$\boxed{\text{Lecture 7}}$ Announcements

Experimental design (project 2a, b) on Wednesday

- Supplies available in office hours today, tomorrow

No lecture Friday

- Quiz 2 is happening this week, so Friday slots are still open

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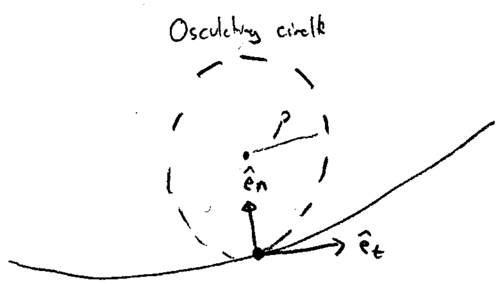
Recap: Tangential - Normal and related



Radius of curvature ρ

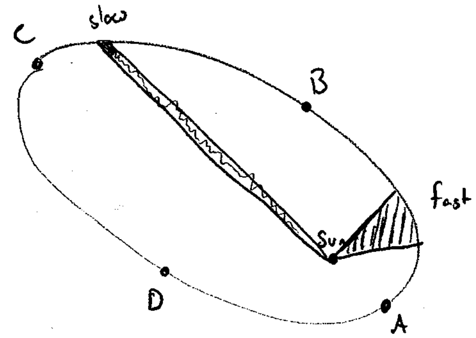
Curvature $\kappa = 1/\rho$

$\vec{v} = \dot{s}\,\hat{e}_t$

$\vec{a} = \ddot{s}\,\hat{e}_t + \frac{\dot{s}^2}{\rho}\,\hat{e}_n$

$\hat{e}_b = \hat{e}_t \times \hat{e}_n$

Example Kepler's 2^{nd} Law



Comet sweeps equal areas in equal time

At each of A, B, C, D, is

	D	A, C	B
a_t	>0	$=0$	<0
a_n	$\boxed{>0}$	$=0$	<0

$$\vec{a} = \ddot{s}\,\hat{e}_t + \frac{\dot{s}^2}{\rho}\,\hat{e}_n$$

Completing the basis

$\hat{e}_t$ Direction of travel

$\hat{e}_n$ Direction of turning

$\hat{e}_b := \hat{e}_t \times \hat{e}_n$ Binormal

A turning-like concept torsion τ rate that basis is twisting

$$\dot{\hat{e}}_b = \frac{\partial}{\partial t}\hat{e}_b = \frac{\partial}{\partial s}\hat{e}_b \frac{\partial s}{\partial t} = \dot{s}\frac{\partial}{\partial s}\hat{e}_b =: -\dot{s}\boxed{\tau\,\hat{e}_n}$$

These two metrics combine in expression for angular velocity

$$\vec{\omega} = \underbrace{\dot{s}\,\tau\,\hat{e}_t}_{\text{Twisting}} + \underbrace{\dot{s}\,\kappa\,\hat{e}_b}_{\text{Turning}}$$

Comparison of three bases

	Cartesian	Polar	TN
Position	$x\hat{\imath} + y\hat{\jmath}$	$r\hat{e}_r$	$\emptyset$
Velocity	$\dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$	$\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$	$\dot{s}\hat{e}_t$
Acceleration	$\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$	$(\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta$	$\ddot{s}\hat{e}_n$

	Cartesian	Polar	TN	
Calculus easy	✓	✗	✗	
P - V relate	✗	✓	✗	Physical intuition
V - A relate	✗	✗	✓	
Requires axes	✓	✓	✗	
Time invariant	✓	✗	✗	
Continuously defined	✓	✓*	✗	

Particle Kinetics

Kinematics = motions $\vec{r}, \vec{v}, \vec{a}$

Kinetics = forces $\vec{F}, \cancel{\vec{M}}$ soon

Tools from statics

$\Sigma \vec{F}$

Free body diagrams

Kinematics

$\vec{a}$

$$\vec{F} = m\vec{a}$$

Two possible approaches to this equation

(1) Method of assumed forces

- All known about $\vec{F}$
- Trajectory can be generated by computing $\vec{a}$

Simulation $\vec{F} \rightarrow \vec{a}$

(2) Method of assumed motion

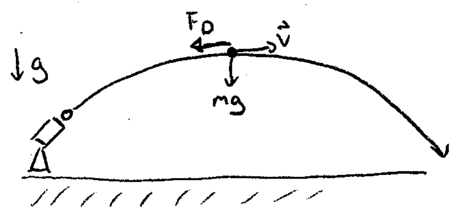
- Motion is observed
- Forces are modeled from this

Measurement $\vec{a} \rightarrow \vec{F}$

George Box : "All models are wrong. Some models are useful."

Example   Assumed forces

Cannon



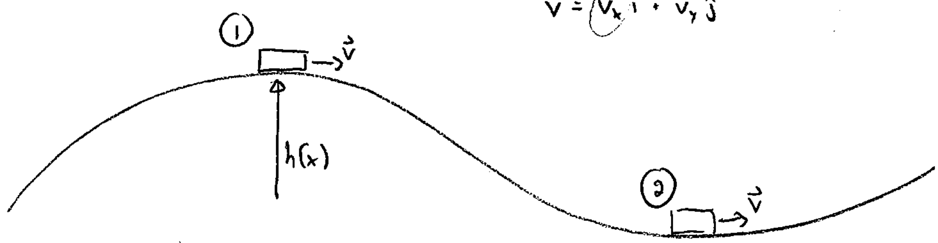
$$\vec{F} = -mg\hat{j} \; (-mv\vec{v})$$

Example   Assumed motions

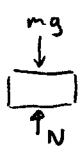
Car on road

Constant

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$



$$h(x) = H\cos\left(\frac{x}{L}\right)$$



Is
$N > mg$
$N = mg$
$N < mg$
Depends.

Solution Steps

① System diagram - coordinates, origins, basis, etc

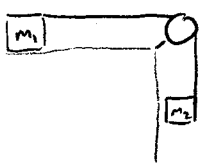
(2a) Free body diagram + Forces

(2b) Kinematics + $\vec{a}$

④ Kinetics $\vec{F} = m\vec{a}$

⑤ Algebra : Solve for any unknowns

Example



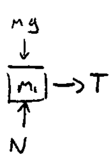
$2m_1 = m_2$

What is $\vec{a}_2$?

$> m_2 g$

$< m_2 g$

$= m_2 g$



$m_1 a_1 \hat{i} = T\hat{i}$

T, a_1, a_2

$m_2 a_2 \hat{j} = T\hat{j} - m_2 g\hat{j}$

$\boxed{a_1 = -a_2}$

$(m_1 + 2m_1)a_2 = -m_1 g$

$\boxed{a_2 = -\frac{2}{3} g}$