Statics - TAM 211

Lecture 34

December 19, 2018

Announcements

- Check ALL of your grades on Blackboard! Report issues
- Prof. H-W office hours
  - Wednesday 7-8pm (Residential College Lobby)
- Upcoming deadlines:
  - Friday (12/21)
    - Written Assignment 13
  - Tuesday (12/25)
    - HW 14
  - Friday (12/28)
    - Written Assignment 14
- Final Exam – computer based
  - Wednesday January 9, 9:00-12:00
  - Instructional Lab Building: D211 (ME students), D331 (CEE students)

http://knowledge.wharton.upenn.edu

Chapter 11: Virtual Work

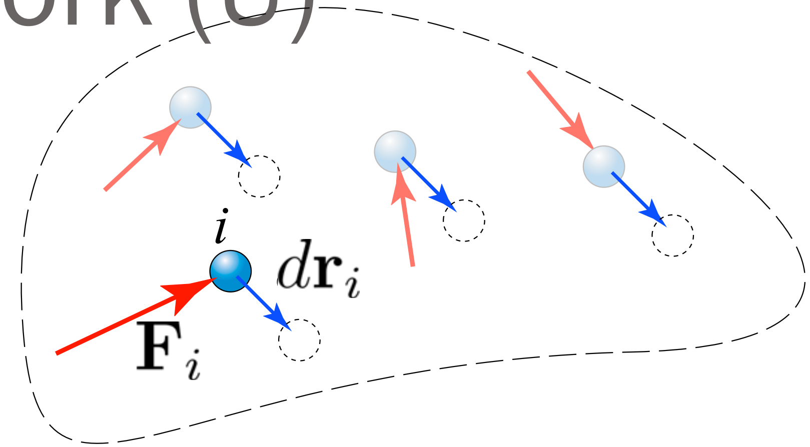
Goals and Objectives

- Introduce the principle of virtual work
- Show how it applies to determining the equilibrium configuration of a series of pin-connected members

Recap: Methods to derive Equations of Equilibrium

- Force-Balance Method
  - $\sum \vec{\boldsymbol{F}} = 0,\ \sum \vec{\boldsymbol{M}} = 0$
- Work-Energy Method (or Virtual Work Method)
  - $\delta U = \sum \left(\vec{\boldsymbol{F}} \cdot \boldsymbol{\delta} \vec{\boldsymbol{r}} \right) + \sum \left(\vec{\boldsymbol{M}} \cdot \delta \vec{\boldsymbol{\theta}} \right) = 0$
- Virtual Work Method is particularly useful for structures with many members, whereas Force-Balance Method needs multiple equations per member

Recap: Definition of Work (U)

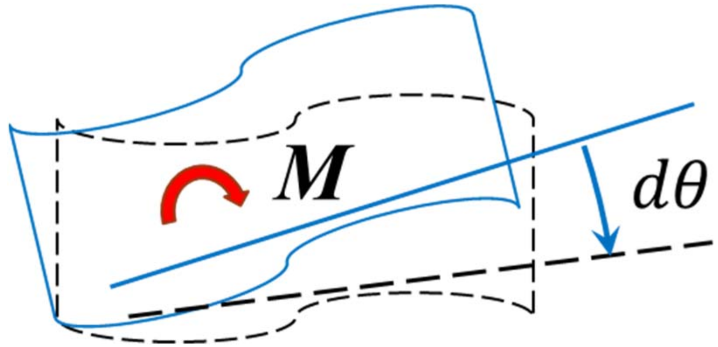


Work of a force

$$dU = \mathbf{F} \cdot d\mathbf{r}$$

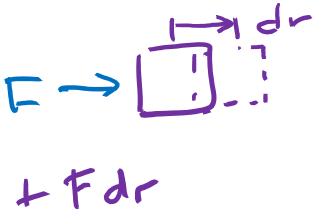
Only force component in direction of displacement does work

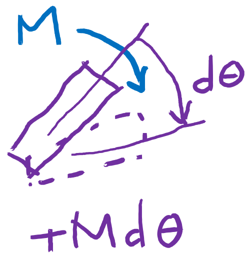
Work of a couple moment



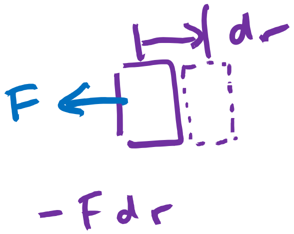
$$dU = M\mathbf{k} \cdot d\theta\,\mathbf{k} = M\,d\theta$$

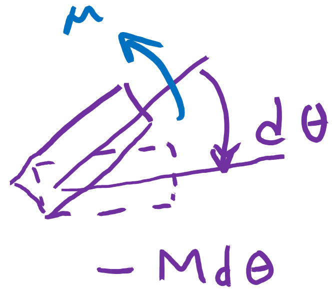
Positive Work: Force (or moment) is in the **same** direction as displacement





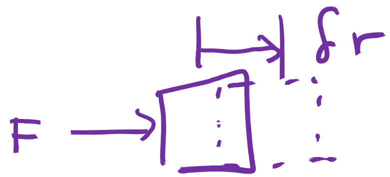
Negative Work: Force (or moment) is in the **opposite** direction as displacement

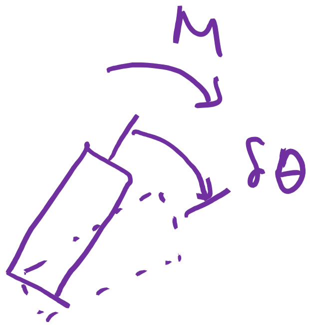




Recap: Virtual Displacements, Virtual Work

Virtual displacement: extremely small displacement. Represented as δr or $\delta\theta$





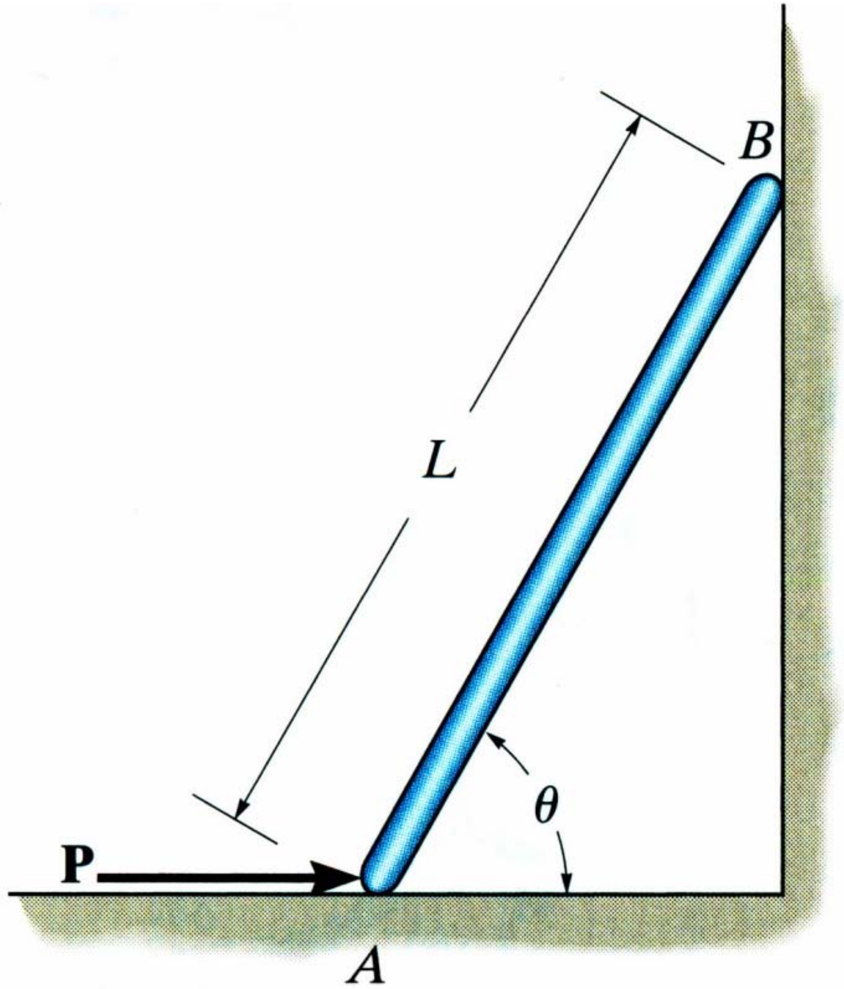
Virtual work : algebraic sum of the virtual work done by all the forces and couple moments acting on the body is zero for any virtual displacement of the body. Represented as δU.

$$\delta U = 0$$

$$\delta U = \Sigma\left(\vec{\boldsymbol{F}} \cdot \delta\vec{\boldsymbol{r}}\right) + \Sigma\left(\vec{\boldsymbol{M}} \cdot \delta\vec{\boldsymbol{\theta}}\right) = 0$$

For 2D:

$$\delta U = \Sigma\left(\vec{\boldsymbol{F}} \cdot \delta\vec{\boldsymbol{r}}\right) + \Sigma(M\,\delta\theta) = 0$$

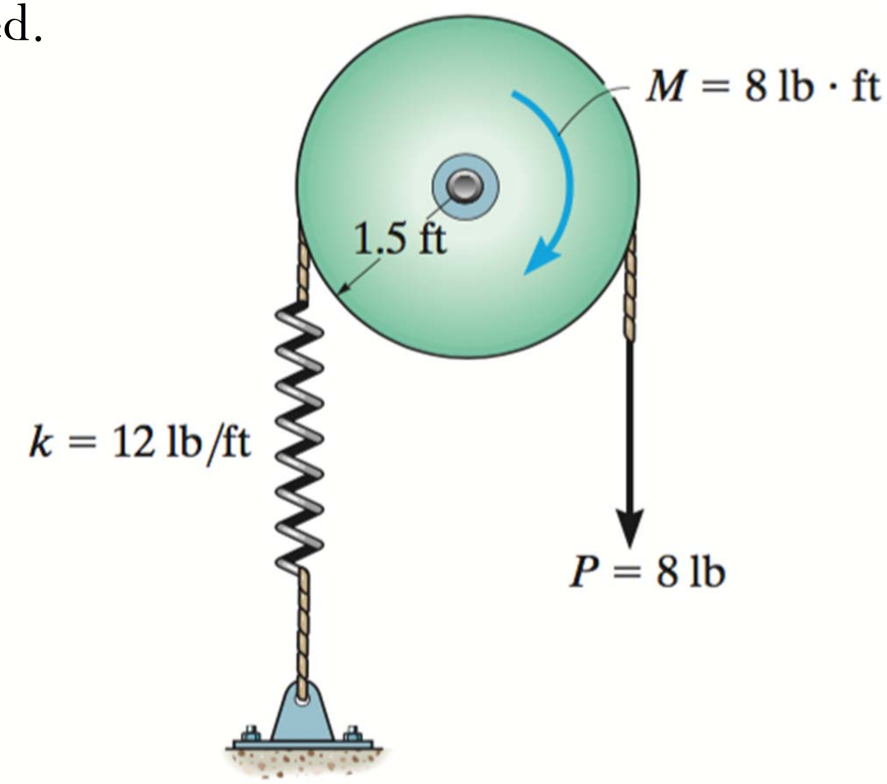


The thin rod of weight W rests against the smooth wall and floor. Determine the magnitude of force P needed to hold it in equilibrium.

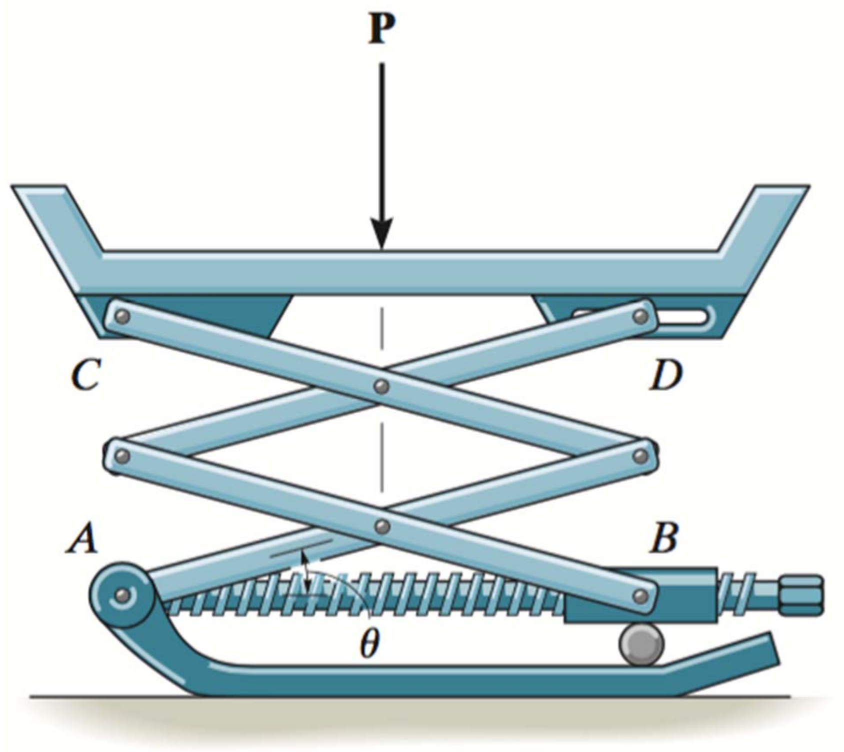
Procedure for Analysis

1. Draw FBD of the entire system and provide coordinate system
2. Sketch the "deflected position" of the system
3. Define position coordinates measured from a <u>fixed</u> point and select the parallel line of action component and <u>remove forces that do no work</u>
4. <u>Differentiate</u> position coordinates to obtain virtual displacement
5. Write the virtual work equation and express the virtual work of each force/ couple moment
6. Factor out the comment virtual displacement term and solve

Disk of 10 lb is subjected to a vertical force P = 8 lb and a couple moment M = 8 lb ft. Determine the disk's rotation θ if the end of the spring wraps around the periphery of the disk as the disk turns. The spring is originally unstretched.



The scissors jack supports a load **P**. Determine axial force in the screw necessary for equilibrium when the jack is in the position shown. Each of the four links has a length *l* and is pin-connected at its center. Points *B* and *D* can move horizontally.



Determine the mass of A and B required to hold the 400 g desk lamp in balance for any angles θ and ϕ. Neglect the weight of the mechanism and the size of the lamp. Assume that pins are frictionless.