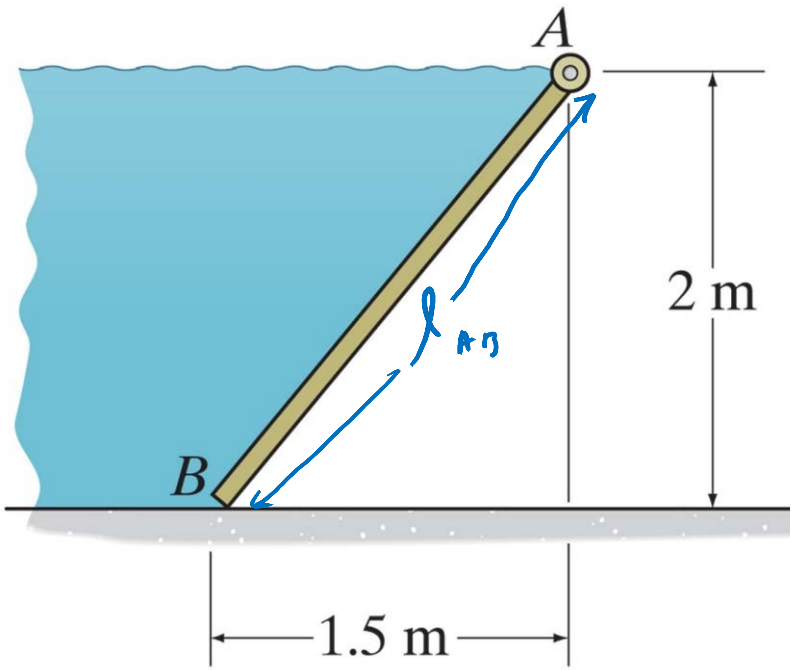
$$W_f = \gamma \cdot \forall = \gamma A_{ABD} \cdot b$$

$$\Sigma F_y = F_{Ry} = \text{Trapezoid}$$

$$\Sigma F_z = F_{Rz} + W_f$$

$$F_R = \sqrt{F_{Ry}^2 + (F_{Rz} + W_f)^2}$$

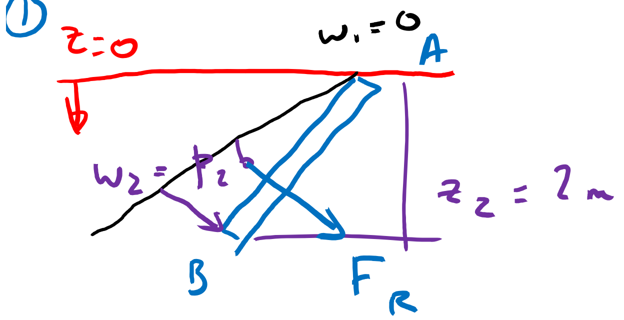
Determine the magnitude of the resultant hydrostatic force acting on the gate AB. The gate has width 1.5m.



$b = 1.5\,m$

2 approaches :

①

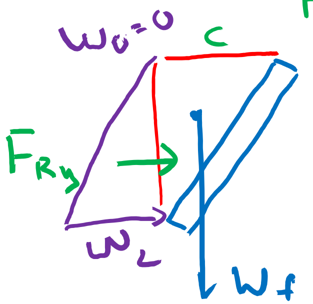


Triangle

$F_R = \frac{1}{2}(l_{AB})\, p_z b$

$l_{AB} = \sqrt{c^2 + z_2^2}$

②



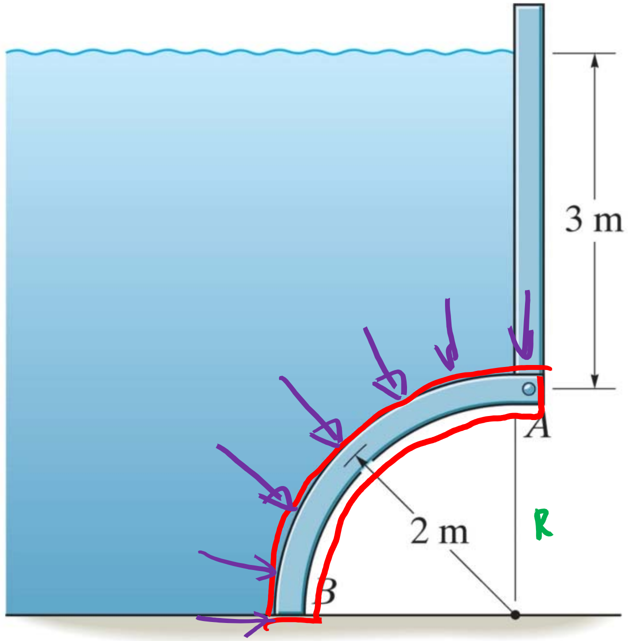
$F_{Rz} = 0,\ w(0) = 0$

$\Sigma F_y = F_{Ry} = \frac{1}{2} w_2 (z_2)$

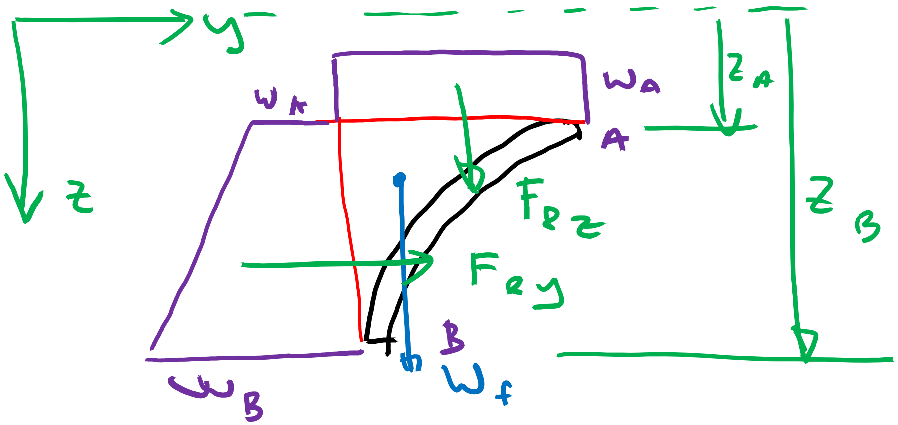
$\Sigma F_z = W_f = \gamma \cdot \forall = \gamma \cdot \frac{z_2 c}{2} \cdot b$

The arched surface AB is shaped in the form of a quarter circle. If it is 8 m long, determine the horizontal and vertical components of the resultant force caused by the water acting on the surface.

$b = 8\,m$



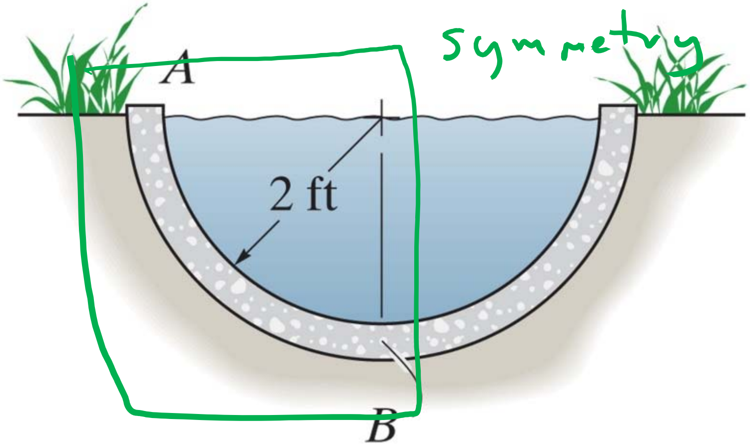
2nd approach



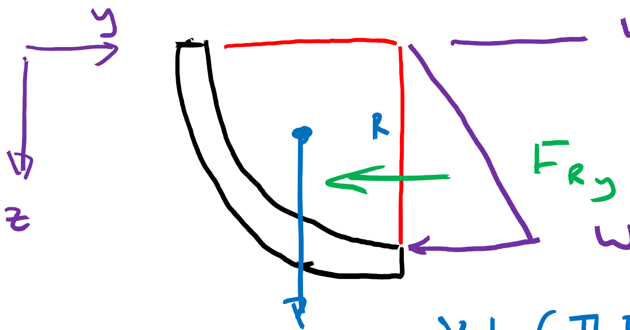
$\Sigma F_y : F_{Ry}$ (trapezoid)

$\Sigma F_z : F_{Rz}$ (rect) $+ W_f$, $W_f = \gamma \forall = \gamma [A_{square} - A_{\frac{1}{4} circle}] b$

$= \gamma b \left[R^2 - \frac{\pi R^2}{4} \right]$



The semicircular drainage pipe is filled with water. Determine the resultant force that the water exerts on the side AB of the pipe per foot of pipe length. The specific weight of the water is $\gamma = 62.4\ \text{lb/ft}^3$



$w(0) = 0$

$w(B) = p_B\, b \qquad b = ?$

$\boxed{\dfrac{w}{b}}$

$$W_f = \gamma\, b \left(\frac{\pi R^2}{4}\right) \Rightarrow \boxed{\frac{W_f}{b} = \frac{\gamma \pi R^2}{4} = 196.6\ \frac{\text{lb}}{\text{ft}}}$$

$$F_{Ry} = \frac{1}{2} w_B R = \frac{1}{2}(\gamma R\, b) R = \frac{1}{2}\gamma R^2 b \Rightarrow \boxed{\frac{F_{Ry}}{b} = \frac{\gamma R^2}{2} = 124.8\ \frac{\text{lb}}{\text{ft}^2}}$$

$$\boxed{\frac{F_{Ry}}{b}, \quad \frac{W_f}{b}}$$

$$F_R = \sqrt{F_{Ry}^2 + W_f^2} \Rightarrow \boxed{\frac{F_R}{b} = \sqrt{\left(\frac{F_{Ry}}{b}\right)^2 + \left(\frac{W_f}{b}\right)^2}}$$