Lecture 3 Announcements

Numerical Integration

Using force to describe acceleration $F(t) = m a(t)$

Integrate to find velocity and position

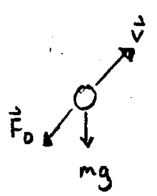
$$v(t) = \int_0^t a(\tau)\, \partial\tau + v_0$$

$$r(t) = \int_0^t v(\tau)\, \partial\tau + r_0$$

What if these integrals are too hard?

Example: Quadratic drag $\vec{F}_D = -c v \vec{v}$

* Opposes motion
* Scales like v^2



$$\vec{a} = \frac{\Sigma \vec{F}}{m} = f(\vec{v})$$

$$\vec{v}(t) = \int_0^t \vec{a}(\tau)\, \partial\tau = \int_0^t f(\vec{v})\, \partial\tau$$

These are <u>hard</u>

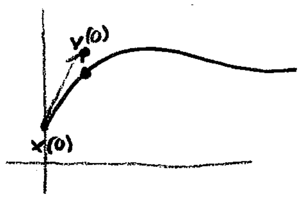
Numerical Integration

Independent variable: t

Initial conditions: $x(0)$
$v(0)$
NOT $a(0)$ – compute from others

State variables: $x(t)$
$v(t)$

Time step: Δt

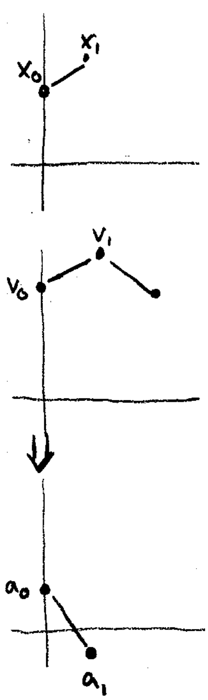


Update rule:

$$x(\Delta t) \approx x(0) + \Delta t\, v(0)$$

$$v(\Delta t) \approx v(0) + \Delta t\, a(0)$$

(More complicated rules also exist)



$$x_1 = x_0 + \Delta t\, v_0 \qquad x_2 = x_1 + \Delta t\, v_1$$

$$\Uparrow \qquad\qquad \uparrow$$

$$v_1 = v_0 + \Delta t\, a_0 \qquad v_2 = v_1 + \Delta t\, a_1$$

$$\Uparrow \qquad\qquad \uparrow$$

$$a_0 = f(x_0, v_0) \qquad a_1 = f(x_1, v_1)$$