Statics - TAM 211

Lecture 31

December 10, 2018

Chap 10.1, 10.2, 10.4, 10.8

Announcements

- Check ALL of your grades on Blackboard! Report issues
- Prof. H-W office hours
  - Monday 3-5pm (Room C315 ZJUI Building)
  - Wednesday 7-8pm (Residential College Lobby)
- Upcoming deadlines:
  - Tuesday (12/11)
    - HW 12
  - Friday (12/14)
    - Written Assignment 12
  - Friday (12/14) all in Teaching Building A418-420
    - **9:00 am:** Quiz 6, On paper. Chapter 9 (CoG thru Fluid Pressure)
    - 10:00 am: Discussion section for ALL students
    - No lecture to accommodate quiz 6 testing time

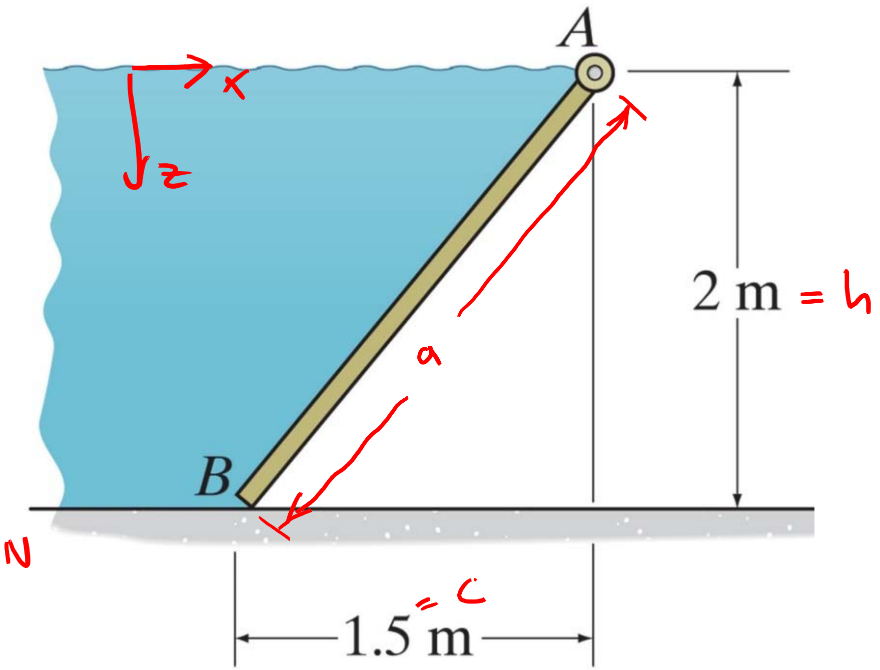
Chapter 9 Part II – Fluid Pressure

Chap 9.5

Recap of examples

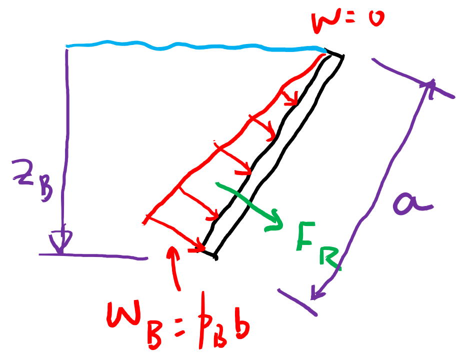
Determine the magnitude of the resultant hydrostatic force acting on the gate AB. The gate has width 1.5m.

$b = 1.5\,m$



2 solution approaches:

① Perpendicular load:

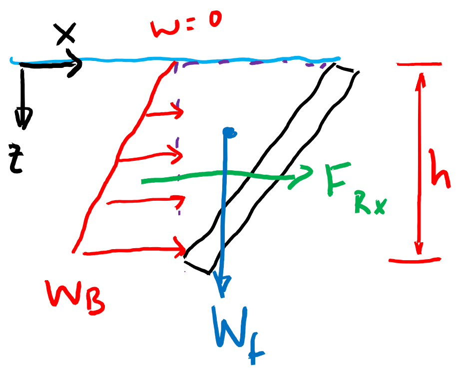


Triangular load

$$F_R = \frac{W_B\, a}{2} = \frac{p_B\, b\, a}{2}$$

$$F_R = \frac{\rho g z_B\, b\, a}{2} = 36.8\,kN$$

② Separate into x, z components:



Triangle load:

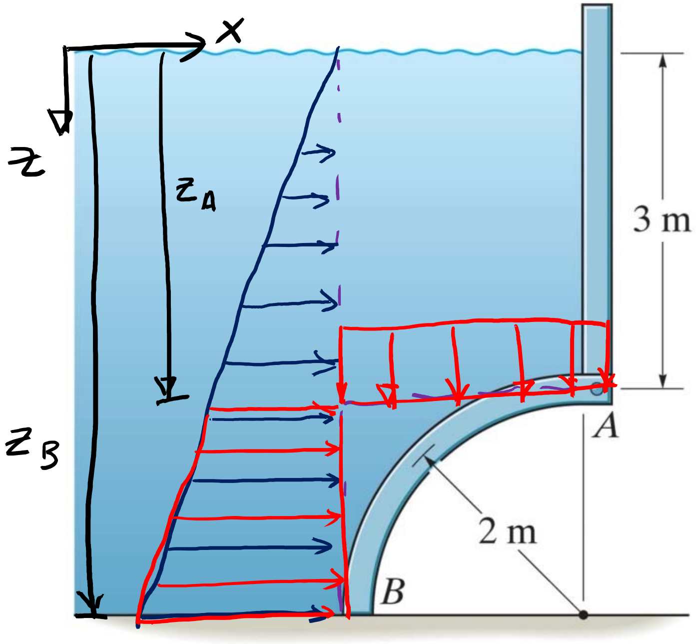
$$F_{Rx} = \frac{W_b\, h}{2} = \frac{\rho g b h^2}{2}, \quad W_f = \gamma \cdot Vol = \rho g A_{tri.} b = \rho g \frac{ch}{2} b$$

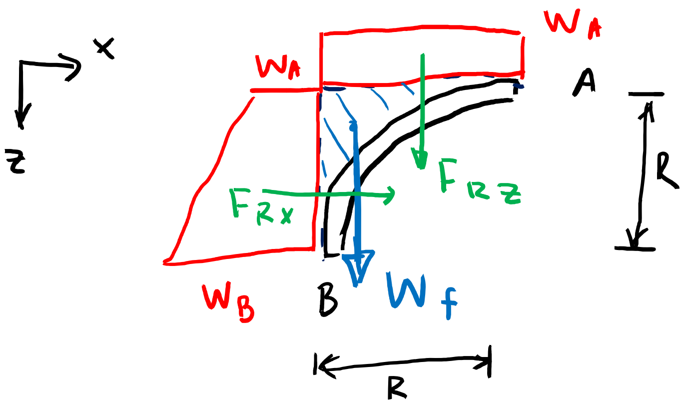
$$F_R = \sqrt{F_{Rx}^2 + W_f^2} = \frac{\rho g b h}{2}\sqrt{h^2 + c^2}$$

$$F_R = \frac{\rho g h b a}{2}$$ ✓ same as before since $h = z_B$

The arched surface AB is shaped in the form of a quarter circle. If it is 8 m long, determine the horizontal and vertical components of the resultant force caused by the water acting on the surface.

$b = 8\,m$





Rectangle: $F_{Rz} = W_A R = p_A b R = \rho g z_A b R = \underline{470.9\ kN}$

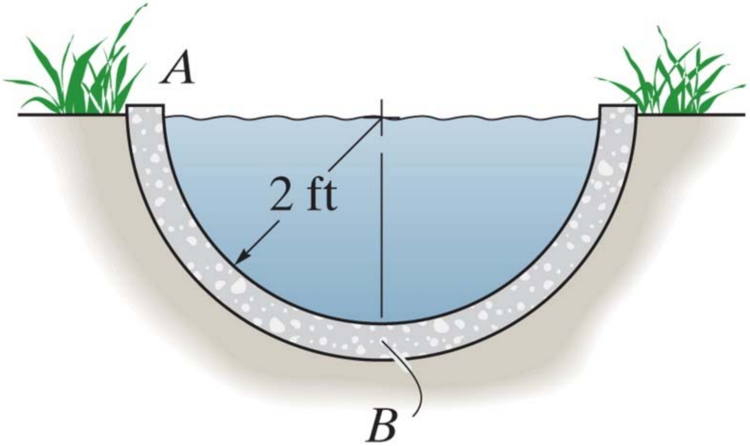
Weight of water: $W_f = \gamma \forall = \rho g A b$, $A = R^2 - \frac{\pi R^4}{2} \Rightarrow W_f = \underline{67.4\ kN}$

Trapezoid: $F_{Rx} = \frac{1}{2} R (w_A + w_B) = \frac{R}{2} b (p_A + p_B) = \underline{627.8\ kN}$

$\therefore \Sigma F_{vert} = F_{Rz} + W_f \Rightarrow \boxed{F_{vert} = 538.3\ kN}$

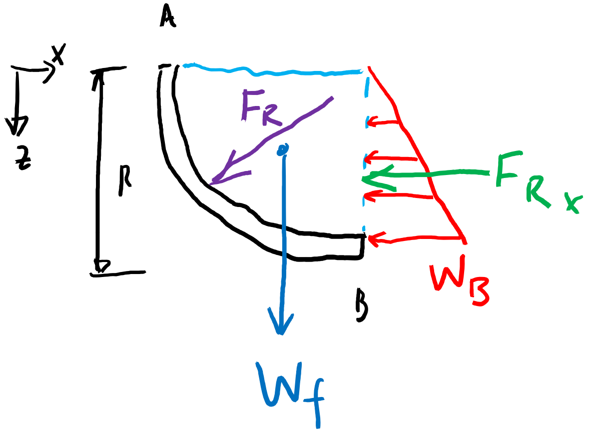
$\Sigma F_{hor} = F_{Rx} \Rightarrow \boxed{F_{hor} = 627.8\ kN}$

$F_R = \sqrt{F_V^2 + F_H^2} = \underline{827.0\ kN}$



The semicircular drainage pipe is filled with water. Determine the resultant force that the water exerts on the side AB of the pipe per foot of pipe length. The specific weight of the water is $\gamma = 62.4\ \text{lb/ft}^3$

b



$$F_{Rx} = \frac{W_B R}{2} = \frac{\overset{\gamma R}{p_B}\, b R}{2} = \frac{\gamma R^2 b}{2}$$

Triangle

$$\frac{F_{Rx}}{b} = \frac{\gamma R^2}{2} = \boxed{124.8\ \frac{\text{lb}}{\text{ft}}}$$

$$W_f = \gamma \forall = \gamma A b = \gamma\left(\frac{\pi R^2}{4}\right) b$$

$$\frac{W_f}{b} = \frac{\gamma \pi R^2}{4} = \boxed{196.6\ \frac{\text{lb}}{\text{ft}}}$$

$$F_R = \sqrt{F_{Rx}^2 + W_f^2} = \frac{\gamma R^2 b}{2}\sqrt{1 + \frac{\pi}{2}}$$

$$\frac{F_R}{b} = \frac{\gamma R^2}{2}\sqrt{1 + \frac{\pi}{2}}$$

specific weight

density

$$\gamma = \rho g$$

$$p = \rho g z$$

$$= \gamma z$$

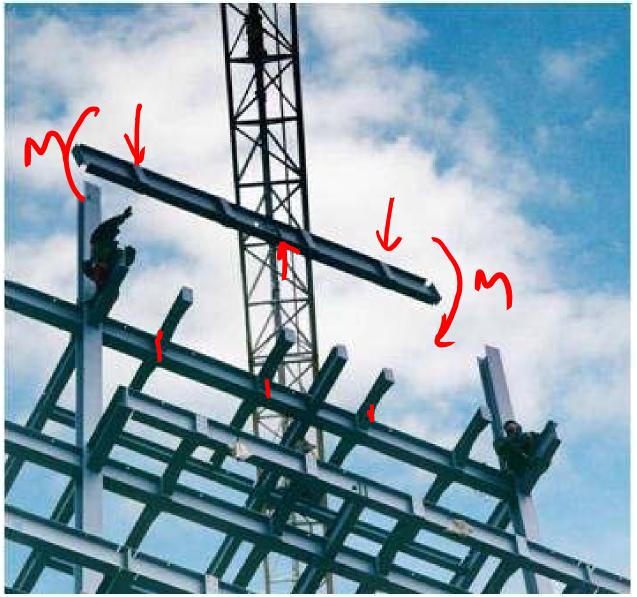
$$p = \gamma R$$

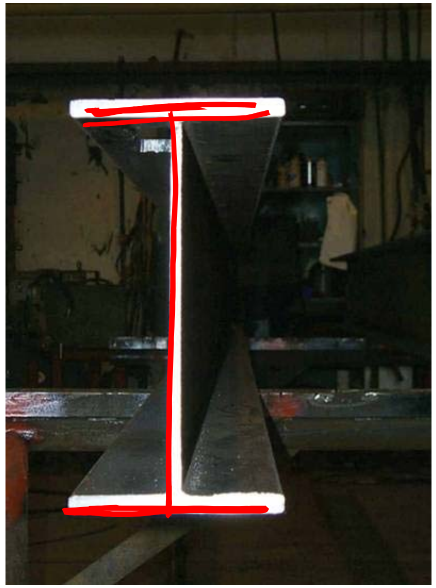
Chapter 10: Moments of Inertia

Goals and Objectives

- Understand the term "moment" as used in this chapter
- Determine and know the differences between
    - First/second moment of area
    - Moment of inertia for an area
    - Polar moment of inertia
    - Mass moment of inertia
- Introduce the parallel-axis theorem.
- Be able to compute the moments of inertia of composite areas.

Applications





Many structural members like beams and columns have cross sectional shapes like an I, H, C, etc.

Why do they usually not have solid rectangular, square, or circular cross sectional areas?

R = R

What primary property of these members influences design decisions?

Applications

Many structural members are made of tubes rather than solid squares or rounds. **Why?**

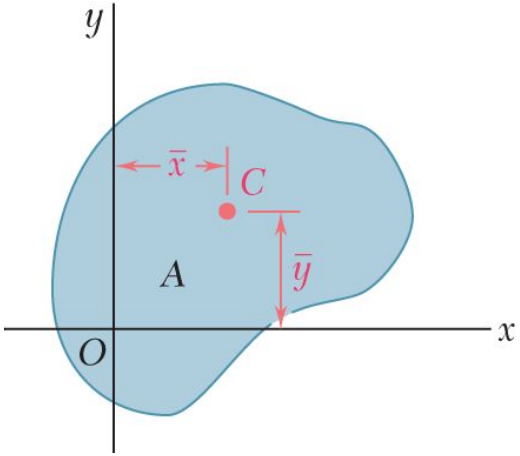
This section of the book covers some parameters of the cross sectional area that influence the designer's selection.

First moment of an area (centroid of an area)

- The first moment of the area A with respect to the x-axis is given by $Q_x = \int_A y\, dA$

- The first moment of the area A with respect to the y-axis is given by $Q_y = \int_A x\, dA$

- The centroid of the area A is defined as the point C of coordinates and, which satisfies the relation

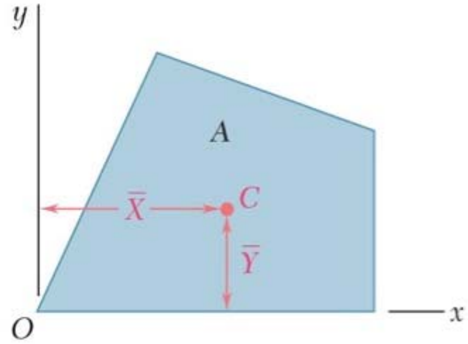


Centroid

First moment of Area

$$\int_A x\, dA = A\, \bar{x} \approx Q_x$$

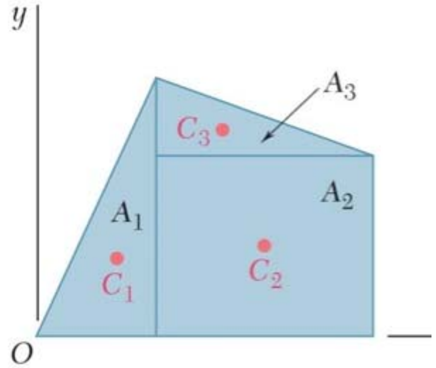
$$\int_A y\, dA = A\, \bar{y}$$



- In the case of a composite area, we divide the area A into parts

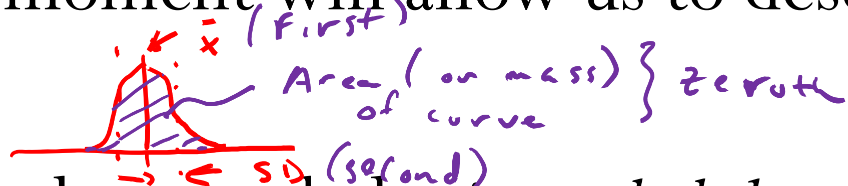
$$A_{total}\, \bar{X} = \sum_i A_i\, \bar{x}_i \qquad A_{total}\, \bar{Y} = \sum_i A_i\, \bar{y}_i$$

=



Terminology: the term **moment** in this module refers to the mathematical sense of different "measures" of an area or volume.

- The *zeroth* moment is the total mass.
- The *first* moment (a single power of position) gave us the centroid.
- The *second* moment will allow us to describe the "width."



- An analogy that may help: in *probability* the first moment gives you the mean (the center of the distribution), and the second is the standard deviation (the width of the distribution).

Second moment of area (a.k.a. Area moment of inertia)

Areas Moment of Inertia is the property of a deformable body that determines the moment (couple moment) needed to obtain a desired curvature about an axis. Moment of inertia depends on the shape of the body and may be different around different axes of rotation.

- The moment of inertia of the area A with respect to the x-axis is given by

$$I_x = \int_A y^2 \, dA$$

- The moment of inertia of the area A with respect to the y-axis is given by

$$I_y = \int_A x^2 \, dA$$

- The moment of inertia of the area A with respect to the origin O is given by (Polar moment of inertia)

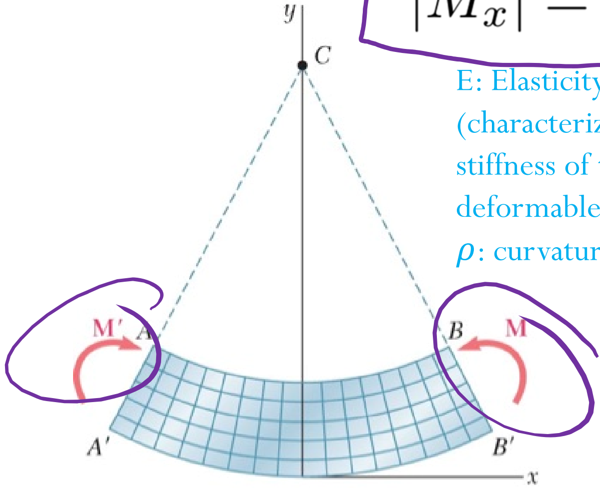
$$J_o = \int_A r^2 \, dA = \int_A (x^2 + y^2) \, dA = I_y + I_x$$

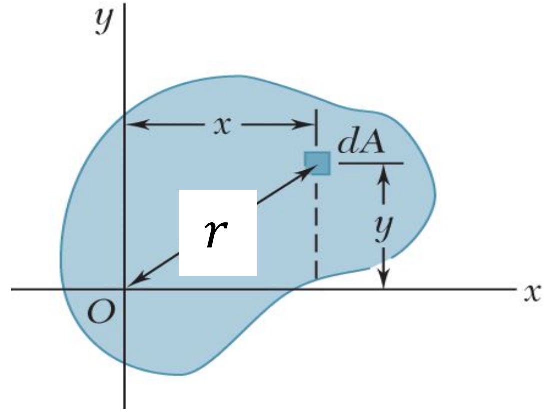
Moment-curvature relation:

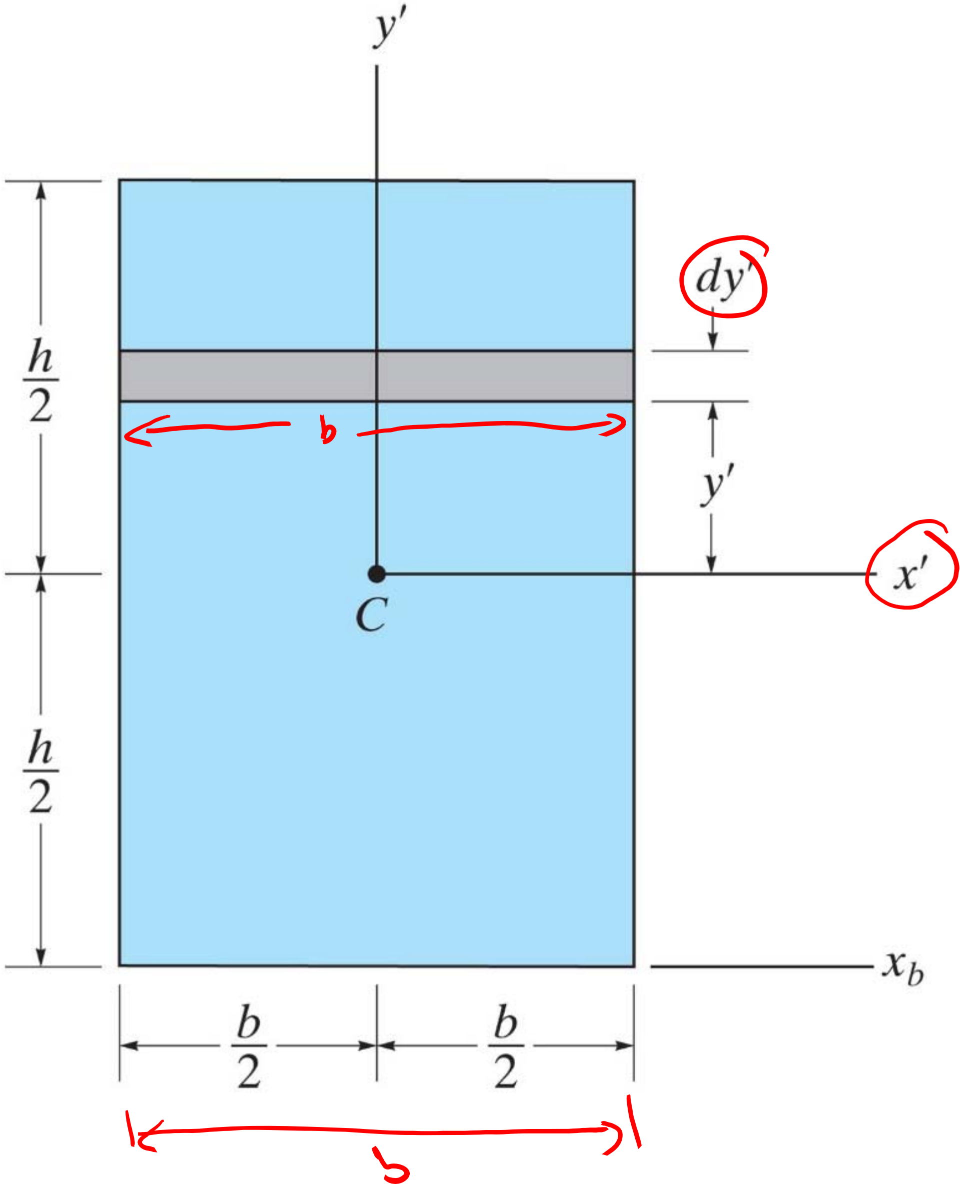
$$|M_x| = \frac{E I_x}{\rho}$$

E: Elasticity modulus (characterizes stiffness of the deformable body)

ρ: curvature







Determine the moment of inertia for the rectangular area shown w.r.t. the centroidal axis x'.

$$I_{x'} = \int_A (y')^2 \, dA$$

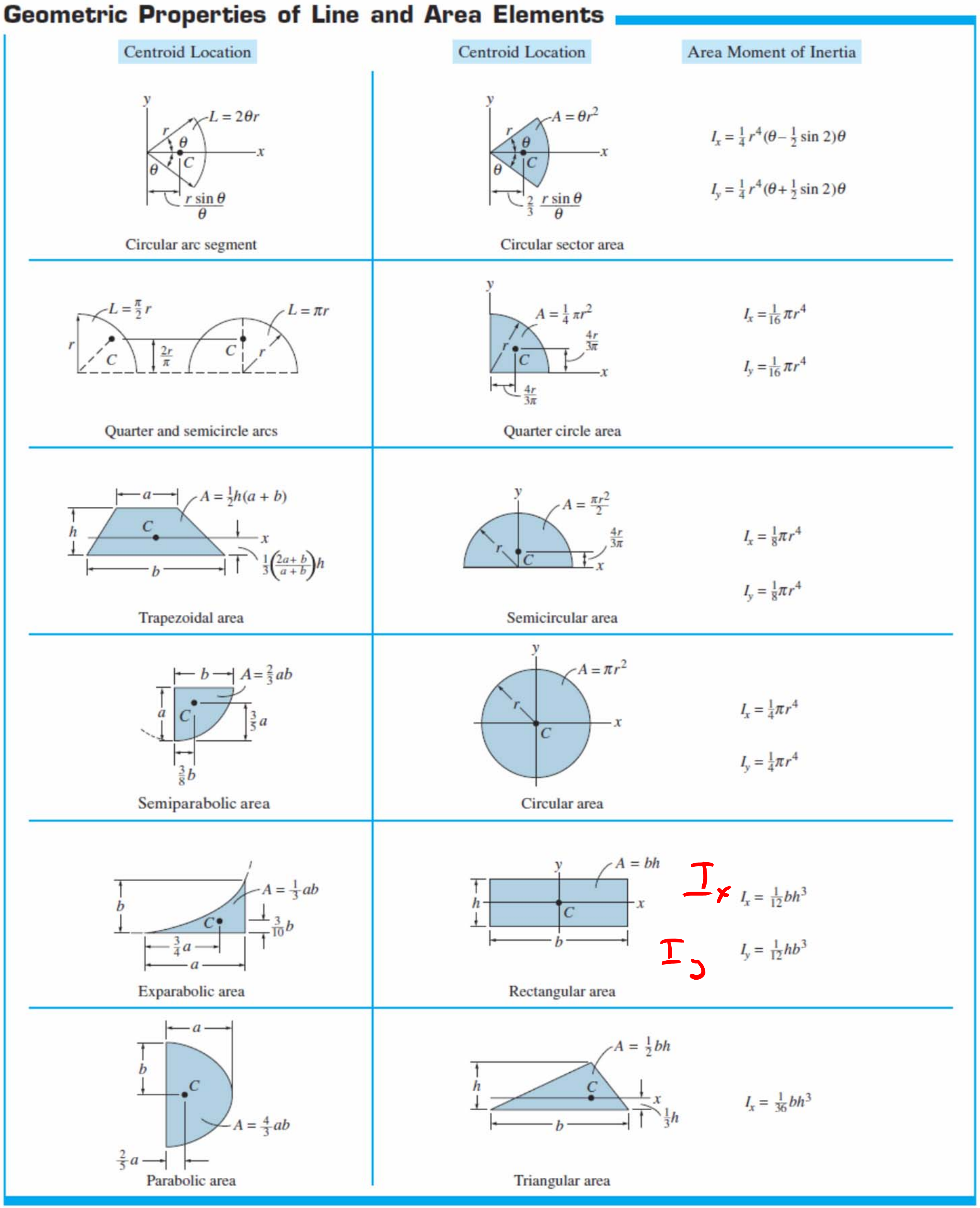
$$= \int_{-h/2}^{h/2} (y')^2 (b \, dy')$$

$$= b \int_{-h/2}^{h/2} (y')^2 \, dy'$$

$$= \frac{b}{3} (y')^3 \Big|_{-h/2}^{h/2}$$

$$\boxed{I_{x'} = \frac{1}{12} b h^3}$$

From inside back cover of Hibbler textbook



Geometric Properties of Line and Area Elements

Centroid Location	Centroid Location	Area Moment of Inertia
$L = 2\theta r$; $\frac{r \sin\theta}{\theta}$ Circular arc segment	$A = \theta r^2$; $\frac{2}{3}\frac{r \sin\theta}{\theta}$ Circular sector area	$I_x = \frac{1}{4} r^4(\theta - \frac{1}{2}\sin 2)\theta$ $I_y = \frac{1}{4} r^4(\theta + \frac{1}{2}\sin 2)\theta$
$L = \frac{\pi}{2} r$; $L = \pi r$; $\frac{2r}{\pi}$ Quarter and semicircle arcs	$A = \frac{1}{4}\pi r^2$; $\frac{4r}{3\pi}$ Quarter circle area	$I_x = \frac{1}{16}\pi r^4$ $I_y = \frac{1}{16}\pi r^4$
$A = \frac{1}{2}h(a+b)$; $\frac{1}{3}\left(\frac{2a+b}{a+b}\right)h$ Trapezoidal area	$A = \frac{\pi r^2}{2}$; $\frac{4r}{3\pi}$ Semicircular area	$I_x = \frac{1}{8}\pi r^4$ $I_y = \frac{1}{8}\pi r^4$
$A = \frac{2}{3}ab$; $\frac{3}{5}a$; $\frac{3}{8}b$ Semiparabolic area	$A = \pi r^2$ Circular area	$I_x = \frac{1}{4}\pi r^4$ $I_y = \frac{1}{4}\pi r^4$
$A = \frac{1}{3}ab$; $\frac{3}{4}a$; $\frac{3}{10}b$ Exparabolic area	$A = bh$ Rectangular area	I_x $I_x = \frac{1}{12}bh^3$ I_y $I_y = \frac{1}{12}hb^3$
$A = \frac{4}{3}ab$; $\frac{2}{5}a$ Parabolic area	$A = \frac{1}{2}bh$; $\frac{1}{3}h$ Triangular area	$I_x = \frac{1}{36}bh^3$

Note that equation sheets will be provided for quizzes/exam.

Parallel axis theorem

- Often, the **moment of inertia** of an area is known for an axis passing through the **centroid**; e.g., *x'* and *y'*:
- The moments around other axes can be computed from the known $I_{x'}$ and $I_{y'}$:

$$I_x = \int_{\text{area}} (y' + d_y)^2 \, dA$$

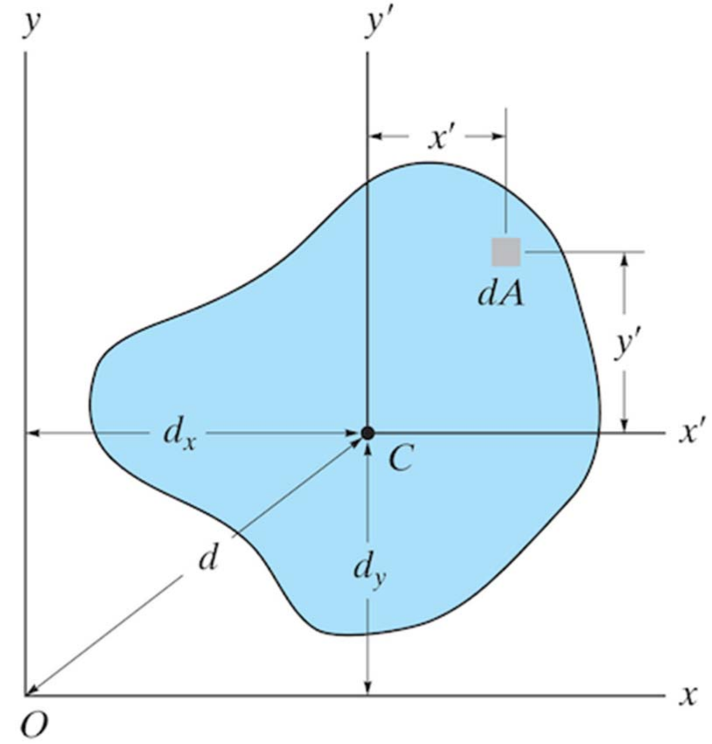
$$= \int_{\text{area}} (y')^2 \, dA + 2d_y \int_{\text{area}} y' \, dA$$

$$+ d_y^2 \int_{\text{area}} dA$$

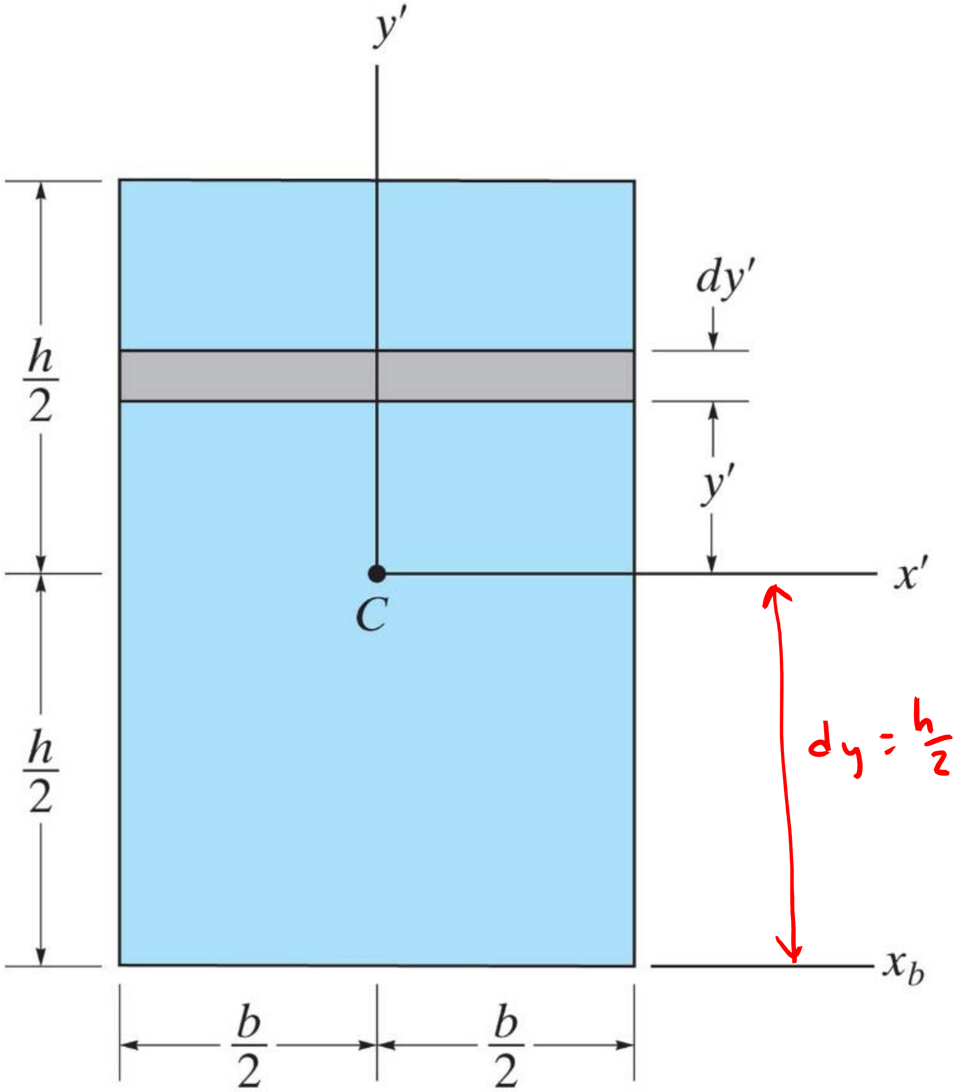
$$= \boxed{I_{x'} + Ad_y^2 = I_x}$$

$$\boxed{I_y = I_{y'} + Ad_x^2}$$

$$J_O = J_C + A(d_x^2 + d_y^2) = \boxed{J_C + Ad^2 = J_O}$$



Note: the integral over y' gives zero *when done through the centroid axis*.



Determine the moment of inertia for the rectangular area shown w.r.t. the axis passing through the base of the rectangle x_b.

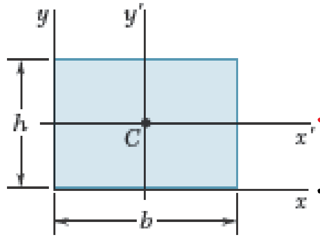
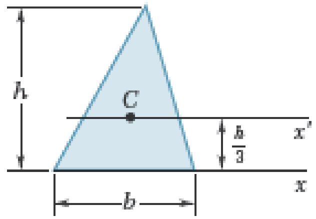
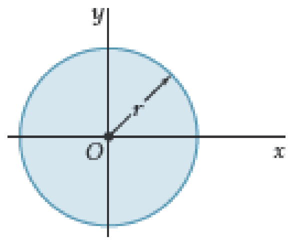
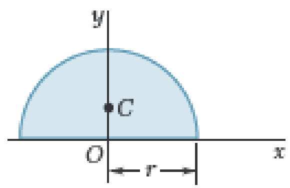
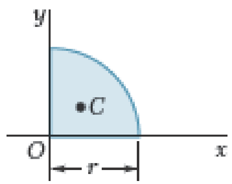
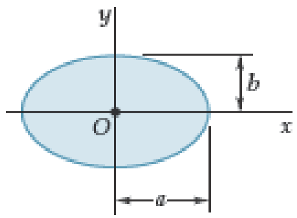
$$I_{x_b} = I_{x'} + A(d_y)^2$$

? $= \frac{h}{2}$

$$= \frac{1}{12} b h^3 + (bh)\left(\frac{h}{2}\right)^2$$

$$\boxed{I_{x_b} = \frac{1}{3} b h^3}$$

Area Moments of Inertia
for common shapes

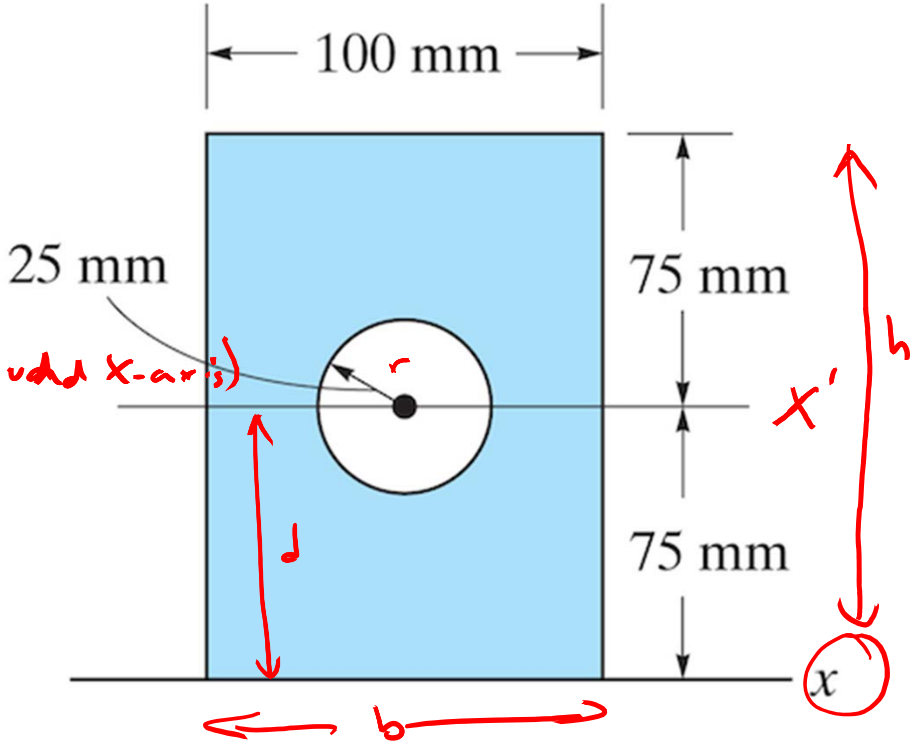
Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Note that equation sheets will be provided for quizzes/exam.

Moment of inertia of composite

- If individual bodies making up a **composite** body have individual areas *A* and moments of inertia *I* computed through their centroids, then the **composite area** and **moment of inertia** is a sum of the individual component contributions.
- This requires the **parallel axis theorem**
- Remember:
  - The position of the centroid of each component **must** be defined with respect to the **same origin**.
  - It is allowed to consider **negative areas** in these expressions. Negative areas correspond to holes/missing area. **This is the one occasion to have negative moment of inertia**.

Determine the moment of inertia for the shaded area about the x-axis.



2 segments : □ − ○

Not around the centroid axis (want around x-axis)

⇒ Parallel Axis Theorem

$$\boxed{I_x = I_{x'} + Ad^2}$$

Rect : $I_{x_R} = I_{x'_{Rect}} + A_{rect}(d)^2$

$$= \boxed{\frac{1}{12}bh^3} + (bh)\,d^2$$

Circle : $I_{x_c} = I_{x'_c} + A_c d^2$

$$= \boxed{\frac{1}{4}\pi r^4} + (\pi r^2)d^2$$

$$I_{x_{composite}} = I_{x_R} - I_{x_c} = 101(10^6)\ \text{mm}^4$$