Statics - TAM 211

Lecture 34

December 19, 2018

Announcements

- Check ALL of your grades on Blackboard! Report issues
- Prof. H-W office hours
  - Wednesday 7-8pm (Residential College Lobby)
- Upcoming deadlines:
  - Friday (12/21)
    - Written Assignment 13
  - Tuesday (12/25)
    - HW 14
  - Friday (12/28)
    - Written Assignment 14

http://knowledge.wharton.upenn.edu

- Final Exam – computer based
  - Wednesday January 9, 9:00-12:00
  - Instructional Lab Building: D211 (ME students), D331 (CEE students)

Chapter 11: Virtual Work

Goals and Objectives

- Introduce the principle of virtual work
- Show how it applies to determining the equilibrium configuration of a series of pin-connected members

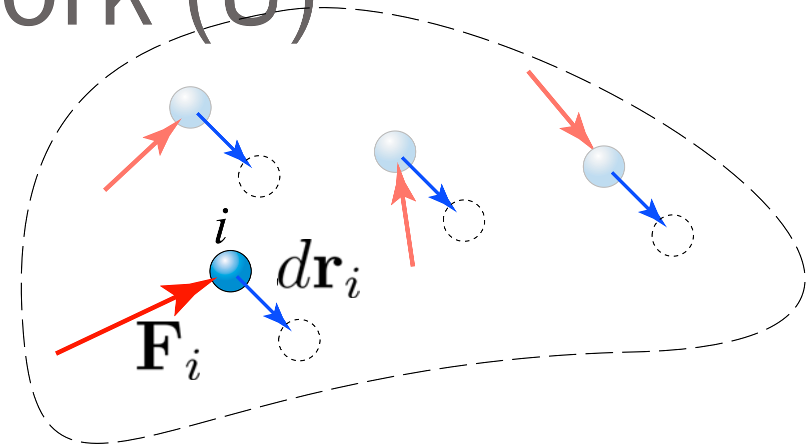
Recap: Methods to derive Equations of Equilibrium

- Force-Balance Method
  - $\sum \vec{\boldsymbol{F}} = 0, \sum \vec{\boldsymbol{M}} = 0$

- Work-Energy Method (or Virtual Work Method)
  - $\delta U = \sum \left(\vec{\boldsymbol{F}} \cdot \boldsymbol{\delta} \vec{\boldsymbol{r}} \right) + \sum \left(\vec{\boldsymbol{M}} \cdot \delta \vec{\boldsymbol{\theta}} \right) = 0$

- Virtual Work Method is particularly useful for structures with many members, whereas Force-Balance Method needs multiple equations per member

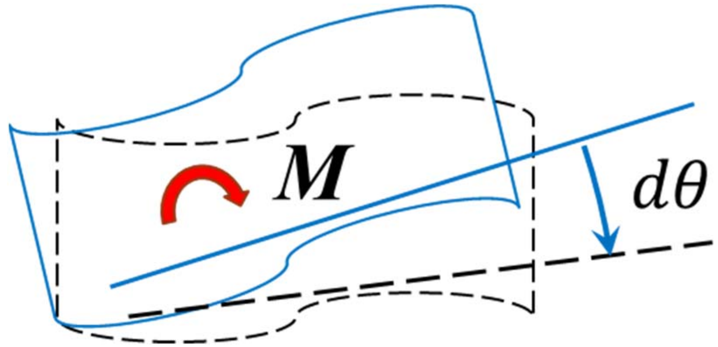
Recap: Definition of Work (U)



Work of a force

$$\boxed{dU = \mathbf{F} \cdot d\mathbf{r}}$$

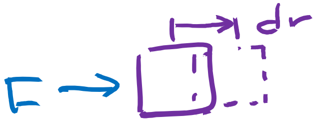
Only force component in direction of displacement does work



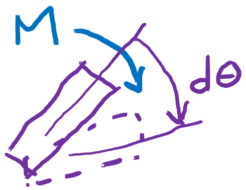
Work of a couple moment

$$\boxed{dU = M\mathbf{k} \cdot d\theta\,\mathbf{k} = M\,d\theta}$$

Positive Work: Force (or moment) is in the **same** direction as displacement

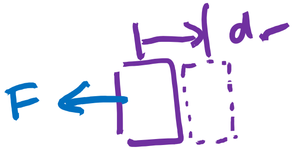


+Fdr

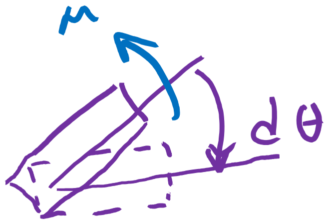


+Mdθ

Negative Work: Force (or moment) is in the **opposite** direction as displacement



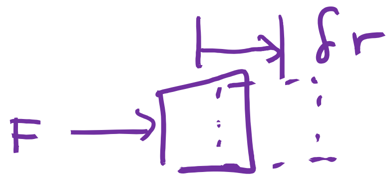
−Fdr

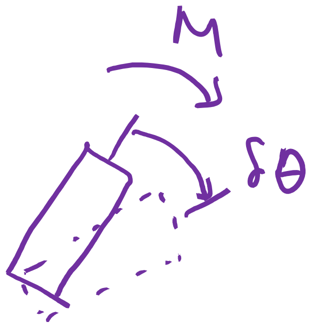


−Mdθ

Recap: Virtual Displacements, Virtual Work

Virtual displacement: extremely small displacement. Represented as δr or $\delta\theta$





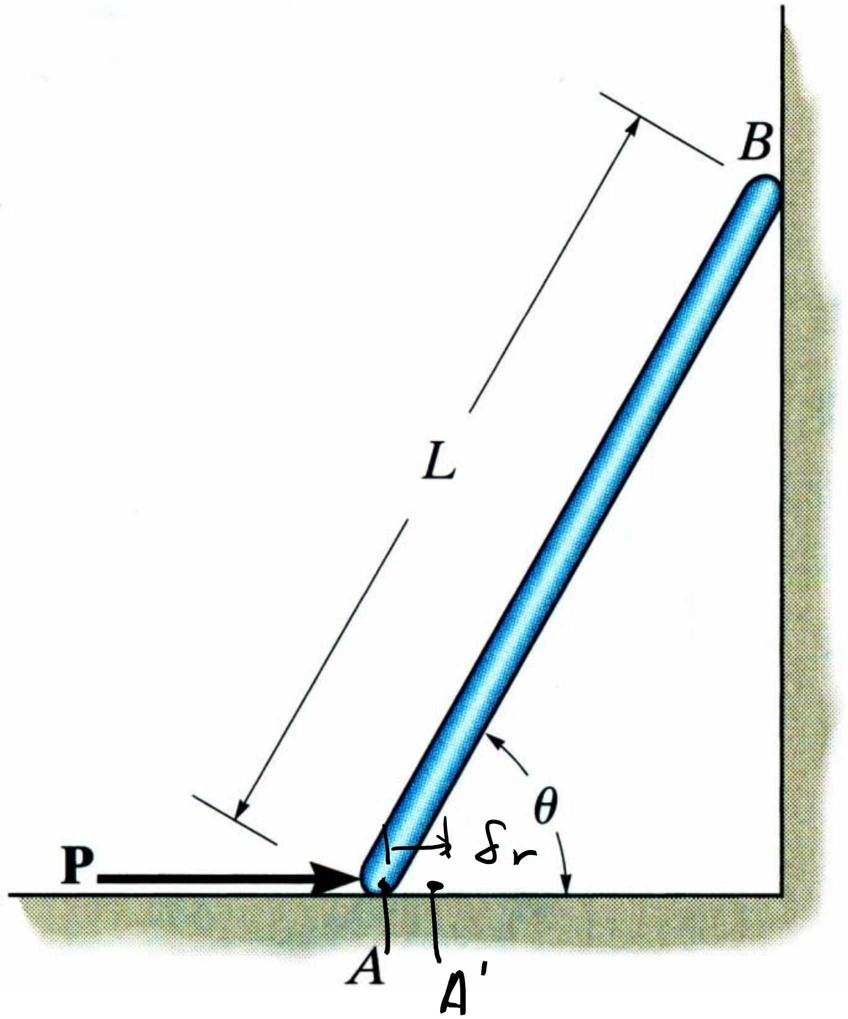
Virtual work : algebraic sum of the virtual work done by all the forces and couple moments acting on the body is zero for any virtual displacement of the body. Represented as δU.

$$\delta U = 0$$

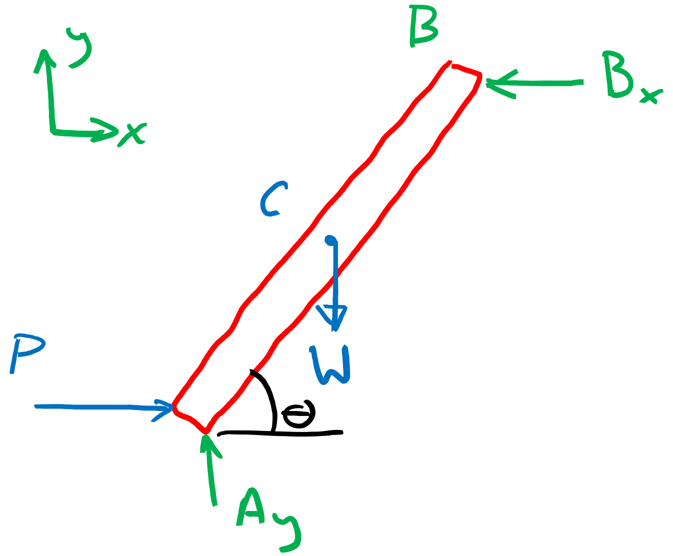
$$\boxed{\delta U = \Sigma\left(\vec{\boldsymbol{F}} \cdot \delta\vec{\boldsymbol{r}}\right) + \Sigma\left(\vec{\boldsymbol{M}} \cdot \delta\vec{\boldsymbol{\theta}}\right) = 0}$$

For 2D:

$$\delta U = \Sigma\left(\vec{\boldsymbol{F}} \cdot \delta\vec{\boldsymbol{r}}\right) + \Sigma(M\,\delta\theta) = 0$$



The thin rod of weight W rests against the smooth wall and floor. Determine the magnitude of force P needed to hold it in equilibrium.



UNKNOWNS: P, A_y, B_x

Need 3 eqns:

Force - Balance method

$\Sigma F_x = 0$

$\Sigma F_y = 0$

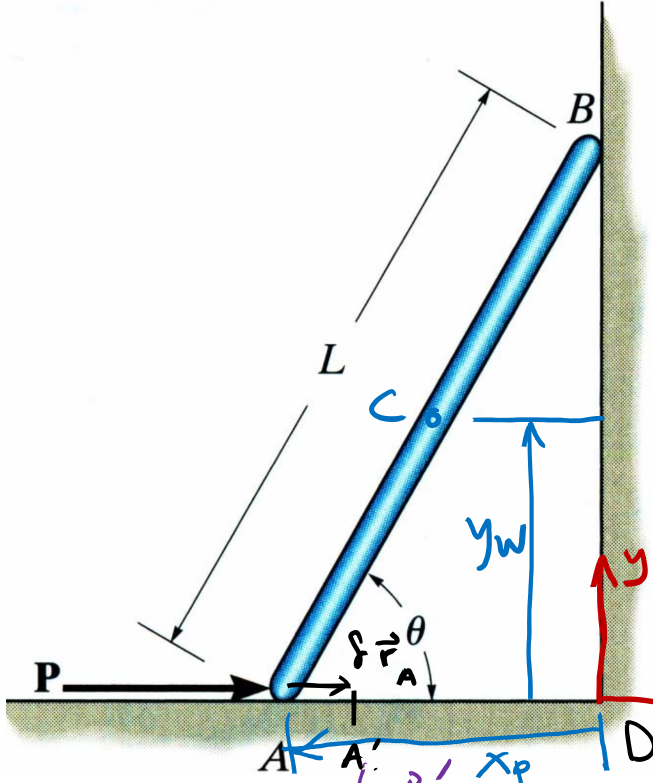
$\Sigma M_? = 0$

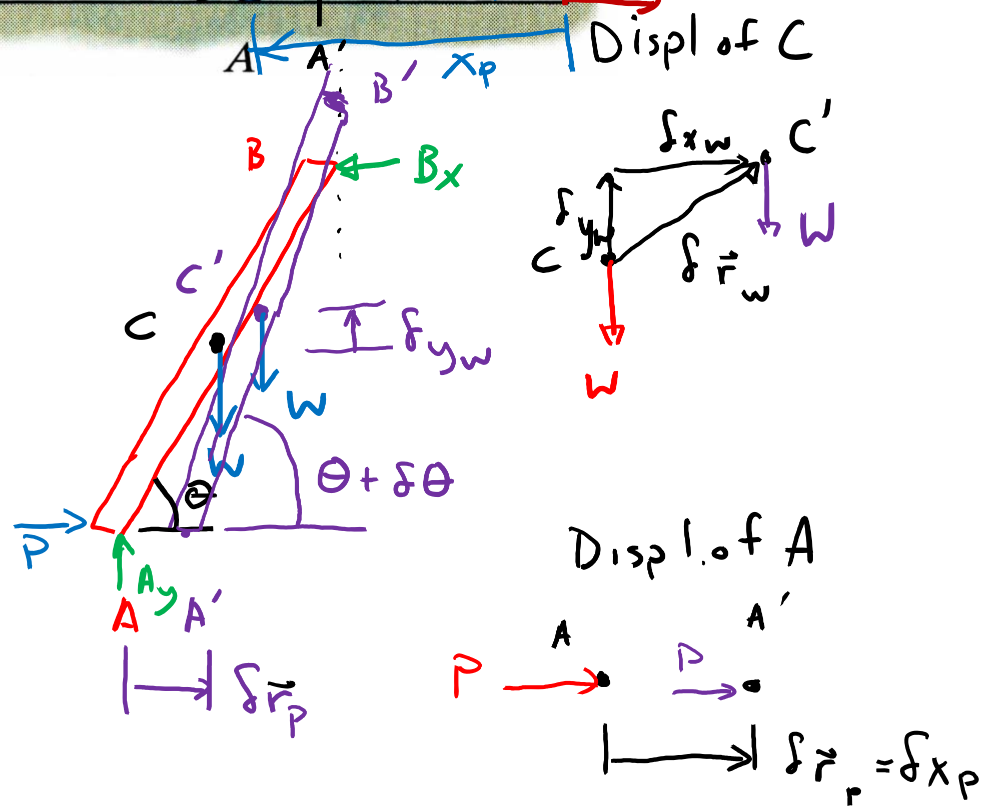
Procedure for Analysis

1. Draw FBD of the entire system and provide coordinate system
2. Sketch the “deflected position” of the system
3. Define position coordinates measured from a <u>fixed</u> point and select the parallel line of action component and <u>remove forces that do no work</u>
4. <u>Differentiate</u> position coordinates to obtain virtual displacement
5. Write the virtual work equation and express the virtual work of each force/ couple moment
6. Factor out the comment virtual displacement term and solve

Thin rod of weight *W* rests against smooth wall and floor. Determine magnitude of force *P* needed to hold it in equilibrium.

1. Draw FBD. 2. Sketch “deflected position”.
3. Define position coordinates from <u>fixed</u> point and select line of action component; <u>remove forces that do no work.</u> 4. <u>Differentiate</u> position coordinates to obtain virtual displacement. 5. Write virtual work equation and express virtual work of each force or couple moment. 6. Factor out common virtual displ. term and solve.





③ Define coordinate system & fixed pt.

$x_p = -L\cos\theta$

$y_w = \frac{L}{2}\sin\theta$

④ $\delta x_p = L\sin\theta\,\delta\theta$

$\delta y_w = \frac{L}{2}\cos\theta\,\delta\theta$

⑤ $\delta U = \delta(\vec F\cdot\delta\vec r) = 0$

$(\vec P\cdot\delta\vec r_p) + (\vec W\cdot\delta\vec r_w) = 0$

$(P\hat i\cdot\delta x_p\hat i) + (-W\hat j\cdot(\delta x_w\hat i + \delta y_w\hat j)) = 0$

$P\,\delta x_p - W\,\delta y_w = 0$

$P(L\sin\theta\,\delta\theta) - W(\frac{L}{2}\cos\theta\,\delta\theta) = 0$

$(PL\sin\theta - W\frac{L}{2}\cos\theta)\,\delta\theta = 0$

since $\delta\theta \neq 0$

$\therefore PL\sin\theta - \frac{WL}{2}c\theta = 0$

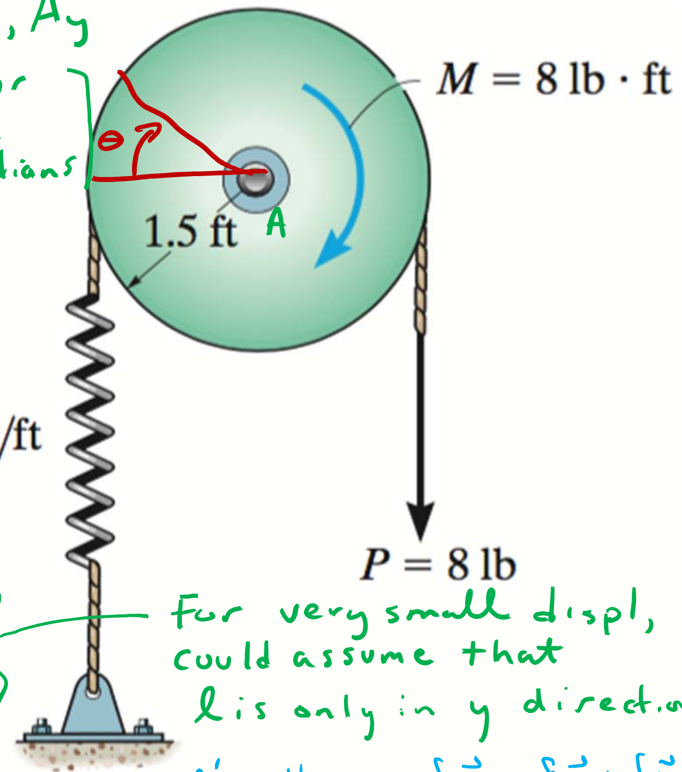
$\boxed{P = \frac{W}{2\tan\theta}}$

Disk of $\boxed{W = 10\text{ lb}}$ is subjected to a vertical force P = 8 lb and a couple moment M = 8 lb ft. Determine disk's rotation θ if the end of the spring wraps around the periphery of the disk as the disk turns. The spring is originally unstretched.

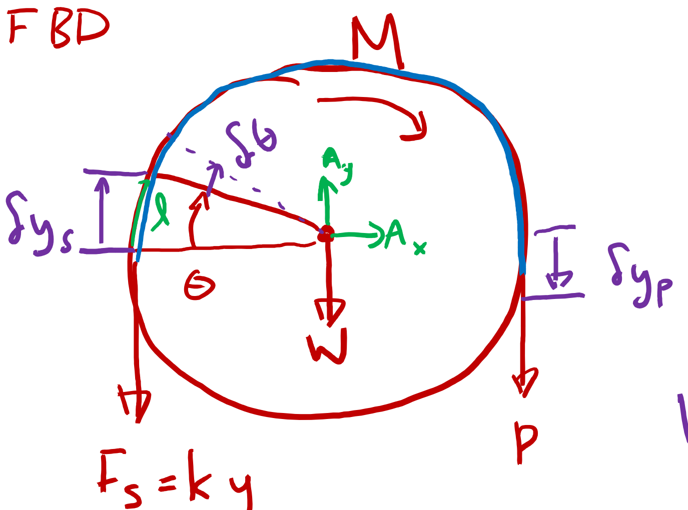
KNOWN: W, M, P

UNKNOWNS: $F_s(y,\theta)$, A_x, A_y

Arc length: $l = r\theta$ [for θ in radians]



FBD



$\delta y_s = \delta y_p$ same cable

Weight W does no work (pinned) no displ.

$l = r\theta = y$
$\delta y = r\,\delta\theta$

For very small displ, could assume that l is only in y direction.

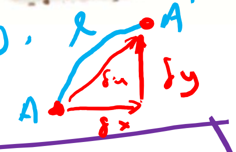
$$\delta U = \sum \vec{F}\,\delta\vec{r} + M\delta\theta = 0$$

$$-F_s\,\delta y_s + P\,\delta y_p + M\,\delta\theta = 0$$

$$-F_s(r\,\delta\theta) + P(r\,\delta\theta) + M\,\delta\theta = 0$$

$$\underbrace{(-F_s r + P r + M)}_{=0}\,\delta\theta = 0$$

Alternatively, if A moves to A'

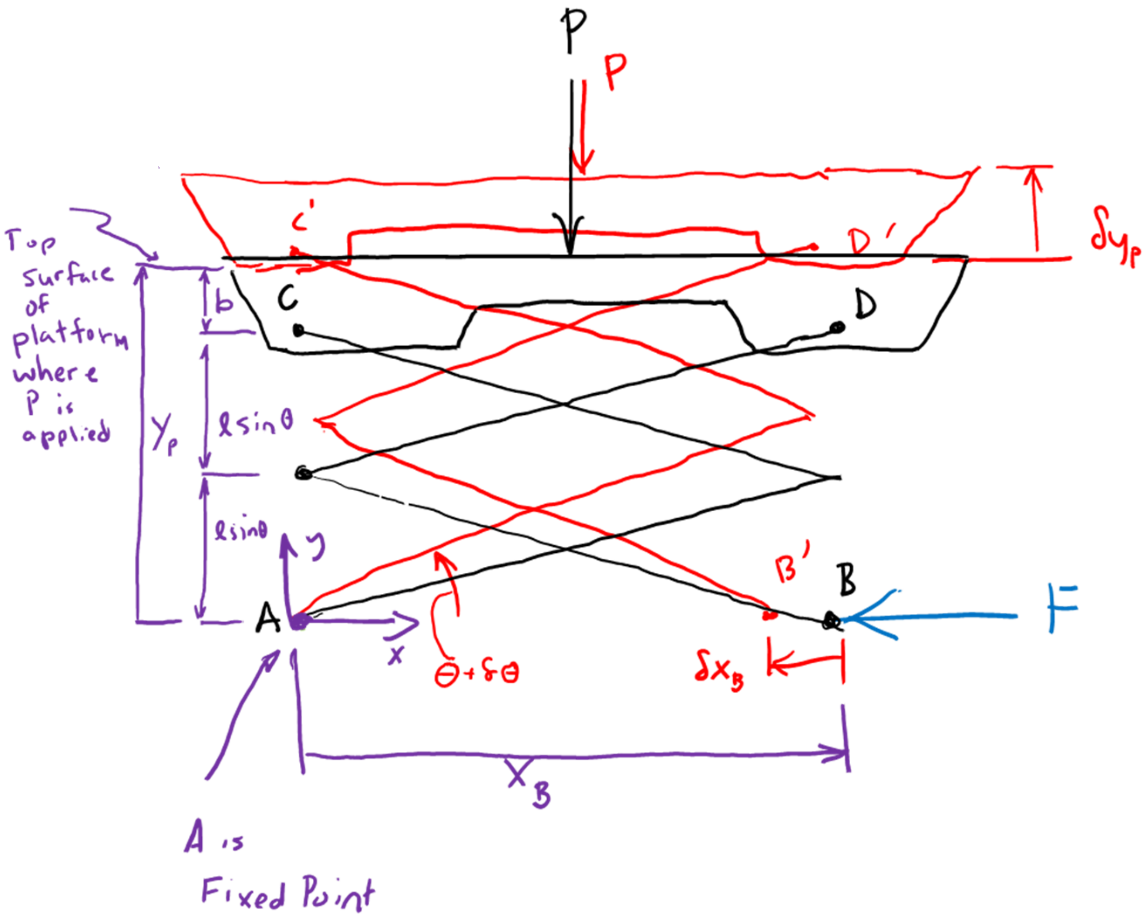


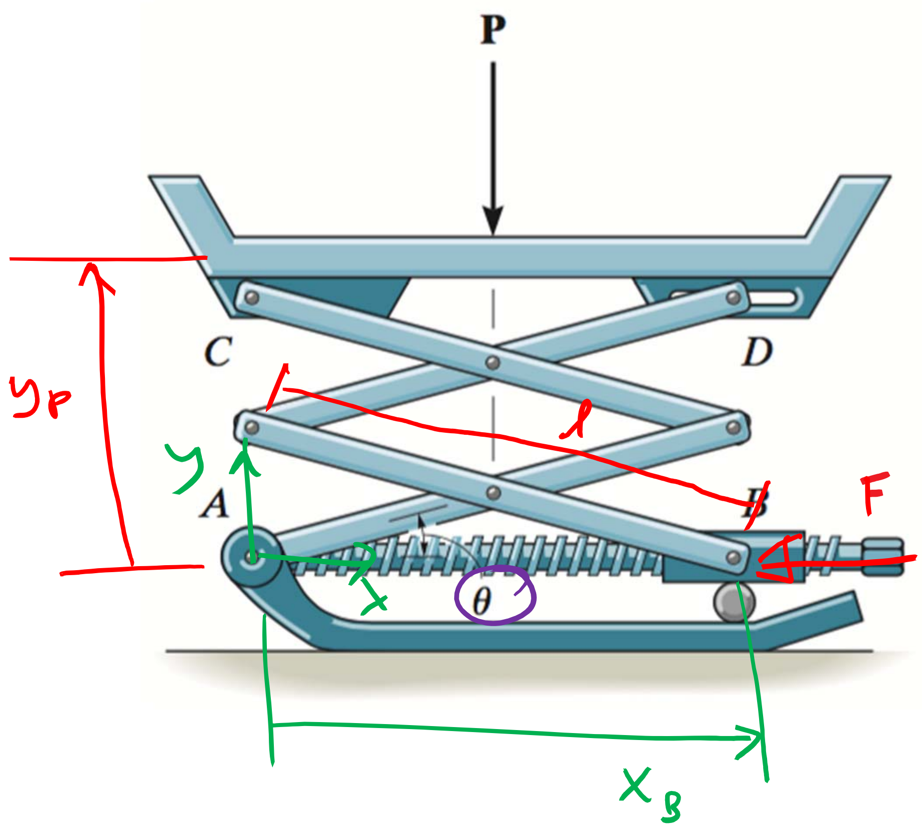
then $\delta\vec{u} = \delta\vec{x} + \delta\vec{y}$ but only δy does work wrt F_s

$$F_s = \frac{Pr + M}{r} = ky = k(r\theta) \Rightarrow \boxed{\theta = \frac{Pr + M}{r^2 k}}$$ radians

ΣM_A: $M - F_s r + Pr = 0$ Compare to force-balance equilibrium eqn method ✓ same

The scissors jack supports a load **P**. Determine axial force in the screw necessary for equilibrium when the jack is in the position shown. Each of the four links has a length *l* and is pin-connected at its center. Points *B* and *D* can move horizontally.

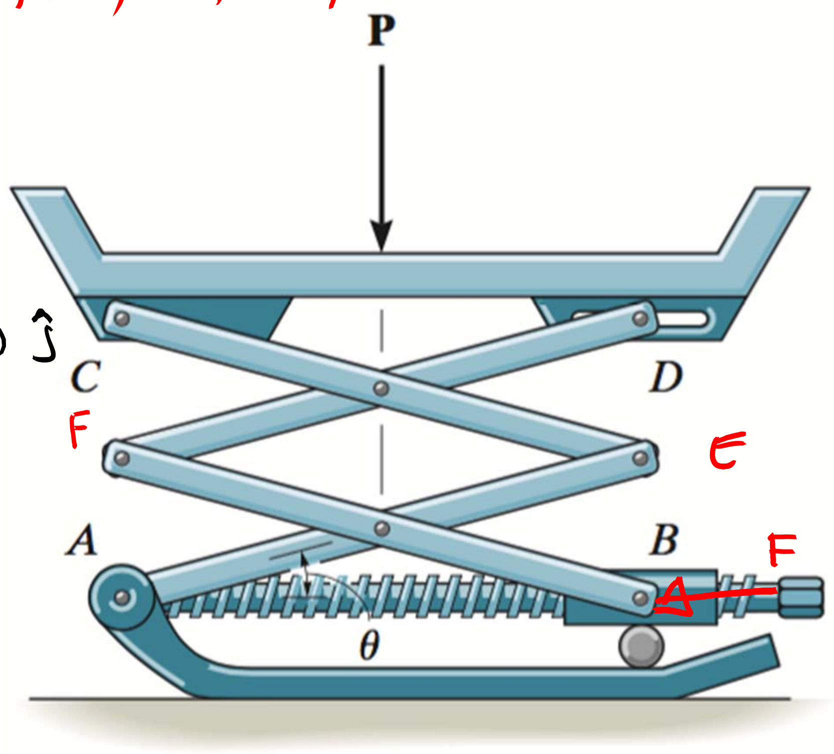




The scissors jack supports a load **P**. Determine axial force in the screw necessary for equilibrium when the jack is in the position shown. Each of the four links has a length *l* and is pin-connected at its center. Points *B* and *D* can move horizontally.

F-B method Need to look at each member: AE, BF, EC, FD, Platform

$\Rightarrow$ Need many equations to solve for F.



V-W method:

$$\vec{x}_B = l\cos\theta\,\hat{i} \Rightarrow \delta\vec{x}_B = -l\sin\theta\,\delta\theta\,\hat{i}$$

$$\vec{y}_P = (2l\sin\theta + b)\,\hat{j} \Rightarrow \delta\vec{y}_P = (2l\cos\theta\,\delta\theta + 0)\,\hat{j}$$

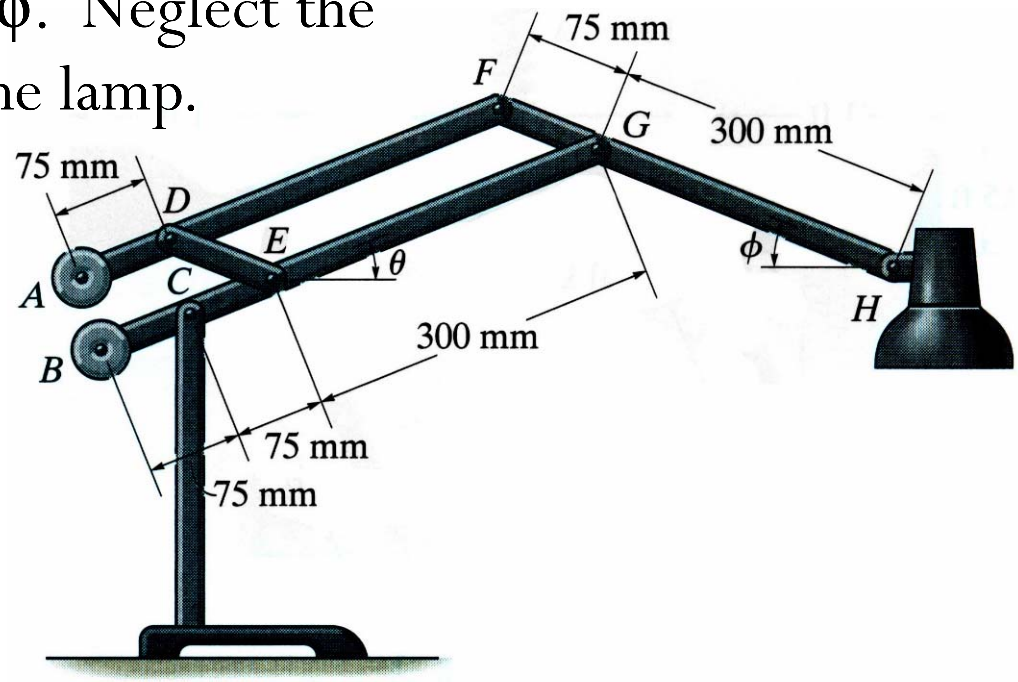
$$\delta U = \vec{F}\cdot d\vec{x}_B + \vec{P}\cdot\delta\vec{y}_P = 0$$

$$\vec{F} = -F\hat{i}, \quad \vec{P} = -P\hat{j}$$

$$\delta U = -F(-l\sin\theta\,\delta\theta) + (-P)(2l\cos\theta\,\delta\theta) = 0$$

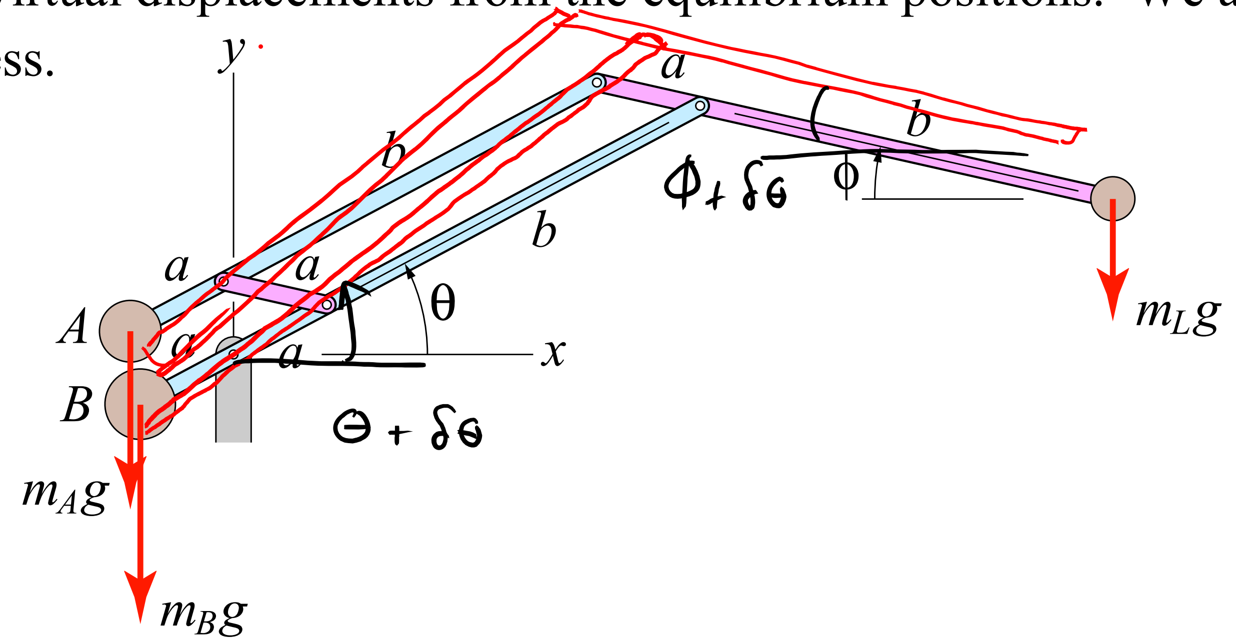
$$\boxed{F = \frac{2P\cos\theta}{\sin\theta}}$$

Determine the mass of A and B required to hold the 400 g desk lamp in balance for any angles θ and ϕ. Neglect the weight of the mechanism and the size of the lamp. Assume that pins are frictionless.



Solution

Use the principle of virtual work. This problem has two degrees of freedom: θ and ϕ. Let $\delta\theta$ and $\delta\phi$ be the virtual displacements from the equilibrium positions. We assume that all the pins are frictionless.



For equilibrium, we require

$$\delta W = -m_L g\, \delta y_L - m_A g\, \delta y_A - m_B g\, \delta y_B = 0$$

Here,

$$y_L = (a+b)\sin\theta - b\sin\phi \quad \Rightarrow \quad \delta y_L = (a+b)\cos\theta\,\delta\theta - b\cos\phi\,\delta\phi$$

$$y_A = a\sin\phi \quad \Rightarrow \quad \delta y_A = a\cos\phi\,\delta\phi$$

$$y_B = -a\sin\theta \quad \Rightarrow \quad \delta y_B = -a\cos\theta\,\delta\theta$$

Substitution gives

$$\delta W = \left(-m_L g(a+b)\cos\theta + m_B g\, a\cos\theta\right)\delta\theta$$

or

$$+\left(+m_L g\, b\cos\phi - m_A g\, a\cos\phi\right)\delta\phi = 0$$

$$\delta W = \left(-m_L g(a+b) + m_B g\, a\right)\cos\theta\,\delta\theta$$

$$+\left(+m_L g\, b - m_A g\, a\right)\cos\phi\,\delta\phi = 0\,.$$

If this relation is to hold for arbitrary $\delta\theta$ and $\delta\phi$, for the general case where $\cos\theta \neq 0$ and $\cos\phi \neq 0$, we require

$$\boxed{m_A = \frac{b}{a} m_L \quad \text{and} \quad m_B = \frac{a+b}{a} m_L\,.}$$

For $a = 75$ mm, $b = 300$ mm, and $m_L = 400$ g,

$$\boxed{\begin{aligned} m_A &= \frac{300}{75}(400\text{ g}) = 1.6\text{ kg}\,, \\ m_B &= \frac{75+300}{75}(400\text{ g}) = 2.0\text{ kg}\,. \end{aligned}}$$